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Renormalization of Horava Gravity

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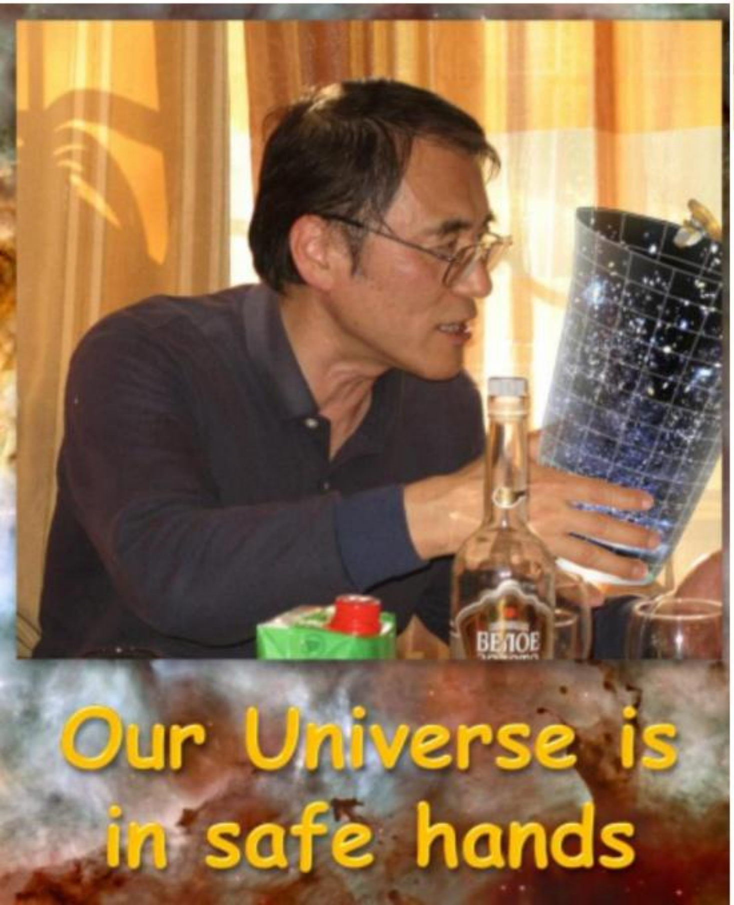
Our Universe is in safe hands

Sweet moments …



and the fate of the Universe!

Those Russians . . .
Professor of cosmology!

Plan

Introduction:

towards local, unitary, perturbatively UV renormalizable QG

Horava-Lifshitz gravity

Problems with renormalization:

BPH renormalization and “regularity” of propagators
covariance of UV counterterms

“Regular” propagators and gauge fixing conditions

Renormalization of gauge theories in background-field approach

BRST structure of renormalization and field reparametrization

Asymptotic freedom of (2+1)-dimensional Horava gravity

Summary and outlook:

with D. Blas
M. Herrero-Valea
S. Sibiryakov
and C. Steinwachs

Phys. Rev. D 93, 064022 (2016), arXiv:1512.02250;
arXiv:1705.03480; PRL 119,211301 (2017), arXiv:1706.06809

$$S_{EH} = \frac{M_P^2}{2}\int dtd^dx\ R$$

nonrenormalizable

$$\Rightarrow \frac{M_P^2}{2}\int dtd^dx\ (h_{ij}\Box h_{ij} + h^2\Box h + \dots)$$

Stelle (1977)

$$\int (M_P^2 R + R_{\mu\nu}R^{\mu\nu} + R^2)$$

$$\Rightarrow \int (M_P^2 h_{ij}\Box h_{ij} + h_{ij}\Box^2 h_{ij} + \dots)$$

dominates at $k \gg M_P$

The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)
Avramidi, Barvinsky (1985)

But has ghost poles $\Rightarrow$ no unitary interpretation

Never give up

Horava (2009)

$$\underbrace{\int dt\, d^dx}_{\propto\, b^{-(z+d)}} \left(\dot{h}_{ij}\dot{h}_{ij} - h_{ij}(-\Delta)^z h_{ij} + \ldots\right)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\,, \qquad t \mapsto b^{-z} t \quad \Rightarrow \quad h_{ij} \mapsto b^{(d-z)/2} h_{ij}$$

Critical theory in $z = d$

LI is necessarily broken. We want to preserve as many symmetries, as possible

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \quad \Rightarrow \quad \gamma_{ij} \quad N^i, \qquad i = 1, \ldots, d$$

$$t \mapsto \tilde{t}(t) \quad \Rightarrow \quad N$$

We consider projectable models with *N=1* in what follows

Foliation preserving diffeomorphisms $x^i \mapsto \tilde{x}^i(\mathbf{x},t)\ ,\quad t \mapsto \tilde{t}(t)$

ADM metric decomposition

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\ ,\qquad i,j = 1,\dots,d$$

space dimensionality

Scaling transformations and scaling dimensions

$$x^i \to \lambda^{-1} x^i, \quad t \to \lambda^{-z} t, \quad N^i \to \lambda^{z-1} N^i, \quad \gamma_{ij} \to \gamma_{ij},$$
$$[x] = -1,\ [t] = -z,\ [N^i] = z-1,\ [\gamma_{ij}] = 0, \qquad [K_{ij}] = z.$$

Horava gravity action

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma} N \Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}(\gamma)\Big)$$
$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Potential term

$$\mathcal{V}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R^{ij} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$
$$+\nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \dots$$

Extra structures in non-projectable theory, reduction of structures for detailed balance case

$$\text{Deg of div} \int \frac{d^{d+1}p}{\left(p^2\right)^N} = d + 1 - 2N = \text{physical dimensionality}$$

$$\Downarrow \qquad p = (\omega, \mathbf{k}), \;\; p^2 \to \omega^2 + \mathbf{k}^{2z}$$

$$\text{Deg of div} \int \frac{d\omega\, d^d k}{\left(\omega^2 + \mathbf{k}^{2z}\right)^N} = z + d - 2zN = \text{scaling dimensionality}$$

physical dimensionality $\neq$ scaling dimensionality

Counting degree of divergences and dimensionalities

$$\mathrm{Tr}\ \ln\Big(-\partial_t^2+(-\Delta)^z+\dots\Big)\Big|_{\mathrm{div}}$$

$$=-\int_{\epsilon}^{\infty}\frac{ds}{s}\,e^{-s\,[-\partial_t^2+(-\Delta)^z+\dots]}\Bigg|_{\mathrm{div}}=\int d\tau\,d^dx\,\gamma^{1/2}\sum\frac{\nabla^{2k}R^n\partial_t^rK^p}{\epsilon^D}$$

$$D=\frac{d+z-2(n+k)-(p+r)z}{2z}$$

degree of divergence

$$[\,s\,]=[\,\epsilon\,]=-2z$$

$$D\le 0,\quad p\ge 2\ \Rightarrow z\ge d$$

$$\boxed{z=d}$$

critical value

Log divergent potential terms

$$r+p=0,\ D=1-\frac{k+n}{d}=0\ \Rightarrow k+n=d,\ \ [\,\nabla^{2k}R^n\,]=2d$$

$$\mathcal{V}^{(d=2)}(\gamma)=2\Lambda-\eta R+\mu_1R^2$$

$$\mathcal{V}^{(d=3)}(\gamma)=2\Lambda-\eta R+\mu_1R^2+O(R^3,R\nabla^2R)$$

So what about renormalizability

or why power-counting is not enough ?

Divergences are local and are removed by local counterterms of $dim \leq 2d$ that are already present in the action

this is not guaranteed $\Longleftrightarrow$ gauge invariance

$$\int \prod_{l=1}^{L} d^{d+1}k^{(l)}\ \mathcal{F}_n(k)\ \prod_{m=1}^{M} \frac{1}{\left(P^{(m)}(k)\right)^2} \quad \Longrightarrow$$

$$\int \prod_{l=1}^{L} d\omega^{(l)} d^d k^{(l)}\ \mathcal{F}_n(\omega, \mathbf{k})\ \prod_{m=1}^{M} \frac{1}{A_m\left(\Omega^{(m)}(\omega)\right)^2 + B_m\left(\mathbf{K}^{(m)}(\mathbf{k})\right)^{2z}}$$

Generalization of BPHZ renormalization theory (subtraction of subdivergences) works only for $\underbrace{A_m > 0 \text{ and } B_m > 0}_{\text{depends on gauge fixing}}$ (or $A_m < 0$ and $B_m < 0$)

depends on gauge fixing

Toy model: d=2

$$S = \frac{1}{2G}\int dt\, d^2x \big(K_{ij}K^{ij} - \lambda K^2 - \mu R^2\big)$$

propagates a single scalar that is well-behaved at $\lambda < 1/2$ or $\lambda > 1$

We need to fix the gauge:

linear combination of the fields

$$\mathcal{L}_{gf} = \frac{\sigma}{2G} F^i \mathcal{O}_{ij} F^j$$

invertible operator

not to spoil p.c. $\Rightarrow$ $[\mathcal{L}_{gf}] = 4$

If $\mathcal{O}_{ij}$ is local $\Rightarrow$ $[\mathcal{O}_{ij}] \geq 0$ $\Rightarrow$ $[F^i] \leq 2$

$$F^i = N^i, \qquad \mathcal{O}_{ij} = \delta_{ij}$$

$$\Downarrow$$

$$\mathcal{P}_\mathrm{s}(\omega, p) = \frac{1}{\omega^2 + 4\mu\frac{1-\lambda}{1-2\lambda}\,p^4}$$

$$\langle h_{ij}(p) h_{kl}(-p)\rangle = \frac{4\kappa^2(1-\lambda)}{1-2l}\delta_{ij}\delta_{kl}\, P_s(p)$$

$$+\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}\right)\frac{2\kappa^2}{\omega^2}$$

$$\sim \frac{1}{\omega^2}$$

$$+16\kappa^2\mu\left[\frac{1-\lambda}{1-2\lambda}\left(\delta_{ij}p_k p_l + p_i p_j \delta_{kl}\right)p^2 - p_i p_j p_k p_l\right]\frac{P_s(p)}{\omega^2}$$

$$\frac{O(p^4)P_s(p)}{\omega^2} \sim \frac{1}{\omega^2}$$

$$\Rightarrow \quad (\ldots) \times |\, t \,|\, \delta^{(2)}(\mathbf{x})$$

Analogy: Coulomb gauge in QED and YM theory

What is the analogue of relativistic gauges?

Regular gauges for HG

$\mathcal{L}_{gf}$ must be non-local

Use guidance from the relativistic case

$$F^\mu = \partial_\nu h_{\nu\mu} + \dots \quad \Rightarrow \quad F^i = \dot{N}^i + \partial_j h_{ji} + \dots$$

For HG put additional spatial derivatives on h_{ij} to preserve the homogeneous scaling. For d=2:

$$F^i = \dot{N}^i + c_1 \Delta \partial_j h_{ji} + c_2 \Delta \partial_i h + c_3 \partial_i \partial_j \partial_k h_{jk}$$

$$\Rightarrow \quad [F^i] = 3 \quad \Rightarrow \quad [\mathcal{O}_{ij}] = -2$$

$$\mathcal{O}_{ij} = -\big(\delta_{ij}\Delta + \xi \partial_i \partial_j\big)^{-1}$$

The choice

$$F^i = \dot{N}^i + \frac{1}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_k h_{jk} - \frac{\lambda}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_j h$$

decouples N^i from h_{ij} in the quadratic action

→ regular propagators for all fields (including Faddeev--Popov ghosts)

two free gf. parameters $\sigma,\ \xi$

Straightforward generalization to $d > 2$, e.g.

$$\mathcal{O}^{d=3}_{ij} = \Delta^{-1}\big(\delta_{ij}\Delta + \xi\partial_i\partial_j\big)^{-1}$$

Diagrammatics in brief

- induction in the number of loops
- subdivergences are cancelled by counterterms introduced at the previous steps

Anselmi, Halat (2007)

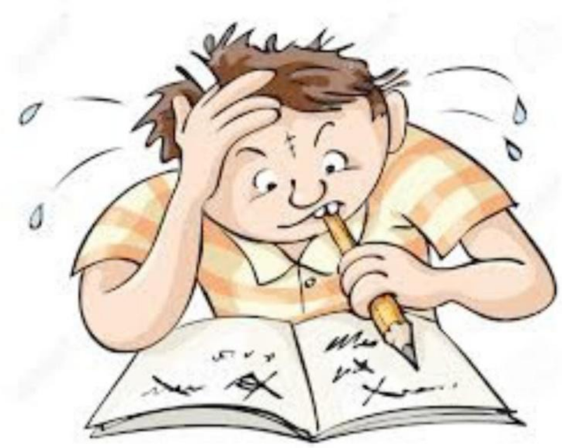
- introduce the degree of divergence $\mathcal{D}$ defined as the scaling of the diagram under stretching the loop momenta and frequencies $k_{loop} \mapsto b\, k_{loop}$, $\omega_{loop} \mapsto b^d\, \omega_{loop}$
- regular propagators ⇨ diagrams with $\mathcal{D} < 0$ converge
- diags. with $\mathcal{D} > 0$ require local counterterms of scaling dimension at most $2d$

What about gauge invariance ?

this is tricky ...

GI is explicitly broken by the gauge-fixing. Instead, we have to rely on the Slavnov-Taylor identities $\Longleftrightarrow$ BRST symmetry.

The latter gets deformed at each loop order and requires non-linear field renormalization to restore ...



Use the background-field method

(why it works is in the second part of the talk)

$$\gamma_{ij} = \bar{\gamma}_{ij} + h_{ij} \;, \quad N^i = \bar{N}^i + n^i$$

covariantize everything with respect to $\bar{\gamma}_{ij}$:

$$\mathcal{O}_{ij} = -\left(\bar{\Delta}\bar{\gamma}^{ij} + \xi\bar{\nabla}^i\bar{\nabla}^j\right)^{-1}$$ etc.

NB. Nonlocality can be resolved by introducing an auxiliary field

effective action is manifestly invariant w.r.t. background gauge transformations

- one-loop counterterms are manifestly gauge-invariant
- at higher loops, the renormalization of quantum fields is fixed

Abbott (1981), Barvinsky, Vilkovisky (1988)
Grassi (1996), Anselmi (2014)
Barvinsky, Blas, Herrero-Valea, Steinwachs, S.S. (to appear)

Non-Projectable model

one more variable $N = 1 + \phi$ + one more equation

still TT + a single scalar

Good: $\omega_s^2 \propto +k^2$ at $k \to 0$

Bad: at $k \to \infty$

$$\langle \phi\phi \rangle = \text{regular} + \frac{1}{k^{2d}}$$

present even in $\sigma\xi$ - gauges
physical: shows up in the interaction of local sources

Blas, Pujolas, S.S. (2011)
Blas, S.S. (2011)

Outlook

- Projectable HG is renormalizable in arbitrary number of spacetime dimensions. Remains true with addition of Lifshitz matter
- Key tools: gauges leading to regular propagators
  + background field method
- To do: explicit computation of quantum corrections. Is the theory asymptotically free or runs into a Landau pole ?
- Toy model to study the role of (spatial) diffeomorphisms in quantum gravity. E.g. does absence of local observables imply non-locality ?
- New ideas are needed to address renormalizability of the NP model

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Summary and outlook:

Renormalization of gauge theories in background-field approach

Task: to prove that counterterms are covariant local functionals of the original gauge field

Gauge-breaking and ghost terms – counterterms to them?

Background covariant gauge conditions: success of the one-loop approximation – what is beyond?

Preservation of the BRST structure and counterterms covariance by 1) UV renormalization and 2) gauge field reparametrization (*DeWitt, Tuytin-Voronov, Batalin-Vilkovisky, Kallosh, Arefieva-Faddeev-Slavnov, Abbot, Henneaux et al*)

40+ years old topic! So what is new here?

Background field extension of the BRST operator

Inclusion of generating functional sources into the gauge fermion

⇓

BRST structure of renormalization via decoupling of the background field

Quantum corrected gauge fermion is a generating functional of the field reparameterization

No power counting or use of field dimensionalities

Extension to Lorentz symmetry violating theories

Extension to (nonrenormalizable) effective field theories

BRST formalism

Gauge theory:

$$\varphi = \varphi^a, \qquad S = S[\varphi], \qquad \frac{\delta S}{\delta \varphi^a} R^a_\alpha = 0$$

Generators of gauge transformations:

$$R^a_\alpha = R^a_\alpha(\varphi), \qquad \delta_\epsilon \varphi^a = R^a_\alpha \epsilon^\alpha,$$

$$R^a_\alpha \frac{\delta R^b_\beta}{\delta \varphi^a} - R^a_\beta \frac{\delta R^b_\alpha}{\delta \varphi^a} = C^\gamma_{\alpha\beta} R^b_\gamma$$

structure constants

DeWitt summation rule: $a = (A, x), \qquad F^a \Psi_a \equiv \int dx\, F^A(x) \Psi_A(x)$

Feynman-DeWitt-Faddeev-Popov functional integral

$$e^{-W[J]} = \int d\varphi\, e^{-S[\varphi]\underbrace{-\frac{1}{2}\chi^{\alpha}O_{\alpha\beta}\chi^{\beta}}_{\text{gauge-breaking term}}-J\varphi}\, \underbrace{\left(\det O_{\alpha\beta}\right)^{1/2}}_{\text{measure}} \det\underbrace{\left(\frac{\delta\chi^{\alpha}}{\delta\varphi^{a}}\,R^{a}_{\beta}\right)}_{\text{Faddeev-Popov operator}}$$

$$S[\varphi] \to \Sigma[\Phi] = S[\varphi] + \overbrace{b_{\alpha}\chi^{\alpha}(\varphi) - \frac{1}{2}O^{\alpha\beta}\,b_{\alpha}b_{\beta}} - \bar{\omega}_{\alpha}\left(\frac{\delta\chi^{\alpha}}{\delta\varphi^{a}}\,R^{a}_{\beta}\right)\omega^{\beta}$$

$$\varphi^{a} \to \Phi = \varphi^{a}, \underbrace{\omega^{\alpha}, \bar{\omega}_{\alpha}, b_{\alpha}}$$

BRST functional integral

$$e^{-W[J]} = \int d\Phi\, e^{-\Sigma[\Phi]-J\varphi}$$

$$\Sigma[\,\Phi\,] = S[\,\varphi\,] + \boldsymbol{s}\,\Psi[\,\Phi\,]$$

BRST action

$$\boldsymbol{s} = (\boldsymbol{s}\Phi)\frac{\delta}{\delta\Phi}, \qquad \boldsymbol{s}^2 = 0$$

nilpotent BRST operator

BRST transformations of Φ

$$\boldsymbol{s}\Phi: \quad \boldsymbol{s}\varphi^a = R^a_\alpha(\varphi)\,\omega^\alpha\,, \quad \boldsymbol{s}\omega^\alpha = \frac{1}{2}C^\alpha_{\beta\gamma}\,\omega^\beta\omega^\gamma,$$
$$\boldsymbol{s}\bar{\omega}_\alpha = b_\alpha, \quad \boldsymbol{s}b_\alpha = 0\,.$$

$$\Psi[\,\Phi\,] = \bar{\omega}_\alpha\Big(\chi^\alpha(\varphi) - \frac{1}{2}O^{\alpha\beta}\,b_\beta\Big)$$

gauge fermion

gauge conditions

gauge-fixing matrix

Assumptions on the class of theories

generators: linear $\delta_\varepsilon \varphi^a = R^a{}_\alpha(\varphi)\,\varepsilon^\alpha, \quad R^a{}_\alpha(\varphi) = P^a{}_\alpha + R^a{}_{b\alpha}\varphi^b$

closed algebra $\left[\delta_\varepsilon, \delta_\eta\right]\varphi^a = \delta_\varsigma \varphi^a \quad \varsigma^\alpha = C^\alpha{}_{\beta\gamma}\varepsilon^\beta \eta^\gamma$

ireducible $R^a{}_\alpha \varepsilon^\alpha = 0 \;\Rightarrow\; \varepsilon^\alpha = 0$

Examples:

YM: $\delta_\varepsilon A^i_\mu = f^{ijk} A^j_\mu \varepsilon^k + \partial_\mu \varepsilon^i$

GR: $\delta_\varepsilon g_{\mu\nu} = \varepsilon^\lambda \partial_\lambda g_{\mu\nu} + g_{\mu\lambda}\partial_\nu \varepsilon^\lambda + g_{\nu\lambda}\partial_\mu \varepsilon^\lambda$

Also higher-derivative gravity, also non-relativistic (Lifshitz) theories

Counterexample:

Supergravity (the algebra does not close off-shell)

Background gauge-fixing

- choose g.f. function $\chi^\alpha(\varphi,\phi) = \chi^\alpha_a(\phi)(\varphi-\phi)^a$ to be

  invariant under BGT: $\delta_\varepsilon \varphi^a = R^a{}_\alpha(\varphi)\varepsilon^\alpha$ $\quad \delta_\varepsilon \phi^a = R^a{}_\alpha(\phi)\varepsilon^\alpha$

  the same

- promote $\boldsymbol{s} \mapsto \boldsymbol{Q} = \boldsymbol{s} + \Omega^a \dfrac{\delta}{\delta\phi^a}$

  anticommuting auxiliary field,
  controls dependence of g.f. on background

g.f. term at tree level: $\boldsymbol{Q}\Psi_0$

auxiliary (anti-)fields coupled to $\boldsymbol{s}\varphi^a$, $\boldsymbol{s}\omega^\alpha$

antighost

Lagrange multiplier

$$\Psi_0 = -\underbrace{\left(\gamma_a - \bar\omega_\alpha \chi^\alpha_a(\phi)\right)}_{\hat\gamma_a}(\varphi-\phi)^a + \zeta_\alpha\omega^\alpha - \frac{1}{2}\bar\omega_\alpha O^{\alpha\beta}(\phi) b_\beta$$

Introduction of sources

backgrpound source

$$\mathcal{J} = \underbrace{J_a,\ \bar\xi_\alpha,\ \xi^\alpha,\ y^\alpha}_{\text{sources for } \Phi = \varphi^a, \omega^\alpha, \bar\omega_\alpha, b_\alpha},\ \underbrace{\gamma_a,\ \zeta_\alpha}_{\text{sources for } \boldsymbol{s}\varphi^a, \boldsymbol{s}\omega^\alpha},\ \Omega^a$$

$$\exp\left\{-\frac{1}{\hbar}W_0[\mathcal{J}]\right\}$$

$$= \int d\Phi \exp\left\{-\frac{1}{\hbar}\Big(\Sigma_0 + J_a(\varphi^a - \phi^a) + \bar\xi_\alpha\omega^\alpha + \xi^\alpha\bar\omega_\alpha + y^\alpha b_\alpha\Big)\right\}$$

$$\Sigma_0[\Phi, \mathcal{J}] = S[\varphi] + \boldsymbol{Q}\,\Psi_0[\Phi, \mathcal{J}]$$

$$\Psi_0[\varphi, \omega, \bar\omega, b, \phi, \gamma, \zeta] = \bar\omega_\alpha\Big(\chi^\alpha_a(\phi)(\varphi^a - \phi^a) - \frac{1}{2}O^{\alpha\beta}(\phi)\, b_\beta\Big)$$

$$-\gamma_a(\varphi^a - \phi^a) + \zeta_\alpha\omega^\alpha$$

More on background gauge transformations – linear representation of the gauge group

$$\delta_\varepsilon \varphi^a = R^a{}_\alpha(\varphi)\,\varepsilon^\alpha \ , \qquad \delta_\varepsilon \phi^a = R^a{}_\alpha(\phi)\,\varepsilon^\alpha$$

$$\delta_\varepsilon(\varphi^a - \phi^a) = \frac{\delta R^a{}_\alpha}{\delta \varphi^b}(\varphi^b - \phi^b)\varepsilon^\alpha$$

fundamental representation

$$\delta_\varepsilon \chi^\alpha \equiv \frac{\delta \chi^\alpha}{\delta \varphi^a}\delta_\varepsilon \varphi^a + \frac{\delta \chi^\alpha}{\delta \phi^a}\delta_\varepsilon \phi^a = -C^\alpha{}_{\beta\gamma}\chi^\beta \varepsilon^\gamma$$

adjoint representation

B.g.t. of sources:

$$\delta_\varepsilon \gamma_a = -\gamma_b R^b{}_{a\alpha}\varepsilon^\alpha \ , \qquad \delta_\varepsilon \omega^\alpha = -C^\alpha{}_{\beta\gamma}\,\omega^\beta \varepsilon^\gamma$$

$$\delta_\varepsilon \zeta_\alpha = \zeta_\beta C^\beta{}_{\alpha\gamma}\varepsilon^\gamma, \qquad \delta_\varepsilon \Omega^\alpha = R^a{}_{b\alpha}\Omega^b \varepsilon^\alpha$$

$$\Downarrow$$

$$\delta_\varepsilon \Psi_0 = 0, \quad \delta_\varepsilon \Sigma_0 = 0$$

Renormalization at a glance

generating functional $$W[J] = -\hbar \log \int d\Phi \exp\left[-\frac{1}{\hbar}(S[\Phi] + J\Phi)\right]$$

mean fields, effective action $$\langle\Phi\rangle = \frac{\delta W}{\delta J}\ , \qquad \Gamma[\langle\Phi\rangle] = W - J\langle\Phi\rangle$$

subtraction $$S_{\mathrm{ren}} = S_0[\Phi] - \hbar\,\Gamma^{1-\mathrm{loop}}_{\mathrm{div}}[\Phi] - \hbar^2\Gamma^{2-\mathrm{loop}}_{\mathrm{div}}[\Phi] - \ldots$$

local

Apply it to a gauge theory:

L-th order renormalization:

quantum field

$$\Sigma_L[\Phi, \phi, \gamma, \zeta, \Omega] = \Sigma_{L-1} - \hbar^L \Gamma_{L,\infty}[\Phi, \phi, \gamma, \zeta, \Omega]$$

Main result

L-th order generating functional:

reparameterized fields

$$\exp\Big\{-\frac{1}{\hbar}W_L[\mathcal{J}]\Big\} = \int d\Phi \exp\Big\{-\frac{1}{\hbar}\Big(\Sigma_L + J_a(\tilde{\varphi}^a_L - \phi^a) + \bar{\xi}_\alpha \tilde{\omega}^\alpha_L + \xi^\alpha \bar{\omega}_\alpha + y^\alpha b_\alpha\Big)\Big\}$$

BRST structure of the renormalized action

$$\Sigma_L[\Phi,\phi,\gamma,\zeta,\Omega] = S_L[\varphi] + \boldsymbol{Q}\,\Psi_L[\Phi,\phi,\gamma,\zeta,\Omega]$$

only original gauge field

Renormalized gauge fermion

$$\Psi_L = \hat{\Psi}_L[\varphi,\omega,\phi,\hat{\gamma},\zeta,\Omega] - \frac{1}{2}\bar{\omega}_\alpha O^{\alpha\beta}(\phi) b_\beta$$ L-th loop order

$$\hat{\Psi}_0 = -\hat{\gamma}_a(\varphi^a - \phi^a) + \zeta_\alpha \omega^\alpha$$ tree level

Local reparameterization of quantum fields to composite operators including external sources

$$\tilde{\varphi}^a_L = \tilde{\varphi}^a_L(\varphi, \omega, \phi, \hat{\gamma}, \zeta, \Omega) \qquad \tilde{\omega}^\alpha_L = \tilde{\omega}^\alpha_L(\varphi, \omega, \phi, \hat{\gamma}, \zeta, \Omega)$$

Gauge fermion is a generating function of the field redefinition

$$\tilde{\varphi}^a_L - \phi^a = -\frac{\delta \Psi_L}{\delta \gamma_a}\,, \qquad \tilde{\omega}^\alpha_L = \frac{\delta \Psi_L}{\delta \zeta_\alpha}$$

Applies to (non-renormalizable) EFT --nonlinear dependence on sources γ and ζ

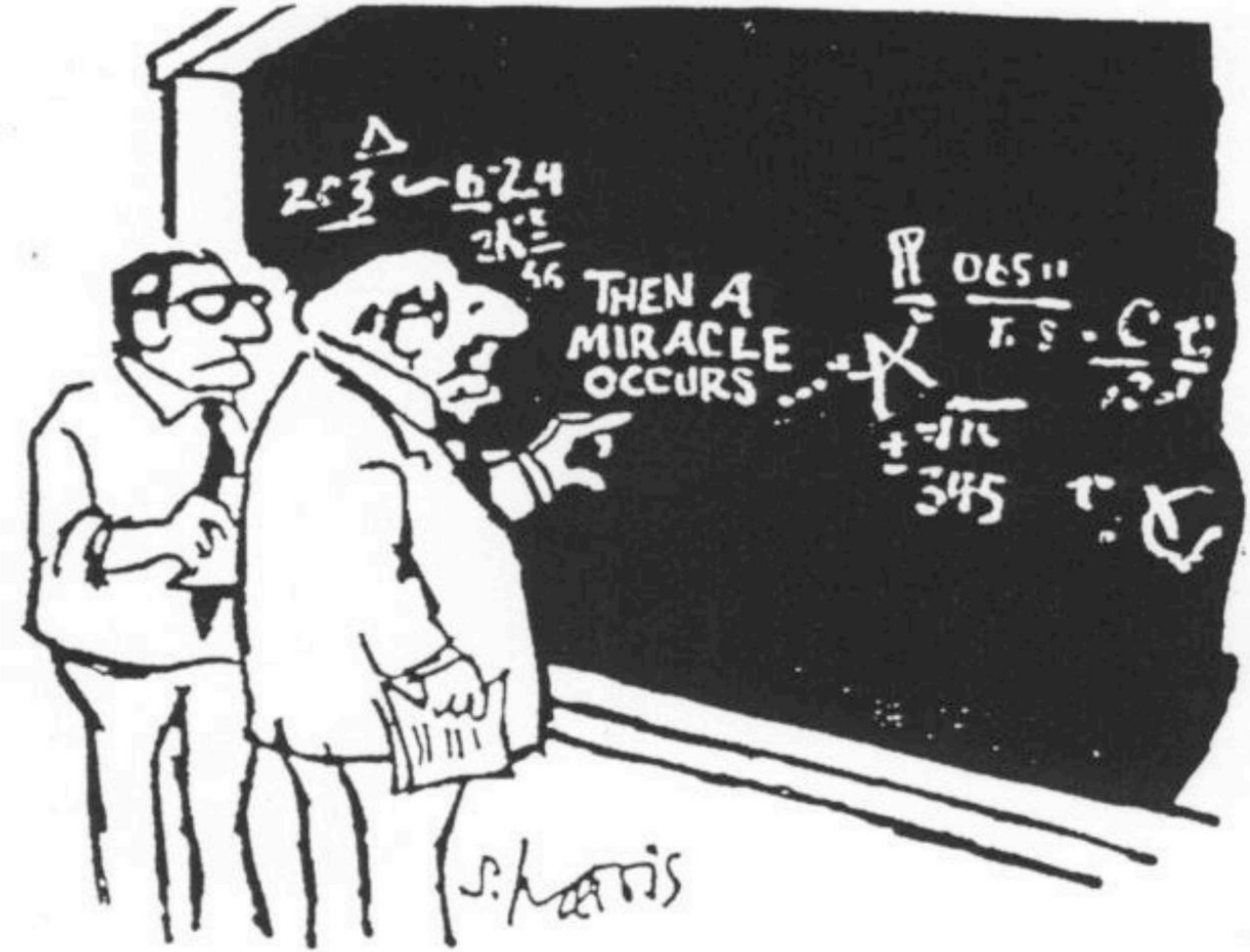
For renormalizable theories:

$$\hat{\Psi}_L = -\hat{\gamma}_a U_L{}^a(\varphi,\phi) + \zeta_\alpha \omega^\beta V_{L\beta}^{\alpha}(\varphi,\phi)$$

linear in γ and ζ

$$\tilde{\varphi}^a_L = \phi^a + U_L{}^a(\varphi,\phi)\,, \quad \tilde{\omega}^\alpha_L = V_{L\beta}^{\alpha}(\varphi,\phi)\,\omega^\beta$$

independent
of other sources



I think you should be a little
more specific, here in Step 2

Slavnov-Taylor and Ward identities

BRST $\rightarrow$ $$\left[-J\frac{\delta}{\delta\gamma}+\bar{\xi}\frac{\delta}{\delta\zeta}+\xi\frac{\delta}{\delta y}+\Omega\frac{\delta}{\delta\phi}\right]W=0$$

b.g,t, $\rightarrow$ $$\left[\delta_\varepsilon\phi\frac{\delta}{\delta\phi}+\delta_\varepsilon\mathcal{J}\frac{\delta}{\delta\mathcal{J}}\right]W=0$$

all sources

$\mathcal{J}=J,\,\bar{\xi},\,\xi,\,y,\,\gamma,\,\zeta,\,\Omega$

reduced effective action

$$W\rightarrow\Gamma\rightarrow\widehat{\Gamma}=\Gamma-\left(b_\alpha-\Omega^a\bar{\omega}_\alpha\frac{\delta}{\delta\phi^a}\right)\left(\chi_a^\alpha(\varphi-\phi)^a-\frac{1}{2}O^{\alpha\beta}b_\beta\right)$$

$$\widehat{\Gamma}=\widehat{\Gamma}[\varphi,\omega,\phi,\widehat{\gamma},\zeta,\Omega],\quad \widehat{\gamma}_a=\gamma_a-\bar{\omega}^\alpha\chi^a_\alpha,\quad \frac{\delta\widehat{\Gamma}}{\delta b}=0$$

Slavnov-Taylor and Ward identities for $\widehat{\Gamma}$

Decoupling of background fields in Γ_{div}

trivial cohomology of $\Omega \frac{\delta}{\delta \phi}$

Batalin and Vilkovisky (1985)

$$\widehat{\Gamma}_{\infty} = \boldsymbol{S}[\varphi] + \Lambda[\varphi, \omega, \widehat{\gamma}, \zeta] + \boldsymbol{Q}_{+}\Upsilon$$

gauge invariant single field counterterm

$$\frac{\delta \boldsymbol{S}}{\delta \varphi^a} R^a{}_{\alpha}(\varphi) = 0$$

ghost term

$$\Lambda \Big|_{\omega=0} = 0$$

BRST exact term (not yet the original one!)

Applies to nonrenormalizable EFT within ***gradient*** *expansion*

Truncate in number of derivatives + locality + fermionic statistics of Ω ⇒ $0 \leq k \leq K$

Structure of Λ

ST+W identities: $$(\boldsymbol{q}_0 + \boldsymbol{q}_1)\,\Lambda = 0$$

$$(\boldsymbol{q}_0)^2 = (\boldsymbol{q}_1)^2 = \boldsymbol{q}_0\boldsymbol{q}_1 + \boldsymbol{q}_1\boldsymbol{q}_0 = 0$$

$$\underbrace{\boldsymbol{q}_0 = \frac{\delta S}{\delta \varphi^a}\frac{\delta}{\delta \hat{\gamma}_a} - \hat{\gamma}_a R^a{}_\alpha(\varphi)\frac{\delta}{\delta \zeta_\alpha},}\qquad \boldsymbol{q}_1 = -\frac{1}{2}C^\gamma{}_{\alpha\beta}\omega^\alpha\omega^\beta\frac{\delta}{\delta\omega^\gamma}$$

***Kozhul-Tate differential** has a trivial cohomology under the assumption of **local completeness** and **irreducibility** of gauge generators for*

$$\Lambda\Big|_{\omega=\hat{\gamma}=\zeta=0} = 0$$

Batalin, Vilkovisky (1985)
Henneaux (1991)
Vandoren, Van Proeyen (1994)

$$\boldsymbol{q}_0 X = 0,\ \ X\Big|_{\omega=\hat{\gamma}=\zeta=0} = 0$$

$$\Rightarrow X = \boldsymbol{q}_0 Y$$

local

$$\Lambda = \sum_{k=1}^{\infty}\omega^{\alpha_1}\ldots\omega^{\alpha_k}\Lambda^{\{k\}}_{[\alpha_1,\ldots,\alpha_k]}$$

$$\Gamma_{L,\infty} = \boldsymbol{S}_L[\varphi] + \boldsymbol{Q}_+\Upsilon_L$$

L-th order subtraction and $Q_+ \to Q$ *transition via field redefinition*

$$\Sigma_L[\Phi,\phi,\gamma,\zeta,\Omega] = \Sigma_{L-1} - \hbar^L \Gamma_{L,\infty} + \mathcal{O}(\hbar^{L+1})$$

gauge fermion renormalization

$$\Psi_L = \Psi_{L-1} - \hbar^L \boldsymbol{\Upsilon}_L$$

field redefinition $\Phi \to \Phi'$:

gauge invariant counterterm

$$\Sigma_L[\Phi,\phi,\gamma,\zeta,\Omega] = \Big[S_{L-1} - \hbar^L \boldsymbol{S}_L + \boldsymbol{Q}\Psi_L \Big]_{\Phi\to\Phi'}$$

$$\varphi^a - \phi^a = -\frac{\delta\Psi_L(\Phi')}{\delta\gamma_a}\,, \quad \omega^\alpha = \frac{\delta\Psi_L(\Phi')}{\delta\zeta_\alpha}$$

Gauge fermion is a generating function of the field redefinition

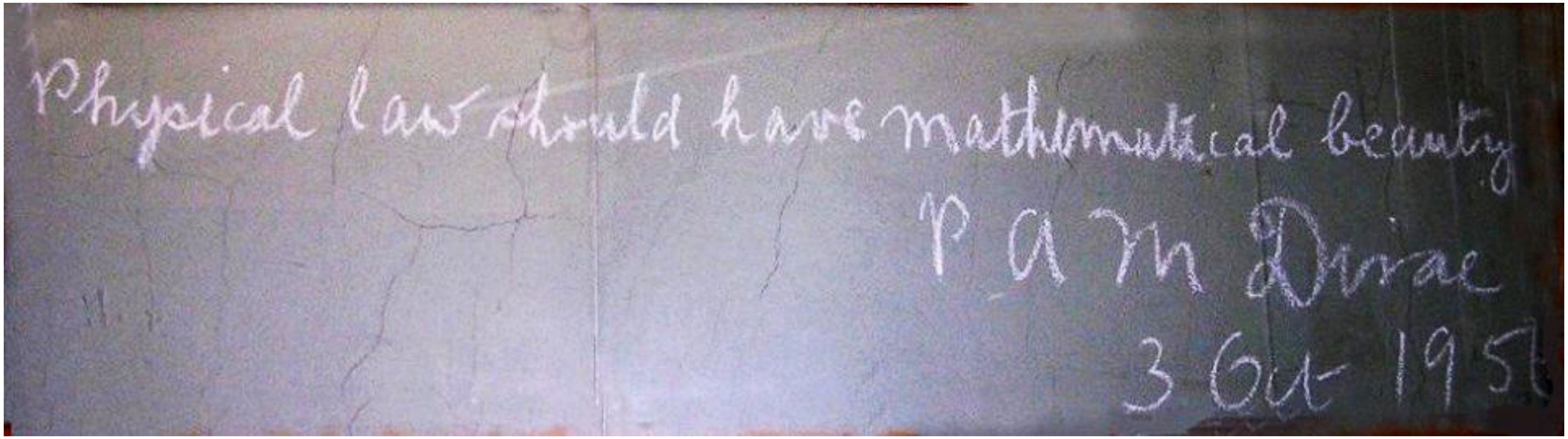
The power and beauty of nilpotent BRST charge

$$\boldsymbol{Q} \to \boldsymbol{Q}_{\text{ext}} = \boldsymbol{s} + \Omega\frac{\delta}{\delta\phi} - J\frac{\delta}{\delta\gamma} + \bar{\xi}\frac{\delta}{\delta\zeta} + \xi\frac{\delta}{\delta y}, \qquad \boldsymbol{Q}_{\text{ext}}^2 = 0$$

$$\Psi \to \boldsymbol{\Psi}_{\text{ext}} \equiv \Psi + y\bar{\omega}$$

$$\Sigma \to \Sigma_{\text{ext}} = \Sigma - J\frac{\delta\Psi}{\delta\gamma} + \bar{\xi}\frac{\delta\Psi}{\delta\zeta} + \xi\bar{\omega} + yb = S + \boldsymbol{Q}_{\text{ext}}\boldsymbol{\Psi}_{\text{ext}}$$

$$e^{-W/\hbar} = \int d\Phi\, e^{-(S+\boldsymbol{Q}_{\text{ext}}\boldsymbol{\Psi}_{\text{ext}})/\hbar}$$



Example: 2D O(N) gauge model

$$S[\varphi] = \frac{1}{2g^2}\int \mathrm{d}^2x \left\{\frac{1}{\varphi^2}\left[\delta_{ij} - \frac{\varphi_i\varphi_j}{\varphi^2}\right]\partial_\mu\varphi^i\partial^\mu\varphi^j\right\}, \quad i = 1, ...N$$

Abelian gauge invariance $\quad \delta_\varepsilon\varphi^i(x) = \varphi^i(x)\,\varepsilon(x)$

One-loop renormalization

$$S_1[\varphi] = \left(\frac{1}{2g^2} + \hbar\frac{N-2}{4\pi(d-2)}\right)\int \mathrm{d}^2x \left\{\frac{1}{\varphi^2}\left[\delta_{ij} - \frac{\varphi_i\varphi_j}{\varphi^2}\right]\partial_\mu\varphi^i\partial^\mu\varphi^j\right\}$$

$$\varphi^i \mapsto \tilde{\varphi}^i_1 = \varphi^i - \frac{\hbar}{4\pi(2-d)}\left[\frac{\phi^2(\varphi^2+\phi^2)}{(\varphi\cdot\phi)^2}\varphi^i - \frac{2\varphi^2}{(\varphi\cdot\phi)}\phi^i\right]$$

essentially nonlinear

Conclusions and Outlook

- Background field method is not only a convenient calculational tool, but is also efficient for general analysis of the structure of renormalization

  cf. Grassi (1996), Anselmi (2014)

- BRST structure (gauge invariance) is preserved by renormalization for non-anomalous theories whose gauge algebra:

  i) has linear generators

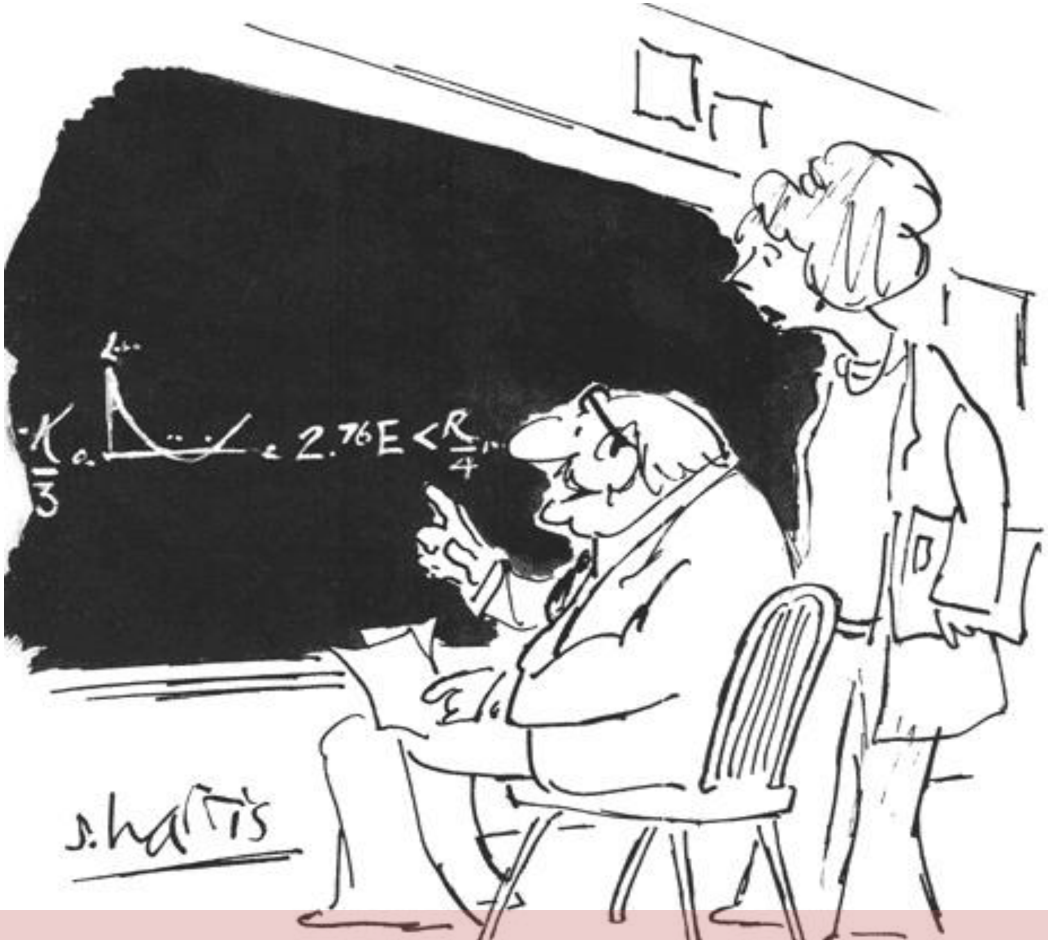
  ii) closes off-shell *can be relaxed (?)*

  iii) is locally complete

  iv) is irreducible *can be relaxed*

- Generalizations: open algebras, supersymmetry, composite operators, anomalies

The power and beauty of the nilpotent BRST operator



"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."

Introduction:

towards local, unitary, perturbatively UV renormalizable QG

Horava-Lifshitz gravity

Problems with renormalization:

BPH renormalization and “regularity” of propagators
covariance of UV counterterms

“Regular” propagators and gauge fixing conditions

Renormalization of gauge theories in background-field approach

BRST structure of renormalization and field reparametrization

Asymptotic freedom of (2+1)-dimensional Horava gravity

Summary and outlook:

the power and beauty of the nilpotent BRST operator

Asymptotic freedom in (2+1)-dimensions

$$S = \frac{1}{2G}\int dt\, d^2x\; N\sqrt{\gamma}\;\left(K_{ij}K^{ij} - \lambda K^2 + \mu R^2\right)$$

$$\Gamma \to \Gamma + \varepsilon \int dt\, d^2x\; \sqrt{\gamma}\;\left[K_{ij}K^{ij} - \lambda K^2 - \mu R^2\right]$$

$$\delta G = -2G^2\varepsilon, \quad \delta\lambda = 0, \quad \delta\mu = -4G\mu\varepsilon$$

Essential coupling constants: $\lambda. \quad \mathcal{G} \equiv \frac{G}{\sqrt{\mu}}$

background split

$$\gamma_{ij} \to \gamma_{ij} + h_{ij}, \qquad N_i = 0 + n_i$$

gauge-fixing term
σ, ξ – free parameters

$$S_{\mathsf{gf}} = \frac{\sigma}{2G}\int dt\, d^2x\, \sqrt{\gamma}\; F_i\, \mathcal{O}^{ij} F_i$$

$$F_i = \partial_t n_i + \frac{1}{2\sigma}\,\mathcal{O}^{-1}_{ij}(\nabla^k h^j_k - \lambda\nabla^j h)$$

$$\mathcal{O}^{ij} = -[\gamma_{ij}\Delta + \xi\nabla_i\nabla_j]^{-1}$$

localization of the kinetic term by auxilairy field π

$$\frac{\sigma}{2G}\int dt\, dt^2x\sqrt{\gamma}\;\partial_t n_i \frac{-1}{\gamma_{ij}\Delta + \xi\nabla_i\nabla_j}\partial_t n_j \mapsto$$

$$\frac{1}{2G}\int dt\, d^2x\sqrt{\gamma}\;\left(-\frac{1}{2\sigma}\pi^i\,\mathcal{O}^{-1}_{ij}\pi^j - i\pi^i\partial_t n_i\right)$$

action of ghosts:

$$S_{\mathsf{gh}} = -\int dt\, d^2x\sqrt{\gamma}\,\bar{c}^i\Big[\partial_t\left(\gamma_{ij}\partial_t c^j\right) - \frac{1}{2\sigma}\Delta^2(\gamma_{ij}c^j) - \frac{1}{2\sigma}\Delta\nabla_k\nabla_i c^k$$
$$+\frac{\lambda}{\sigma}\Delta\nabla_i\nabla_j c^j - \frac{\xi}{2\sigma}\left(\nabla_i\nabla_j\Delta c^j + \nabla_i\nabla_j\nabla_k\nabla^j c^k - 2\lambda\nabla_i\Delta\nabla_j c^j\right)\Big]$$

Diagrammatic technique in terms of $h_{ij},\ n_i,\ \pi_i,\ c^i,\ \bar{c}^i$

Propagators $\propto\ \mathcal{P}_{\mathrm{s}}(\omega,p)=\dfrac{1}{\omega^2+4\mu\frac{1-\lambda}{1-2\lambda}\,p^4}$

$$\sigma=\frac{1-2\lambda}{8\mu(1-\lambda)}$$

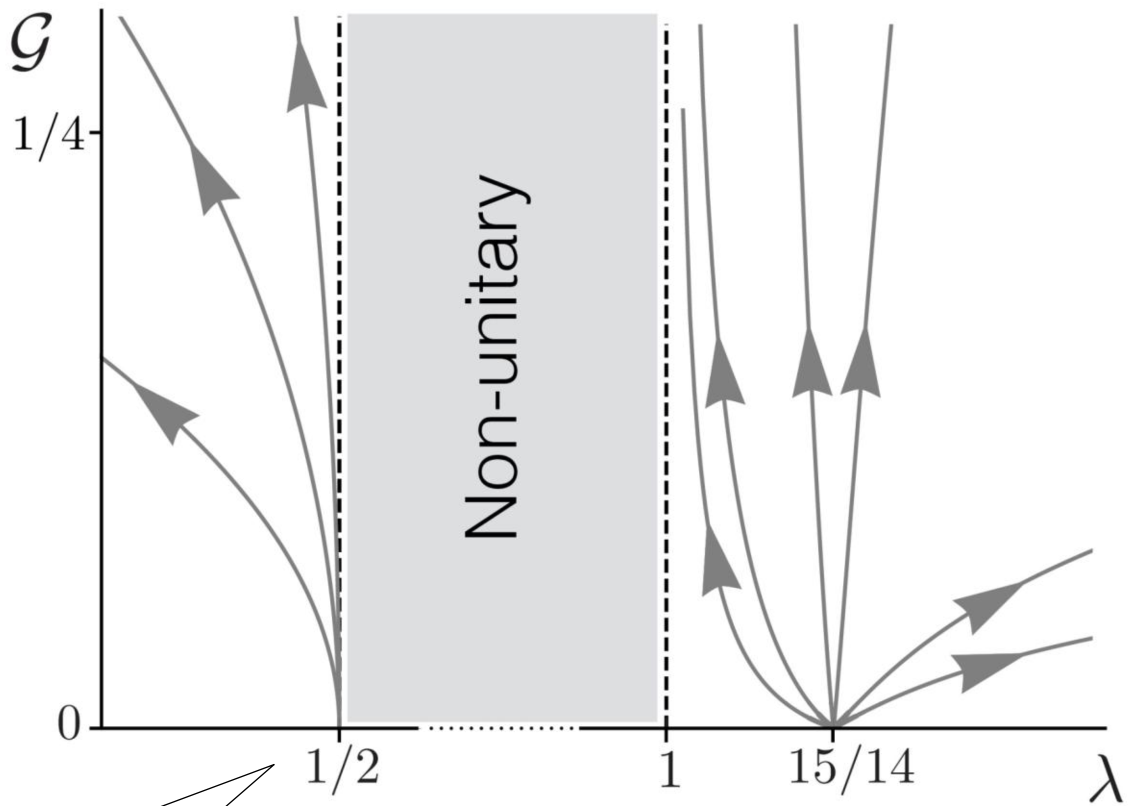
$$\xi=-\frac{1-2\lambda}{2(1-\lambda)}$$

particular gauge parameters

$$\beta_\lambda=\frac{15-14\lambda}{64\pi}\sqrt{\frac{1-2\lambda}{1-\lambda}}\,\mathcal{G}$$

$$\beta_{\mathcal{G}}=-\frac{(16-33\lambda+18\lambda^2)}{64\pi(1-\lambda)^2}\sqrt{\frac{1-\lambda}{1-2\lambda}}\,\mathcal{G}^2$$

Renormalization flows:



strongly coupled f. point

$$\mathcal{G} \to \tilde{\mathcal{G}} = \frac{G}{\sqrt{1-2\lambda}}$$

$$\beta_{\tilde{\mathcal{G}}} = -\frac{(1-2l)^2}{64\pi(1-l)^{3/2}}\,\tilde{\mathcal{G}}^2$$

AF UV fixed point

DEAR, MISAO!

MANY PRODUCTIVE YEARS, GIFTED STUDENTS
AND HAPPINESS TO YOUR FAMILY!

And let your best seminal work be still ahead!