Singularities

in the Universe
in BH

Energy dominance
conditions

$\varepsilon > 0$

$\varepsilon + p > 0$

$\varepsilon + 3p > 0$?

$p = -\varepsilon$ for de Sitter

80-90 th

- Starobinsky

$V(\varphi)$, $R^2 \to p \simeq -\varepsilon$

$$\varepsilon + 3p < 0$$

nonsingular Universe?

- Markov (82)

limiting density

$$3\left(\frac{\dot{a}}{a}\right)^2 = \varepsilon\left(1 - \frac{\varepsilon}{\varepsilon_{lim}}\right)$$

$$\varepsilon \propto \frac{1}{a^3}$$

???

MB (92)

$$S = \int \left(-\frac{1}{2} R + \lambda \left(4 R_{\mu\nu} R^{\mu\nu} - R^2 \right) + \right.$$
$$\left. + V(\lambda) + \dots \right) \dots \Rightarrow$$

nonsingular isotropic Universe

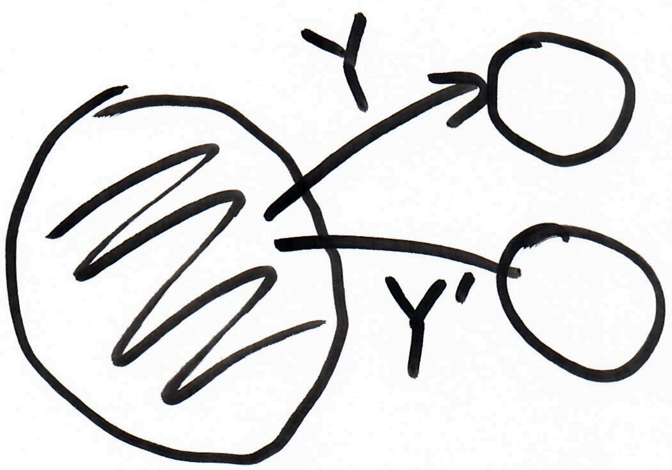
nonsingular 2d BH

<u>but</u>

What about Kasner, 4d. B.H. ?

90th - Strings, LQG...

NCG



$Y = Y^A \Gamma_A$

$Y' = Y'^B \Gamma_B$

$$Z = \frac{1}{2}(Y+1)(Y'+1)$$

$$\langle Z, (DZ)^4 \rangle = \gamma$$

$$\Downarrow$$

$$\sqrt{g} = \varepsilon_{\dots}\varepsilon^{\dots} Y \partial Y \partial Y \partial Y \partial Y$$

$$\Downarrow$$

- cosmol. const as integratio[n] const, 4 volume is quantize[d]
- 3 volume is quantised

$Y^5 = \varepsilon^{\mu}\varphi$, $\partial_\mu \varphi \partial^\mu \varphi = 1$!

$x^4 = \varepsilon t^{\mu}_{D}$

$8\pi G = 1$

$$S = \int \left(-\frac{1}{2} R + \lambda \left((\partial\phi)^2 - 1 \right) + \right.$$
$$\left. + f(\Box\varphi) \right) \sqrt{-g}\, d^4x$$

$$g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi = 1.$$

- Synchronous coordinates

$$ds^2 = dt^2 - \gamma_{ik}\, dx^i dx^k$$

- $\Box\varphi = \frac{\dot{\gamma}}{2\gamma}$!

$$\begin{array}{c} \Box\varphi \\ \shortparallel \\ \chi \end{array}$$

f - ?

$$f(\underset{\overset{\shortparallel}{\chi}}{\Box\varphi}) = 1 - \sqrt{1 - \frac{(\Box\varphi)^2}{\varepsilon_m}} + \dots$$

- $$f(\chi) = \chi_m^2 \left(1 + \frac{1}{3}\left(\frac{\chi}{\chi_m}\right)^2 - \sqrt{\frac{2}{3}}\,\frac{\chi}{\chi_m} \arcsin\left(\sqrt{\frac{2}{3}}\,\frac{\chi}{\chi_m}\right) - \sqrt{1 - \frac{2}{3}\frac{\chi^2}{\chi_m^2}} \right)$$

$$\varkappa^i_k = \gamma^{im}\,\dot{\gamma}_{mk}$$

i-k Einstein eqs.

$$\varkappa^i_k = \frac{1}{3}\varkappa\,\delta^i_k + \frac{\lambda^i_k}{\sqrt{\gamma}}$$

λ^i_k: const, no sp. curv.

$$\varkappa = \frac{\dot{\gamma}}{\gamma}$$

0-0 eq. $\Rightarrow$

$$\frac{1}{12}\left(\frac{\dot{\gamma}}{\gamma}\right)^2 = \varepsilon\left(1 - \frac{\varepsilon}{\varepsilon_m}\right)$$

$\varepsilon_m \to 2\chi_m^2$

where

$$\varepsilon = \frac{\lambda^i_k\lambda^k_i}{\sqrt{\gamma}} + \frac{C}{\sqrt{\gamma}} + T^0_0$$

C: mimetic

Friedmann Universe

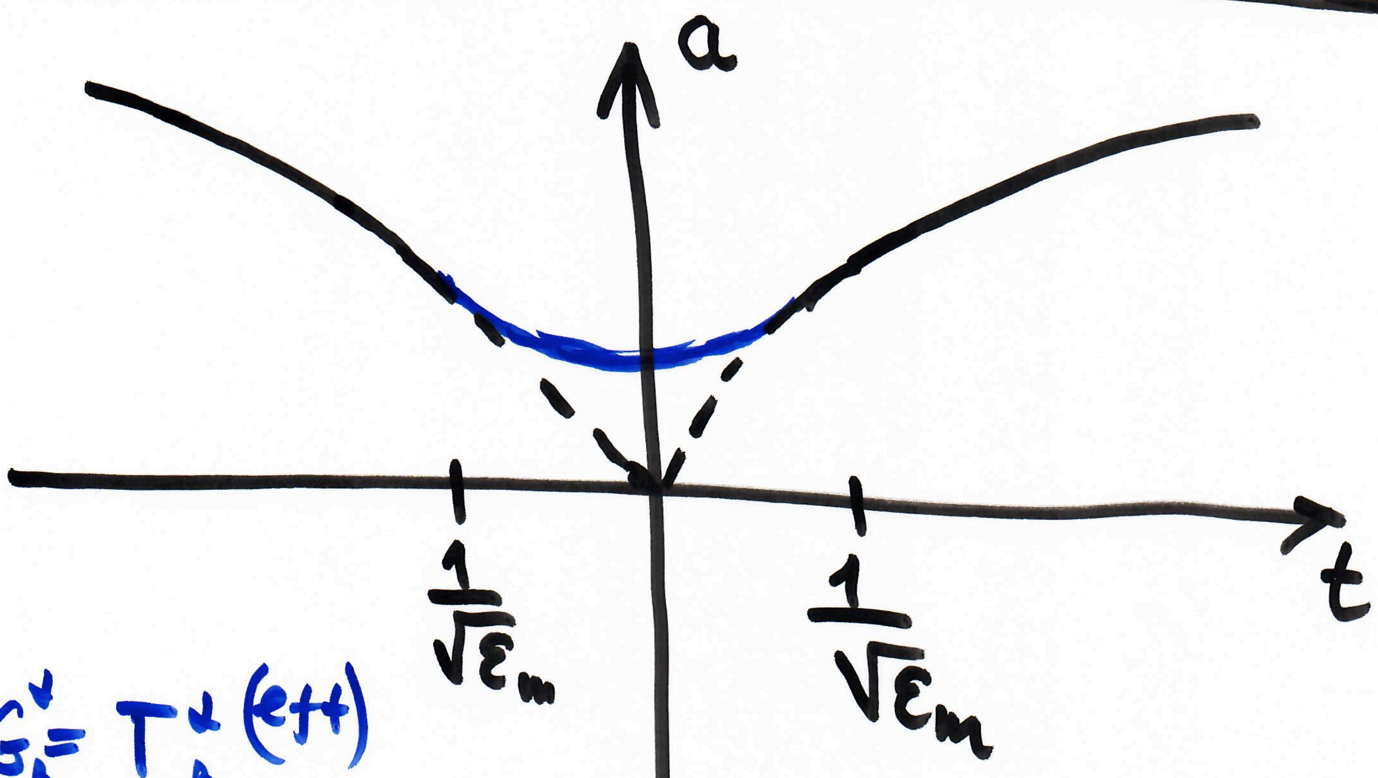
$$ds^2 = dt^2 - a^2(t)\,\delta_{ik}\,dx^i dx^k$$

$$\gamma = a^6 \,, \quad \lambda^i_k = 0$$

add matter with

$$p = +w\varepsilon$$

$$\boxed{a = \left(1 + \frac{3}{4}(1+w)^2 \varepsilon_m t^2\right)^{\frac{1}{3(1+w)}}}$$



$$G^\alpha_\beta = T^\alpha_\beta\,(eff)$$

$$\varepsilon + p < 0 \,, \quad p < 0$$

Kasner Universe

$$ds^2 = dt^2 - t^{2p_1} dx_1^2 - t^{2p_2} dy^2 - t^{2p_3} dz^2$$

$$p_1 + p_2 + p_3 = 1, \quad p_1^2 + p_2^2 + p_3^2 = 1$$

- $R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = -\frac{16}{t^4} p_1 p_2 p_3$
- $\underline{\underline{\gamma \propto t^2}}$

$$\varepsilon = \frac{\lambda^i_k \lambda^k_i}{\gamma} = \frac{\bar{\lambda}^2}{\gamma}$$

$$\left(\frac{\dot{\gamma}}{\gamma}\right)^2 = \frac{3\bar{\lambda}^2}{2\gamma}\left(1 - \frac{\bar{\lambda}^2}{8\varepsilon_m \gamma}\right)$$

$$\Downarrow$$

$$\gamma = \frac{\bar{\lambda}^2}{8\varepsilon_m}\left(1 + 3\varepsilon_m t^2\right)$$

At $t=0$ $\underline{\underline{\gamma \neq 0}}$

$$\gamma_{ik} = \gamma_{(i)} \delta_{ik}$$

$$\gamma_{(i)} = \left[\left(\frac{\bar{\lambda}^2}{8 \varepsilon_m} \right) \left(1 + 3 \varepsilon_m t^2 \right) \right]^{1/3} \times$$

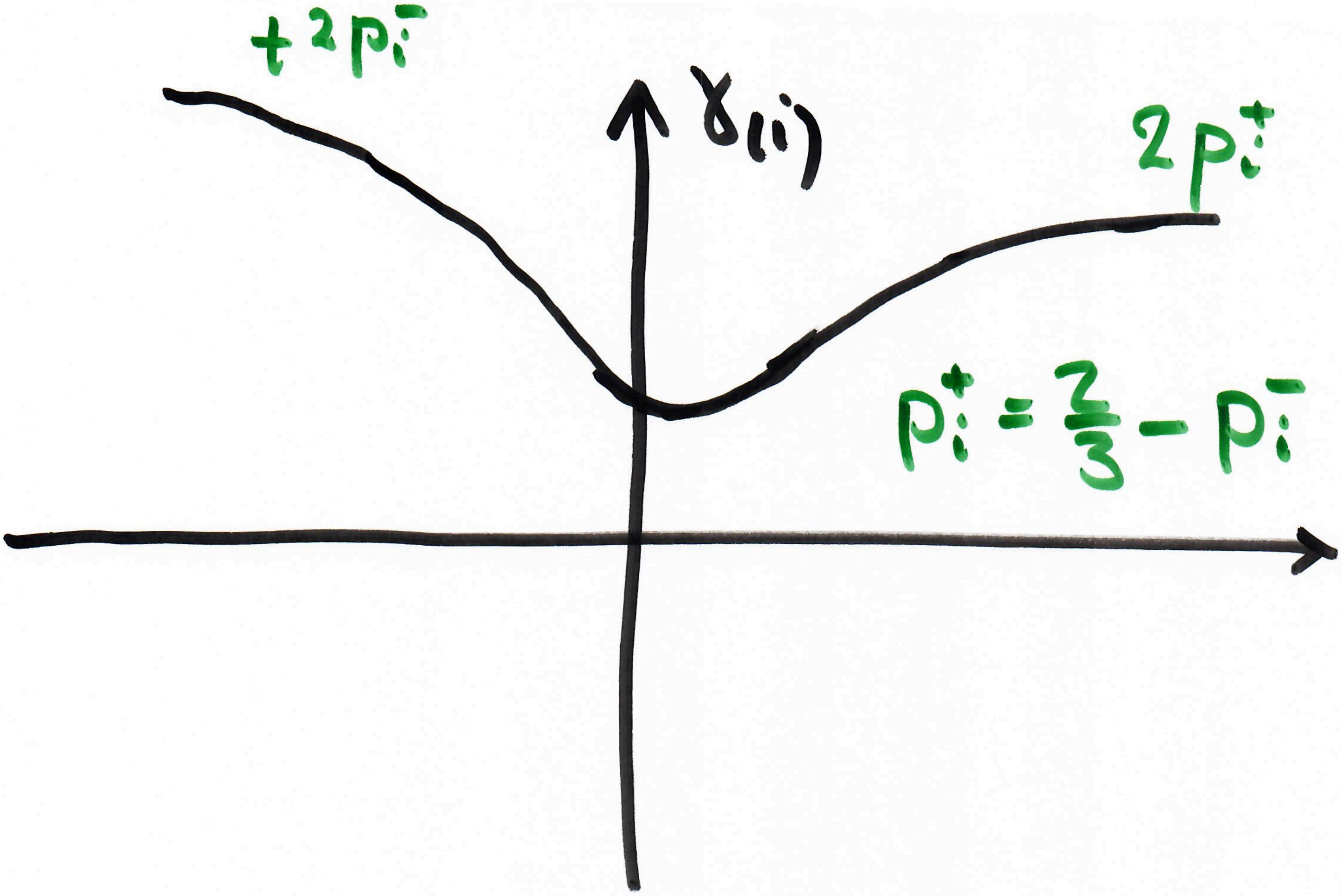
$$\times \exp \left(2 \sqrt{\frac{2}{3}} \frac{\lambda_{(i)}}{\bar{\lambda}} \sinh^{-1} \left(\sqrt{3 \varepsilon_m}\, t \right) \right)$$

<u>Exact solution</u>

$$|t| \gg \frac{1}{\sqrt{\varepsilon_m}}$$

$$\gamma_{(i)} \propto \left(\varepsilon_m t^2 \right)^{p_i^{\pm}}$$

$$\sum_i p_i^{\pm} = 1 \qquad \sum \left(p_i^{\pm} \right)^2 = 1.$$



Kasner with $p_i^- = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$
turns to Kasner with
$p_i^+ = (1, 0, 0) \equiv$ Minkowski

BH

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt_s^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 d\Omega^2$$

$r < r_g$ r is time coordinat

$t_s \to R$ space coord.

Inside BH

$$ds^2 = dt^2 - a^2(t)\, dR^2 - b^2(t)\, d\Omega^2$$

$$t = r_g\left(\arcsin \tau - \tau\sqrt{1-\tau^2}\right)$$

$$a^2(t) = \frac{1-\tau^2(t)}{\tau^2(t)}, \quad b^2 = r_g^2 \tau^4(t)$$

$\varphi(t) = t + \text{const}$?

$\Box\varphi = \frac{\dot{\gamma}}{2\gamma} \to \infty$ (firewall?)

Go to Lamaitre coordinate <u>system</u>

$T = R + \int \frac{\sqrt{1+a^2}}{a} dt^2, \quad \bar{R} = R + \int \frac{dt}{a\sqrt{1+a^2}}$

$$ds^2 = dT^2 - (1+\underset{\substack{\downarrow \\ a(T-R)}}{a^2})d\bar{R}^2 - \beta^2 d\Omega^2$$

$\varphi = T$! (no firewall!)

At late times $\underline{\underline{t \simeq T}}$

Near horizon region

- $\underline{r_g - r \ll r_g}$

$$ds^2 \simeq d\bar{t}^2 - \frac{1}{4}\left(\frac{\bar{t}}{r_g}\right)^2 dR^2 - r_g^2 d\Omega^2$$

Similar to Kasner with

$p_i = (1, 0, 0) \equiv$ Minkowski

Near singul. region

- $r \ll r_g$

$$ds^2 \simeq dt^2 - \left(\frac{t}{t_0}\right)^{-2/3} dR^2 - \left(\frac{t}{t_0}\right)^{4/3} r_g^2 d\Omega^2$$

$\underline{t_0 = 2r_g/3}$

Similar to Kasner with

$p = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

Spatial curvature term from $d\Omega^2$ is relevant only at $r \sim r_g/2$.

In our theory singularity which would happen at $\underline{\underline{t=0}}$ is avoided and "Kasner" with $p^- = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ goes to $p^+ = (1, 0, 0)$

Metric at $t > 1/\sqrt{\varepsilon_m}$

$$ds^2 = dt^2 - Q_0^2 \left(\frac{t}{t_0}\right)^2 dR^2 - \frac{r_g^2}{Q_0} d\Omega^2$$

$$Q_0 = \left(\frac{16}{3} \varepsilon_m r_g^2\right)^{2/3}$$

$$R_{g1} = \frac{r_g}{Q_0^{1/2}} \propto r_g^{1/3}$$

Then

$p = (1,0,0)$ in BH of size $r_g^{1/3}$

$\to p = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right) \to$ next bounce

$\to p = (1,0,0)$ but $R_{g_2} = \dfrac{R_{g_1}}{Q_1} \propto r_g^{1/9}$

et. cet.

Finally we come to BH. of radius $R_{g_n} \propto r_g^{\frac{1}{3^n}}$ ...
and end up at limiting <u>curvature</u>