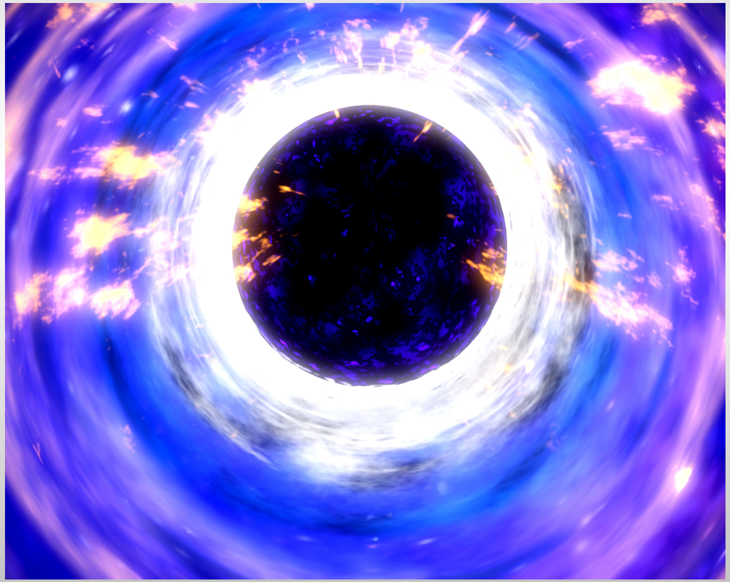
Black Holes and Slow Roll Scalars



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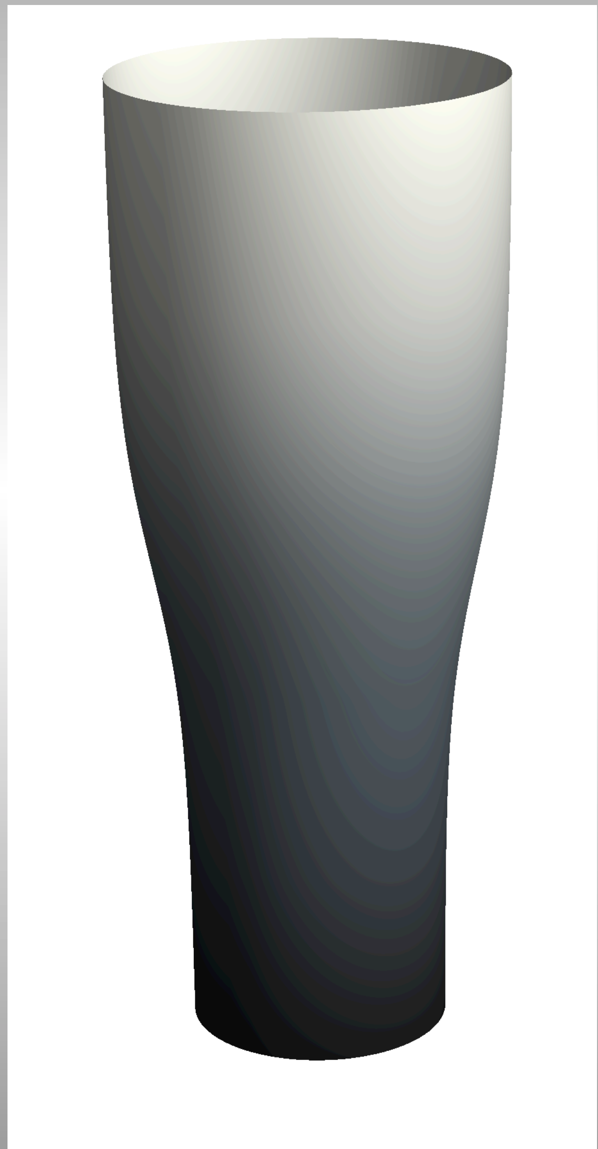
How does a Black Hole respond to a cosmological scalar?

- Relativist's background: thermodynamics with "Λ"?
- Setting up: Some de Sitter stuff
- The rolling scalar: Scalar flow cf Slow Roll
- Back-reaction on the geometry

Original motivation: Thermodynamics with variable Lambda

M_+



Λ_+

M_-

Λ_-

Thermodynamics

A black hole has temperature, entropy, and satisfies a first law:

$$\delta M = T\delta S$$

Can derive this by varying the Schwarzschild potential:

$$\delta f(r_+ + \delta r_+) = -\frac{2\delta M}{r_+} + \frac{2M}{r_+^2}\delta r_+ = 0$$

But we are used to

$$dU = TdS - pdV$$

Thermodynamics and Lambda

Now vary the Schwarzschild potential with lambda:

$$\delta f(r_+ + \delta r_+) = -\frac{\delta\Lambda}{3} r_+^2 - \frac{2\delta M}{r_+} + f'(r_+)\delta r_+ = 0$$

Rearrange to

$$\delta M = T\delta S - \frac{4\pi r_+^3}{3}\delta\left(\frac{\Lambda}{8\pi}\right)$$

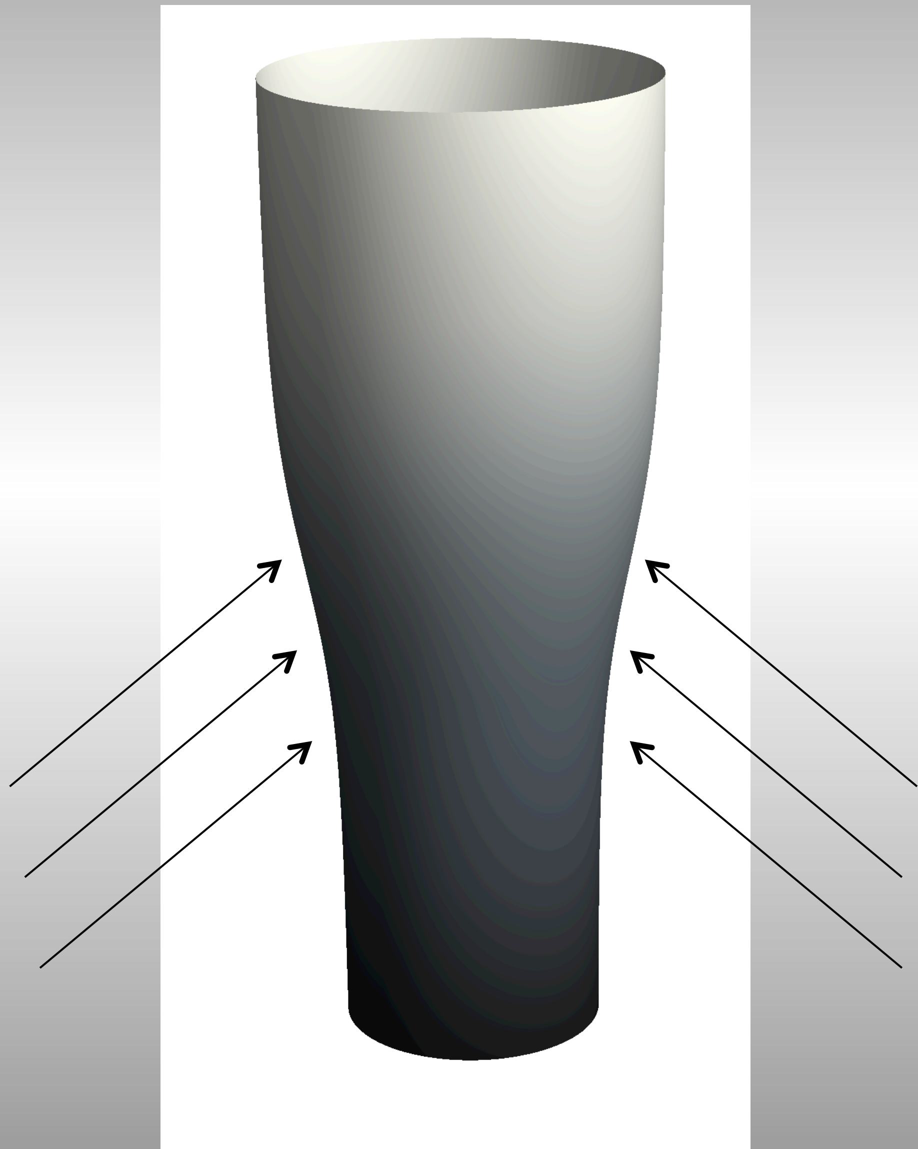
Enthalpy

Thermodynamic Volume

Thermodynamic Pressure

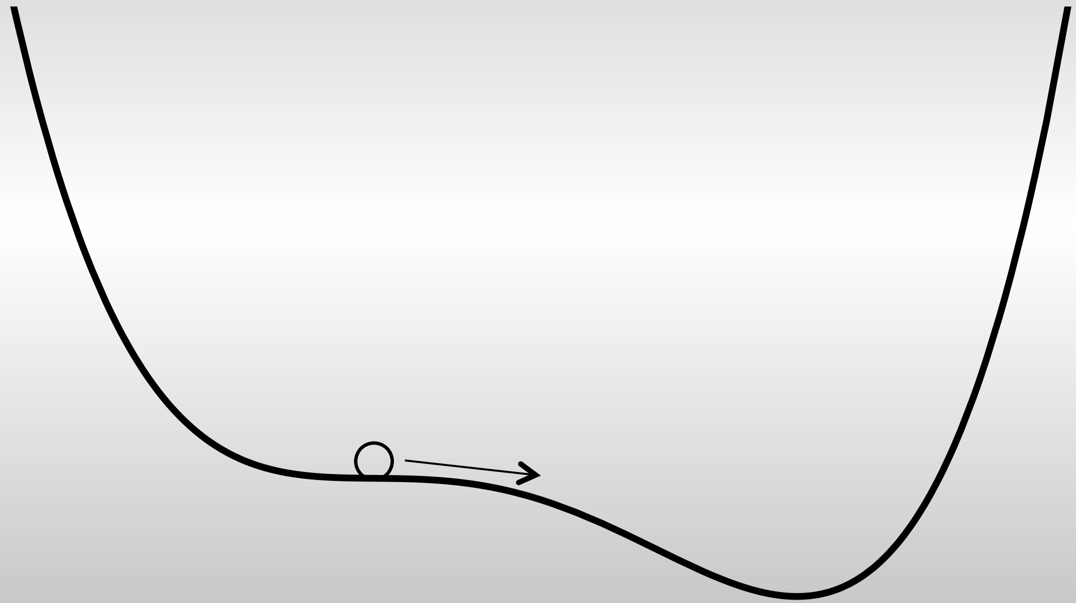
How to vary Lambda:

M_+ $\uparrow$ Φ $\uparrow$ M_-



Λ_+ $\uparrow$ Φ $\uparrow$ Λ_-

Analogous to RG Flow in ADS/CFT – use a scalar to change Lambda



But this is Familiar..!

Varying Lambda

The idea of a varying Lambda is very familiar – in slow roll inflation, lambda varies gradually, while our universe is quasi-de Sitter.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left[\frac{\dot{\phi}^2}{2} + W(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial W}{\partial \phi}$$

Small slow-roll parameters ensure that inflation in maintained:

$$\varepsilon = \frac{M_p^2}{2}\frac{W'^2}{W^2} \qquad \Gamma = 2M_p^2\frac{W''}{W}$$

Add Black Hole?

But a key difference is that our geometry is not explicitly time dependent – so what does “slow roll” mean? The Schwarzschild de Sitter solution is not time dependent:

$$ds^2 = f\,dt^2 - \frac{dr^2}{f} - r^2 d\Omega_{II}^2$$

where

$$f = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

Static de Sitter

Without the black hole, have a “static patch” de Sitter potential:

$$f = 1 - H^2 r^2$$

– not the familiar (flat) cosmological coordinates

$$d\tau^2 - e^{2H\tau} d\mathbf{x}^2$$

The transformation to cosmological time is nontrivial:

$$\tau_{cos} = t_s + \frac{1}{2H}\log(1 - H^2 r_s^2)$$

$$\rho_{cos} = \frac{r_s e^{-Ht_s}}{\sqrt{1 - H^2 r_s^2}}$$

And with black hole, even this relative simplicity is lost.

Black Hole Approximation

With a black hole, intuition is that the geometry is approximately SDS, the scalar still slow-rolls, but that this produces a sub-leading effect on the background black hole geometry. The spacetime slides from one Lambda to a lower one, and the black hole accretes a little mass.

$$\phi = \phi_0 + \delta\phi_{SR}$$

$$g_{\mu\nu} = g_{0\,\mu\nu} + \delta g_{SR\,\mu\nu}$$

SDS $\quad \mathcal{O}\left(\delta\phi_{SR}\right)^2$

Scalar Field Eqn

Idea is to turn e.o.m for Φ

$$\frac{\phi_{,tt}}{f} - \frac{1}{r^2}\left(r^2 f \phi_{,r}\right)_{,r} = -\frac{\partial W}{\partial \phi}$$

into something like a slow roll equation by assuming $\Phi = \Phi(T)$, where

$$T = t + \xi(r)$$

T is constructed so that Φ is regular at both horizons, with only in(out) going modes at black hole (cosmological) horizon.

Substitute in:

$$\frac{1}{r^2}\left(r^2 f \xi'\right)' \dot{\phi} - \frac{\ddot{\phi}}{f}\left(1 - f^2 \xi'^2\right) = \frac{\partial W}{\partial \phi}$$

Dropping second term, and remember $\Phi = \Phi(T)$, we must have

$$\frac{1}{r^2}\left(r^2 f \xi'\right)' = -3\gamma$$

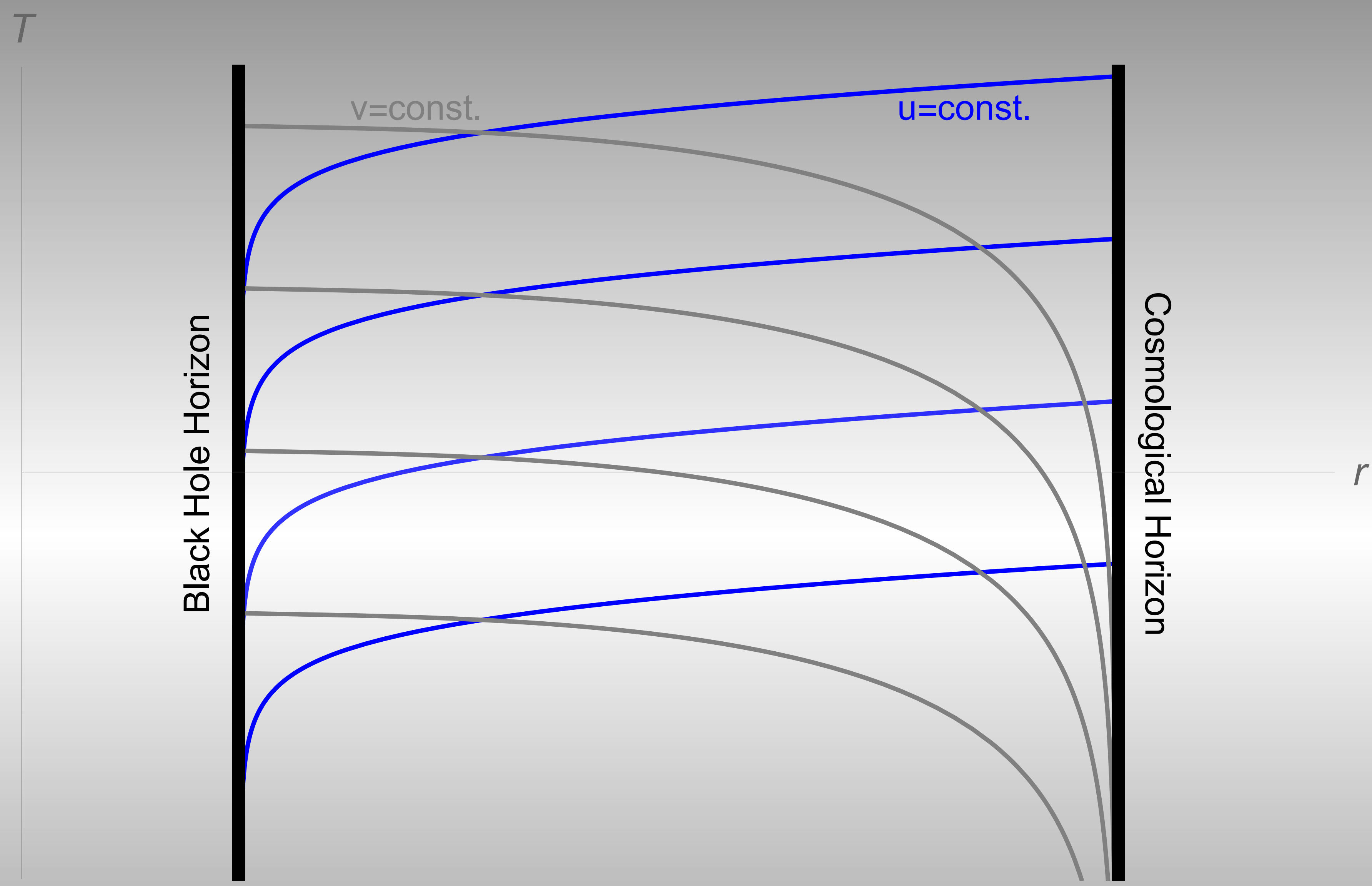
γ constant, and hence

$$\xi' = \frac{1}{f}\left(-\gamma r + \frac{\beta}{r^2}\right)$$

Find γ and β by regularity: Φ(T) must be ingoing on event horizon and outgoing on cosmological horizon. Final answer gives T:

$$T = t - \frac{1}{2\kappa_c}\log\left|\frac{r - r_c}{r_c}\right| + \frac{1}{2\kappa_b}\log\left|\frac{r - r_b}{r_b}\right|$$
$$+ \frac{r_b r_c}{r_c - r_b}\log\frac{r}{r_0} + \left(\frac{r_c}{4\kappa_b r_b} - \frac{r_b}{4\kappa_c r_c}\right)\log\left|\frac{r - r_n}{r_n}\right|$$

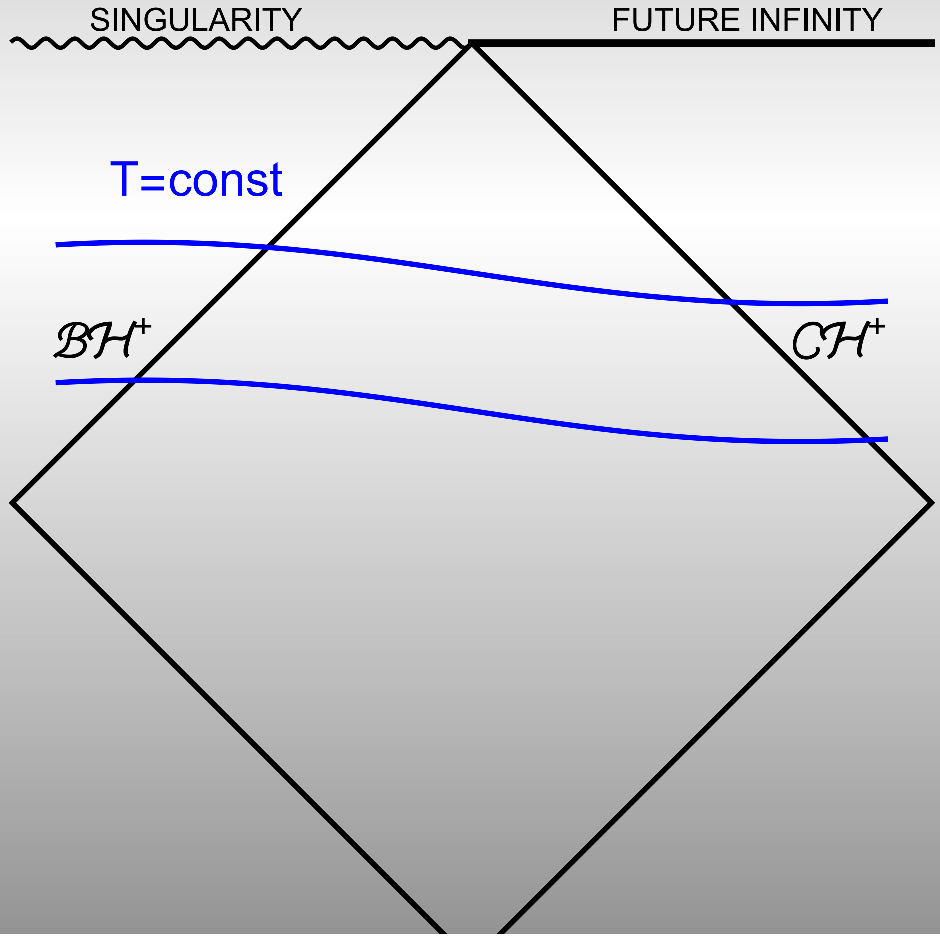
For those familiar with Kruskals, T looks like V at the black hole horizon (r_b) and U at the cosmological horizon (r_c)



T looks like an Eddington-Finkelstein coord on each horizon, at r_h a fn of v, and at r_c a fn of U.

The T Coordinate

The T coordinate is timelike at each horizon, and could be a cosmological time asymptotically.



Phi Equation

The phi equation is now a standard slow-roll type

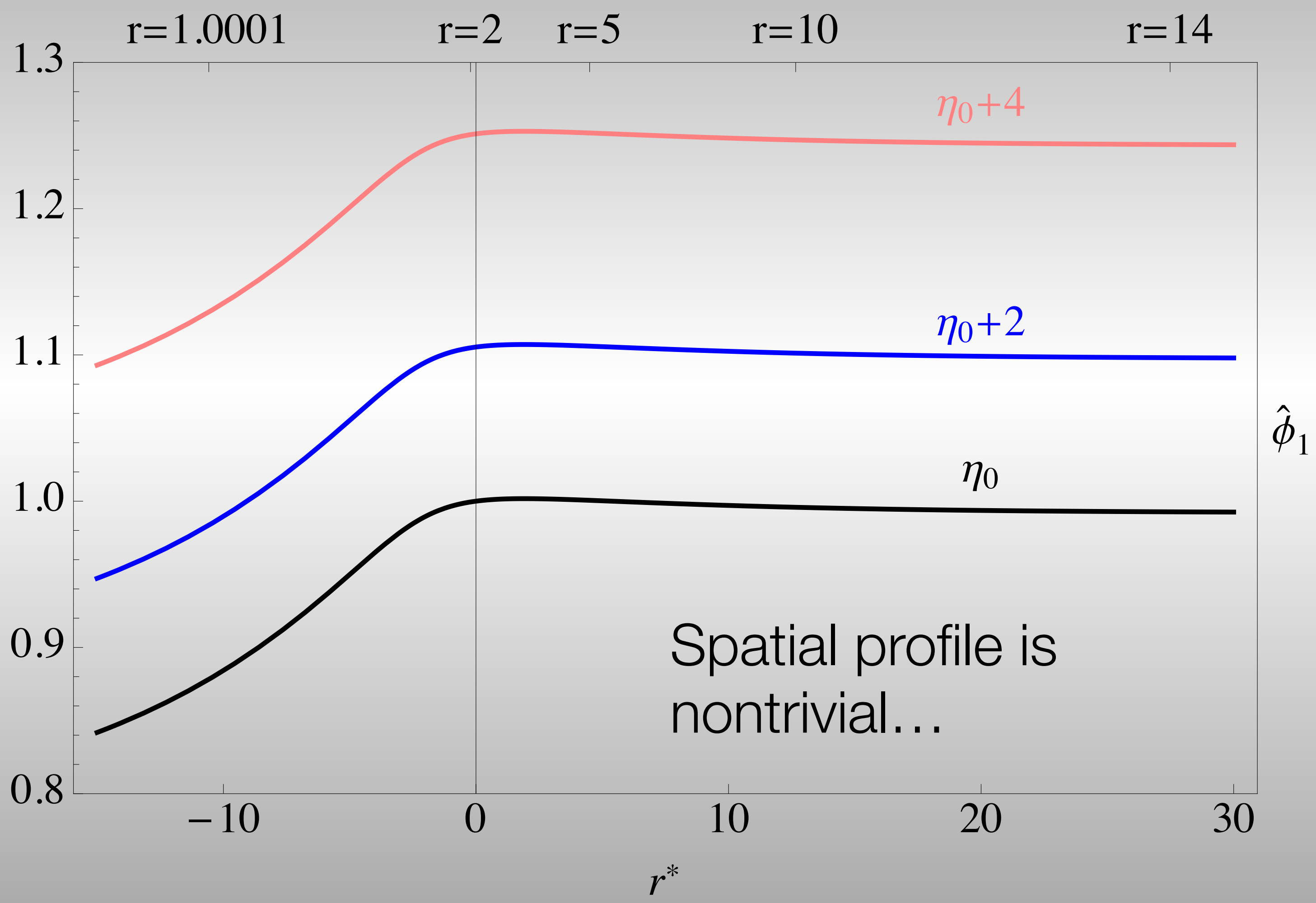
$$3\gamma\dot{\phi}(T) = -\frac{\partial W}{\partial \phi}$$

but with friction parameter modified from H:

$$\gamma = \frac{r_c^2 + r_h^2}{r_c^3 - r_h^3} = \frac{A_{TOT}}{3V}$$

Physical effect of black hole is to add friction to roll, or to slow down the scalar.

Phi Profile



Spatial profile is nontrivial…

Back-Reaction

Given this Eddington-Finkelstein behaviour, look at SDS metric in (T,r) coords:

$$ds^2 = f(r,T)\,dT^2 - 2h(r,T)\,dT dr - \frac{dr^2}{f}\left(1-h^2\right) - r^2 d\Omega^2$$

The energy momentum of the scalar has 2 independent cpts:

$$T_{TT} = \left(W(\phi) + \frac{1+h^2}{2f}\dot{\phi}^2\right)|g_{TT}|\;,$$

$$T_{ab} = \left(-W(\phi) + \frac{1-h^2}{2f}\dot{\phi}^2\right)g_{ab}$$

Which we relate to the Einstein tensor:

$$G_{TT} = \left[\frac{1}{r^2}(1 - f - rf') - \frac{h\dot{f}}{rf}\right] |g_{TT}|$$

$$G_{rr} = \left[-\frac{1}{r^2}(1 - f - rf') + \frac{h\dot{f}}{rf} + \frac{2\dot{h}}{r(1-h^2)}\right] g_{rr}$$

$$G_{rT} = \left[-\frac{1}{r^2}(1 - f - rf') - \frac{(1-h^2)\dot{f}}{rhf}\right] g_{rT}$$

$$G_{\theta\theta} = \left[\frac{f''}{2} + \frac{f'}{r} - \frac{h'\dot{f}}{2f} + \frac{\dot{h}f'}{2f} + \frac{\dot{h}}{r} + \dot{h}' + \frac{1}{2}\left(\frac{(h^2-1)}{f}\right)^{\cdot\cdot}\right] g_{\theta\theta} = \frac{G_{\phi\phi}}{\sin^2\theta}$$

? Slow Roll ?

Need to have control of the slow-roll approximation to identify the key dependences in these equations.

The scalar equation is straightforward to see,

$$\frac{1-h^2}{f}\ddot{\phi} - \frac{(r^2 h)'}{r^2}\dot{\phi} = -W'(\phi)$$

But the equivalent of Friedmann is:

$$\left[\frac{1}{r^2}(1-f-rf') + \frac{(1-h^2)\dot{f}}{rhf}\right] = \frac{1}{M_p^2}\left(W(\phi) - \frac{1-h^2}{2f}\dot{\phi}^2\right)$$

Slow Roll with a Black Hole

Taking the same general slow roll requirements, these now depend on position:

$$\frac{1-h^2}{f}\dot{\phi}^2 \ll W \quad , \qquad \frac{1-h^2}{f}\ddot{\phi} \ll \frac{1}{r^2}\left|\left(r^2 h\right)' \dot{\phi}\right|$$

As usual in slow-roll, take background values of metric functions, and can bound these r-dependent background functions to the usual slow roll type parameters

$$\varepsilon = M_p^2 \frac{W'^2}{W^2} \ll 1 \qquad \Gamma = M_p^2 \frac{W''}{W} \ll 1$$

This allows us to solve the Einstein equations to leading order in the slow-roll parameters.

- TT + Tr:
$$\dot{f} = -rh\,\frac{\dot{\phi}^2}{M_p^2}$$

implies

$$f(r,T) = f_0(r) + \delta f(r,T) - rh_0 \int \frac{\dot{\phi}^2}{M_p^2}$$

Where δf is order εΓ, but slowly varying. We can then integrate the φ kinetic energy:

$$\int_{T_0}^{T} \dot{\phi}^2 dT' = -\frac{1}{3\gamma}\left\{W(\phi(T)) - W(\phi(T_0)\right\} = -\frac{\delta W}{3\gamma}$$

And end up with a familiar expression:

$$f(r,T) = 1 - \frac{\Lambda(T)}{3M_p^2}r^2 - \frac{2GM(T)}{r} + \delta f$$

With

$$\Lambda(T) = W[\phi(T)] \qquad M(T) = M_0 - 4\pi\beta\frac{\delta W}{3\gamma}$$

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A somewhat finicky argument shows that delta f is transient, and of sub-leading order ($\varepsilon\Gamma$) to the changes in Λ and M.

We can solve for h(r,T) as well, and we find that to leading order, for a slow roll scalar the black hole geometry takes its "Schwarzschild" form in the scalar T-coordinate (regular on both horizons) but with Λ and M now time varying.
We get a remarkably simple expression for the time-dependence of the horizon areas:

$$\dot{A}_h = \frac{A_h}{|\kappa_h|} \frac{\dot{\phi}^2}{M_p^2}$$

Thermodynamics

We can find exact, differential forms of the various thermodynamic first laws:

- De Sitter patch:

$$|\kappa_b|\dot{A}_b + |\kappa_c|\dot{A}_c + V\dot{\Lambda} = 0$$

- Black hole first law:

$$\frac{\dot{M}}{M_p^2} - |\kappa_b|\dot{A}_b + V_b\dot{\Lambda} = 0$$

Which of course begs the question of temperature..

Dynamical Temperature

Hayward et al suggested a dynamical temperature

$$\kappa_{dyn} = \frac{1}{2} \star d \star dr$$

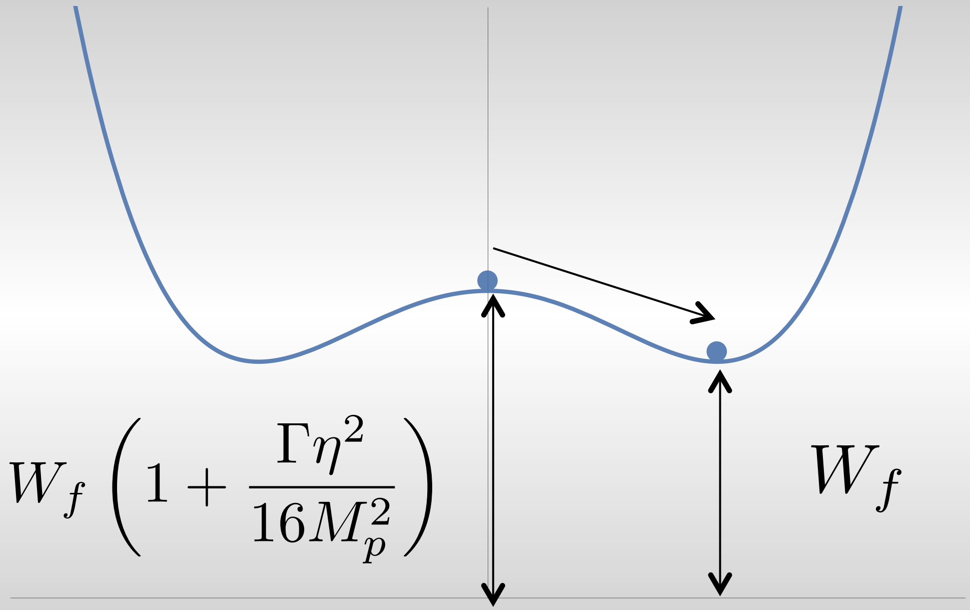
Which we can calculate for our solution

$$\kappa_{dyn}(T) = (f' + \dot{h}) = \kappa_b(T) + \mathcal{O}(\varepsilon\Gamma)$$

i.e. the instantaneous temperature of the time-dependent SdS potential.

Explicit Example

Double well potential



Cosmological soln:

$$\phi^2 = \frac{\eta^2}{2}\left[1 + \tanh\left(\frac{H_f \Gamma}{4}\tau\right)\right]$$

At black hole horizon in terms of E-F advanced time:

$$\kappa_h T \simeq \ln(2\kappa_h v) = \ln \hat{v}$$

And write $a = \dfrac{\Gamma H_i^2}{2\gamma\kappa_h}$ so that $\phi^2 = \eta^2 \dfrac{\hat{v}^a}{1+\hat{v}^a}$

Then can extract the behaviour of the horizon, directly depending on the gravitational strength of the scalar

$$\Delta = \frac{\eta^2}{M_p^2}$$

Horizon Growth

Horizon growth depends primarily on Δ but the rate of growth determined by the slow roll friction parameter

$$\mathcal{A} = 4\pi B = \mathcal{A}_0 \left(1 - \frac{a\Delta\hat{v}}{8} \mathcal{I}[\hat{v}, a] \right)$$

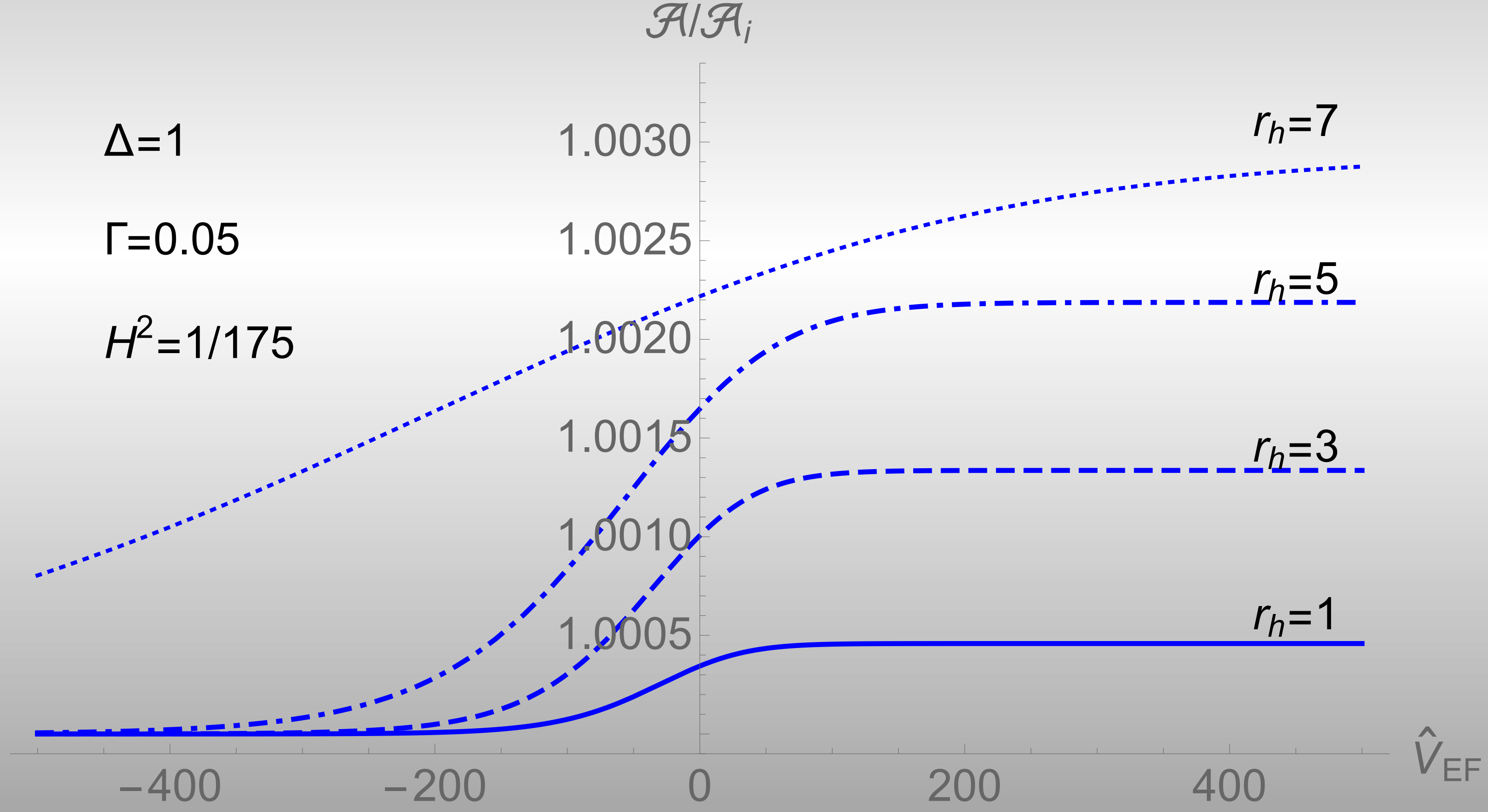
(Can integrate horizon behaviour exactly in null coord system)

$$\mathcal{I}[\hat{v}, a] = -\int_{\hat{v}}^{\infty} \frac{y^a(2 + y^a)dy}{y^2(1 + y^a)^2}$$

$$= -\frac{(1 + a(1 + \hat{v}^a))}{a\hat{v}(1 + \hat{v}^a)} + \frac{1 + a}{a\hat{v}} \left(1 - \ {}_2F_1 \left[1, \frac{1}{a}; \frac{1 + a}{a}; -\hat{v}^{-a} \right] \right)$$

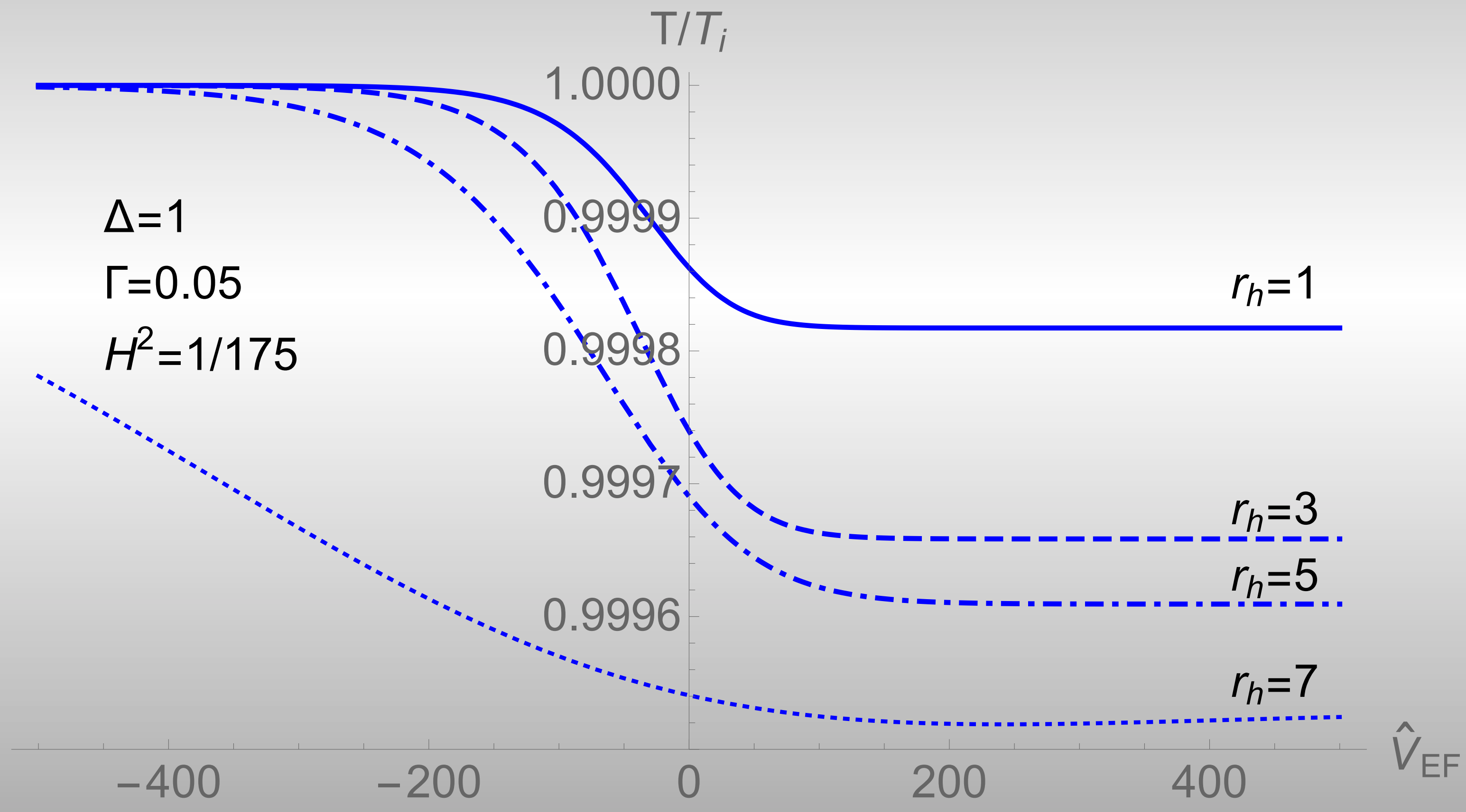
Horizon Growth

Horizon growth depends primarily on Δ but the rate of growth determined by the slow roll friction parameter



Dynamical T:

And temperature variation:



Summary

- Have generalised slow-roll description to non-homogeneous black hole background.
- The friction parameter for the scalar is increased by the black hole
- Checked the first law – holds dynamically during the flow.
- Explored dynamical temperature
- The black hole geometry is to a very good approximation quasi-Schwarzschild de Sitter.