Nonlocal Teleparallel Gravity

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01 March 2018

Based on S. Bahamonde, S. Capozziello, M. Faizal and R. C. Nunes, Eur. Phys. J. C **77** (2017) no.9, 628

Outline

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices;
  Latin letters $\rightarrow$ tangent space indices; ${E_a}^\mu$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^\mu {e^n}_\mu = \delta^n_m$ and ${E_m}^\nu {e^m}_\mu = \delta^\nu_\mu$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu$

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Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called “Weitzenböck connection”:

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_{\mu} e^{a}{}_{\nu} = E_a{}^{\rho}(\partial_{\mu} e^{a}{}_{\nu} + w^{a}{}_{b\mu} e^{b}{}_{\nu}) \, .$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu} \, .$$

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Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu} \,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

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Curvature in Teleparallel gravity

$$R^{a}{}_{b\mu\nu}(\omega^{a}{}_{b\mu}) = \partial_{\mu}\omega^{a}{}_{b\nu} - \partial_{\nu}\omega^{a}{}_{b\mu} + \omega^{a}{}_{c\mu}\omega^{c}{}_{b\nu} - \omega^{a}{}_{c\nu}\omega^{c}{}_{b\mu} \equiv 0 \, .$$

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Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

$$S_{\rm TEGR} = \int \left[-T + 2\kappa^2 \mathcal{L}_{\rm m}\right] e\, d^4x\,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e_a^\mu) = \sqrt{-g}$, $\mathcal{L}_{\rm m}$ matter Lagrangian and $T = \frac{1}{4}T^\rho{}_{\mu\nu}T_\rho{}^{\mu\nu} + \frac{1}{2}T^\rho{}_{\mu\nu}T^{\nu\mu}{}_\rho - T^\lambda{}_{\lambda\mu}T_\nu{}^{\nu\mu}$.

- T and the scalar curvature R differs by a boundary term B as $R = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

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Two different ways of understanding gravity

Equivalence on their field equations

VERY IMPORTANT POINT: TEGR has the same equations as GR, so **CLASSICALLY** it is impossible to make any observation to distinguish between them.

Validity of TEGR

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Teleparallel gravity vs General Relativity

Two completely equivalent ways of understanding gravity:

Connections and strength fields

G.R. $\Longrightarrow$ Levi-Civita connection $\Longrightarrow$ Curvature with vanishing torsion
TEGR $\Longrightarrow$ Weitzenböck connection $\Longrightarrow$ Torsion with vanishing curvature (flat).

How gravity is explained in both theories?

GR $\Longrightarrow$ Geometry (curvature of space-time) $\Longrightarrow$ geodesic equations
TEGR $\Longrightarrow$ Forces $\Longrightarrow$ Force equations as Maxwell eqs (no geodesic eq.).

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Gauge structure

GR $\Longrightarrow$ NO (only diffeomorphism)
TEGR $\Longrightarrow$ Gauge theory of the translations

Must have the equivalence principle?

GR $\Longrightarrow$ YES
TEGR $\Longrightarrow$ Can survive with or without

Can we separate inertia with gravity?

GR $\Longrightarrow$ NO (mixed) $\Longrightarrow$ No tensorial expression for the gravitational energy-momentum density
TEGR $\Longrightarrow$ YES $\Longrightarrow$ gravitational energy-momentum density is a tensor.

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What to do?

Since both theories predict the same classical experiments, but they are different conceptually and physically, how can we know which theory is the correct one?

Nonlocality

- Since TEGR and GR are conceptually different, they are expected to produce different quantum effects.
- Many quantum gravity proposals (string theory, loop quantum gravity) $\Rightarrow$ Intrinsic extended structure in the geometry of spacetime $\Rightarrow$ Effective nonlocal behavior for spacetime.
- Then, first order corrections from quantum gravity might produce nonlocal deformations of GR.
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Deser-Woodard Nonlocal Gravity

- There exists different nonlocal models based on standard GR. One very interesting one has the following action (S. Deser and R. P. Woodard, Phys. Rev. Lett. **99** (2007) 111301)

DW Nonlocal action

$$\mathcal{S} = \mathcal{S}_{\rm GR} + \frac{1}{2\kappa}\int d^4x\,\sqrt{-g(x)}\,R(x)f\Big((\Box^{-1}R)(x)\Big) + \mathcal{S}_{\rm m}\,.$$

- Here f is an arbitrary function which depends on the retarded Green function evaluated at the curvature scalar R and $\mathcal{G}[f](x)$ is a nonlocal operator which can be written in terms of the Green function $G(x,x')$ as

$$\mathcal{G}[f](x) \quad = \quad (\Box^{-1}f)(x) = \int d^4x'\,\sqrt{-g(x')}f(x')G(x,x')\,.$$

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Deser-Woodard Nonlocal Gravity - Properties

- It has been shown that DW NG in its **standard form** is (R. P. Woodard, Found. Phys. **44** (2014) 213):
  - Ghost-free and stable (f must satisfy a condition).
  - Do not propagate extra degrees of freedom.
  - Can mimic dark energy without a Λ (specific distortion function).
  - It is consistent with Solar system constraints.
  - Acasual (due to the advanced Green function).
- It is possible to **localised the action** by introducing two auxiliary field $\phi = \Box^{-1}R$ and $\xi = -f'(\phi)R$, which gives a causal theory but contains ghost (can be avoided for some f at some very special times). (S. Nojiri and S. D. Odintsov (2008), S. Nojiri, S. D. Odintsov, M. Sasaki and Y. l. Zhang (2011))

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Nonlocal Teleparallel gravity - A way to contrast GR with TEGR

- Quantum corrected nonlocal GR can be written as

Nonlocal Quantum correction action

$$\mathcal{S}_1 = \mathcal{S}_{\text{GR}} + \mathcal{S}_{\text{GRNL}}.$$

- Similarly, in TEGR, one can consider

Teleparallel Nonlocal Quantum correction action

$$\mathcal{S}_2 = \mathcal{S}_{\text{TEGR}} + \mathcal{S}_{\text{TEGRNL}}.$$

- It is not possible to differentiate classically between $\mathcal{S}_{\text{GR}}$ and $\mathcal{S}_{\text{TEGR}}$, but the quantum corrections to these theories $\mathcal{S}_{\text{GRNL}}$ and $\mathcal{S}_{\text{TEGRNL}}$ are different.
- The effects might be used to experimentally discriminate between these two theories.

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Nonlocal Quantum correction action

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- TEGR **DOES NOT REQUIRE** the equivalence principle, and it has been argued that quantum effects can cause the violation of the Equivalence Principle.
- Violation of the Equivalence Principle can be related to a violation of the Lorentz symmetry $\rightarrow$ this is also broken at the UV scale in various approaches to quantum gravity (e.g. non-commutative, Horava, etc.) $\rightarrow$ Some teleparallel gravity theories break the Lorentz invariance.
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where now f is a function of the Green function evaluated at the torsion scalar T.

- Since $R = -T + B$, the boundary term B will produce that the above action would be different than standard curvature DW nonlocal gravity.

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Nonlocal Teleparallel Gravity - Possible experiments

Since nonlocal TEGR gravity is different than DW nonlocal gravity, one could be able to make some experiments to distinguish between TEGR and GR at some scales, for example:

- Violation of Weak Equivalence Principle (SR-POEM project with more precision) $\rightarrow$ At some scales, it can be violated and nonlocal TEGR can be used as a experimental test to know which of these theories is more correct.
- Photon time delay $\rightarrow$ Both nonlocal theories will produce different photon time delays $\rightarrow$ One can measure the round trip time of a bounced radar beam off the surface of Venus.
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