The Formation of Vortical Motion in Cosmic Large Scale Structure

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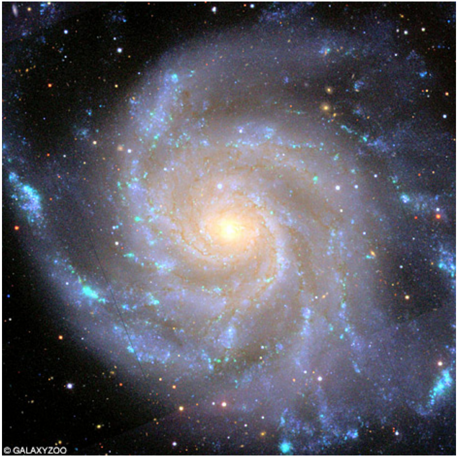
Outline



NGC 5457

Most galaxies in the Universe rotate.

The rotation axes of neighboring galaxies are correlated.

New observations find alignments of jets in radio galaxies at $z = 1$ out to (10-20) Mpc
(A. Taylor & P. Jagannathan (2016)).

Can these vortical motions be explained within standard ΛCDM?

Can we learn something about cosmology by observing them?

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Introduction: vorticity in cosmology

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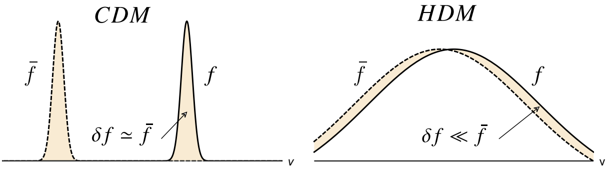
In a fluid description shell (orbit)-crossing is a singular process while in phase space it is regular.

N-body simulations can accommodate shell crossing without problem, they are actually nothing else than a poor woman's Vlasov equation solver.

One might think that a perturbative approach to the Vlasov equation could be successful but...



... the flow of CMD is very cold. Contrary to the case of hot dark matter, a perturbative treatment using the Vlasov equation is not adequate for CDM.

But one can go to higher moments of the Vlasov equation, beyond the 0th and first moments which yield the continuity and Euler equations for perfect fluids.

$$\begin{aligned} \partial_t\delta + \nabla((1+\delta)\mathbf{v}) &= 0\,, \\ \left(\partial_t + v^i\partial_i\right) v_j + \mathcal{H}v_j + \partial_j\Phi + \frac{1}{\rho}\partial_i(\rho\sigma_{ij}) &= 0\,, \\ (\partial_t + v^k\partial_k)\sigma^{ij} + 2\mathcal{H}\sigma^{ij} + \sigma^{ik}\partial_k v^j + \sigma^{jk}\partial_k v^i &= 0 \\ \sigma^{ijk} &= 0 \end{aligned}$$

The curl of the Euler eqn. then gives, $\boldsymbol{\omega} = \boldsymbol{\nabla}\wedge\mathbf{v}$,

$$\frac{\partial\boldsymbol{\omega}}{dt} + \mathcal{H}\boldsymbol{\omega} - \nabla\wedge[\mathbf{v}\wedge\boldsymbol{\omega}] = -\nabla\wedge\left(\frac{1}{\rho}\nabla(\rho\sigma)\right)\,.$$

To lowest order in perturbation theory, the velocity dispersion take the form $\sigma_{ij} = \frac{\sigma_0}{3}a^{-2}\delta_{ij}$.

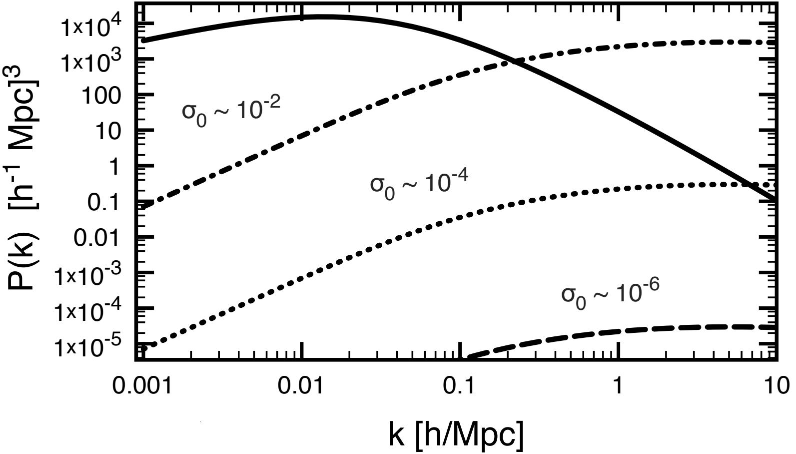
We have solved the vorticity equation to lowest non-vanishing order (Cusin, Tansella & RD, 2017).

$$\langle\omega_i^{(2)}(\mathbf{k},t)\omega_j^{(2)\,*}(\mathbf{k}',t)\rangle = (2\pi)^3\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)\delta(\mathbf{k}-\mathbf{k}')P_\omega(k,t)\,.$$

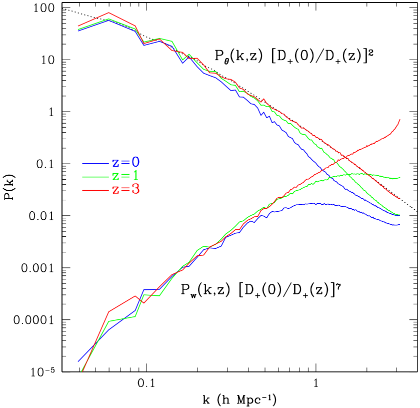
$$P_\omega(k) = \frac{1}{9}\frac{\sigma_0^2 D_+(t)}{\mathcal{H}_0^2\Omega_m}\int\frac{d^3\mathbf{w}}{(2\pi)^3}\left(\frac{\mathbf{w}\cdot(\mathbf{k}-\mathbf{w})}{w^2|\mathbf{k}-\mathbf{w}|^2}\right)^2|\mathbf{w}\wedge\mathbf{k}|^2\left[2\mathbf{k}\cdot\mathbf{w}-k^2\right]^2P_\delta(w)P_\delta(|\mathbf{k}-\mathbf{w}|)$$

$$P_\omega(k,t)\stackrel{k\to 0}{\to}\ k^4D_+(t)$$

The rotational velocity spectrum, $P_R = k^{-2} P_\omega$ compared to the gradient velocity spectrum $P_G = k^{-2} P_\theta$, $\theta = \boldsymbol{\nabla} \cdot \mathbf{v}$.



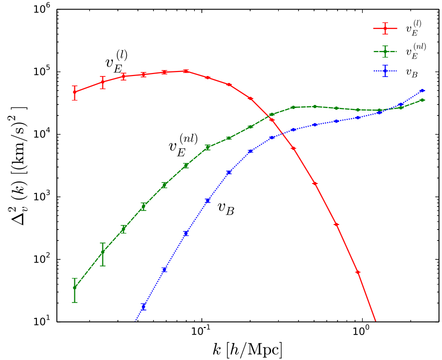
From
Pueblas & Scoccimarro '09
Using a Delauny tessellation for the velocity field.



The vorticity and divergence spectra, P_ω and P_θ. They find a slope $P_\omega \propto k^{2.5}$ and time dependence $P_\omega \propto D_+^7$.

The results shown are from a $L = 256$Mpc simulations with $N = 512^3$ particles using Gadget-2 with softening length 0.04.

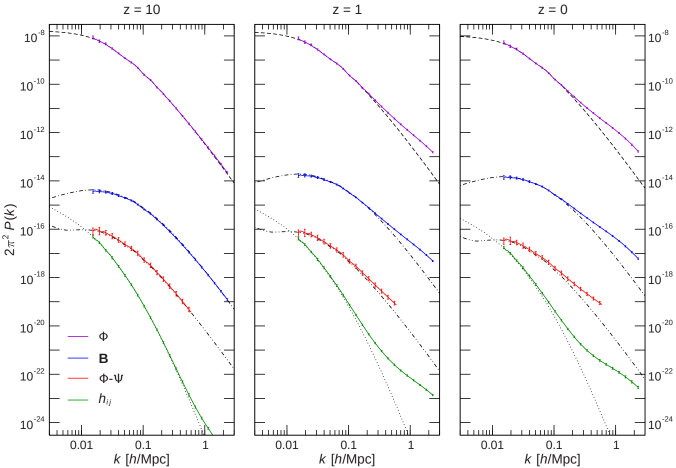
From
Zhu, Yu &Pen '17



The gradient and rotational velocity spectra, $k^3P_G(k)$ and $k^3P_R(k)$. They find a slope $P_R \propto k^0$. It is not clear whether these are spectra are Fourier transforms from Eulerian or Lagrangian coordinates.

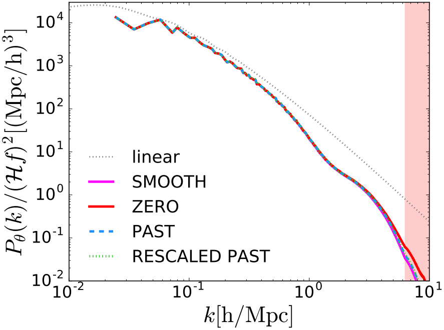
The results shown are from a $L = 600$Mpc simulations with $N = 1024^3$ particles on a 512^3 grid using CUBEP^{3}M using multi-grid techniques to compute the displacement field and velocity (in Lagrangian coordinates).

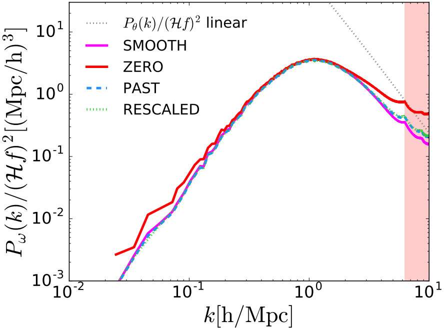
We performed N-body simulations using gevolution.
Gevolution is a relativistic PM N-body code using a weak field approximation of the metric, which computes all 6 degrees of freedom of the gravitational field (Adamek, Daverio, RD, Kunz (2016)).



Vorticity with gevolution: velocity reconstruction

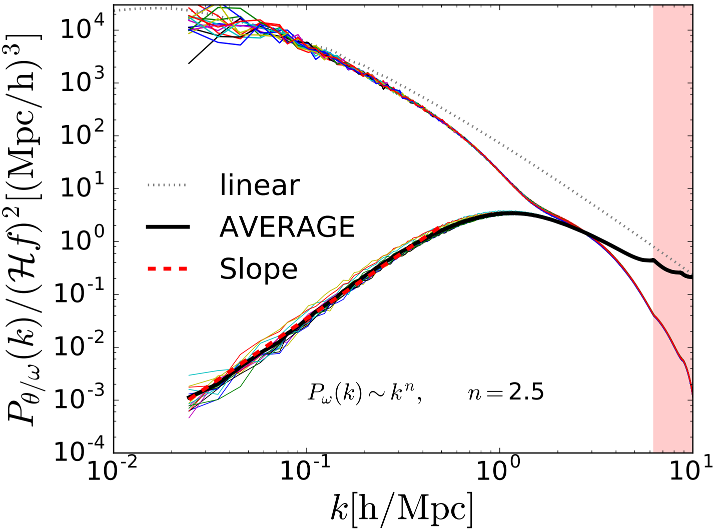
We (Jelic-Cimek, Lepori & RD, in preparation) have tested different velocity reconstruction methods which are in good agreement.



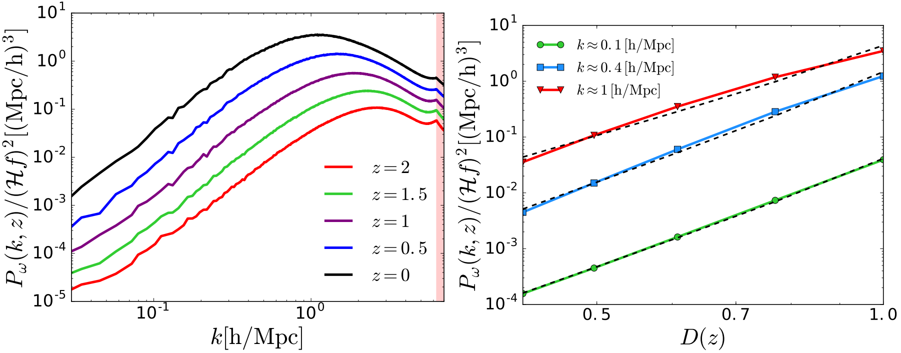


Vorticity with gevolution: vorticity power spectrum

The resulting spectrum behaves as $k^{2.5}$ on large scales.



Vorticity with gevolution: time dependence of vorticity



The resulting spectrum behaves as a^{γ} on large scales with $\gamma \simeq 5$.

Observation of vorticity

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Like gradient velocity, vortical velocity leads to redshift space distortions. But these only determine the radial component of the velocity, hence cannot distinguish between gradient and vortical motion.

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Gradient RSD: $P(k,\mu) = \left(1 + \frac{f}{b}\mu^2\right)^2 P_g(k)$

Rotational RSD: $P_{\mathrm{RSD}\,\omega}(k,\mu) = \mathcal{H}^{-2}\mu^2(1-\mu^2)P_\omega(k)$

In real space: $\xi(r,\mu) = \xi_0(r) + \xi_2(r)P_2(\mu) + \xi_4(r)P_4(\mu)$

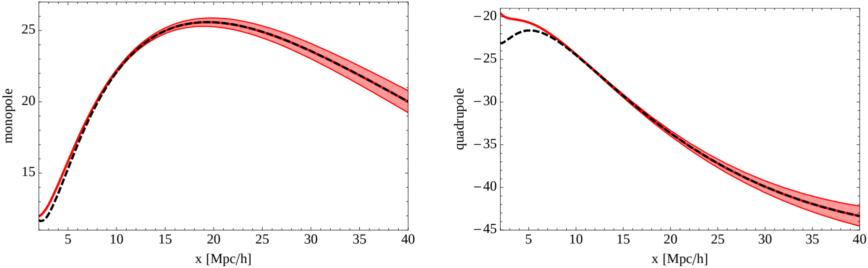
$$\xi_n(r) = \frac{\mathcal{H}^{-2}}{2\pi^2}\int P_\omega(k) j_n(kr) k^2 dk$$

RSD monopole & quadrupole

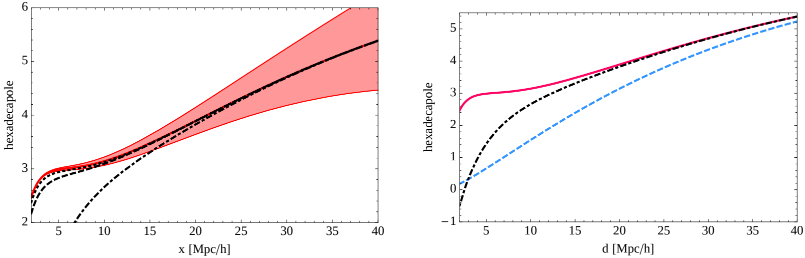
Constraints for vorticity from structure formation:

$$P_\omega(k,z) = A_V k^2 D_+^7(z) \frac{(k/k_*)^{n_\ell}}{[1+(k/k_*)]^{n_\ell+n_s}}$$

$$n_\ell = 1.3\,, \quad n_s = 4.3\,, \quad k_* = 0.7\,h/\mathrm{Mpc}\,, \quad A_V \simeq 10^{-5}(\mathrm{Mpc}/h)^3\,.$$



(from Bonvin, RD, Koshravi, Kunz, Sawicki, 2017)
Red region: Scalar signal with error for SKA at $\bar{z} = 0.35$. Black dashed: including vorticity with $A_V = 5 \times 10^{-3}$.



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Left: Red region: Scalar signal with error for SKA at $\bar{z} = 0.35$. Black: including vorticity with $A_V = 3 \times 10^{-5}$ (dotted line), $A_V = 10^{-4}$ (dashed line) and $A_V = 10^{-3}$ (dot-dashed line). Right: non-linear scalar, linear scalar, non-linear scalar+vector.

Constraints on A_V from an SKA like survey for A_V using data from $x \in [x_{\rm min}, 40 {\rm Mpc}/h]$, $z \in [0.1, 2]$, $\Delta z = 0.1$.

$x_{\rm min}$ [Mpc/h]	mono	quad	hexa	total
2	3.7×10^{-5}	4.2×10^{-6}	8.7×10^{-7}	8.7×10^{-7}
10	9.4×10^{-4}	2×10^{-3}	7.1×10^{-5}	7.1×10^{-5}
20	7.2×10^{-2}	4.6×10^{-2}	1.6×10^{-3}	1.6×10^{-3}

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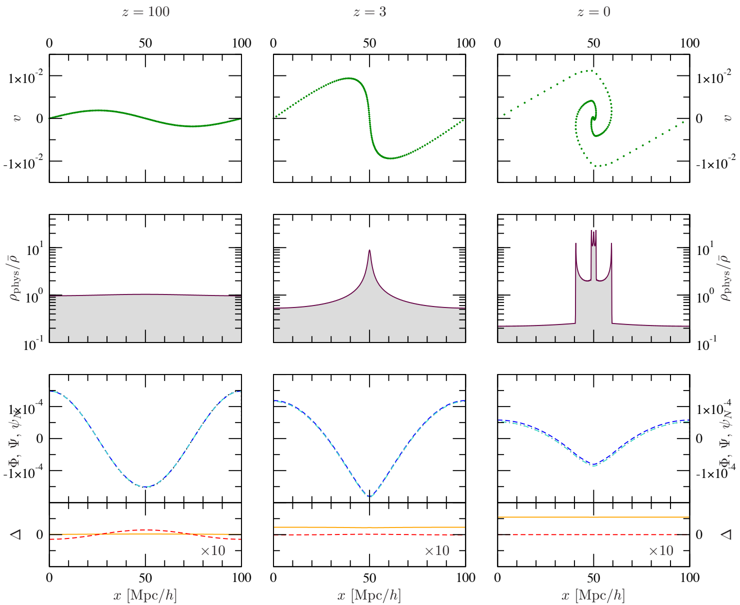
Thank You !

Shell crossing of a plane wave



Resolution dependence of vorticity