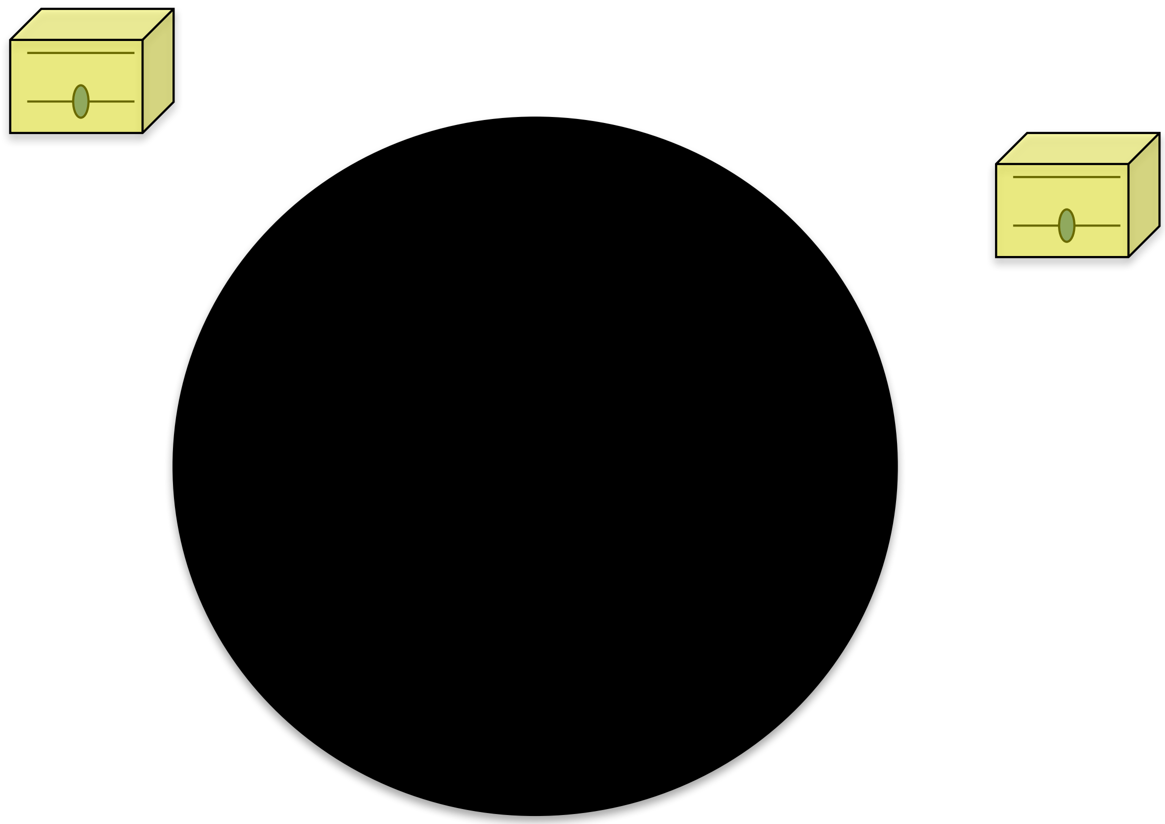
Harvesting Entanglement Near Black Holes



Robert B. Mann
L. Henderson R. Hennigar K. Ng A. Smith J. Zhang
E. Martin-Martinez

Entanglement Harvesting

- Extracting correlations from the vacuum
- Uncorrelated detectors can become correlated after a finite time depending on their
  - Energy gaps
  - Separation
  - State of motion
- Applications (in principle)
  - Seismology
  - Rangefinding
  - Quantum Key Distribution
  - Extraction from Atoms
- Very little is known about extraction in curved spacetime

Brown/Donnelly/Kempf/RBM/Martin-Martinez NJP16 (2014)105020

Salton/RBM/Menicucci NJP17 (2015) 035001

Ralph/Walk NJP17 (2015) 063008

Pozas-Kerstjens /Martin-Martinez PRD94 (2016) 064074

A Brief History of Entanglement Harvesting

- 1991: Valentini: uncorrelated atoms that are spacelike separated can become correlated via QED vacuum fluctuations

  Valentini PLA153(1991) 321
- 2003: Reznik rediscovers and quantifies this effect

  Reznik FndPhy 33 (2003) 167
- 2009: VerSteeg/Menicucci: investigate non-local correlations between detectors in de Sitter spacetime → first curved-spacetime study

  Ver Steeg/Menicucci PRD 79 (2009) 044027
- 2011: Harvesting Named: Quantum field correlations swapped with accelerating detectors

  Salton/RBM/Menicucci NJP17 (2015) 035001
- 2013: Sustainable Harvesting → entanglement farming

  Martin-Martinez /Brown/Donnelly PRA88 (2013) 052310
- 2017: Harvesting outside of black holes

  Henderson/Hennigar/RBM/Smith/Zhang (2018) 1712.10018

Quantum Detectors

S-Y Lin, B.L.Hu PRD73 (2006) 124018
PRD76 (2007) 064008

$$S_I = \lambda_0 \int d\tau \int d^4 x Q(\tau)\Phi(x)\delta^4\left(x^\mu - z^\mu(\tau)\right)$$

Vacuum

$$S = \underbrace{\frac{m_0}{2}\int d\tau\left[\left(\partial_\tau Q\right)^2 - \Omega_0^2 Q^2\right]}_{\text{detector}} - \underbrace{\int d^4 x\sqrt{-g}\,\frac{1}{2}\left(\nabla\Phi(x)\right)^2}_{\text{field}} + \underbrace{S_I}_{\text{interaction}}$$

detector field interaction

Cavity

$$\hat{H} = \Omega_d \hat{a}_d^\dagger \hat{a}_d + \frac{dt}{d\tau}\sum_n \omega_n \hat{a}_n^\dagger \hat{a}_n + H_I$$

$$H_I = \lambda(\tau)(\hat{a}_d e^{-i\Omega\tau} + \hat{a}_d^\dagger e^{i\Omega\tau})\sum_n \left(\hat{a}_n u_n[x(\tau),t(\tau)] + \hat{a}_n^\dagger u_n^*[x(\tau),t(\tau)]\right)$$

Provide an operational means of probing the quantum character of spacetime

E.G. Brown, E. Martin-Martinez, N. Menicucci, RBM PRD87 (2013) 084062
D. Bruschi, A. Lee, I Fuentes J. Phys A46 (2013) 165303

Hot Accelerating Detectors?

$$T = \frac{a}{2\pi}\left(\frac{\hbar}{k_B c}\right)$$

S.A. Fulling PRD7 (1973) 2850 P.C.W. Davies J Phys A8 (1975) 609
W. G. Unruh PRD14 (1976) 3251

- Unruh effect
  - Geometric Methods + Bogoliubov transformations
  - Eternally accelerating qubit coupled to a quantum field

B. deWitt in *General Relativity: An Einstein Centenary Survey* (CUP 1980)

- Limitations
  - Highly idealized: eternal uniform acceleration, unbounded system, perturbative, model-dependent, …
- What we would like and need to know
  - Finite time and distance effects (cavities, switching)
  - Boundary conditions
  - Non-perturbative effects; non-equilibrium effects
  - Entanglement, Non-locality of correlations
- Interplay with curved spacetime and gravity?

Detector Response Outside Black Holes

- BTZ Black holes
  - Static and Rotating
- Schwarzschild Black Holes
- Schwarzschild AdS Black Holes
- All for various boundary conditions, detector trajectories

Hodgkinson/Louko PRD86 (2012) 064031

Hodgkinson/Louko/Ottewill PRD89 (2014) 104002

Ng/Hodgkinson/Louko/ RBM/Martin-Martinez PRD90 (2014) 064003

field

$$H_{\text{int}} = c\chi(\tau)\mu(\tau)\phi(x(\tau))$$

switch

monopole operator

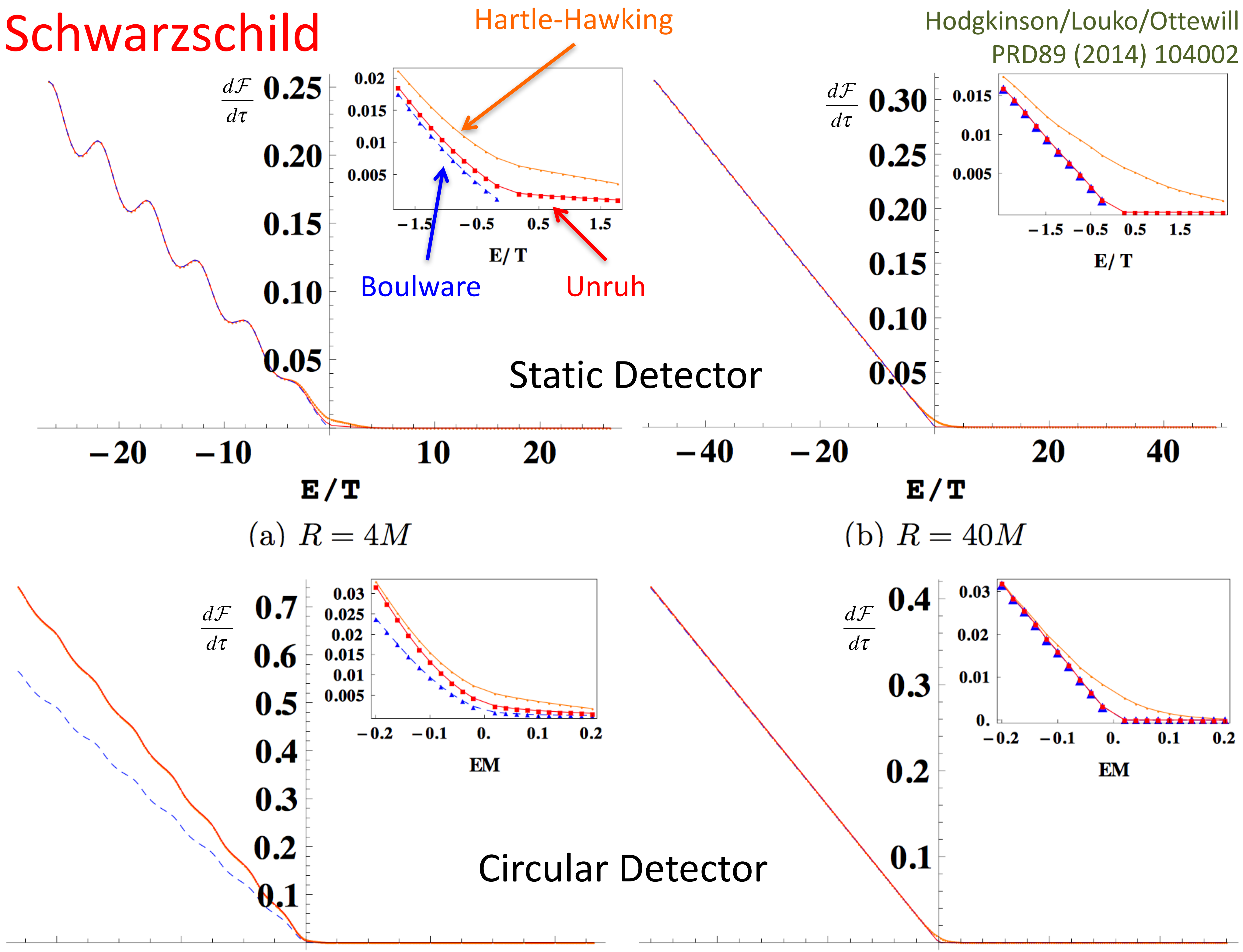
$$P(E) = c^2 \left|\langle 0_d | \mu(0) | E \rangle\right|^2 \mathcal{F}(E)$$

Wightman function

$$\mathcal{F}(E) = \Re\left[\int_{-\infty}^{\infty} du\,\chi(u)\int_0^{\infty} ds\,\chi(u-s)e^{-iEs}W(u,u-s)\right]$$

$$\frac{d\mathcal{F}}{d\tau}(E;M,\ell,\ldots) = \frac{1}{4} + 2\Re\left[\int_0^{\Delta\tau} ds\,e^{-iEs}W(\tau,\tau-s)\right]$$

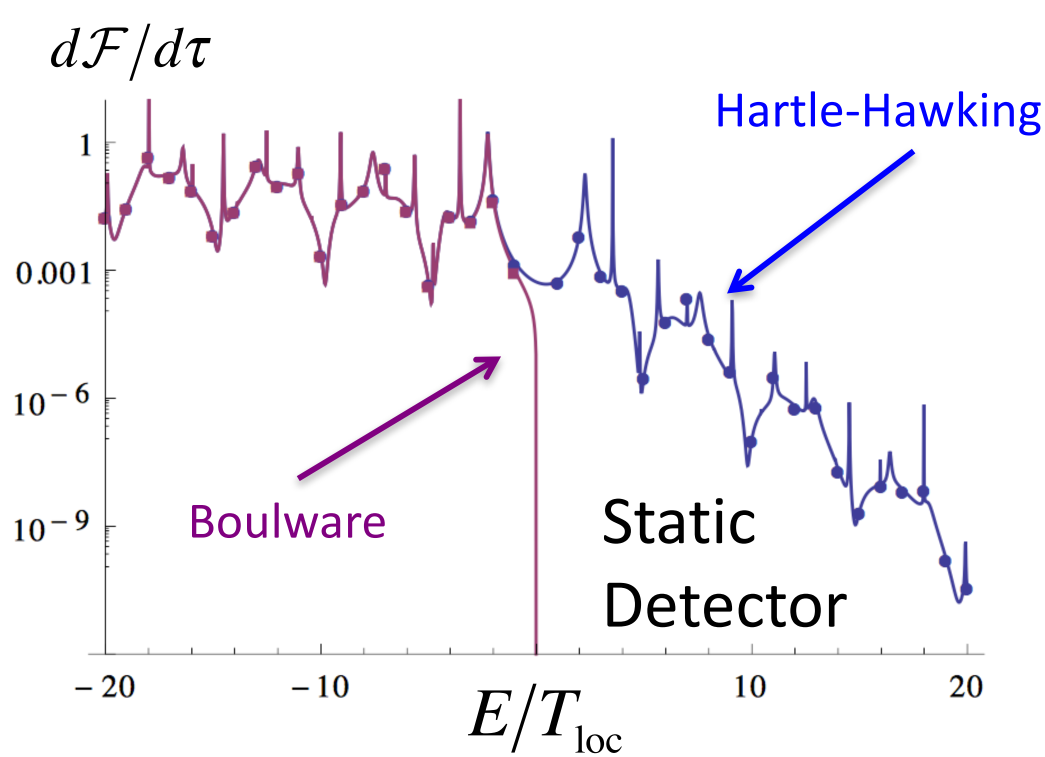
Detector Response Rate

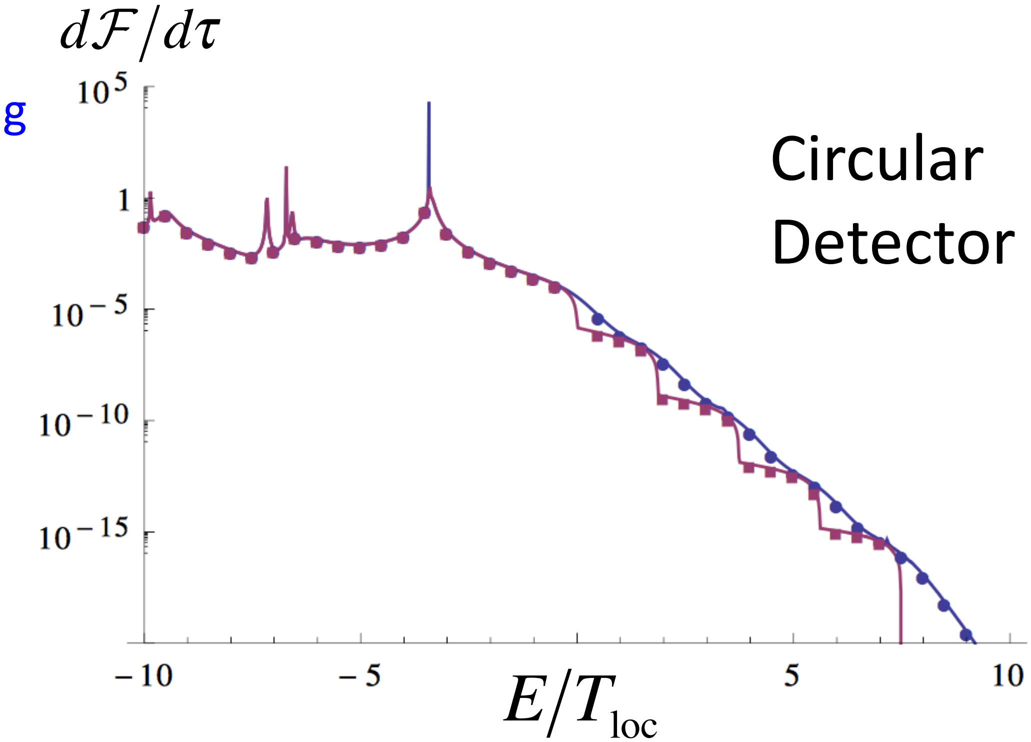


Schwarzschild
Hartle-Hawking
Hodgkinson/Louko/Ottewill
PRD89 (2014) 104002
Boulware
Unruh
Static Detector
Circular Detector
$\frac{d\mathcal{F}}{d\tau}$
E/T
EM
(a) $R = 4M$
(b) $R = 40M$

Detector Response in Schwarzschild-AdS

Ng/ Hodgkinson/Louko/RBM/Martin-Martinez
PRD90 (2014) 064003





- Spikes due to Quasinormal mode resonances
- Visible only when black hole is much smaller than AdS length
- Peaks become higher and sharper as black hole size decreases

Harvesting in Flat Spacetime

- Single Detectors have sensitivity to spacetime geometry and topology
- Can more be learned from Detector correlations?
- Flat Spacetime
  - 2 Static Detectors
  - 2 Accelerating Detectors
  - Detectors in Identified Minkowski Space

Pozas-Kerstjens/Martin-Martinez
PRD92 (2015) 064042

Salton/RBM/Menicucci
NJP17 (2015) 035001

Martin-Martinez /Smith/Terno
PRD93 (2016) 044001

Harvesting Formalism

$$S = -\int d^4x\sqrt{-g}\left[R + \frac{1}{2}\partial_\mu\Phi(x)\partial^\mu\Phi(x) - \xi R\Phi^2(x)\right]$$

$$+\int d\tau\left\{\frac{m_0}{2}\left[\left(\partial_\tau Q\right)^2 - \Omega_0^2 Q^2\right] + \sum_D \lambda_D \int d^4x Q_D(\tau)\Phi(x)\delta^4\left(x^\mu - z_D^\mu(\tau)\right)\right\}$$

$$U = \mathcal{T}e^{-i\int dt\left[\sum_D \frac{d\tau_D}{dt}H_{ID}(\tau_D)\right]}$$

$$H_{ID}(\tau) = \chi_D(\tau)\left[e^{i\Omega_D\tau}\sigma_D^+ + e^{-i\Omega_D\tau}\sigma_D^-\right]\Phi[z_D(\tau)]$$

switcher

monopole operator

$$\chi_D = \exp\left(-\frac{(\tau-\tau_D)^2}{2\sigma_D^2}\right)$$

$$\sigma_D^+ := |1_D\rangle\langle 0_D| = |e_D\rangle\langle g_D|$$

$$\sigma_D^+ := |0_D\rangle\langle 1_D| = |g_D\rangle\langle e_D|$$

$$\rho_{ij} := \mathrm{Tr}_\Phi(U|\Psi\rangle_i\langle\Psi|_i U^\dagger) = \begin{pmatrix} 1-P_A-P_B & 0 & 0 & X \\ 0 & P_B & C & 0 \\ 0 & C^* & P_A & 0 \\ X^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\rho = \begin{pmatrix} 1-P_A-P_B & 0 & 0 & X \\ 0 & P_B & C & 0 \\ 0 & C^* & P_A & 0 \\ X^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$W(x,x') := \langle 0|\phi(x)\phi(x')|0\rangle$$

$$\tau_D = \gamma_D t$$

$$P_D = \lambda^2 \int_{-\infty}^{\infty} d\tau_D \int_{-\infty}^{\infty} d\tau_{D'}\, \chi_D(\tau_D)\chi_{D'}(\tau_{D'}) e^{-i\Omega_D(\tau_D-\tau_{D'})} W\big(x_D(\tau_D), x_D(\tau_{D'})\big) \quad D = A,B$$

Local excitations

$$C = \lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A-\Omega_B\tau_B)} W\big(x_A(t), x_B(t')\big)$$

Local correlations

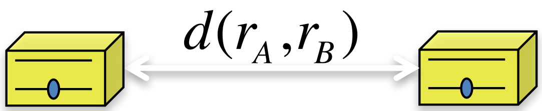
$$X = -\lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \Big[\frac{d\tau_A}{dt'}\frac{d\tau_B}{dt} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_B\tau_B+\Omega_A\tau_A)} W\big(x_A(t'), x_B(t)\big) + \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A+\Omega_B\tau_B)} W\big(x_B(t'), x_A(t)\big)\Big]$$

Non-Local correlations

Concurrence

$$\mathcal{C} = 2\mathcal{N} = \max\{0, |X| - \sqrt{P_A P_B}\} + \mathcal{O}(\lambda^4)$$

Harvesting with Static Detectors



Switching Width

$$\sigma_A = \sigma_B = T$$

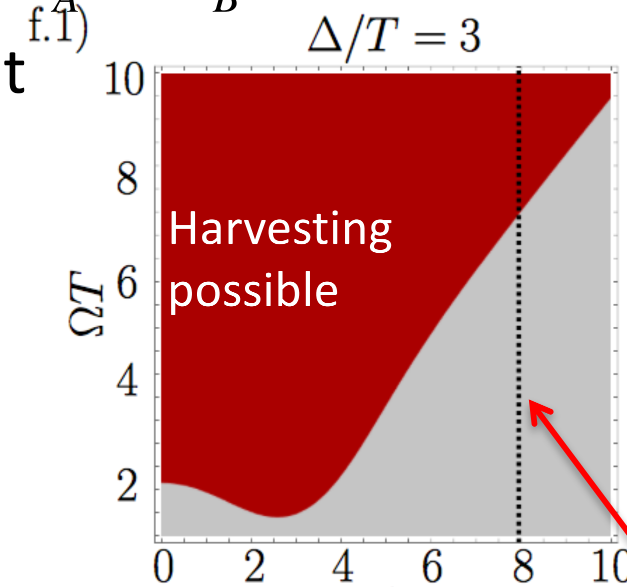
Pozas-Kerstjens/
Martin-Martinez
PRD92 (2015) 064042

Switching Displacement

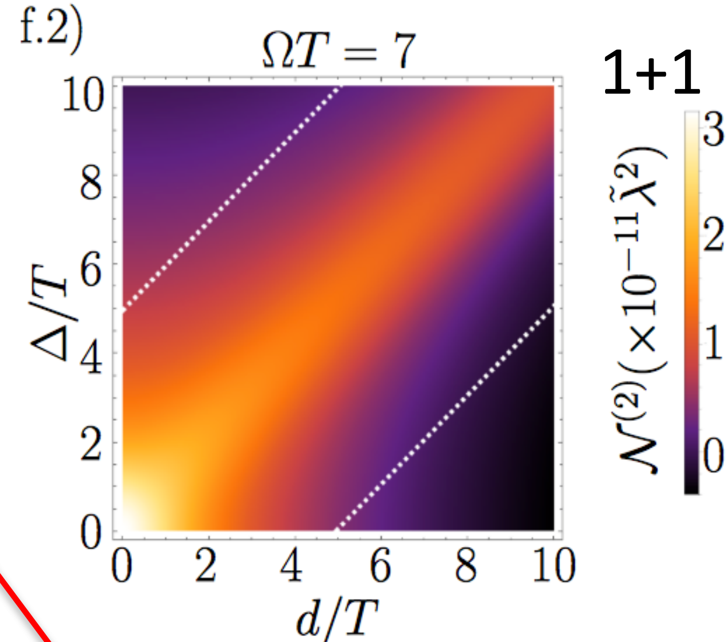
$$\tau_A - \tau_B = \Delta$$

Detector Gap

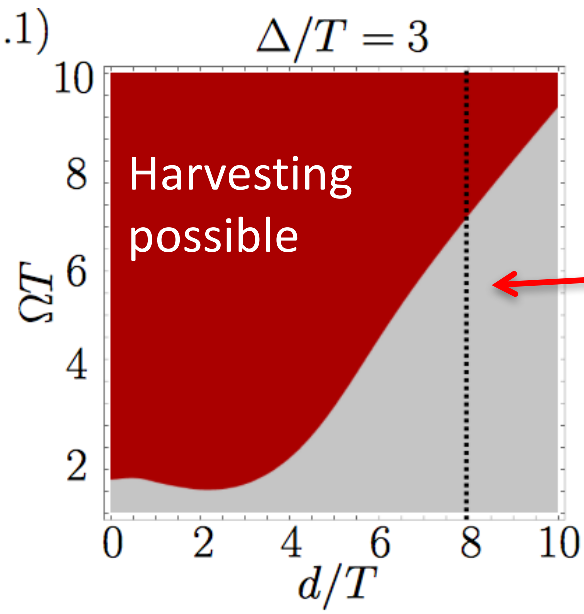
$$\Omega_A = \Omega_B = \Omega$$

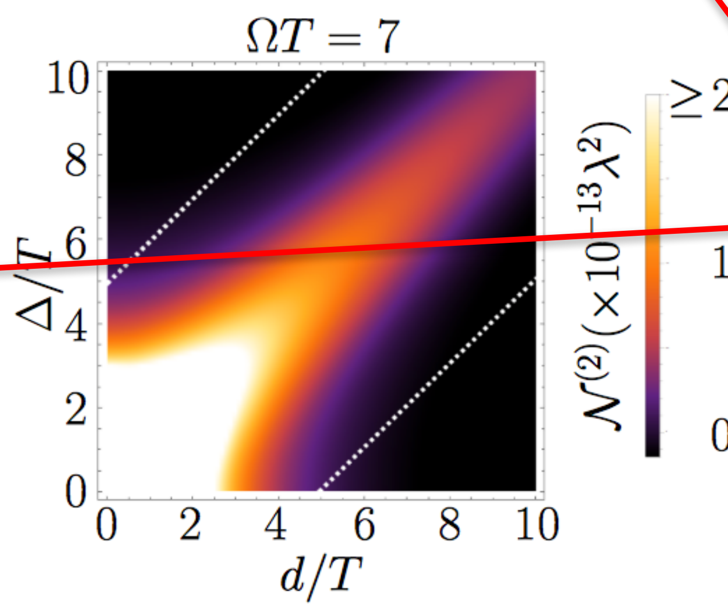


1+1



3+1





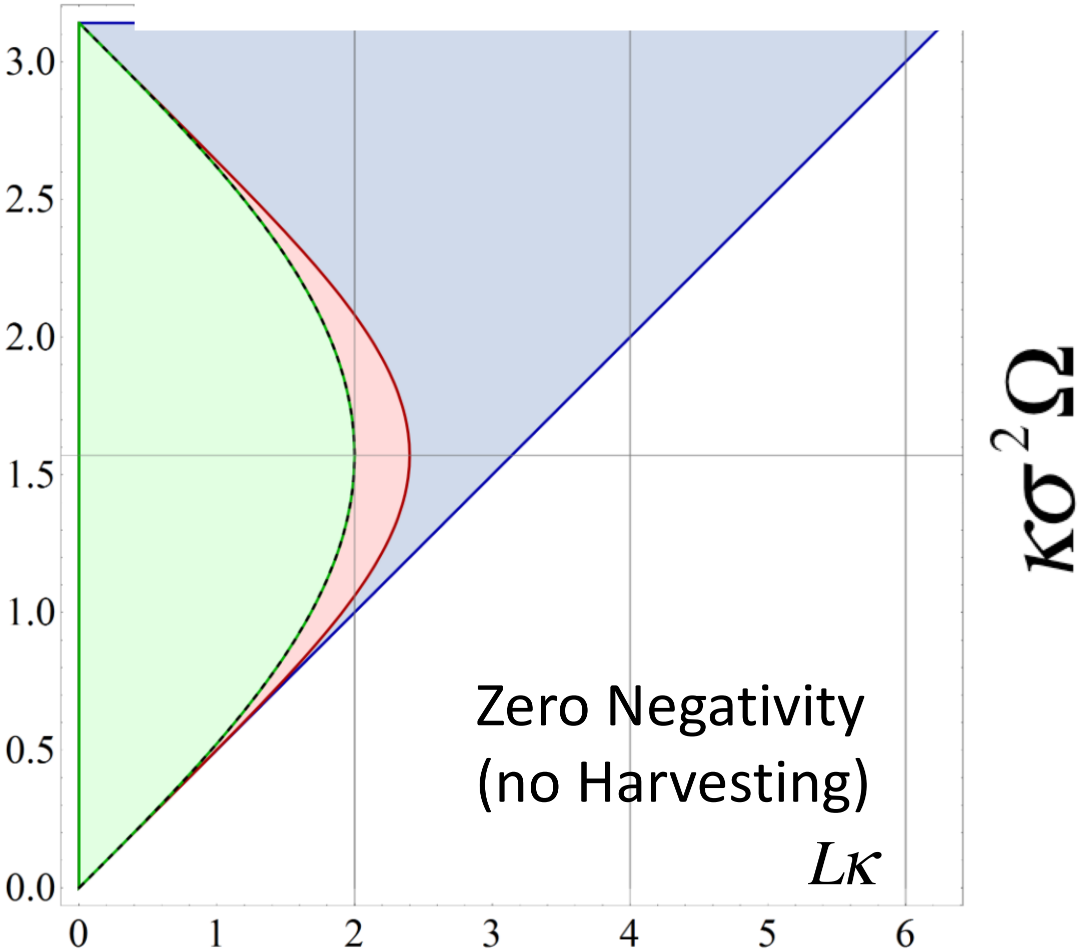
Spacelike separation

General Features of Flat-Space Harvesting

- Entanglement
  - Decreases with increasing separation
  - Increases with increasing gap
  - Weak dependence on spacetime dimension
  - Harvesting Possible at spacelike separation
- Other features studied
  - Pointlike vs. finite size
  - Sudden switching vs. Gaussian Switching

} Gaussian more efficient

Harvesting with Accelerating Detectors



Parallel Acc'n or de Sitter

Inertial detectors in thermal Minkowski

Inertial detectors in vacuum Minkowski

VerSteeg/Menicucci
PRD79 (2009) 044027

Salton/RBM/Menicucci
NJP17 (2015) 035001

Harvesting in Curved Spacetime

- De Sitter spacetime
  - Correlations distinct from thermal case

VerSteeg/Menicucci
PRD79 (2009) 044027

- Anti de Sitter spacetime
  - Single detector: thermal response above threshold
  - Harvesting: ??????

Jennings
CQG 27 (2010) 205005

$$T = \frac{\sqrt{a^2\ell^2 - 1}}{2\pi\ell}$$

- Black Holes
  - Single detector
    - response correlated with BH QNMs
    - sensitive to (hidden) topology
  - Harvesting: ??????

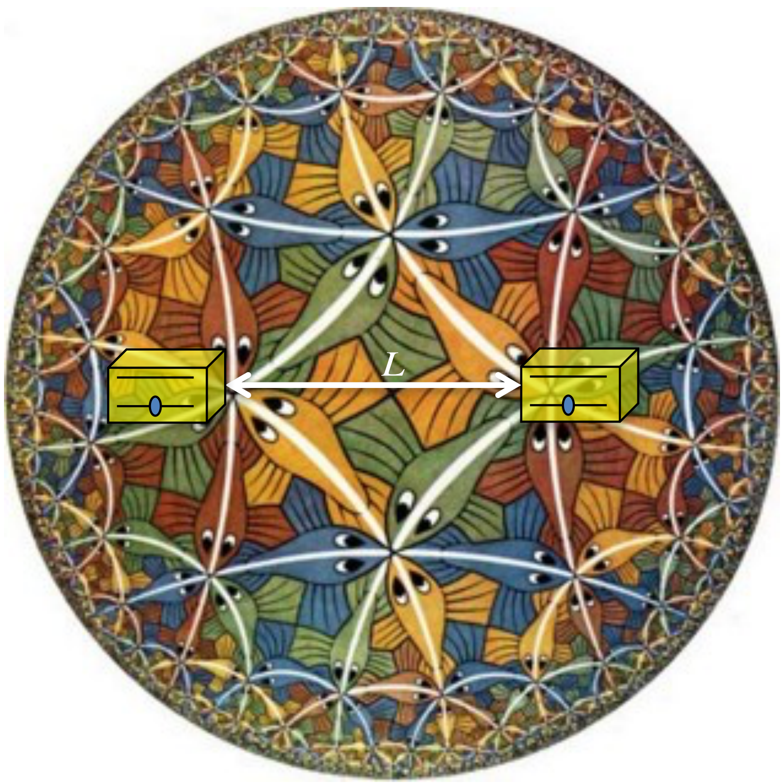
Ng/Hodgkinson/Louko/
Mann/Martin-Martinez
PRD 90 (2014) 064003

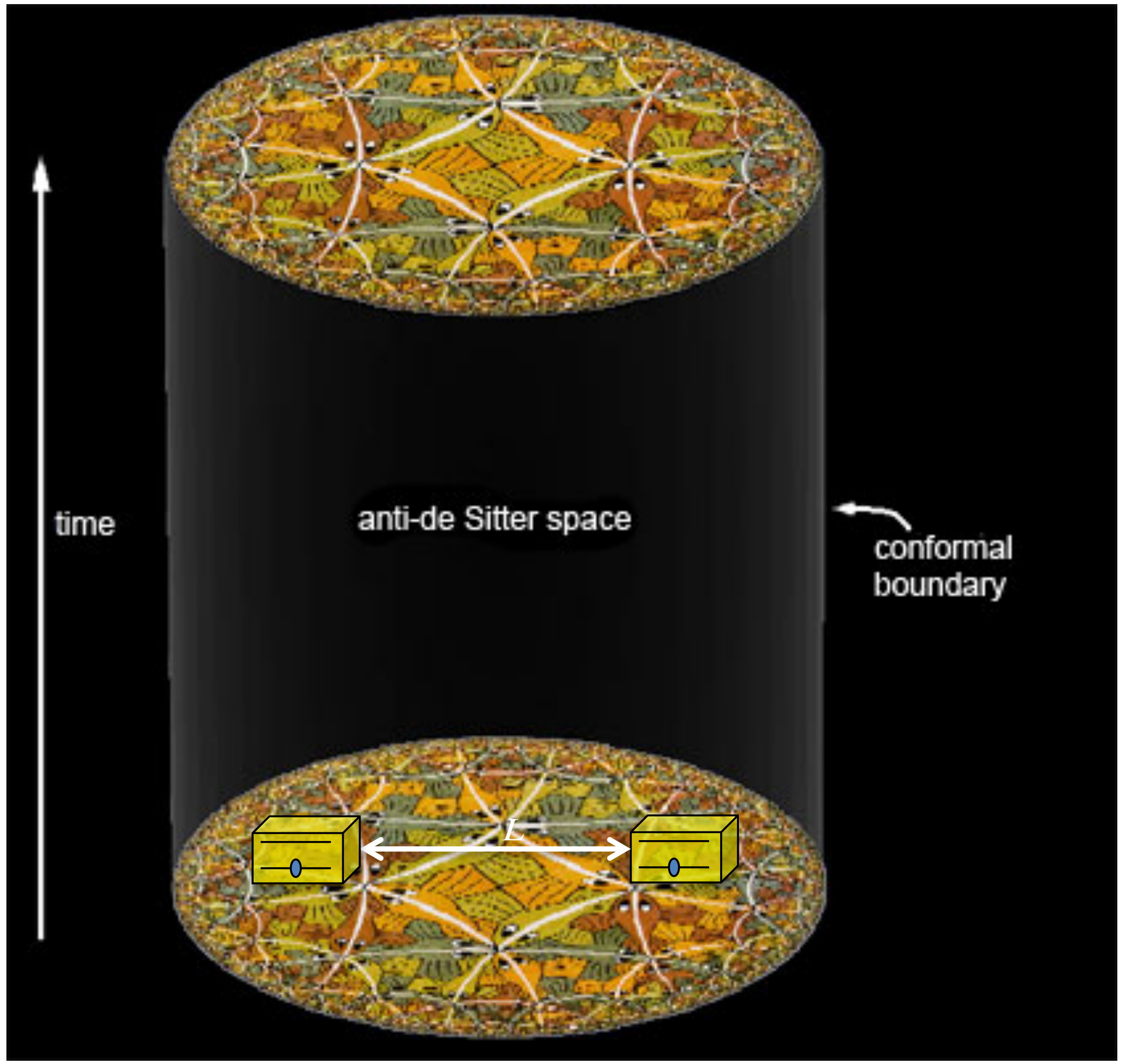
Ng/Mann/Martin-Martinez
PRD 96 (2017) 0850043

Smith/Mann CQG 31 (2014) 082001

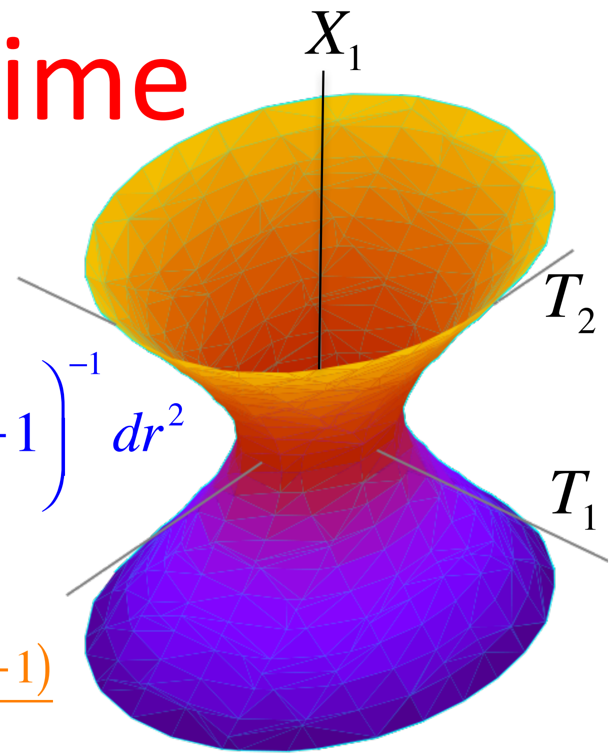
Harvesting in Anti de Sitter Space

2+1: Henderson/Hennigar/Smith/Zhang/RBM 1712.10018
3+1 Ng/Martin-Martinez/RBM (to appear)





Anti de Sitter Spacetime



$$\sum_{J=1}^{D-1} X_J^2 - T_1^2 - T_2^2 = -\ell^2$$

$$ds^2 = -dT_1^2 - dT_2^2 + \sum_{J=1}^{D-1} dX_J^2$$

Hyperboloid in flat spacetime

$$ds^2 = -\left(\frac{r^2}{\ell^2}+1\right)dt^2 + \left(\frac{r^2}{\ell^2}+1\right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2$$

$$\Lambda = -\frac{(D-2)(D-1)}{2\ell^2}$$

Conformal scalar coupling

$$W_{AdS}^{(\zeta)}(x,x') = \frac{1}{4\pi\ell\sqrt{2}}\left(\frac{1}{\sqrt{\sigma_\epsilon(x,x')}} - \frac{\zeta}{\sqrt{\sigma_\epsilon(x,x')+2}}\right)$$

2+1

Geodesic length

$\zeta = 1$ (Dirichlet)

$\zeta = 0$ (Transparent)

$\zeta = -1$ (Neumann)

$$W_{AdS}^{(\zeta=1)}(x,x') = \sum_{\omega=0}^{\infty}\sum_{lm}\frac{1}{2\omega}e^{-i\omega(t-t')}\varphi_{\omega lm}(x)\bar{\varphi}_{\omega lm}(x')$$

3+1

Sum over modes

$$\Phi_{\omega lm}(t,x) = \frac{1}{\sqrt{2\omega}}e^{-i\omega t}\varphi_{\omega lm}(x)$$

2+1 vs 3+1

	2+1	3+1
Constant Curvature	Yes	Yes
Conformally Flat	Yes	Yes
Conformally Coupled Scalar	Yes	Yes
Wightmann Function	Explicit Function of Geodesic length	Sum over Modes
Huygens Principle	No	Yes
Black Hole	From identifying AdS	Not identifying AdS

Results in 2+1 AdS

Henderson/Hennigar/Smith/Zhang/RBM
(to appear)

$$ds^2 = -\left(\frac{r^2}{\ell^2}+1\right)dt^2 + \left(\frac{r^2}{\ell^2}+1\right)^{-1} dr^2 + r^2 d\phi^2$$

Switching Width

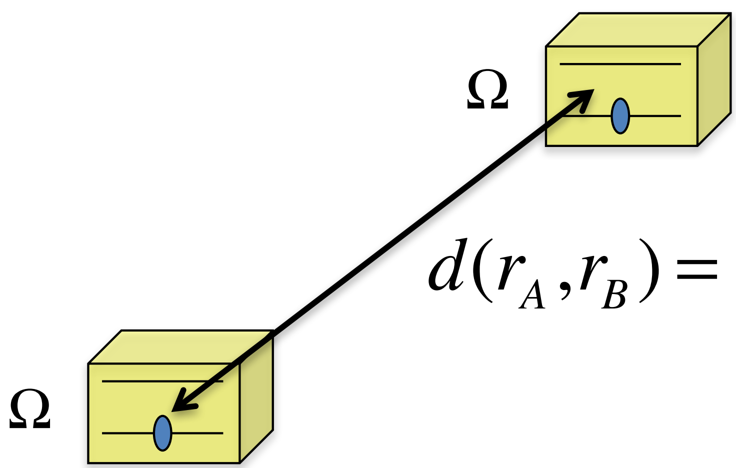
$$\sigma_A = \sigma_B = \sigma$$

Switching Displacement

$$\tau_A = \tau_B = 0$$

Detector Gap

$$\Omega_A = \Omega_B = \Omega$$



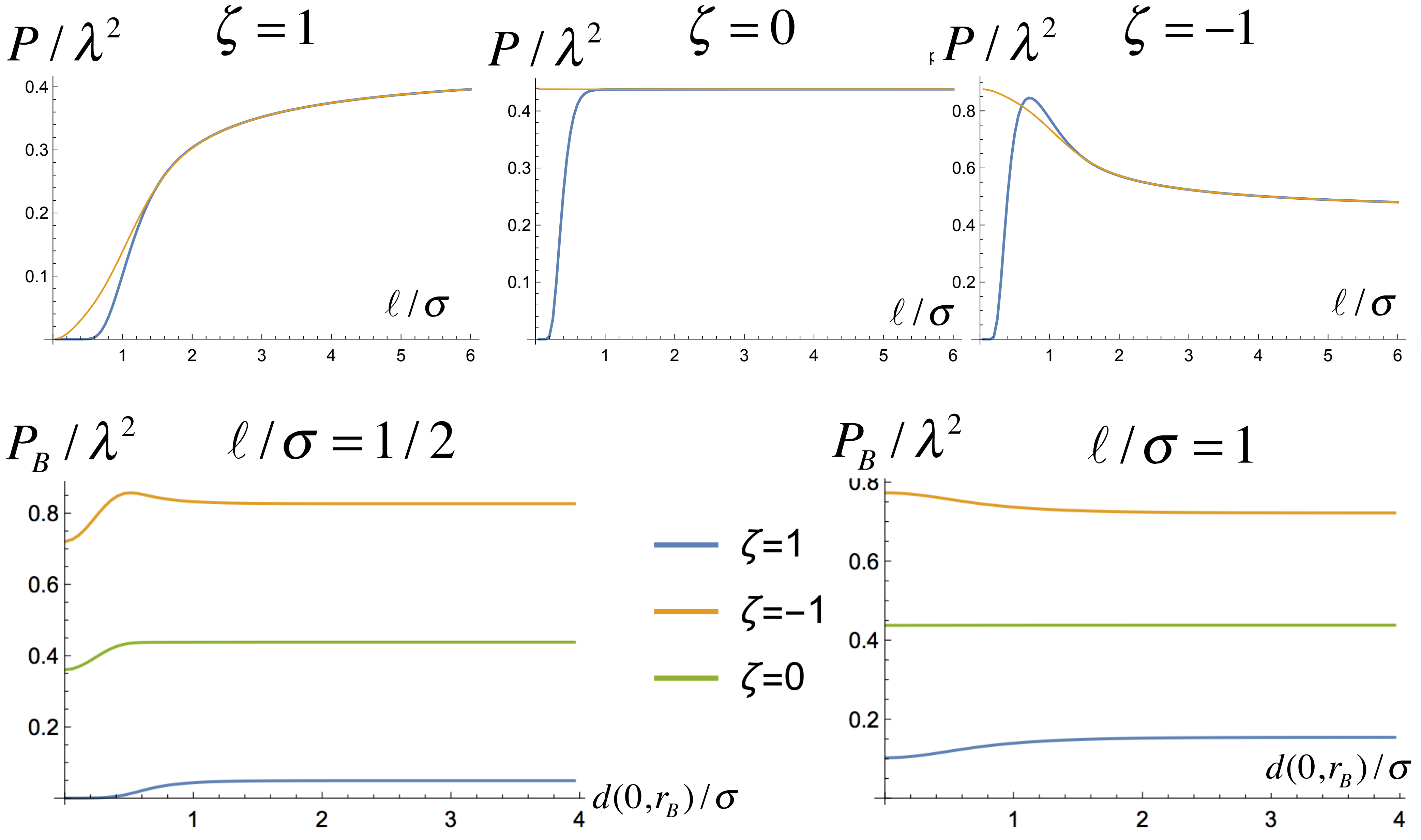
$$d(r_A, r_B) = \ell \ln\left[\frac{r_B + \sqrt{r_B^2 + \ell^2}}{r_A + \sqrt{r_A^2 + \ell^2}}\right]$$

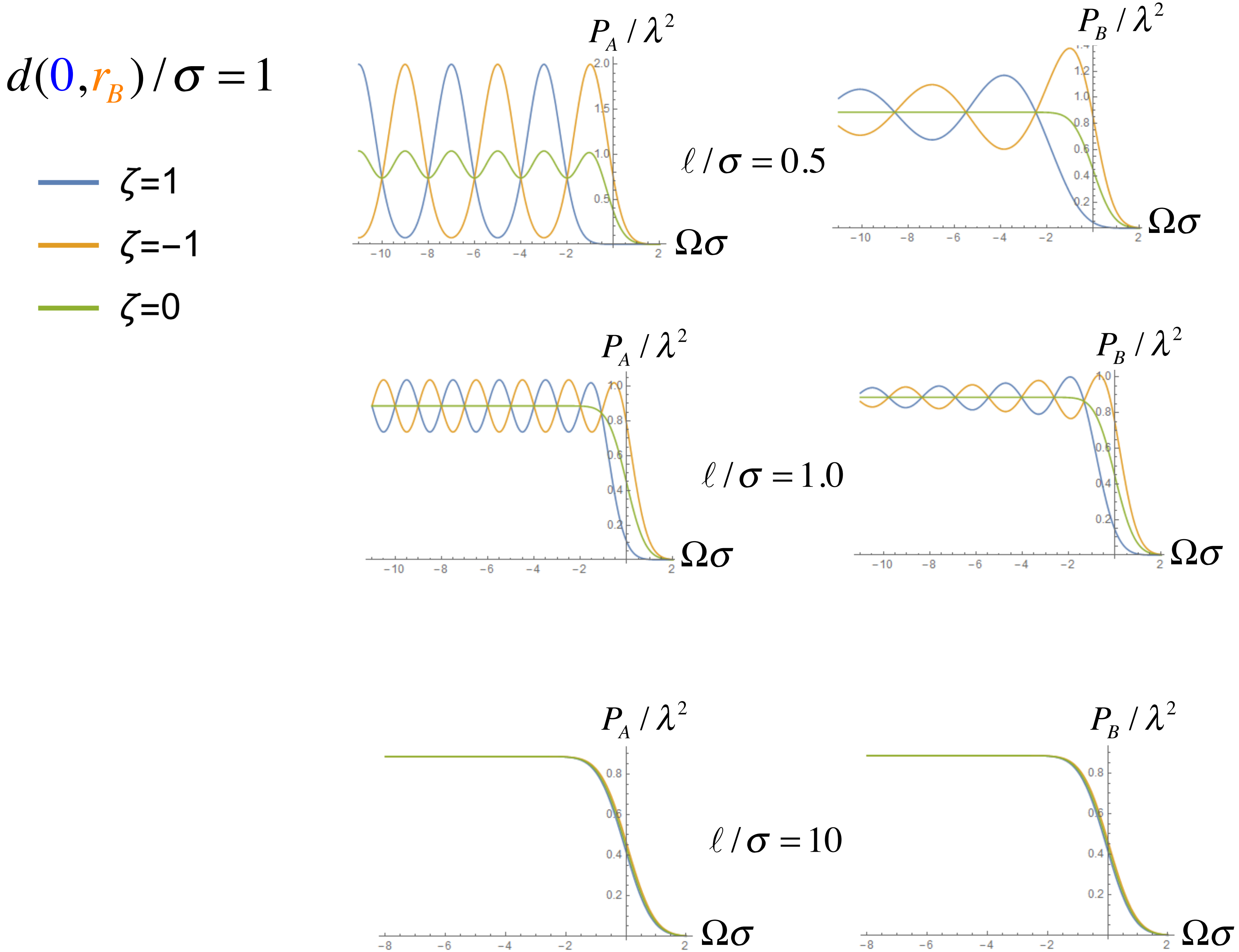
Detector Separation

Detector Excitation

$d(0, r_B)/\sigma = 1$

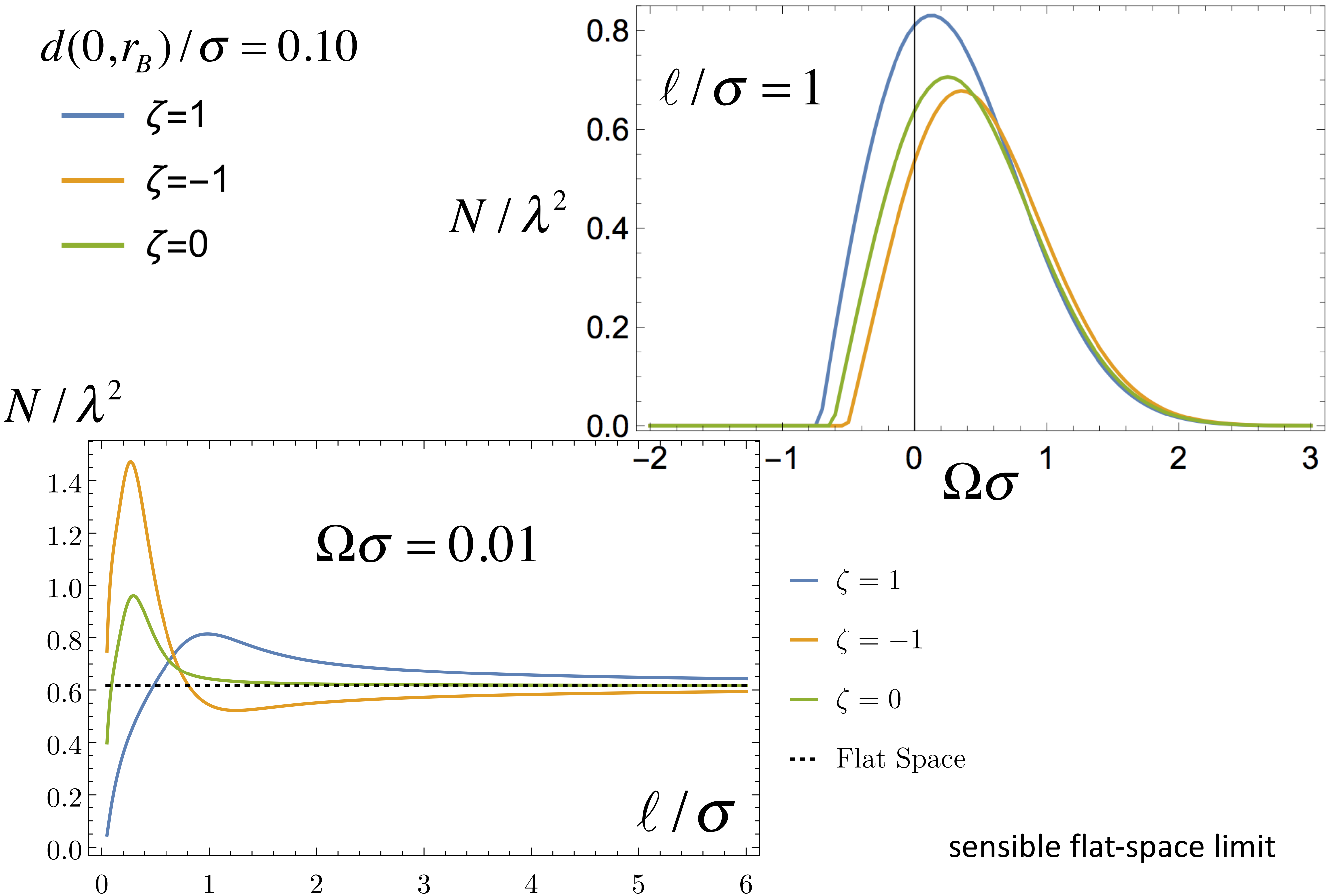
$\Omega\sigma = 0.1$





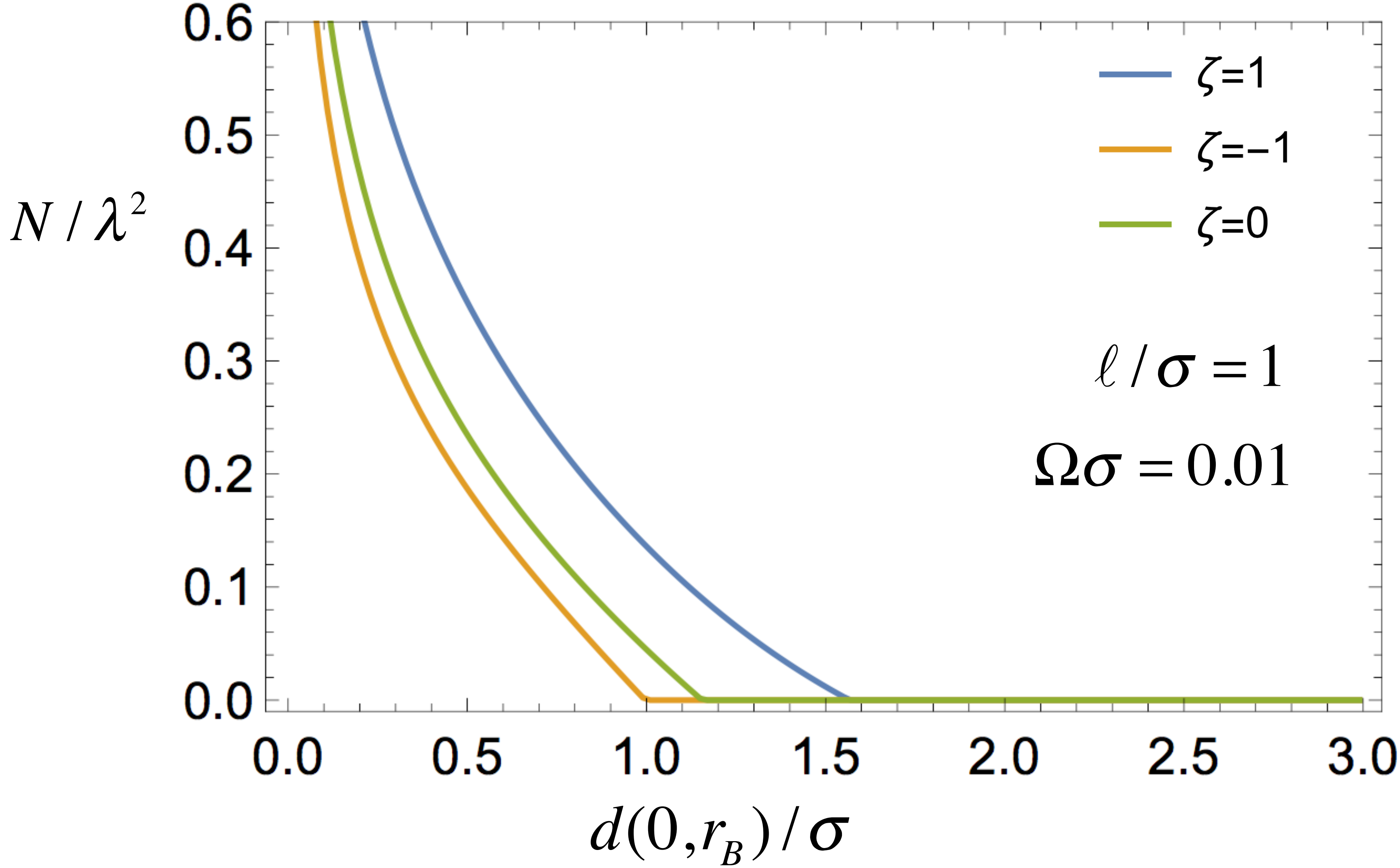
$d(0, r_B)/\sigma = 1$
ζ=1
ζ=−1
ζ=0
P_A/λ^2
P_B/λ^2
$\ell/\sigma = 0.5$
$\ell/\sigma = 1.0$
$\ell/\sigma = 10$
$\Omega\sigma$

Entanglement

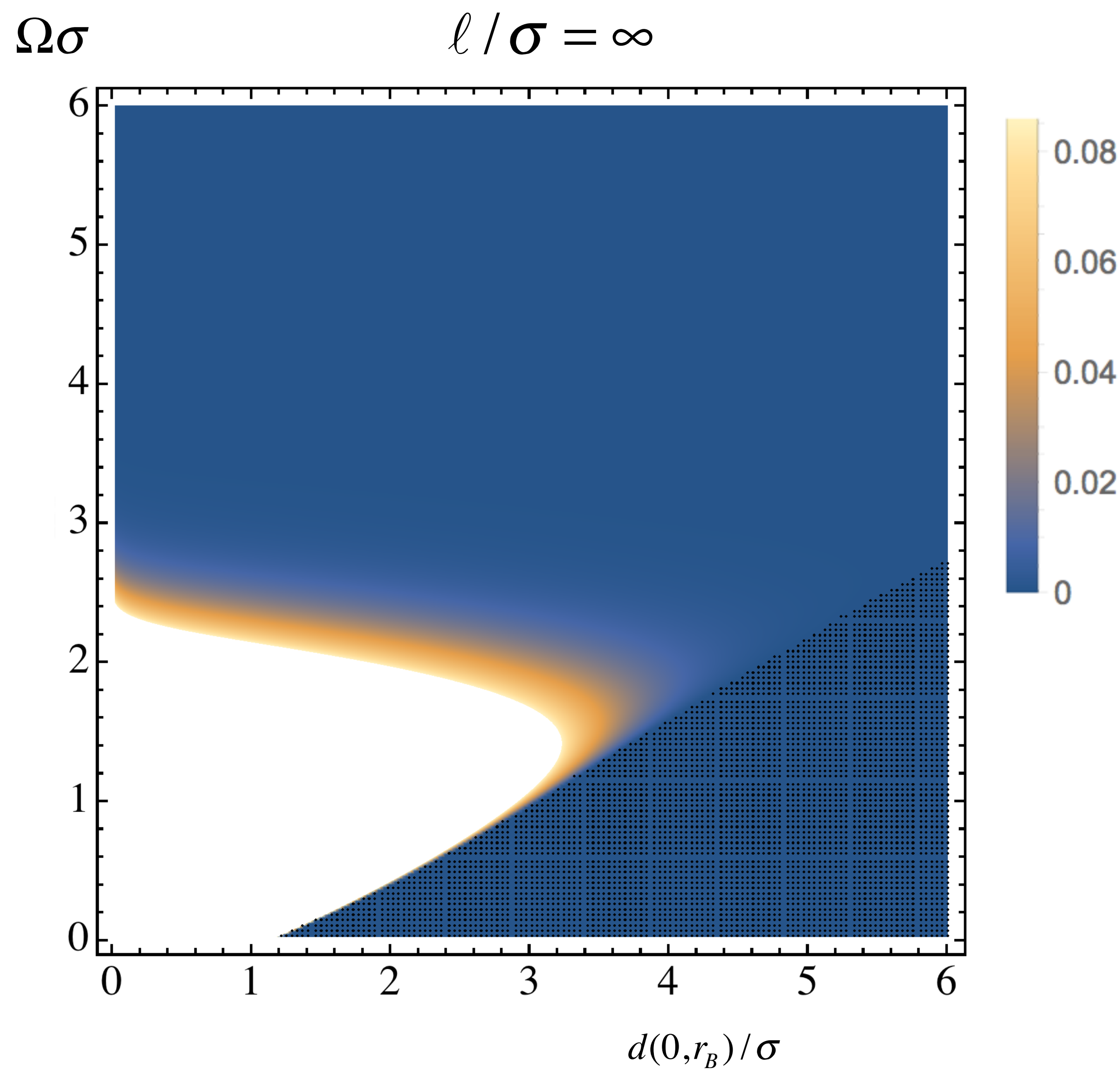


sensible flat-space limit

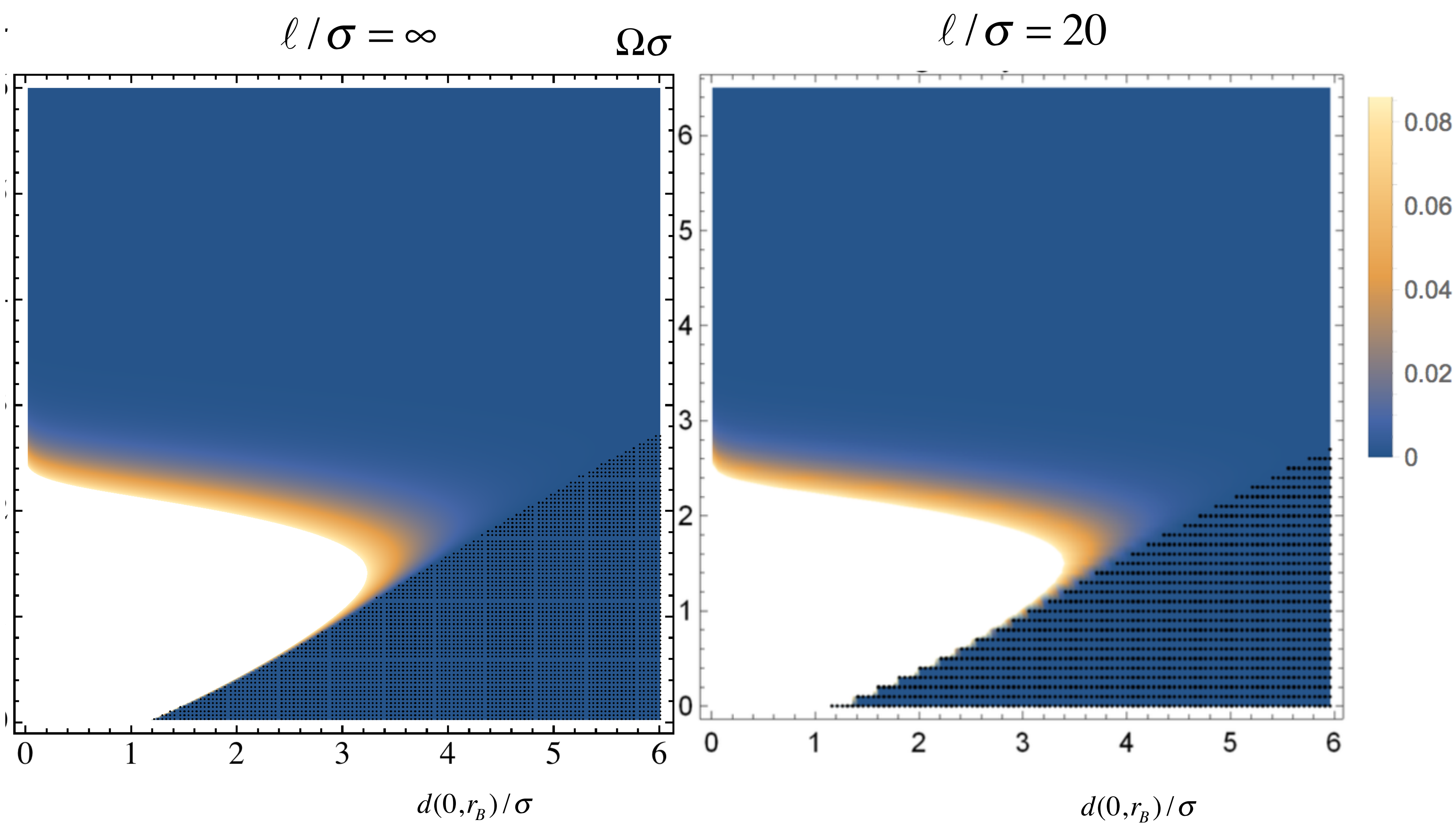
Entanglement Death at Large Separation



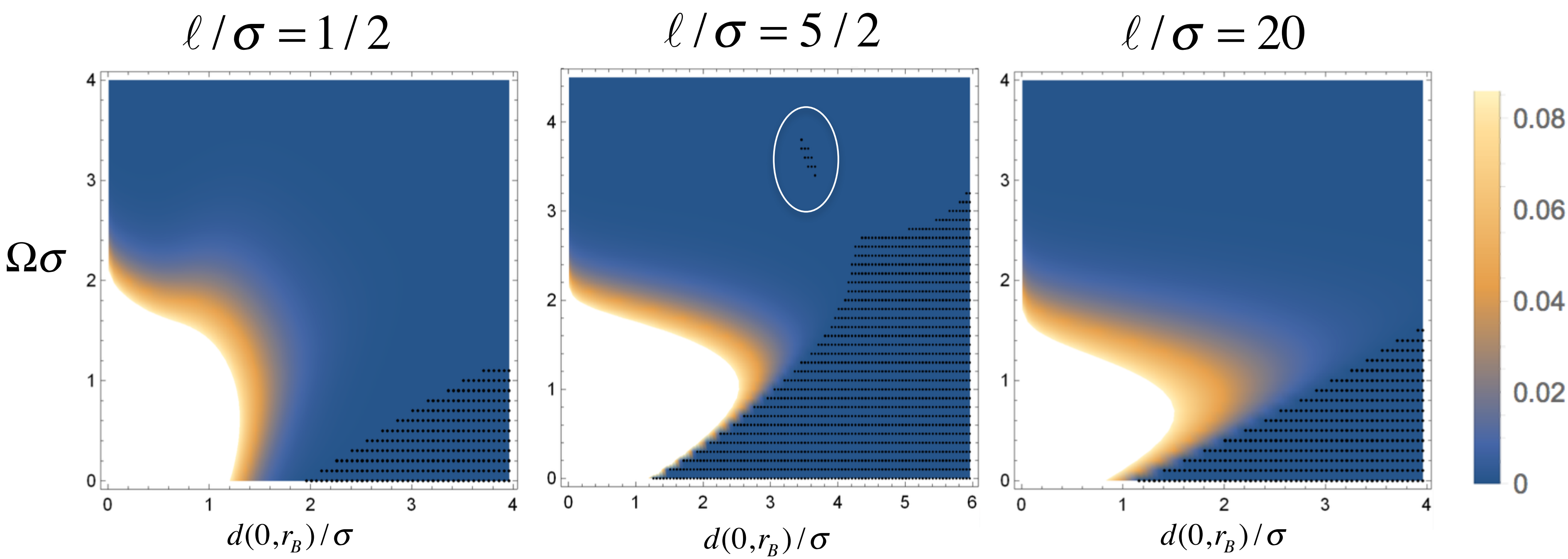
Flat Space Limit



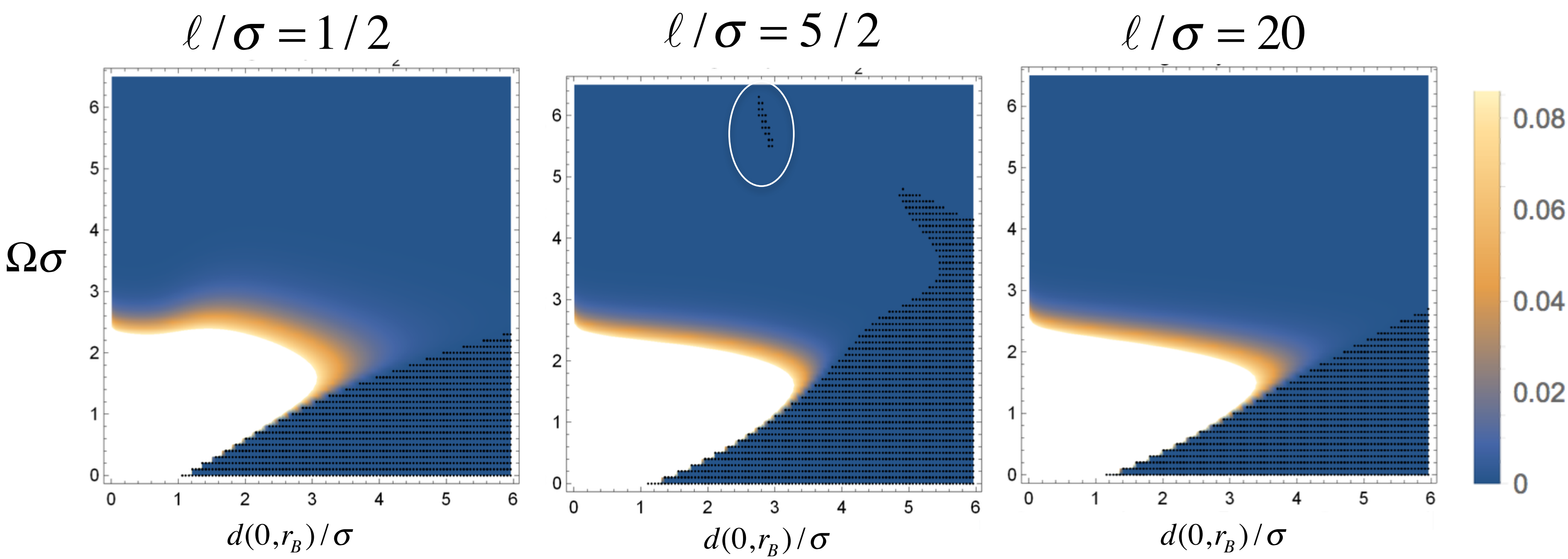
Flat Space Limit



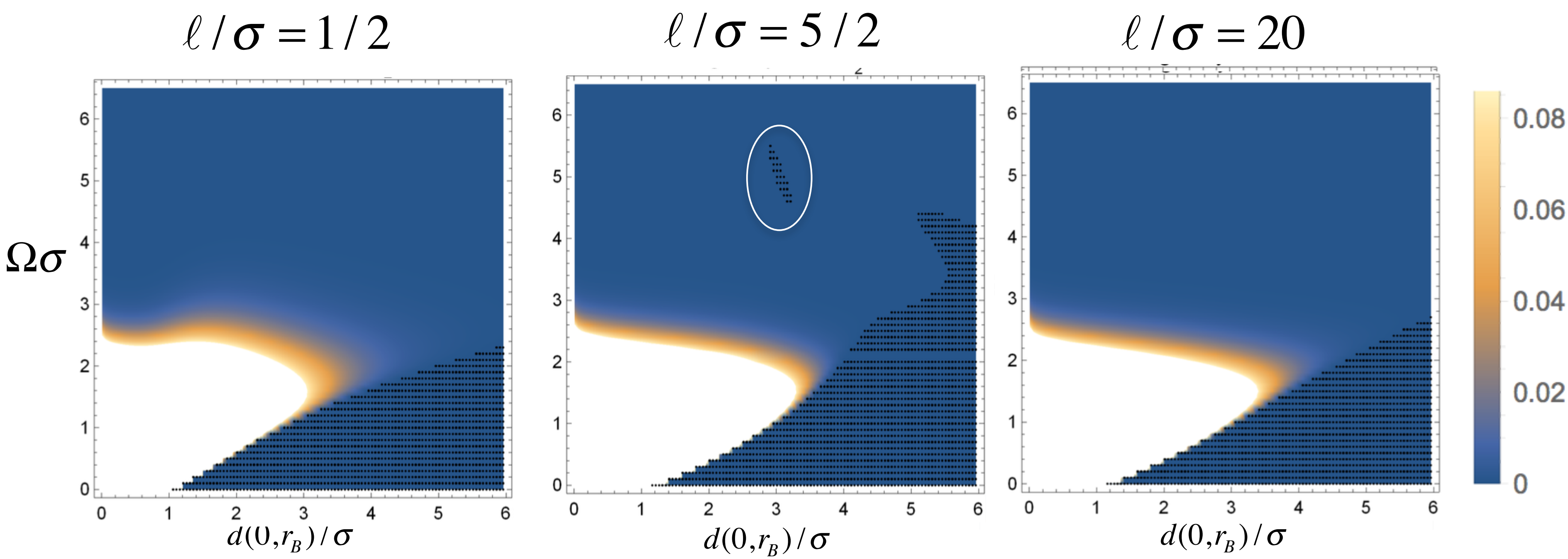
Negativity $\zeta = 1$



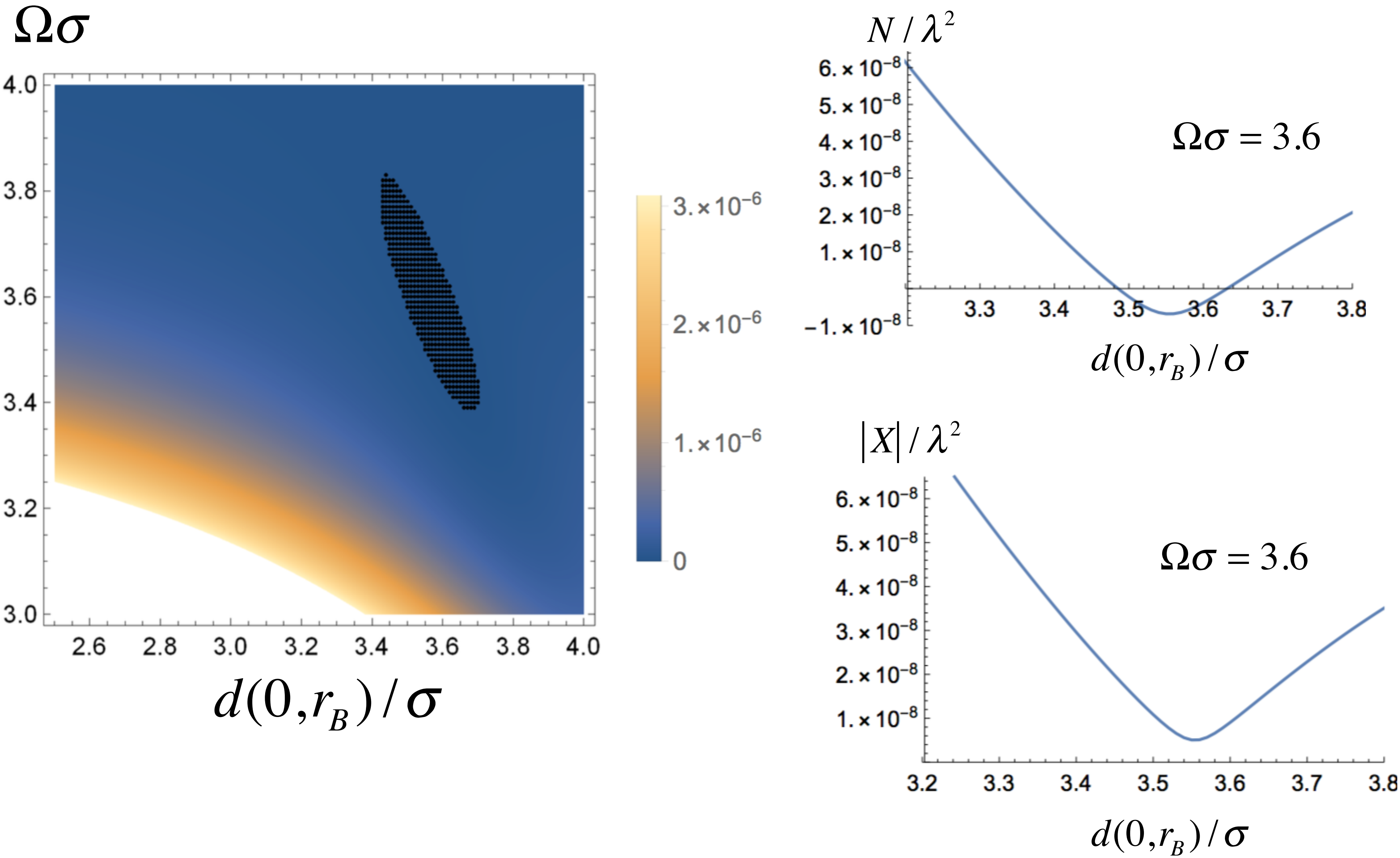
Negativity $\zeta = -1$



Negativity $\zeta = 0$



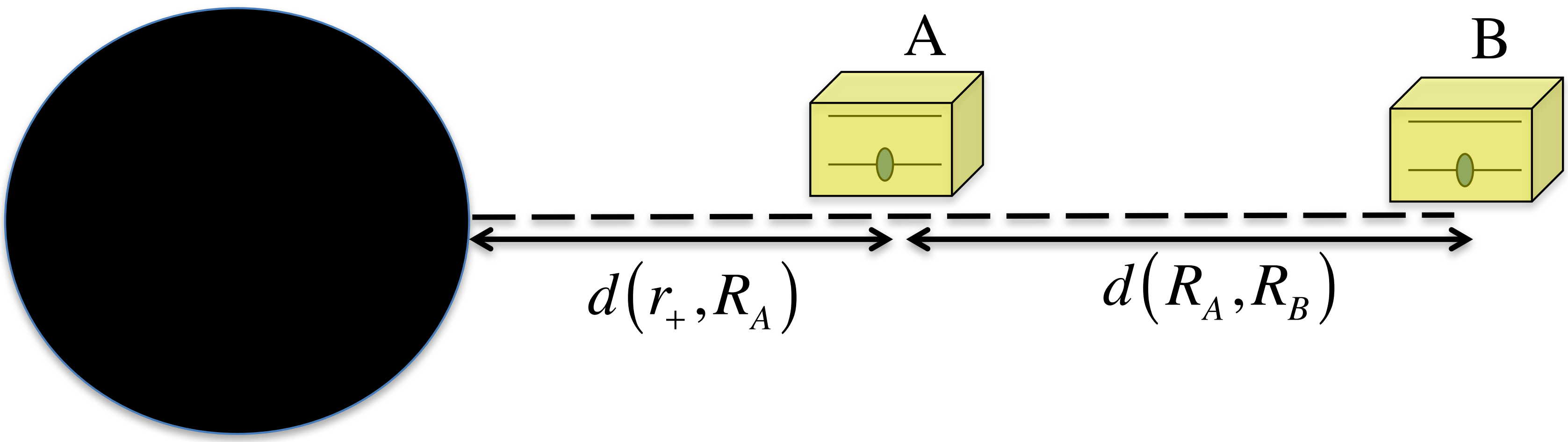
The Dead Zone $\ell / \sigma = 5/2$



Harvesting Near Black Holes

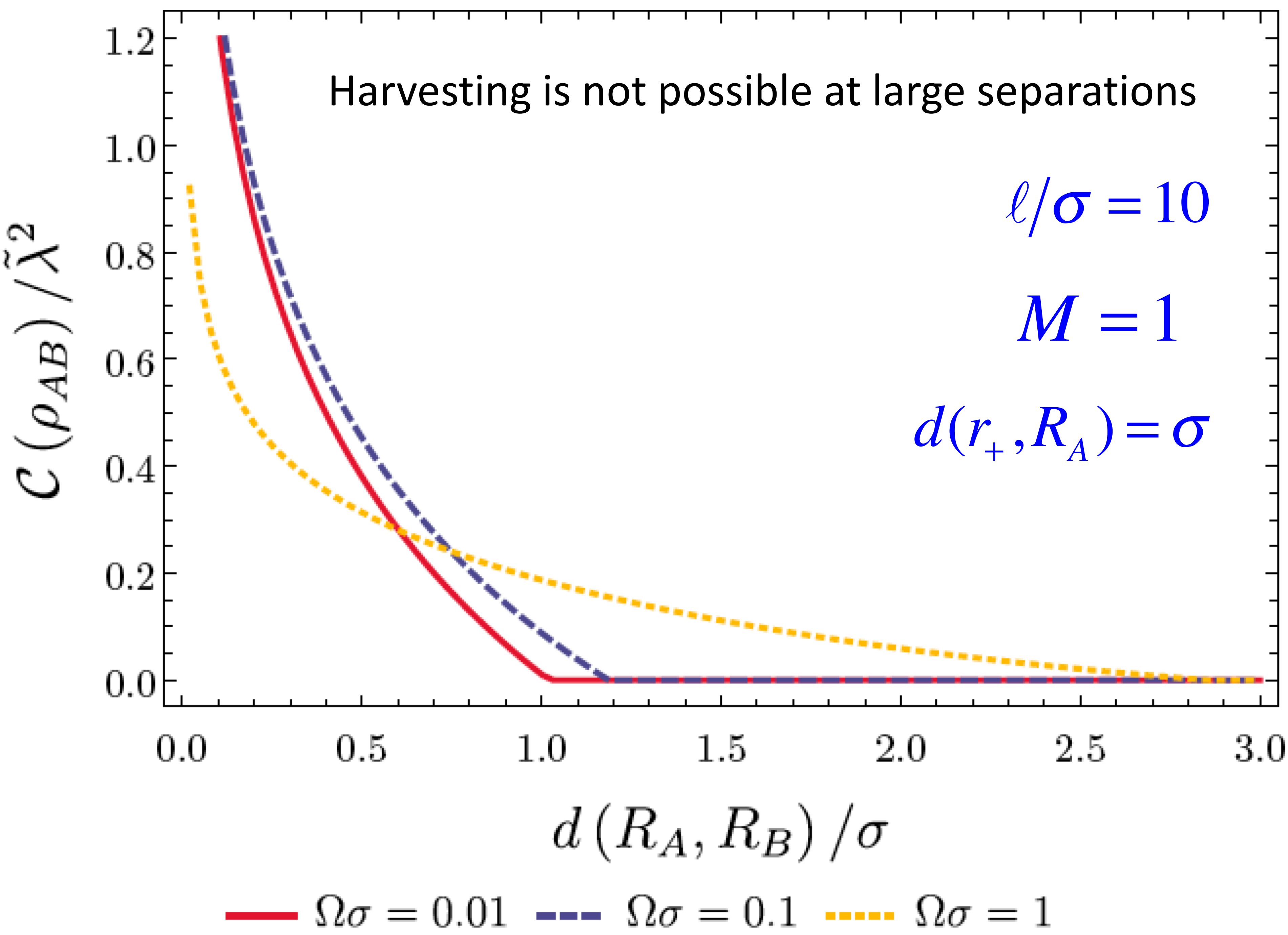
Henderson/Hennigar/Smith/Zhang/RBM
1712.10018

2+1 Black Hole



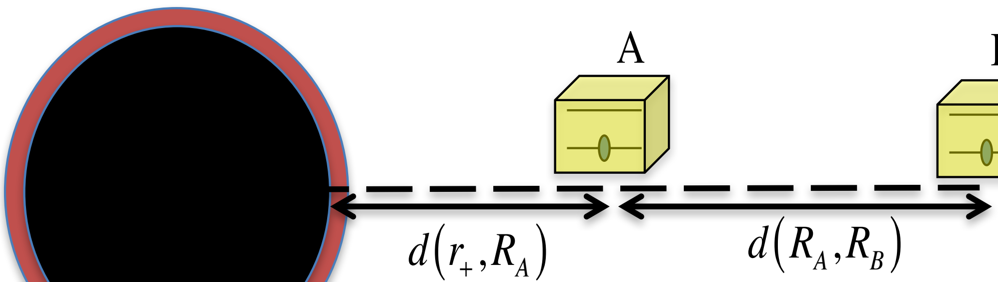
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2$$

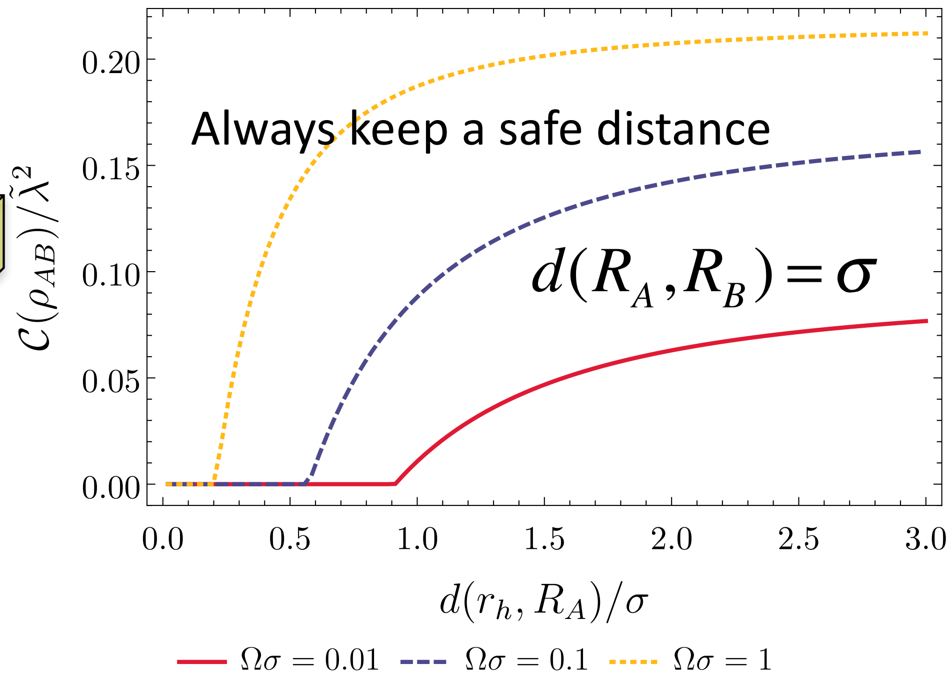
$$f(r) = \left(\frac{r^2}{\ell^2} - M\right)$$



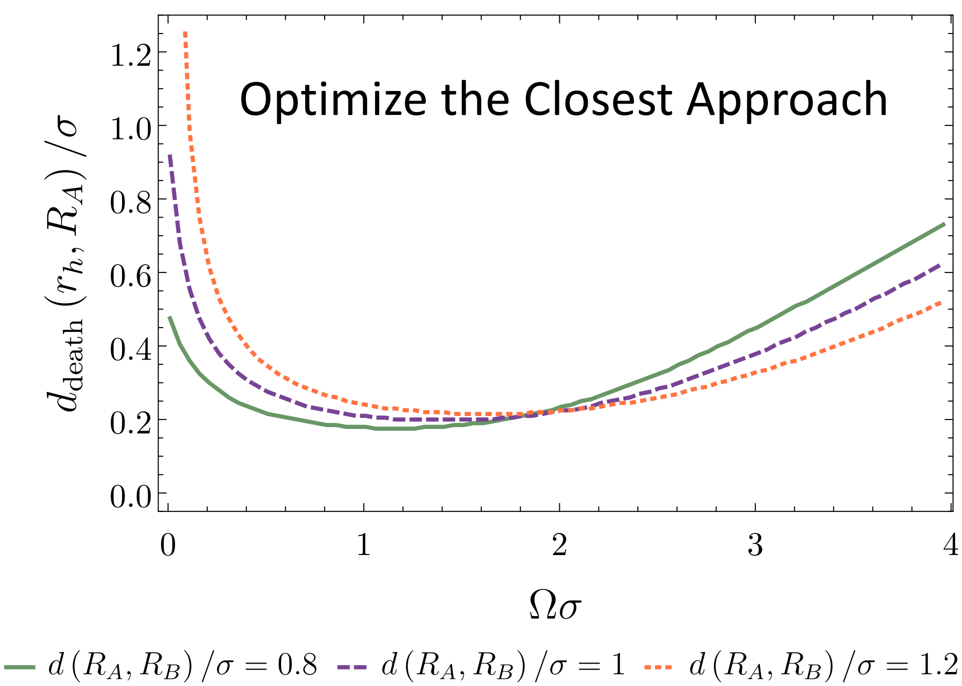
Harvesting is not possible at large separations
$\ell/\sigma = 10$
$M = 1$
$d(r_+, R_A) = \sigma$
$\mathcal{C}(\rho_{AB})/\tilde{\lambda}^2$
$d(R_A, R_B)/\sigma$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\Omega\sigma = 0.01$
$\Omega\sigma = 0.1$
$\Omega\sigma = 1$

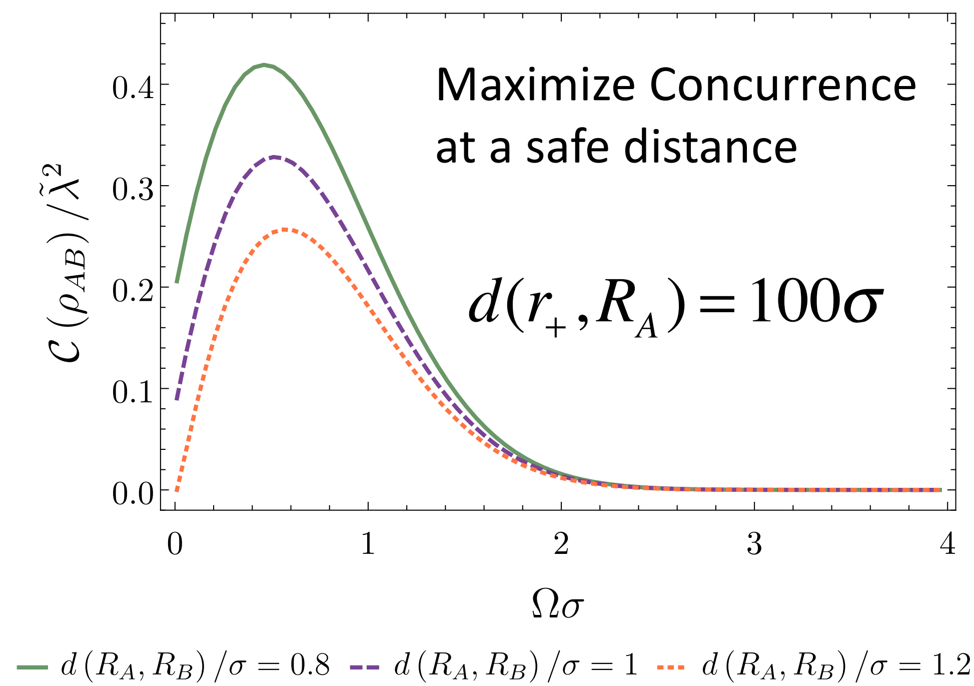
Black Hole Death Zone



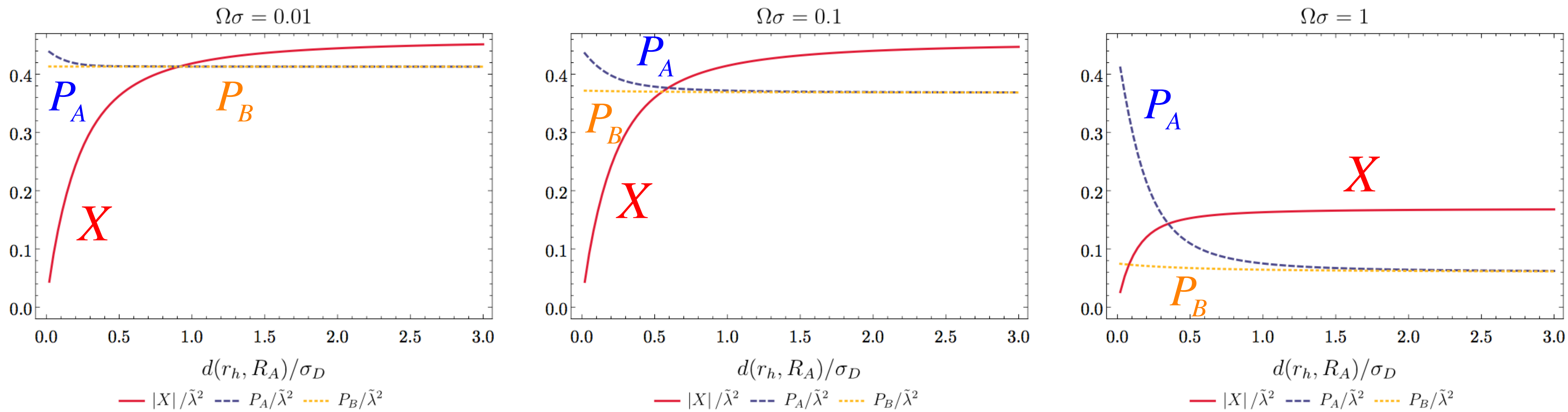


$\ell/\sigma = 10 \quad M = 1$



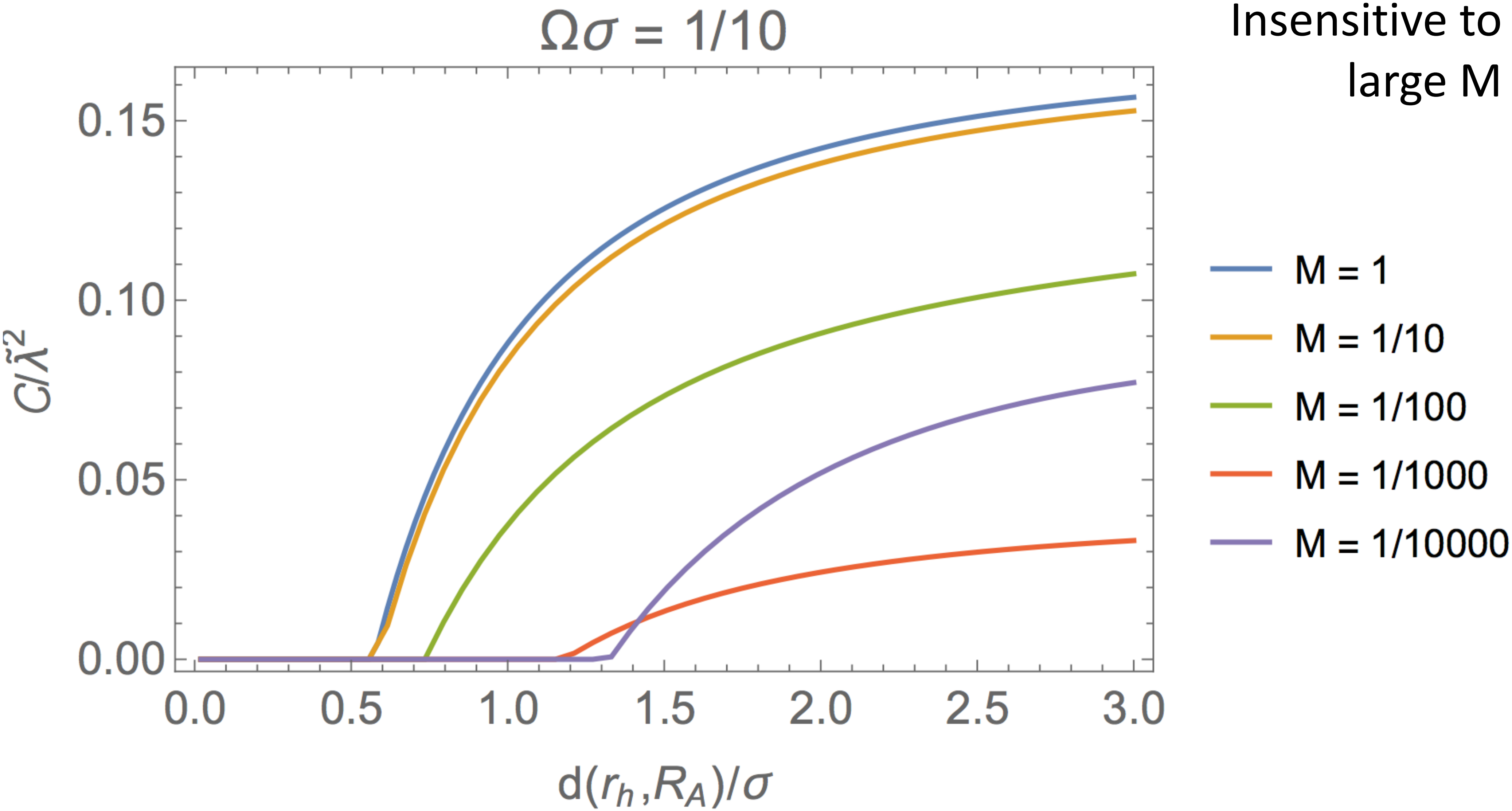


Entanglement Inhibition



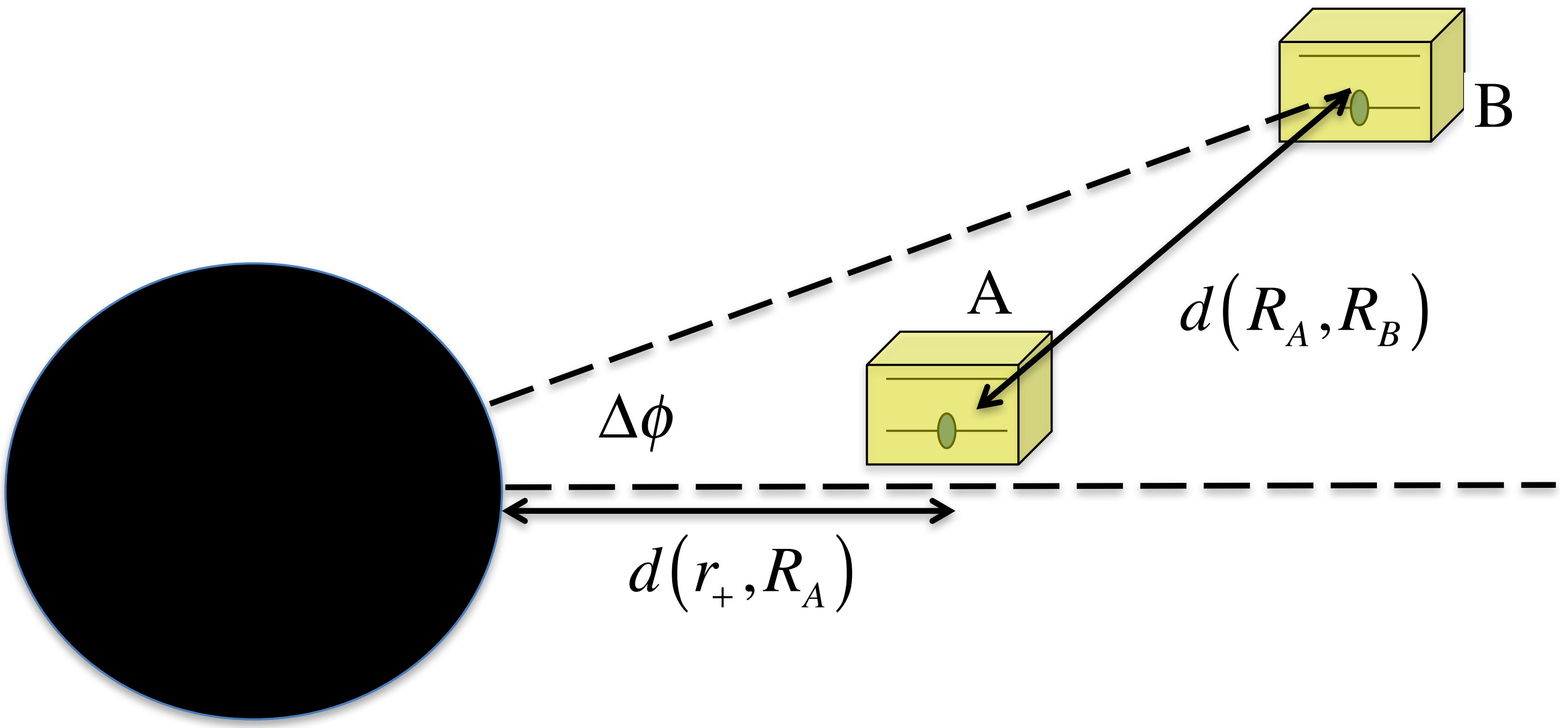
- Competition between increasing local excitations and decreasing non-local correlation
  - Hawking radiation → excitation probability rises
  - Redshift effects → erode correlations
- Competition is sensitive to gap size

Mass Dependence



Insensitive to large M

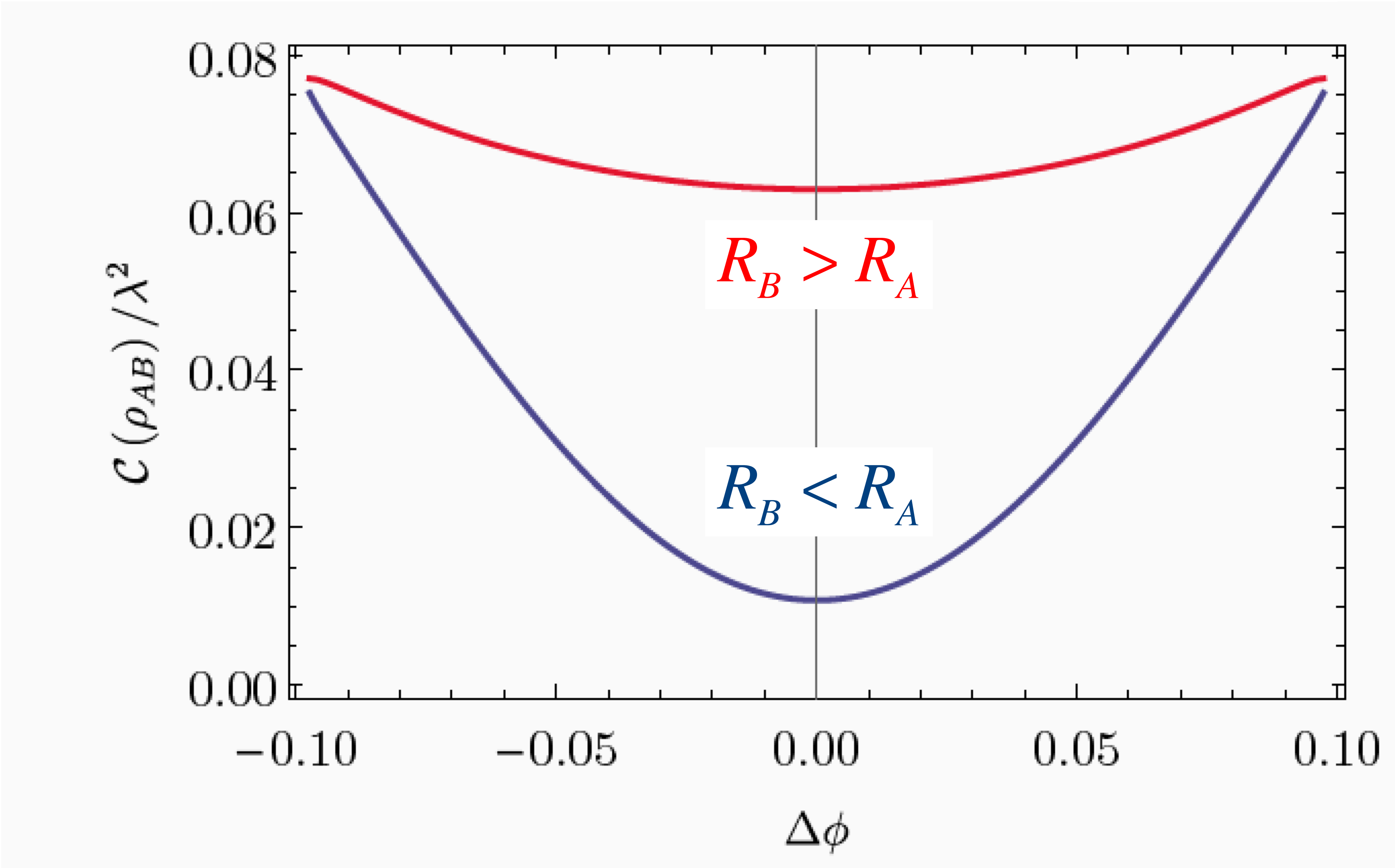
Angular Dependence



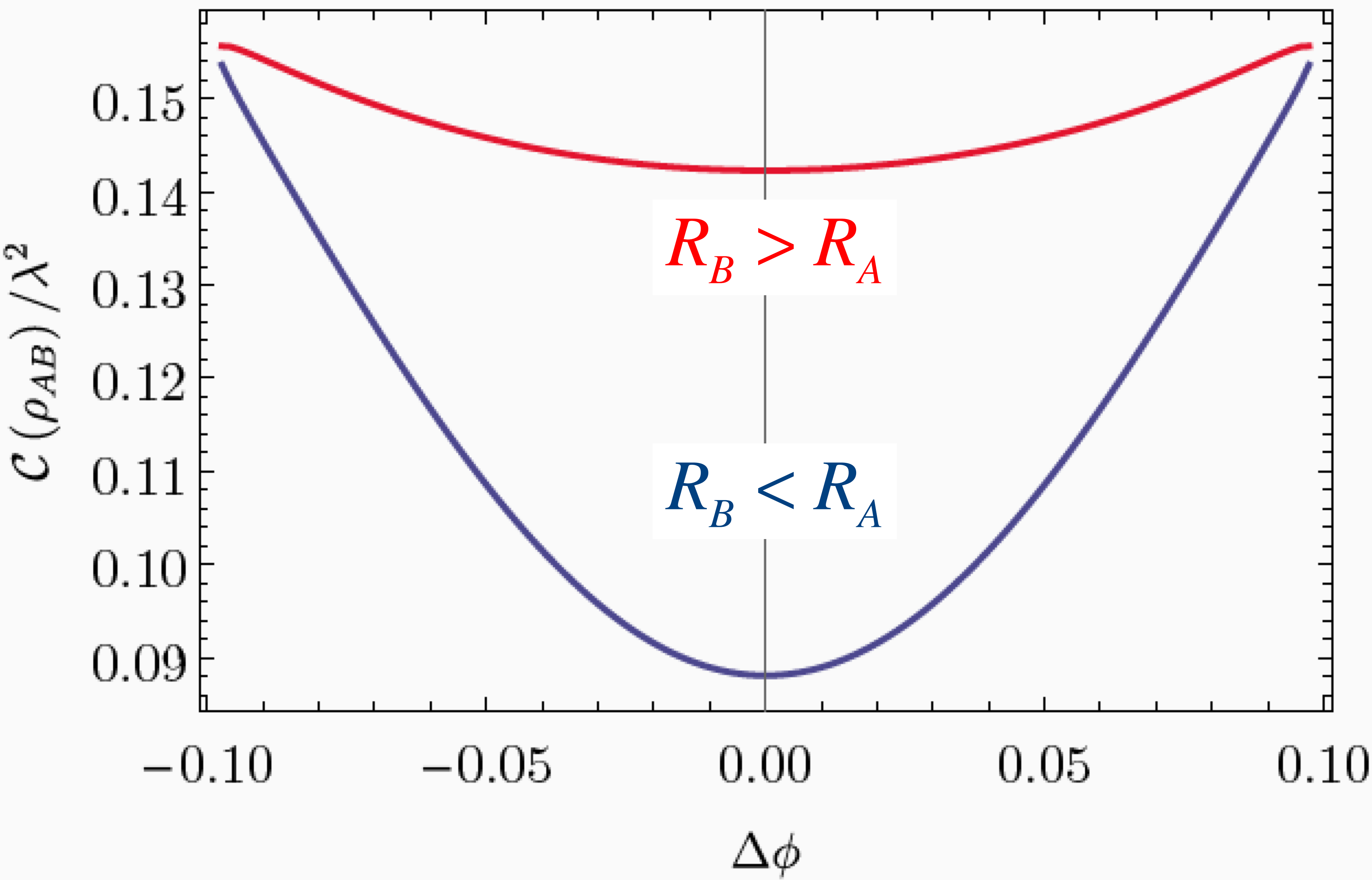
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2$$

$$f(r) = \left(\frac{r^2}{\ell^2} - M\right)$$

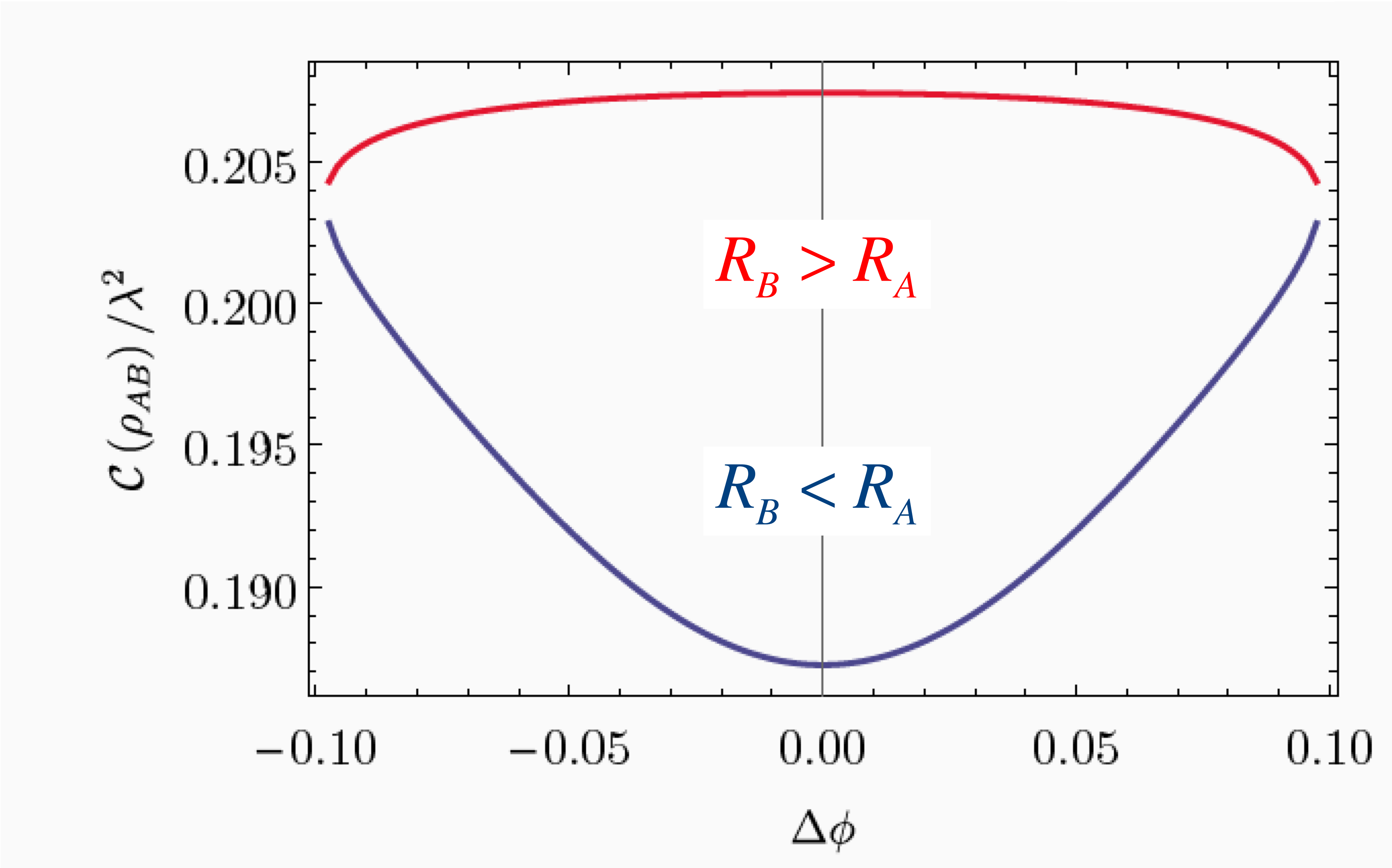
$\Omega\sigma = 0.01$ $M = 1$ $\ell = 10\sigma$ $d(r_+, R_A) = \sigma$ $d(R_A, R_B) = \sigma$



$\Omega\sigma = 0.1$ $\quad M = 1 \quad \ell = 10\sigma \quad d(r_+, R_A) = \sigma \quad d(R_A, R_B) = \sigma$

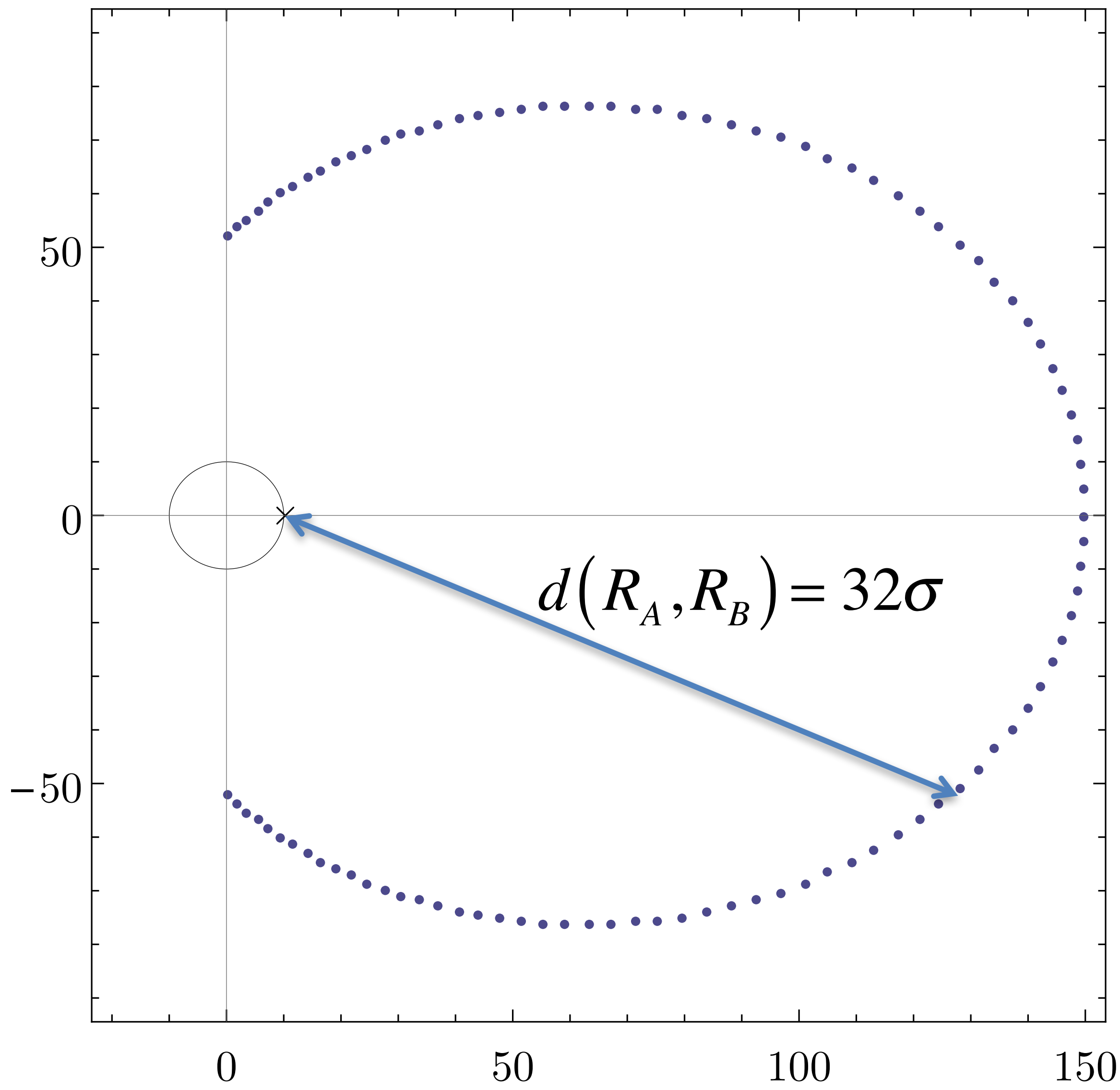


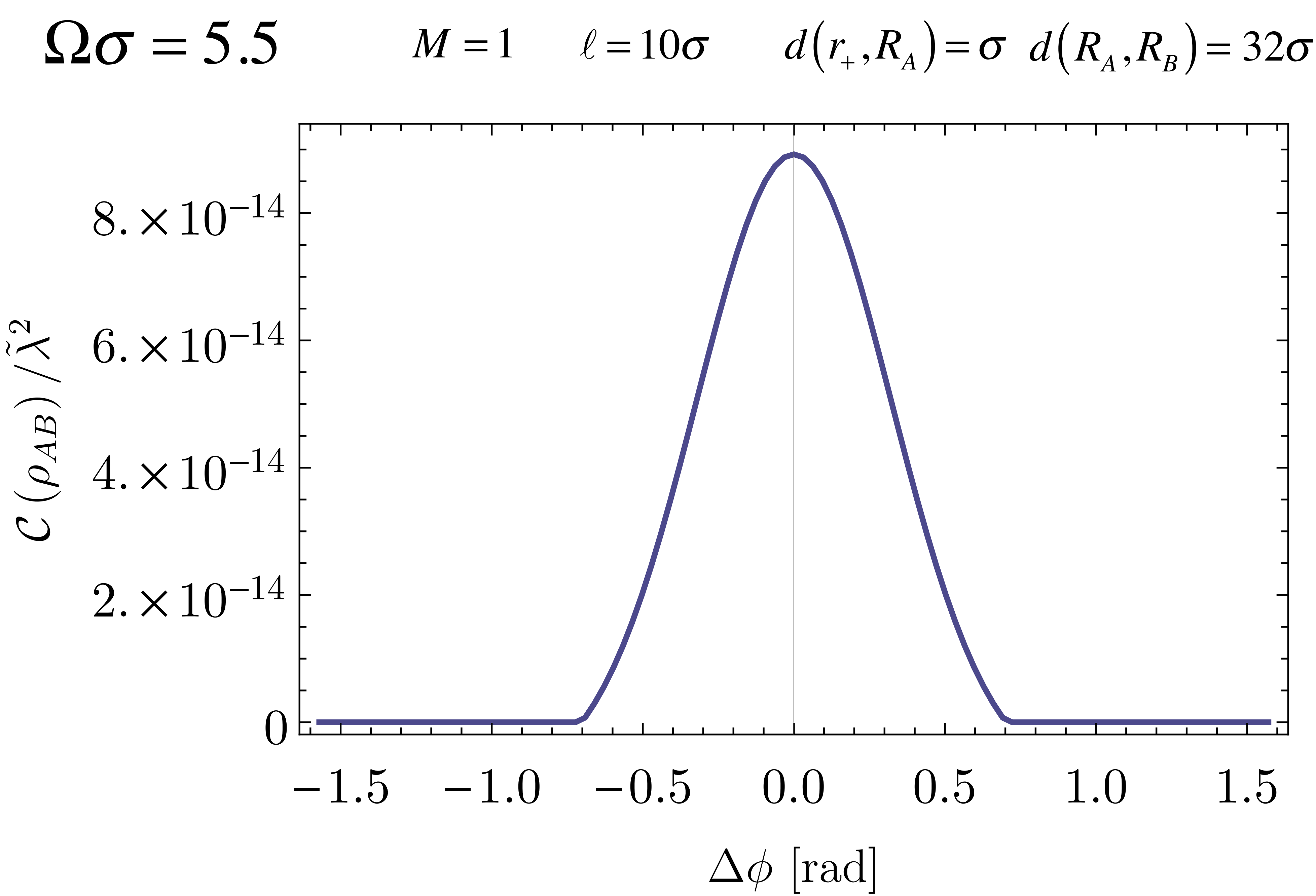
$\Omega\sigma = 1.0$ $M = 1$ $\ell = 10\sigma$ $d(r_+, R_A) = \sigma$ $d(R_A, R_B) = \sigma$



Encompassing the Hole

$d(r_+, R_A) = \sigma$





$\Omega\sigma = 5.5$
$M = 1$ $\ell = 10\sigma$ $d(r_+, R_A) = \sigma$ $d(R_A, R_B) = 32\sigma$
$\mathcal{C}(\rho_{AB})/\tilde{\lambda}^2$
$8.\times10^{-14}$
$6.\times10^{-14}$
$4.\times10^{-14}$
$2.\times10^{-14}$
0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
$\Delta\phi$ [rad]

Results in 3+1 AdS

Ng/Mann/
Martin-Martinez
(to appear)

$$ds^2 = -\left(\frac{r^2}{\ell^2}+1\right)dt^2 + \left(\frac{r^2}{\ell^2}+1\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Switching Displacement

$$\tau_A = 0 \quad \tau_B = \Delta\tau$$

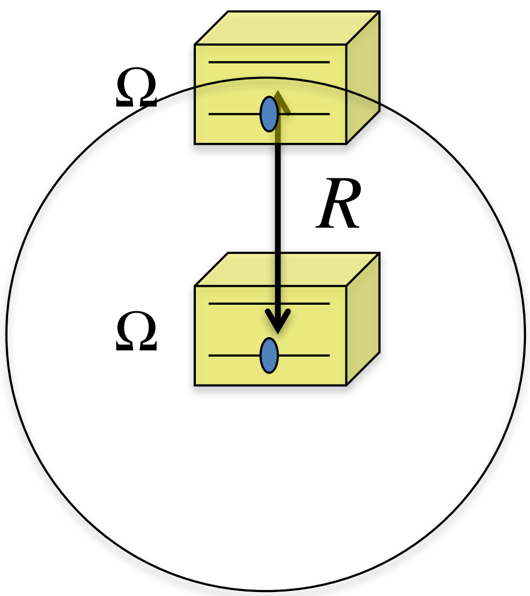
Switching Width

$$\sigma_A = \sigma_B = \sigma$$

Detector Gaps

$$\Omega_A = \Omega_B = \Omega$$

Calibrated wrt coordinate time



$$r = R$$

$$\theta = \pi / 2$$

$$t = \tau$$

$$\phi = \tau / \ell$$

Dirichlet Condition

$$\zeta = 1$$

Geodesic circular orbit: redshift and angular velocity are completely independent of radius

Correlation Relations

For Spacelike Separated Detectors

For any detector motion!

$$X = -\frac{1}{2}\left[C_{BA}(-\Omega_B,\Omega_A) + C_{AB}(-\Omega_A,\Omega_B)\right]$$

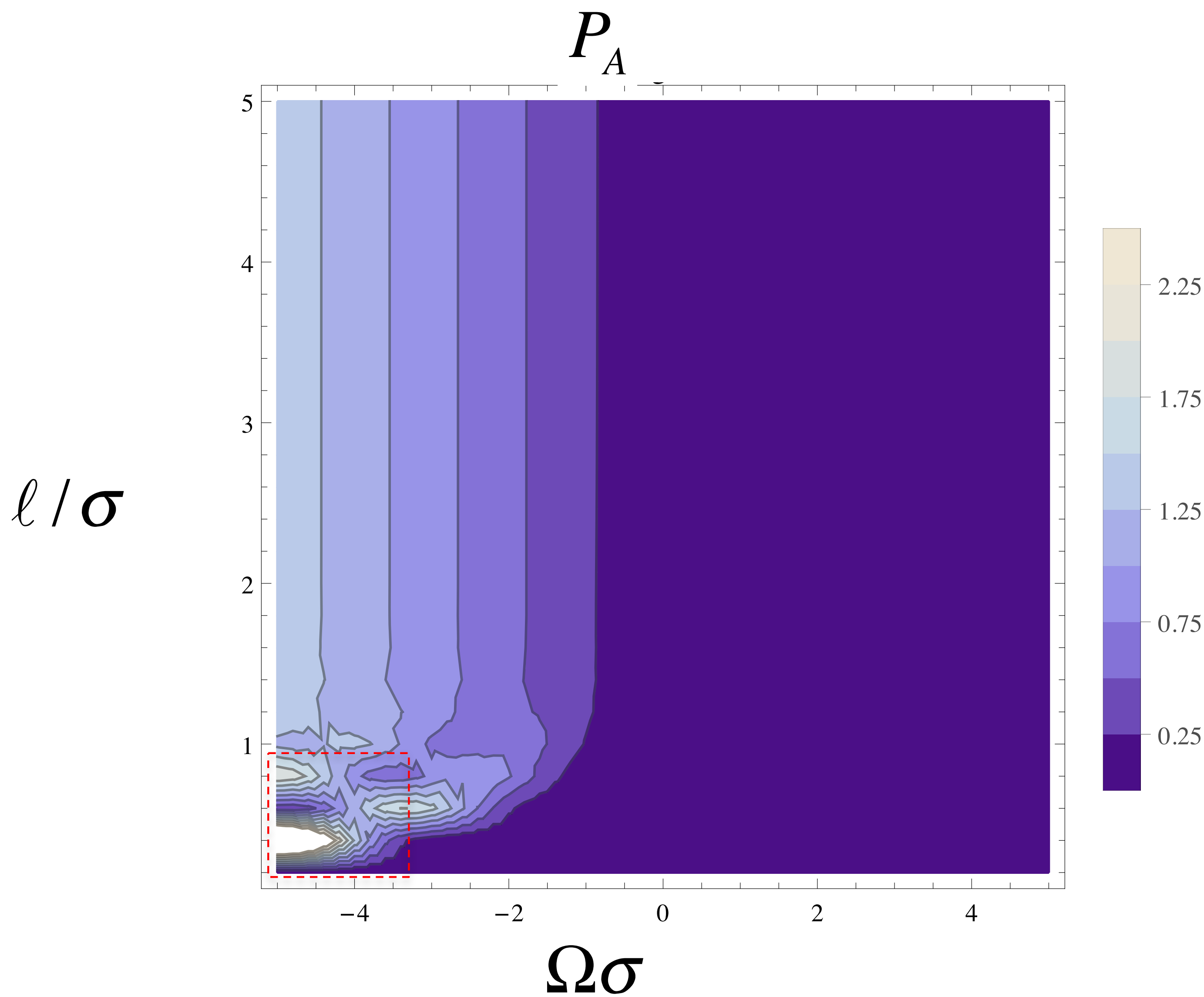
Non-local Correlations

Local Correlations

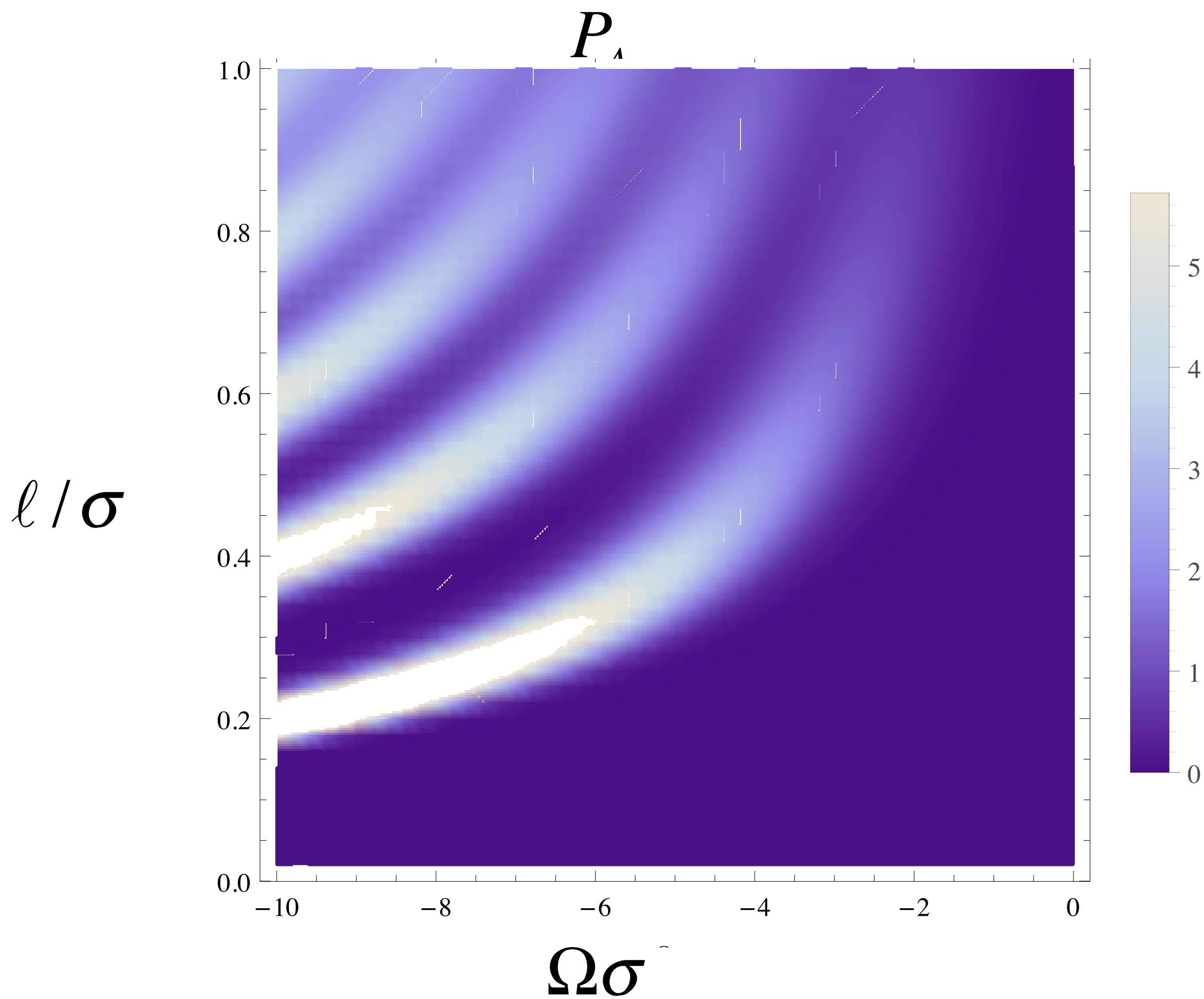
$$C_{AB} = \lambda^2 \sum_n \frac{\pi}{2n+2}\varphi_{2n+2}(x_B)\varphi_{2n+2}(x_A)\hat{\chi}_B^*(2n+2+\Omega_B)\hat{\chi}_A(2n+2+\Omega_A)$$

$$X = \lambda^2 \sum_n \frac{\pi}{2n+2}\varphi_{2n+2}(x_B)\varphi_{2n+2}(x_A)\hat{\chi}_B^*(2n+2+\Omega_B)\hat{\chi}_A(2n+2+\Omega_A)$$

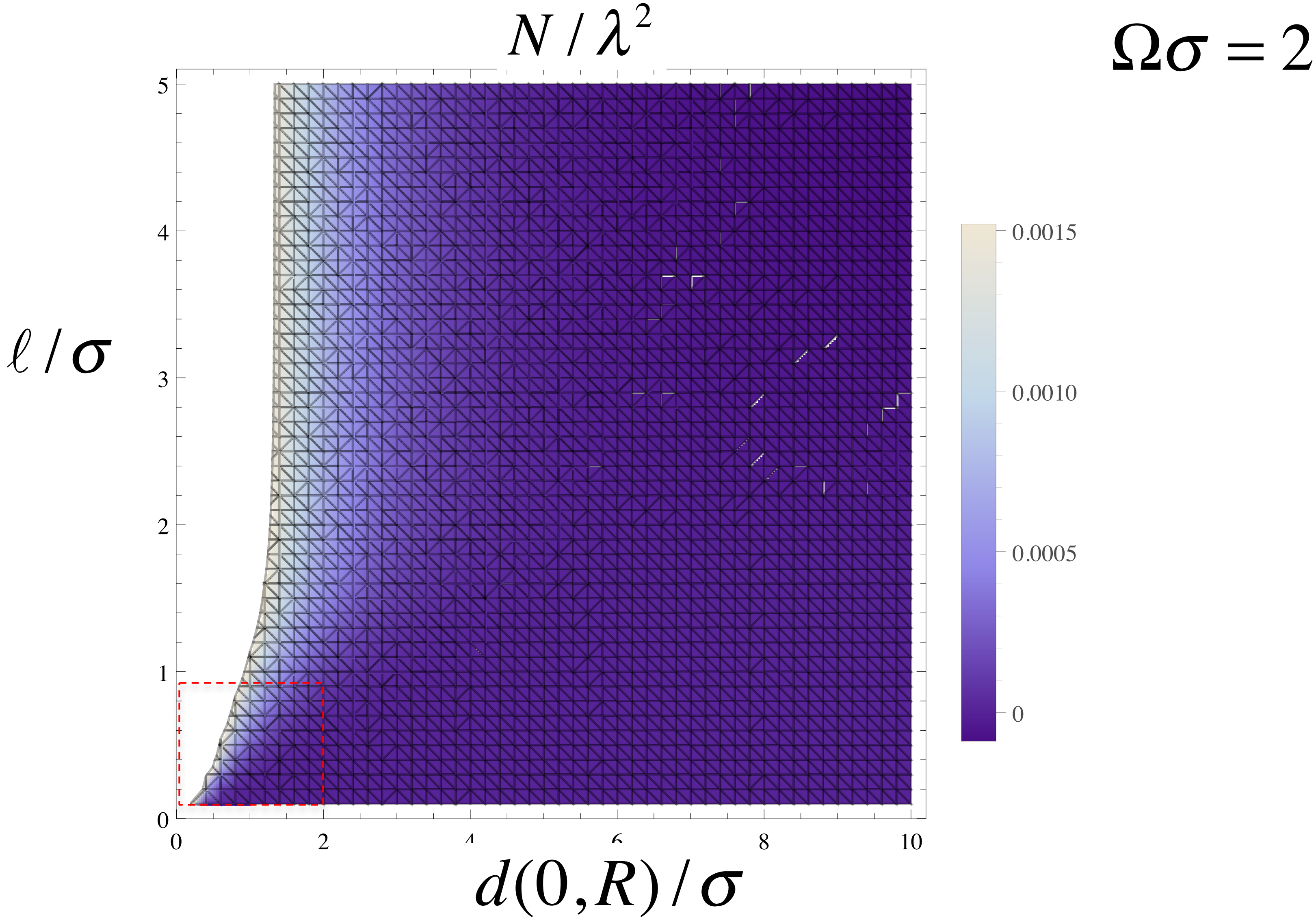
Detector Excitation



Detector Excitation



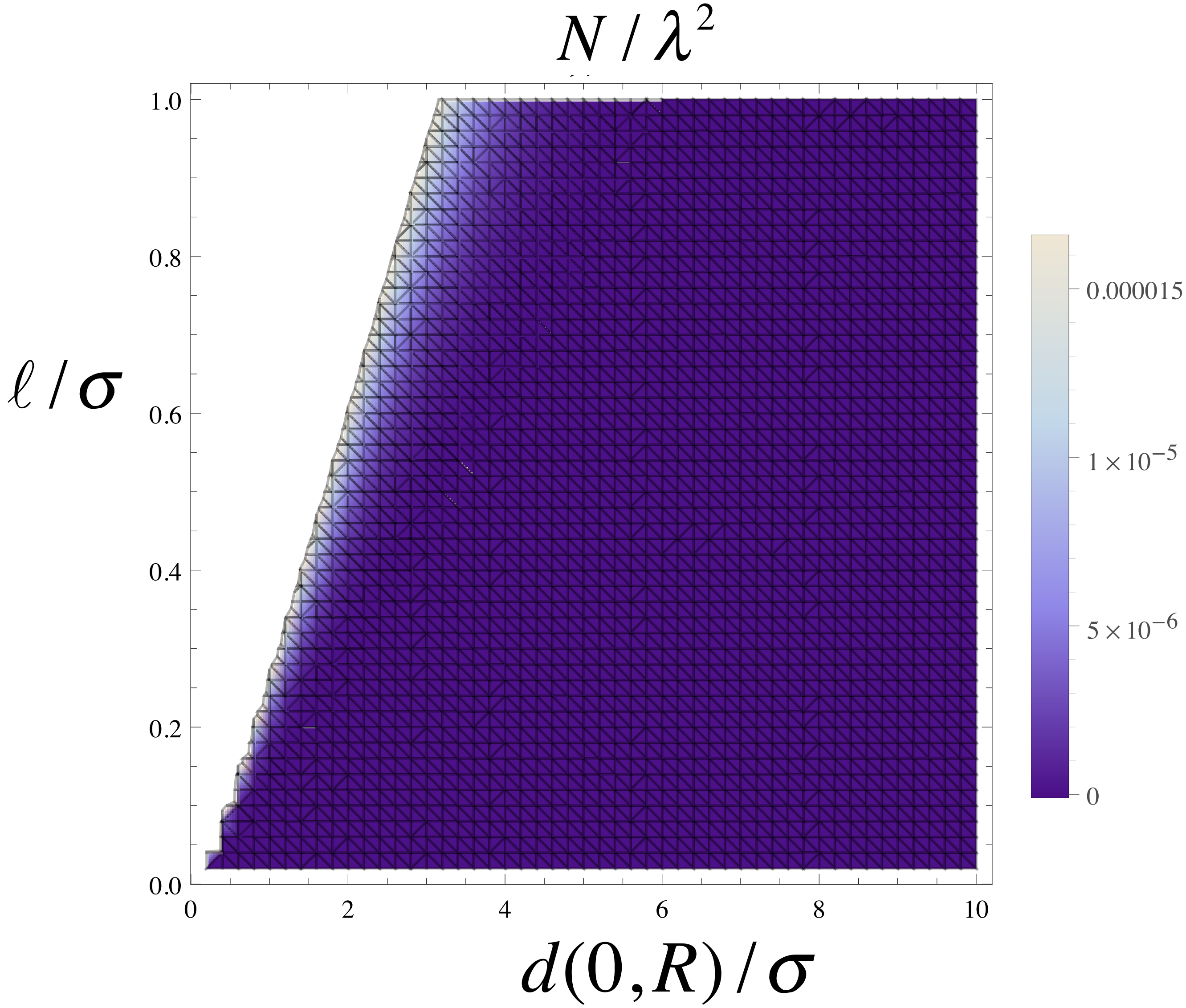
Negativity



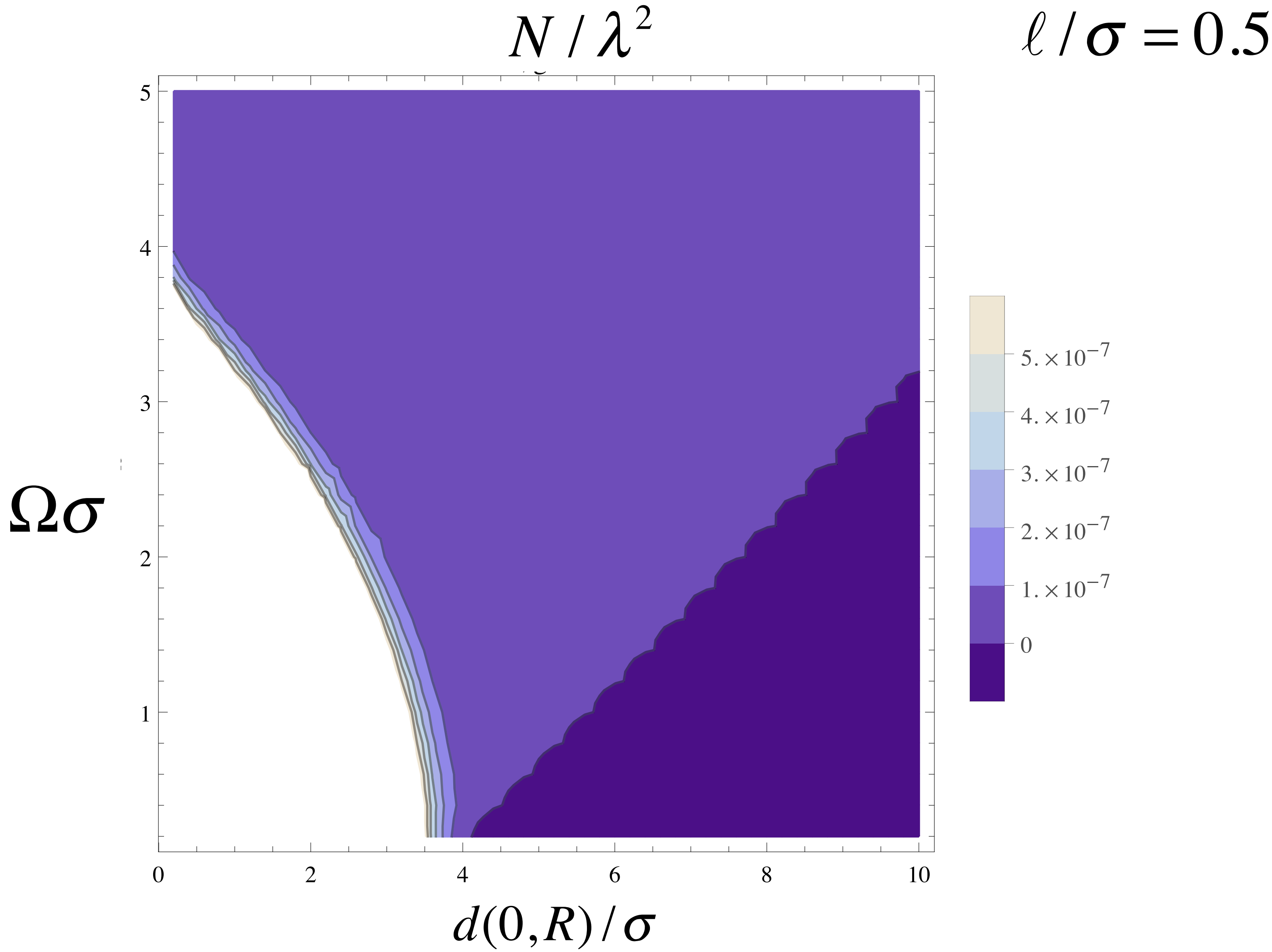
N/λ^2
$\Omega\sigma = 2$
ℓ/σ
$d(0,R)/\sigma$
0
1
2
3
4
5
0
2
4
6
8
10
0.0015
0.0010
0.0005
0

Negativity

$\Omega\sigma = 2$



Gap Dependence



Summary

- Entanglement Harvesting
  - New operational means of probing the vacuum structure of spacetime
- Curved Spacetime features
  - Similar to flat spacetime in several ways
    - Dependence on detector separation, gap, etc.
  - Increasing dependence on curvature scale as it approaches switching width
  - Entanglement "dead zones" → need to be understood
- Black Holes Inhibit Entanglement
  - Competition between enhanced local excitations and and redshift erosion of non-local correlations