Compact binary systems in scalar-tensor theories

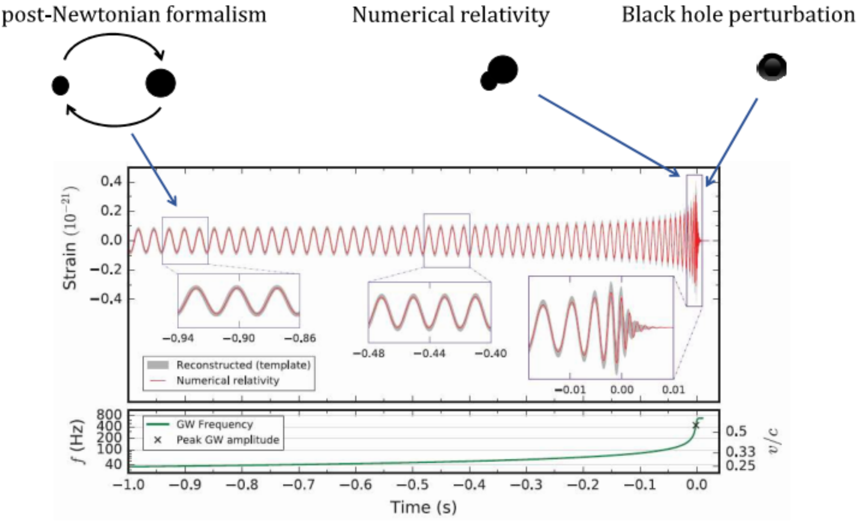
Laura BERNARD (IST, Lisbon)

Gravity and Cosmology 2018

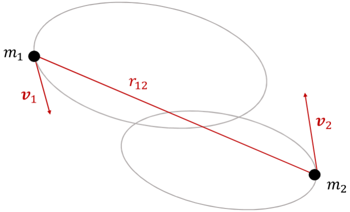
based on arXiv: 1803.10201

grit
gravitation in técnico

centra
multidisciplinary centre for astrophysics



[PRL 116, 241103 (2016)]



Post-Newtonian source

$\longrightarrow$ Slow moving, weakly-stressed compact source

$$\boxed{\epsilon \equiv \frac{v_{12}^2}{c^2} \sim \frac{Gm}{r_{12}c^2} \ll 1}$$

post-Newtonian order : $n\mathrm{PN} = \mathcal{O}\left(\frac{1}{c^{2n}}\right) \equiv \mathcal{O}\left(2n\right)$.

- ▷ First introduced by Jordan, Fierz, Brans and Dicke more than 50 years ago,
- ▷ Only one additional massless scalar field, minimally coupled to gravity.
- ▷ It is the **simplest**, well motivated and most studied alternative theory of gravity,
- ▷ Binary BHs gravitational radiation indistinguishable from GR (Hawking, 1972),
- ▷ But strong deviations from GR are expected for neutron stars (scalarization).

The action

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}g^{\alpha\beta}\partial_\alpha\partial_\beta\phi\right] + S_m\left(\mathfrak{m}, g_{\alpha\beta}\right)$$

- Metric $g_{\mu\nu}$,
- Scalar field ϕ and scalar function $\omega(\phi)$,
- Matter fields $\mathfrak{m}$, minimally coupled to the physical metric,
- No potential or mass for the scalar field.
- No direct coupling between the matter and scalar fields,

Metric (Jordan) frame

- ▷ **Physical metric $g_{\alpha\beta}$** : Scalar field only coupled to the gravitational sector,
- ▷ Frame for physical results and observations.

Conformal (Einstein) frame

$$\tilde{g}_{\mu\nu} = \varphi\, g_{\mu\nu}\,, \qquad \varphi = \frac{\phi}{\phi_0} \qquad \text{with } \phi_0 = \phi(\infty) = \text{cst}$$

- Scalar field only coupled to the matter sector.
- Scalar field and metric decoupling $\Longrightarrow$ BHs are the same as in GR.
- Simpler to do calculations.

In ST theories : **violation of the Strong Equivalence Principle**,

Self-gravitating bodies : $\boldsymbol{M_A(\phi)}$

$$S_{\rm m} = -\sum_A \int \mathrm{d}t\, M_A(\phi)\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

▷ Sensitivities : $s_A = \left.\frac{\mathrm{d}\ln M_A(\phi)}{\mathrm{d}\ln\phi}\right|_0$, and all higher order derivatives,

- Neutron stars : $s_A \sim 0.2$,
- Black holes : $s_A = 1/2$,
- related to the scalar charge $\alpha_A \propto 1 - 2s_A$.

- Equations of motion at 2.5PN [Mirshekari & Will, 2013],
- Tensor gravitational waveform to 2PN [Lang, 2013],
- Scalar waveform to 1.5PN [Lang, 2014] : **starts at -0.5PN**,
- Energy flux to 1PN [Lang, 2014] : **starts at -1PN**,

$$\frac{\mathrm{d}E_{\text{dipole}}}{\mathrm{d}t} = \frac{4m\nu^2}{3rc^3}\left(\frac{\tilde{G}\alpha m}{r}\right)^3 \frac{(s_2 - s_1)^2}{\alpha(4+2\omega_0)}$$

- Equations of motion at 2.5PN [Mirshekari & Will, 2013],
- Tensor gravitational waveform to 2PN [Lang, 2013],
- Scalar waveform to 1.5PN [Lang, 2014] : **starts at -0.5PN**,
- Energy flux to 1PN [Lang, 2014] : **starts at -1PN**,

$$\frac{\mathrm{d}E_{\text{dipole}}}{\mathrm{d}t} = \frac{4m\nu^2}{3rc^3}\left(\frac{\tilde{G}\alpha m}{r}\right)^3 \frac{(s_2 - s_1)^2}{\alpha(4 + 2\omega_0)}$$

What's next

- Flux and gravitational waveform at 2PN : on-going (A. Heffernan, C. Will),

▷ We need the EoM at 3PN .

- **In the near zone :** post-Newtonian expansion

$$\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{h}^{\mu\nu}_m \,, \quad \text{with} \quad \Box \bar{h}^{\mu\nu}_m = 16\pi G \, \bar{\tau}^{\mu\nu}_m ,$$

$$\bar{\psi} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{\psi}_m \,, \quad \text{with} \quad \Box \bar{\psi}_m = -8\pi G \, \bar{\tau}^{(s)}_m$$

- **In the near zone :** post-Newtonian expansion

$$\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{h}^{\mu\nu}_m \,, \quad \text{with} \quad \Box \bar{h}^{\mu\nu}_m = 16\pi G \, \bar{\tau}^{\mu\nu}_m \,,$$

$$\bar{\psi} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{\psi}_m \,, \quad \text{with} \quad \Box \bar{\psi}_m = -8\pi G \, \bar{\tau}^{(s)}_m$$

- **In the wave zone :** multipolar expansion

$$\mathcal{M}(h)^{\alpha\beta} = \sum_{n=1}^{\infty} G^n h^{\alpha\beta}_{(n)} \,, \quad \text{with} \quad \Box h^{\alpha\beta}_{(n)} = \Lambda^{\alpha\beta}_n \left[h_{(1)}, \ldots, h_{(n-1)}; \psi \right] ,$$

$$\mathcal{M}(\psi) = \sum_{n=1}^{\infty} G^n \psi_{(n)} \,, \quad \text{with} \quad \Box \psi_{(n)} = \Lambda^{(s)}_n \left[\psi_{(1)}, \ldots, \psi_{(n-1)}; h \right] ,$$

- **In the near zone :** post-Newtonian expansion

$$\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{h}_m^{\mu\nu}\,, \quad \text{with} \quad \Box \bar{h}_m^{\mu\nu} = 16\pi G\, \bar{\tau}_m^{\mu\nu}\,,$$

$$\bar{\psi} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{\psi}_m\,, \quad \text{with} \quad \Box \bar{\psi}_m = -8\pi G\, \bar{\tau}_m^{(s)}$$

- **In the wave zone :** multipolar expansion

$$\mathcal{M}(h)^{\alpha\beta} = \sum_{n=1}^{\infty} G^n h_{(n)}^{\alpha\beta}\,, \quad \text{with} \quad \Box h_{(n)}^{\alpha\beta} = \Lambda_n^{\alpha\beta}\left[h_{(1)}, \dots, h_{(n-1)}; \psi\right],$$

$$\mathcal{M}(\psi) = \sum_{n=1}^{\infty} G^n \psi_{(n)}\,, \quad \text{with} \quad \Box \psi_{(n)} = \Lambda_n^{(s)}\left[\psi_{(1)}, \dots, \psi_{(n-1)}; h\right],$$

- **Buffer zone** $\Longrightarrow$ matching between the near zone and far zone solutions :

$$\overline{\mathcal{M}(h)} = \mathcal{M}\left(\bar{h}\right) \quad \text{everywhere},$$

$$\overline{\mathcal{M}(\psi)} = \mathcal{M}\left(\bar{\psi}\right) \quad \text{everywhere}.$$

What is the Fokker Lagrangian ?

- Replace the gravitational degrees of freedom by their solution

$$S_{\text{Fokker}}\left[y_A, v_A, \dots\right] = S\left[g_{\text{sol}}\left(y_B, v_B, \dots\right), \phi_{\text{sol}}\left(y_B, v_B, \dots\right); v_A\right]$$

- **Generalized Lagrangian** : dependent on the accelerations.
- Same dynamics as the original action.

What is the Fokker Lagrangian ?

- Replace the gravitational degrees of freedom by their solution

$$S_{\text{Fokker}}\left[y_A, v_A, \dots\right] = S\left[g_{\text{sol}}\left(y_B, v_B, \dots\right), \phi_{\text{sol}}\left(y_B, v_B, \dots\right); v_A\right]$$

- **Generalized Lagrangian** : dependent on the accelerations.
- Same dynamics as the original action.

Why a Fokker Lagrangian ?

- The "$n+2$" method : we need to know the metric at only half the order we would have expected,

$$\mathcal{O}(n+2) \quad \text{instead of} \quad \mathcal{O}(2n)\,.$$

The gravitational part

- Conformal gothic metric $\tilde{\mathfrak{g}}^{\mu\nu} = \sqrt{\tilde{g}}\tilde{g}^{\mu\nu}$,

$$S_{\rm ST} = \frac{c^3\phi_0}{32\pi G}\int \mathrm{d}^4x\Bigg[-\frac{1}{2}\left(\tilde{\mathfrak{g}}_{\mu\sigma}\tilde{\mathfrak{g}}_{\mu\rho} - \frac{1}{2}\tilde{\mathfrak{g}}_{\mu\nu}\tilde{\mathfrak{g}}_{\rho\sigma}\right)\tilde{\mathfrak{g}}^{\lambda\gamma}\partial_\lambda\tilde{\mathfrak{g}}^{\mu\nu}\partial_\gamma\tilde{\mathfrak{g}}^{\rho\sigma}$$
$$+\,\tilde{\mathfrak{g}}_{\mu\nu}\left(\partial_\sigma\tilde{\mathfrak{g}}^{\rho\mu}\partial_\rho\tilde{\mathfrak{g}}^{\sigma\nu} - \partial_\rho\tilde{\mathfrak{g}}^{\rho\mu}\partial_\sigma\tilde{\mathfrak{g}}^{\sigma\nu}\right) - \frac{3+2\omega}{\varphi^2}\tilde{\mathfrak{g}}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\Bigg]$$

▷ gauge-fixing term $-\frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\Gamma}^\mu\tilde{\Gamma}^\nu \longrightarrow$ harmonic coordinates $\partial_\nu h^{\mu\nu} = 0$

The matter part

$$S_{\rm m} = -\sum_A \int \mathrm{d}t\, M_A(\phi)\, c^2\sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

- Solve flat space-time wave equations for the PN potentials.

UV divergences

- At the position of the particles

▷ simple pole $1/\varepsilon$

▷ vanishes through a redefinition of the trajectory of the particles : ok

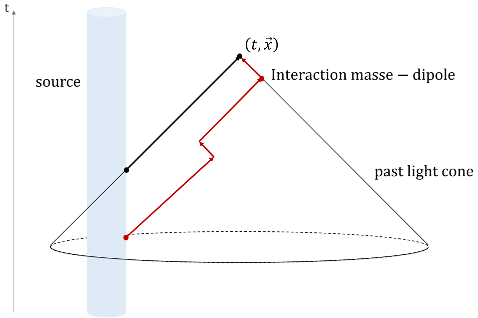
IR divergences

- Divergence of the PN solution at infinity

▷ simple pole $1/\varepsilon$

▷ **does not vanish through a redefinition of the trajectory of the particles !**

▷ New effect in ST theories



- Non-local tail terms in the conservative dynamics at 3PN :

$$L_{\text{tail}} = \frac{2G^2M}{3c^6}(3+2\omega_0)I_i^{(2)}(t)\int_0^{+\infty}\mathrm{d}t\left[\ln\left(\frac{\tau}{\tau_0}\right)-\frac{1}{2\varepsilon}\right]I_i^{(3)}(t-\tau)$$

▷ Exactly compensate the pole $1/\varepsilon$ from the IR divergences.

- New effect in ST theories

3PN equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons. terms}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons. \& local}} + \underbrace{\frac{\mathbf{A}^{\text{tail}}_{3\text{PN}}}{c^6}}_{\text{cons. \& nonlocal}} + \cdots$$

- Confirmation of the previous 2PN result by Mirshekari & Will (2013).

- Renormalisation of the trajectories $\Longleftrightarrow$ the poles disappear : ok
- GR limit : ok
- 2-black-hole limit : ok
- Lorentz invariance : ok

Equations of motion at 3PN in scalar-tensor theories

On-going calculations

- Lorentz-Poincaré symmetry ⟶ **10 conserved quantities**
- to be used in the scalar waveform and the scalar flux at 2PN,

Prospects

▷ Incorporate the tidal effects for neutron stars ⟶ start at 3PN.

- Construct a full IMR waveform,
- Comparison with numerical relativity or self-force results in ST theories.