Some applications of massive gravity in gauge/gravity duality

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Based on: Li-Ming Cao & **Yuxuan Peng**, Phys. Rev. D 92 124052;

Li-Ming Cao, **Yuxuan Peng** & Yun-Long Zhang, Phys. Rev. D 93 124015

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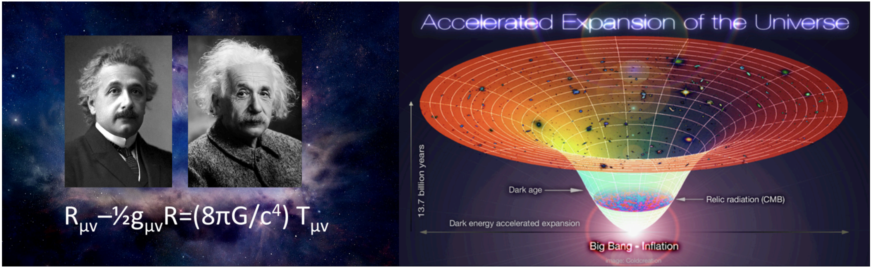
Puzzle of the late-time acceleration of our Universe...



Dark energy: the cosmological constant Λ, scalar field ϕ,...

Modified gravity theories: **Massive gravity** , $f(R)$ gravity, ...

Linear massive gravity theory

General relativity can be described by the following Einstein-Hilbert action,

$$S = \int_{\mathrm{M}} \mathrm{d}^4 x \sqrt{-g} \left(\frac{1}{2\kappa^2} R + L_{\mathrm{m}} \right) . \tag{1}$$

To quadratic order of a small perturbation h_{ab} around Minkowski background

$$S_h = \frac{1}{8\kappa^2} \int \mathrm{d}^4 x \left(2\partial^c h^{ab} \partial_a h_{bc} - 2\partial_a h^{ab} \partial_b h - \partial^c h^{ab} \partial_c h_{ab} + \partial^c h \partial_c h \right) , \tag{2}$$

A natural question:

Massive gravitons? Fierz-Pauli (FP) theory:

$$S_{\mathrm{FP}} = S_h - \frac{1}{8\kappa^2} \int \mathrm{d}^4 x \, m^2 (h^{ab} h_{ab} - h^2) . \tag{3}$$

(Fierz & Pauli, Proc. Roy. Soc. Lond. 1939, A173)

The unique linear theory free of ghost of the massive graviton h_{ab}.

Need for a non-linear theory

A problem of the linear theory

- The $m \to 0$ limit is discontinuous

(van Dam & Veltman, Nucl. Phys. B. 1970, 22 (2), Zakharov, JETP Lett. 1970, 12)

van Dam-Veltman-Zakharov (vDVZ) discontinuity

- Non-linear interactions screen the extra degrees of freedom inside some *Vainshtein* radius R_{V} (Vainshtein mechanism)

(Vainshtein, Phys. Lett. B 1972, 39)

Ghost problem of non-linear theories

One example of non-linear theories

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R + S_{\mathrm{NLFP}}[f,g]\,, \tag{4}$$

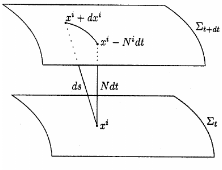
$$S_{\mathrm{NLFP}} = -\frac{1}{8\kappa^2}m^2\int \mathrm{d}^4x\,\sqrt{-f}\,H_{ab}H_{cd}\left(f^{ac}f^{bd} - f^{ab}f^{cd}\right) \tag{5}$$

with $h_{ab} = g_{ab} - \eta_{ab} \to H_{ab} = g_{ab} - f_{ab}$ and f_{ab} is the reference metric.

Problem

- No constraints for the dynamical field
- Hamiltonian unbounded from below

(Boulware & Deser, Phys. Rev. D. 1972, 6)



(Liko & Kauffman)

The extra dof: Boulware-Deser(BD)ghost.

de Rham-Gabadadze-Tolley (dRGT) massive gravity

Non-linear ghost-free massive gravity theory (de Rham, Gabadadze, Tolley Phys. Rev. Lett. 2011, 106: 231101)

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left[R + m^2\sum_{i=0}^{4} c_i\mathcal{U}_i(\mathcal{K})\right], \tag{6}$$

where the tensor$\mathcal{K}$ satisfies

$$\mathcal{K}_a{}^c\mathcal{K}_c{}^b = f_{ac}g^{cb}\,, \tag{7}$$

and it is symmetricm $\mathcal{K}_{ab} = \mathcal{K}_{ba}$. For any matrix of the components of some tensor $X_a{}^b$, $\mathcal{U}_i(X)$ have the form

$$\begin{aligned}
&\mathcal{U}_0 = 1\,, \qquad \mathcal{U}_1 = \mathrm{Tr}(X)\,, \\
&\mathcal{U}_2 = \mathrm{Tr}(X)^2 - \mathrm{Tr}(X^2)\,, \qquad \mathcal{U}_3 = \mathrm{Tr}(X)^3 - 3\mathrm{Tr}(X)\mathrm{Tr}(X^2) + 2\mathrm{Tr}(X^3)\,, \\
&\mathcal{U}_4 = \mathrm{Tr}(X)^4 - 6\mathrm{Tr}(X^2)\mathrm{Tr}(X)^2 + 8\mathrm{Tr}(X^3)\mathrm{Tr}(X) + 3\mathrm{Tr}(X^2)^2 - 6\mathrm{Tr}(X^4)\,.
\end{aligned} \tag{8}$$

The equation of motion is

$$R_{ab} - \frac{1}{2} g_{ab} R + m^2 X_{ab} = 0\,, \tag{9}$$

where

$$\begin{aligned} 2X_{ab} \quad = \quad & -c_0 g_{ab} - c_1(\mathcal{U}_1 g_{ab} - \mathcal{K}_{ab}) - c_2(\mathcal{U}_2 g_{ab} - 2\mathcal{U}_1\mathcal{K}_{ab} + 2(\mathcal{K}^2)_{ab}) \\ & -c_3\big(\mathcal{U}_3 g_{ab} - 3\mathcal{U}_2\mathcal{K}_{ab} + 6\mathcal{U}_1(\mathcal{K}^2)_{ab} - 6(\mathcal{K}^3)_{ab}\big) \\ & -c_4\big(\mathcal{U}_4 g_{ab} - 4\mathcal{U}_3\mathcal{K}_{ab} + 12\mathcal{U}_2(\mathcal{K}^2)_{ab} - 24\mathcal{U}_1(\mathcal{K}^3)_{ab} + 24(\mathcal{K}^4)_{ab}\big)\,. \end{aligned} \tag{10}$$

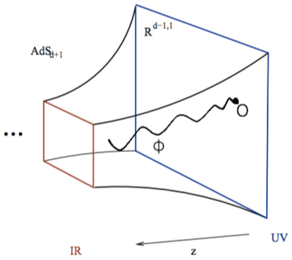
General non-linear ghost-free massive gravity theory constructed in 4 dimensions.

Other models: DGP theory, new massive gravity,...

Dvali et al., Phys. Lett. B 485, 208 747 (2000);

Bergshoeff et al., Phys. Rev. Lett. 102, 201301 755 (2009)

Gauge/Gravity duality



$$\left\langle \exp i \int \phi_{(0)} \mathcal{O} \right\rangle_{FT} = Z_{\text{bulk}}(\phi_{(0)})$$

strongly coupled QFT   classical gravity in asymptotically AdS

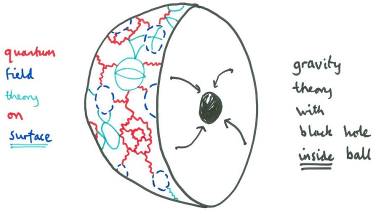
Holographic principle ('tHooft, Salamfest 1993, Susskind, J.Math.Phys. 1995, 36)

AdS/CFT correspondence (Maldacena, Int. J. Theor. Phys. 1999, 38, Gubser et al., Phys. Lett. B 1998, 428, Witten, Adv. Theor. Math. Phys. 1998)

Applications: QCD, superconductivity, entanglement entropy and quantum phase transition...

Tool for studying strongly coupled field theories

Holographic conductivity



(By Tom Brown)

A system with finite temperature and chemical potential

$$S = \frac{1}{2\kappa^2}\int_M d^4x\sqrt{-\mathcal{G}}\left[R+\frac{6}{\ell^2}-\frac{1}{4}F^2\right] \tag{11}$$

with the Reissner-Nordstrom-AdS (RN-AdS) black hole as a solution

$$ds^2 = \frac{\ell^2}{z^2}\left(-f(z)dt^2+\frac{dz^2}{f(z)}+dx^2+dy^2\right), \tag{12}$$

where

$$f(z) = 1-\left(1+\frac{z_+^2\mu^2}{4\ell^2}\right)\left(\frac{z}{z_+}\right)^3+\frac{\mu^2}{4\ell^2}\frac{z^4}{z_+^2}. \tag{13}$$

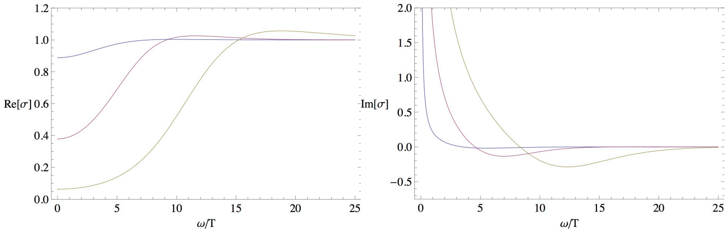


Figure 1: The conductivity obtained in Einstein gravity with divergence in the real part(Hartnoll, Class.Quant. Grav. 2009, 26: 224002.)

To Break the Translational Invariance

- To describe condensed matter systems one should break the translational invariance
- One way is to introduce the holographic lattices

$$S = \frac{1}{2\kappa^2}\int_{\mathrm{M}} \mathrm{d}^4x\sqrt{-\mathcal{G}}\left[R + \frac{6}{\ell^2} - \frac{1}{4}F^2 - \nabla^a\Phi\nabla_a\Phi - V(\Phi)\right] \tag{14}$$

with the near-boundary $z \to 0$ behavior

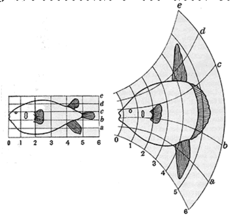
$$\Phi \to z\phi_1 + z^2\phi_2 + \ldots, \qquad \phi_1 \propto cos(kx) \tag{15}$$

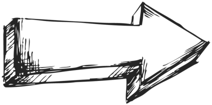
Horowitz et al., J. High Energy Phys. 2012, 07: 168 and a lot more

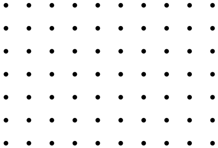
Pay the price of solving PDEs!

Massive gravity: the alternative way

Local symmetry in the bulk $\rightarrow$ global symmetry on the boundary







Diffeomorphism invariance in $(d+1)$ Translational invariance in d

Consider some black brane solution with some background

$$f_{\mu\nu} \propto \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(Vegh, arXiv: 1301.0537)

Features of the model above

- Broken diffeomorphism in x and y directions.
- The so called Drude peak is realized

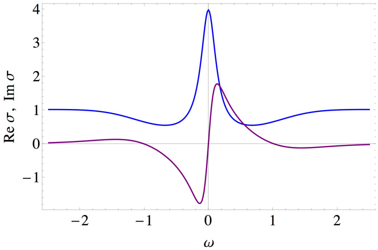


Figure 2: Drude peak obtained by the dRGT model

Related studies

- Dissipation in the boundary fluid

$$\partial_t T^{ti} = -\tau_{\text{rel.}}^{-1} T^{ti} \tag{16}$$

Davison, Phys. Rev. D 2013, 88: 086003

- Equivalence to the holographic lattice

$$\phi \propto cos(kx) \tag{17}$$

Blake et al., Phys. Rev. Lett. (2014) 112: 071602

- Thermodynamics of the black hole solution and the P-V criticality

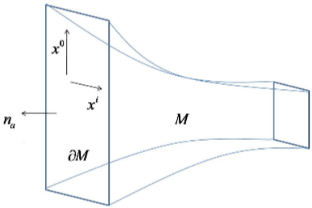
$$\begin{aligned} E &= \Omega + TS + \mu Q \\ &\cdots \end{aligned} \tag{18}$$

Cai et al., Phys. Rev. D 2015, 91 (2): 024032; Xu et al., Phys. Rev. D 2015, 91: 124033

- Transport, stability, and a lot more,...

The Need for Counterterms

The on-shell action also gives the thermodynamic potential, however divergent.



$$S = \frac{1}{2\kappa^2}\int_{\mathrm{M}} \mathrm{d}^{d+1}x\sqrt{-\mathcal{G}}\left[R + \frac{d(d-1)}{\ell^2}\right] - \frac{1}{\kappa^2}\int_{\partial\mathrm{M}} d^d x\sqrt{-\gamma}K\,. \tag{19}$$

$$\mathcal{G}_{ab} = \gamma_{ab} + n_a n_b\,,\quad K_{ab} = -\gamma_a{}^c\nabla_c n_b \tag{20}$$

A important widely used method: boundary counterterms (holographic renormalization)

Counterterms in Einstein gravity

- The action with the boundary counterterms added

$$S_{ct} = -\frac{1}{2\kappa^2}\int_{\partial \mathrm{M}} \mathrm{d}^d x\sqrt{-\gamma}\left[2(d-1)/\ell + \frac{\ell}{d-2}R[\gamma] + \frac{\ell^3}{(d-4)(d-2)^2}(R[\gamma]_{ij}R[\gamma]^{ij} - \frac{d}{4(d-1)}R[\gamma]^2) + ...\right], \tag{21}$$

where $R[\gamma]_{ij}$ is the Ricci tensor on $\partial\mathrm{M}$; the odd number $d \leq 5$.

(Balasubramanian & P. Kraus, Commun. Math. Phys. 1999, 208; de Haro et al., Commun. Math. Phys. 2001, 217;...)

- Invariant integrals on the boundary

Holographic renormalization method

- Set $\ell = 1$, and expand the metric near the boundary $u = 0$

$$\mathrm{d}s^2 = \frac{1}{u^2}\left(\mathrm{d}u^2 + g_{ij}(x,u)\mathrm{d}x^i\mathrm{d}x^j\right), i = 0\,,2\,,3\,\cdots\, d+1\,, \tag{22}$$

$$g_{ij}(x,u) = g_{(0)ij} + g_{(2)ij}u^2 + \cdots + g_{(d)ij}u^d + \cdots \tag{23}$$

$$\gamma_{ij} = \frac{1}{u^2} g_{ij}\,, \tag{24}$$

(Fefferman & Graham, Elie Cartan et les Mathématiques d'aujourd'hui (Astérisque, 1985) 95)

- Regularize the on-shell action at $u = \epsilon$

$$S_{\text{reg}} = \frac{1}{2\kappa^2}\int \mathrm{d}^d x\sqrt{-g_{(0)}}\left[\epsilon^{-d}a_{(0)} + \epsilon^{-d+2}a_{(2)} + \cdots\right] + \mathcal{O}(\epsilon^0)\,. \tag{25}$$

- The divergent part of the counterterms $S_{\text{ct,reg}}$ is

$$-\frac{1}{2\kappa^2}\int \mathrm{d}^d x\sqrt{-g_{(0)}}\left[\epsilon^{-d}a_{(0)} + \epsilon^{-d+2}a_{(2)} + \cdots\right] \tag{26}$$

- re-express the coefficients $a_{(i)}$ by the boundary metric γ_{ij} and other geometric quantities.

Counterterms for the $(3+1)$-dimensional dRGT theory

Li-Ming Cao, **Yuxuan Peng**, Counterterms in massive gravity theory, Phys. Rev. D 92, 124052.

New form of the asymptotic expansions

$$g_{ij}(x,u) = g_{(0)ij} + g_{(1)ij}u + g_{(2)ij}u^2 + \cdots + g_{(d)ij}u^d + \cdots \tag{27}$$

$$\mathcal{K}_{ij} \equiv \frac{1}{u}\xi_{ij} = \frac{1}{u}\left[\xi_{(0)ij} + u\xi_{(1)ij} + \cdots\right], \tag{28}$$

$$\mathcal{U}_1 \equiv u\mu = u\left[\mu_{(0)} + \mu_{(1)}u + \cdots\right] = u\xi^i{}_i, \tag{29}$$

$$\mathcal{U}_2 \equiv u^2\lambda = u^2\left[\lambda_{(0)} + \lambda_{(1)}u + \cdots\right]. \tag{30}$$

Counterterms for the (3 + 1)-dimensional dRGT theory

- The EoM

$$R_{ab} - \frac{1}{2} R \mathcal{G}_{ab} - 3\mathcal{G}_{ab} + m^2 X_{ab} = 0$$
$$2X_{ab} = -c_1 \left(\mathcal{U}_1 \mathcal{G}_{ab} - \mathcal{K}_{ab}\right) - c_2 \left(\mathcal{U}_2 \mathcal{G}_{ab} - 2\mathcal{U}_1 \mathcal{K}_{ab} + 2(\mathcal{K}^2)_{ab}\right) . \tag{31}$$

- Expansion of the on-shell action

$$\begin{aligned} S_{\mathrm{mg,reg}} &= \frac{1}{2\kappa^2} \int \mathrm{d}^3 x \int_{\epsilon} \mathrm{d}u \sqrt{-g} \left(-6 - \frac{m^2 c_1 \mathcal{U}_1}{2}\right) - \frac{1}{\kappa^2} \int \mathrm{d}^3 x \sqrt{-\gamma} K \\ &= \frac{1}{2\kappa^2} \int \mathrm{d}^3 x \left[a_{(0)} \epsilon^{-3} + a_{(1)} \epsilon^{-2} + a_{(2)} \epsilon^{-1} + a_{(3)} \ln \epsilon + \mathcal{O}(\epsilon^0)\right] \end{aligned} \tag{32}$$

Counterterms for the (3 + 1)-dimensional dRGT theory

- (3 + 1) decomposition of the geometric quantities

$$\gamma_a{}^c\gamma_b{}^d R_{ab} \to (R[\gamma]_{ij}, K_{ij}, u)\,, \tag{33}$$

$$G_{ab}n^a n^b \to (R[\gamma]_{ij}, K_{ij}) \tag{34}$$

gives the EoMs

$$\frac{1}{2}\left(g' g^{-1} g'\right)_{ij} - \frac{1}{4}\mathrm{Tr}\left(g^{-1}g'\right) g'_{ij} - \frac{1}{2} g''_{ij} + \frac{1}{2u}\mathrm{Tr}\left(g^{-1}g'\right) g_{ij} + \frac{1}{u} g'_{ij} + R_{ij}(g) = m^2\left[c_1\left(-\frac{\mathcal{U}_1}{4u^2} g_{ij} - \frac{\mathcal{K}_{ij}}{2}\right) + c_2\left(-\mathcal{U}_1\mathcal{K}_{ij} + \mathcal{K}^2_{ij}\right)\right], \tag{35}$$

$$-\frac{u^2}{2}\mathrm{Tr}\left(g^{-1}g''\right) + \frac{u^2}{4}\mathrm{Tr}\left(g^{-1}g' g^{-1} g'\right) + \frac{u}{2}\mathrm{Tr}\left(g^{-1}g'\right) = -\frac{1}{4} m^2 c_1 \mathcal{U}_1\,. \tag{36}$$

where $'$ denotes the derivative with respect to u, and $R_{ij}(g)$ is the Ricci tensor of g_{ij} .

- The relation $a_{(l)} \to (R[g]_{(k)ij}, g_{(k)ij}, \mu_{(k)}, \lambda_{(k)}, \cdots)$ is got by solving EoMs.

Counterterms for the $(3+1)$-dimensional dRGT theory

$$
\begin{aligned}
S_{\mathrm{ct}} = -\frac{1}{\kappa^2}\int \mathrm{d}^3x\sqrt{-\gamma}\,\Bigg[&\frac{2}{\ell}+\frac{1}{2}\ell R[\gamma] \\
&+m^2\left(\frac{1}{4}c_1\ell\,\mathcal{U}_1+(\frac{1}{2}c_2\ell-\frac{1}{32}m^2c_1^2\ell^3)\mathcal{U}_2\right)\Bigg]\,,
\end{aligned}
\tag{37}
$$

Boundary stress-energy tensor

The Brown-York stress-energy tensor is

$$T^{ij} = \frac{2}{\sqrt{-\gamma}} \frac{\delta(S + S_{\rm ct})}{\delta \gamma_{ij}}\,, \tag{38}$$

and it can be divided into two parts

$$T_{ij} = T_{ij}^{\rm EG} + T_{ij}^{\rm mg}\,. \tag{39}$$

The first term already appears in Einstein gravity

$$T_{ij}^{\rm EG} = \frac{1}{\kappa^2}\left[K_{ij} - \gamma_{ij}K - \gamma_{ij}\frac{2}{\ell} + \ell\left(R[\gamma]_{ij} - \frac{1}{2}\,R[\gamma]\gamma_{ij}\right)\right], \tag{40}$$

and the second term comes from massive terms

$$\begin{aligned} T_{ij}^{\rm mg} \quad &= \quad -\frac{m^2}{\kappa^2}\left[\,\frac{1}{4}c_1\ell\left(\mathcal{U}_1\gamma_{ij} - \mathcal{K}_{ij}\right)\right. \\ &\quad \left. + \left(\frac{1}{2}c_2\ell - \frac{1}{32}m^2 c_1^2\ell^3\right)\left(\mathcal{U}_2\gamma_{ij} - 2\mathcal{U}_1\mathcal{K}_{ij} + 2\mathcal{K}_{ik}\mathcal{K}^k{}_j\right)\right]. \end{aligned} \tag{41}$$

The stress-energy tensor of the boundary field theory

$$\langle \hat{T}_{ij} \rangle = \lim_{u\to 0} \frac{T_{ij}}{u}\,. \tag{42}$$

Apply to the solutions

A class of charged black hole solutions in the literature

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2 h_{ij}\mathrm{d}x^i\mathrm{d}x^j\,, \tag{43}$$

$$f(r) = k + \frac{r^2}{\ell^2} - \frac{m_0}{r} + \frac{\mu^2 r_+^2}{4r^2} + \frac{c_1 m^2}{2} r + c_2 m^2\,, \tag{44}$$

$$m_0 = kr_+ + \frac{r_+^3}{\ell^2} + \frac{\mu^2 r_+}{4} + \frac{c_1 m^2}{2} r_+^2 + c_2 m^2 r_+\,. \tag{45}$$

(Cai et al., Phys. Rev. D. 2015, 91 (2): 024032)

Apply to the solutions

- The on-shell action is finite with our counterterms
- While fixing μ, the Euclideanized action is the grand potential

$$\Omega = TS_{\mathrm{E}} = \frac{V}{2\kappa^2}\left(kr_+ - \frac{r_+^3}{\ell^2} + c_2 m^2 r_+ - \frac{1}{4}\mu^2 r_+\right) + \frac{V}{\kappa^2}\left(\frac{m^4 c_1 c_2 \ell^2}{4} - \frac{m^6 c_1^3 \ell^4}{64} + \frac{k m^2 c_1 \ell^2}{4}\right), \tag{46}$$

where V is the volume of the 2-surface with the metric h_{ij}.

- The energy is obtained by

$$E = \Omega + TS + \mu Q \tag{47}$$

The quantities Ω and E are the same as in the literature up to *finite terms*.

Results I

- Counterterms in massive gravity theory are obtained.
- The energy momentum tensor satisfies

$$\hat{D}^i \langle \hat{T}_{ij} \rangle = 0 \tag{48}$$

since this is calculated in the background solution.

- To eliminate the finite terms one can modify the counterterms as

$$S_{\rm ct} = -\frac{1}{\kappa^2} \int \mathrm{d}^3 x \sqrt{-\gamma} \sqrt{\frac{4}{\ell^2} + 2R[\gamma] + m^2 \left(c_1 \mathcal{U}_1 + 2 c_2 \mathcal{U}_2 \right)} \,. \tag{49}$$

Dynamical solution?

With the decomposition

$$\mathcal{G}_{ab} = -t_a t_b + n_a n_b + \sigma_{ab}\,, \tag{50}$$

we have chosen

$$f_{ab} = \frac{1}{2}\big((\mathcal{K}^2)_c{}^c\big)\sigma_{ab}\,, \qquad \mathcal{K}_{ab} = \frac{1}{2}(\mathcal{K}_c{}^c)\sigma_{ab}\,. \tag{51}$$

However if we choose

$$\mathcal{K}_{ab} = y n_a n_b + \frac{1}{2}(\mathcal{K}_c{}^c)\sigma_a{}^b\,. \tag{52}$$

$$\mathrm{d}s^2 = -f(r,v)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2 h_{ij}\mathrm{d}x^i\mathrm{d}x^j\,. \tag{53}$$

$$f(r,v) = k + \frac{r^2}{\ell^2} - \frac{2M(v)}{r} + \frac{q^2(v)}{4r^2} + \frac{1}{2}r_0 c_1 m^2 r + r_0^2 c_2 m^2\,, \tag{54}$$

we have that $\dot{M}(v) = c_1 m^2 \mu(v)$ and $q(v)\dot{q}(v) = -16 r_0 c_2 m^2 \mu(v)$

$$\mathrm{d}s^2 = -f\Big(\frac{\mathrm{d}v}{\mathrm{d}t}\Big)^2\mathrm{d}t^2 + f^{-1}\mathrm{d}r^2 + r^2 h_{ij}\mathrm{d}x^i\mathrm{d}x^j\,, \tag{55}$$

$$D^i T_{it} = -\dot{M}(t) = -c_1 m^2 \mu(t)\,. \tag{56}$$

Dynamical solution?

In our model,

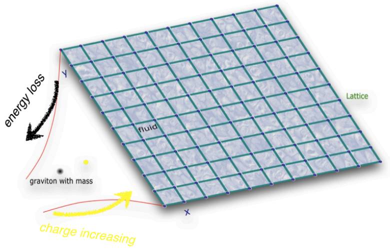


Figure 3: Enery loss of boundary fluid

However incorrect solution.

EoMs and the uniqueness

Li-Ming Cao, **Yuxuan Peng**, Yun-Long Zhang, de Rham-Gabadadze-Tolley massive gravity with degenerate reference metrics, Phys. Rev. D 93, 124015.

- Take $\mathcal{U}_1 = \mathrm{Tr}\mathcal{K}$ as an example.
- The inverse of ${\mathcal{K}_a}^b$ is needed.

$$\begin{aligned} 2\delta\mathrm{Tr}\mathcal{K} &= {\delta_a}^b \delta{\mathcal{K}_b}^a + {\delta_a}^b \delta{\mathcal{K}_b}^a \\ &= (\mathcal{K}^{-1})_a{}^c {\mathcal{K}_c}^b \delta{\mathcal{K}_b}^a + {\mathcal{K}_a}^c (\mathcal{K}^{-1})_c{}^b \delta{\mathcal{K}_b}^a \\ &= (\mathcal{K}^{-1})_a{}^c {\mathcal{K}_c}^b \delta{\mathcal{K}_b}^a + (\mathcal{K}^{-1})_a{}^c \delta{\mathcal{K}_c}^b {\mathcal{K}_b}^a \\ &= (\mathcal{K}^{-1})_a{}^c \delta({\mathcal{K}_c}^b {\mathcal{K}_b}^a) \\ &= \mathcal{K}_{ab}\delta g^{ab} \,, \end{aligned} \tag{57}$$

- The variation of the ith power of $\mathcal{K}$ i.e. ${\mathcal{K}_a}^{c_1} {\mathcal{K}_{c_1}}^{c_2} \cdots {\mathcal{K}_{c_i}}^b$ is

$$\delta\mathrm{Tr}\mathcal{K}^i = \frac{i}{2}(\mathcal{K}^i)_{ab}\delta g^{ab} \,, \quad i = 1\,, \cdots d\,. \tag{58}$$

Results II

- The reference metric and its square-root $\mathcal{K}$ is degenerate in the model studied here.
- In this case, we should apply the *generalized Moore-Penrose pseudo-inverse* of degenerate tensors to derive the EoMs. This fact leads a Birkhoff-type theorem, ruling out the "solution" above.

Summary

- Non-linear ghost-free massive gravity theory: the dRGT theory.
- The dRGT theory has been applied to construct a holographic model with momentum dissipation.
- We derived the boundary counterterms of this model and renormalized the thermodynamical potentials and the boundary stress-energy tensor.
- We also studied the uniqueness of the solutions of this model.
- To study the dissipation, one should study the perturbations around the background.