Constraining Nonlocal Teleparallel cosmology using Noether Symmetries

Kostas Dialektopoulos

University of Naples "Federico II", Naples, Italy

Gravity and Cosmology 2018
Yukawa Institute for Theoretical Physics
Kyoto University

Overview

1. Part I: Introduction
   - Non-local Cosmology
   - Noether Symmetry Approach
2. Part II: Noether Symmetries in GNT cosmology
   - Generalized Nonlocal Teleparallel
   - Nonlocal-Curvature: $R + Rf(\Box^{-1}R)$
   - Nonlocal-Teleparallel: $-T + Tf(\Box^{-1}T)$
3. Conclusions

S. Bahamonde, S. Capozziello, K. F. Dialektopoulos, Eur.Phys.J. C77 (2017) no.11, 722

Nonlocal Theories

The action of the Nonlocal theory[1] is

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa} R \left(1 + f(\Box^{-1} R)\right) + \mathcal{L}_{matter}\right],$$

where $\Box^{-1} F(x) = \int d^4x' F(x') G(x, x')$.

[1] Deser, Woodard: Phys.Rev.Lett. 2007

Nonlocal Theories

The action of the Nonlocal theory[1] is

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{1}{2\kappa}R\left(1+f(\Box^{-1}R)\right)+\mathcal{L}_{matter}\Big],$$

where $\Box^{-1}F(x) = \int d^4x' F(x')G(x,x')$.
In EPJC 77 (2017) 628 they study its teleparallel version, i.e.

$$\mathcal{S} = \int d^4x e\Big[\frac{1}{2\kappa}T\left(f(\Box^{-1}T)-1\right)+\mathcal{L}_{matter}\Big],$$

[1] Deser, Woodard: Phys.Rev.Lett. 2007

Nonlocal Theories

The action of the Nonlocal theory[1] is

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{1}{2\kappa}R\left(1+f(\Box^{-1}R)\right)+\mathcal{L}_{matter}\Big],$$

where $\Box^{-1}F(x)=\int d^4x'F(x')G(x,x')$.
In EPJC 77 (2017) 628 they study its teleparallel version, i.e.

$$\mathcal{S} = \int d^4xe\Big[\frac{1}{2\kappa}T\left(f(\Box^{-1}T)-1\right)+\mathcal{L}_{matter}\Big],$$

and its scalar-tensor representation reads

$$\mathcal{S} = \frac{1}{2\kappa}\int d^4xe\Big[T\left(f(\phi)-1-\theta\right)-\nabla_\mu\theta\nabla^\mu\phi+\mathcal{L}_{matter}\Big].$$

[1] Deser, Woodard: Phys.Rev.Lett. 2007

Nonlocal Teleparallel Cosmology

In flat FRW cosmology,

$$ds^2 = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j \,,$$

it is easy to find the modified FRW eqs:

$$3H^2(1+\theta - f(\phi)) = \frac{1}{2}\dot{\theta}\dot{\phi} + \kappa\rho \,,$$
$$(1+\theta - f(\phi))(3H^2 + 2\dot{H}) = -\frac{1}{2}\dot{\theta}\dot{\phi} + 2H(\dot{\theta} - \dot{f}(\phi)) - \kappa p \,,$$

and the equations for the scalar fields can be written as

$$-6H^2 f'(\phi) + 3H\dot{\theta} + \ddot{\theta} = 0 \,, \quad 3H\dot{\phi} + 6H^2 + \ddot{\phi} = 0 \,.$$

Nonlocal Teleparallel Cosmology

In flat FRW cosmology,

$$ds^2 = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j \,,$$

it is easy to find the modified FRW eqs:

$$3H^2(1+\theta - f(\phi)) = \frac{1}{2}\dot{\theta}\dot{\phi} + \kappa\rho \,,$$
$$(1+\theta - f(\phi))(3H^2 + 2\dot{H}) = -\frac{1}{2}\dot{\theta}\dot{\phi} + 2H(\dot{\theta} - \dot{f}(\phi)) - \kappa p \,,$$

and the equations for the scalar fields can be written as

$$-6H^2 f'(\phi) + 3H\dot{\theta} + \ddot{\theta} = 0 \,, \quad 3H\dot{\phi} + 6H^2 + \ddot{\phi} = 0 \,.$$

EPJC 77 (2017) 628: SNe Ia+BAO+CC+H_0 and they constrain the exponential distortion function.

Generalized Nonlocal Teleparallel Theory

Since

$$R = -T + B \text{ and } \Box^{-1}R = -\Box^{-1}T + \Box^{-1}B\,.$$

we construct the following theory

$$\mathcal{S} = -\frac{1}{2\kappa}\int d^4xeT + \frac{1}{2\kappa}\int d^4xe\Big[\,(c_1 T + c_2 B)\, f\left(\Box^{-1}T, \Box^{-1}B\right)\Big],$$

which we call Generalized Nonlocal Teleparallel Theory.

Generalized Nonlocal Teleparallel Theory

Since

$$R = -T + B \text{ and } \Box^{-1}R = -\Box^{-1}T + \Box^{-1}B\,.$$

we construct the following theory

$$\mathcal{S} = -\frac{1}{2\kappa}\int d^4xeT + \frac{1}{2\kappa}\int d^4xe\Big[(c_1T + c_2B)\,f\left(\Box^{-1}T, \Box^{-1}B\right)\Big],$$

which we call Generalized Nonlocal Teleparallel Theory.
Localized version: introduce four auxiliary fields ϕ, ψ, θ and ζ, to get

$$\begin{aligned}\mathcal{S} = \quad & -\frac{1}{2\kappa}\int d^4x\, eT + \\ & +\frac{1}{2\kappa}\int d^4x\, e\Big[(c_1T + c_2B)f(\phi, \varphi) - \partial_\mu\theta\partial^\mu\phi - \\ & -\theta T - \partial_\mu\zeta\partial^\mu\varphi - \zeta B\Big] + \int d^4x\, e\, L_m\,.\end{aligned}$$

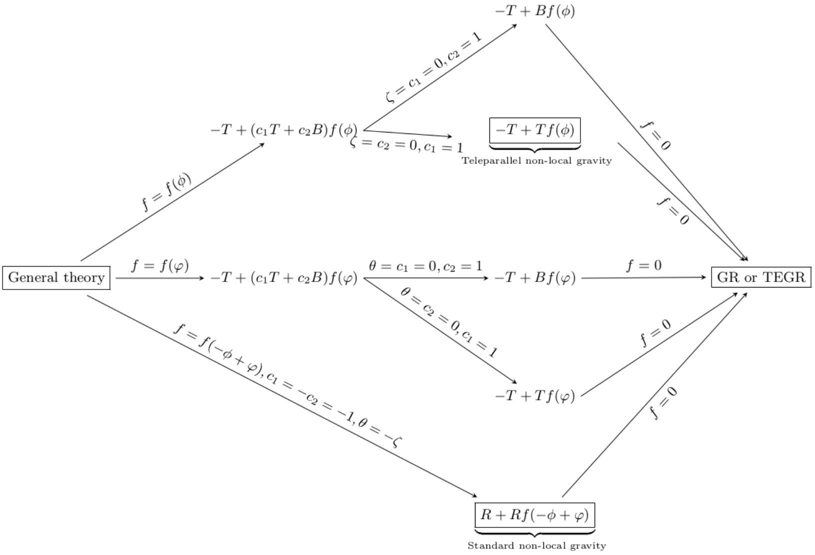

FIG. 1: This diagram shows how one can recover different theories of gravity from the scalar-field representation to the standard representation. Note that $\phi = \Box^{-1} T$ and $\varphi = \Box^{-1} B$ so that $-\phi + \varphi = \Box^{-1} R$.

Noether Symmetry Approach

Let

$$\bar{t} = t + \epsilon\xi(t, x^k)\,,\;\; \bar{x} = x + \epsilon\eta^i(t, x^k)\,,$$

be infinitesimal one-parameter point transformations and

$$X = \xi(t, x^k)\partial_t + \eta^i(t, x^k)\partial_i\,,$$

their generator.

Noether Symmetry Approach

Let

$$\bar{t} = t + \epsilon\xi(t, x^k)\,, \;\; \bar{x} = x + \epsilon\eta^i(t, x^k)\,,$$

be infinitesimal one-parameter point transformations and

$$X = \xi(t, x^k)\partial_t + \eta^i(t, x^k)\partial_i\,,$$

their generator.
Let $L = L(t, x^k, \dot{x}^k)$ be a Lagrangian describing a dynamical system

$$E_i L = 0 \Rightarrow \frac{d}{dt}\frac{\partial}{\partial \dot{x}^i}L - \frac{\partial}{\partial x^i}L = 0\,,$$

Noether Symmetry Approach

Noether's Theorem

Iff there exists an $f = f(t, x^k, \dot{x}^k)$ such that

$$X^{[1]}L + L\frac{d\xi}{dt} = \frac{df}{dt},$$

then the Euler-Lagrange equations remain invariant under X.

Noether Symmetry Approach

Noether's Theorem

Iff there exists an $f = f(t, x^k, \dot{x}^k)$ such that

$$X^{[1]}L + L\frac{d\xi}{dt} = \frac{df}{dt}\,,$$

then the Euler-Lagrange equations remain invariant under X.

Integral of motion

If X is a Noether symmetry of the dynamical system, there corresponds a function

$$\phi(t, x^k, \dot{x}^k) = f - \xi\left(\dot{x}^i\frac{\partial L}{\partial \dot{x}^i} - L\right) - \eta^i\partial_i L\,,$$

which is a conserved quantity.

Noether Symmetries in GNT cosmology

From the action of the theory we want to deduce a point like Lagrangian. We want to study cosmology so we consider:

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)\,,$$

and the traces of the torsion tensor and the boundary term take the form

$$T = -6H^2\,, B = -18H^2 - 6\dot{H}\,.$$

Noether Symmetries in GNT cosmology

From the action of the theory we want to deduce a point like Lagrangian. We want to study cosmology so we consider:

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)\,,$$

and the traces of the torsion tensor and the boundary term take the form

$$T = -6H^2\,, B = -18H^2 - 6\dot{H}\,.$$

The action now becomes

$$\mathcal{S} \sim \int dta^3\Big[-6\frac{\dot{a}^2}{a^2}\left(c_1 f(\phi,\varphi) - \theta - 1\right) - \\ -6\left(2\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a}\right)\left(c_2 f(\phi,\varphi) - \zeta\right) - \dot{\theta}\dot{\phi} - \dot{\zeta}\dot{\varphi}\Big].$$

Noether Symmetries in GNT cosmology

and the point like Lagrangian is given by

$$\mathcal{L} = 6a\dot{a}^2(\theta + 1 - c_1 f(\phi,\varphi)) + 6a^2\dot{a}(c_2\dot{f}(\phi,\varphi) - \dot{\zeta}) - a^3\dot{\theta}\dot{\phi} - a^3\dot{\zeta}\dot{\varphi}\,.$$

The generator of infinitesimal transformations is givan by

$$X = \xi(t,x^\mu)\partial_t + \eta^i(t,x^\mu)\partial_i\,,$$

where $x^\mu = (a,\theta,\phi,\varphi,\zeta)$ and the vector η^i is

$$\eta^i(t,x^\mu) = (\eta^a,\eta^\theta,\eta^\phi,\eta^\varphi,\eta^\zeta)\,.$$

Noether Symmetries in GNT cosmology

The point-like canonical Lagrangian is:

$$\begin{aligned}\mathcal{L} =& 6c_2 a^2 \dot{a}\dot{\phi} f_\phi(\phi,\varphi) + 6c_2 a^2 \dot{a}\dot{\varphi} f_\varphi(\phi,\varphi) - 6c_1 a\dot{a}^2 f(\phi,\varphi) - \\ &- 6a^2\dot{a}\dot{\zeta} + 6a\theta\dot{a}^2 + 6a\dot{a}^2 - a^3\dot{\zeta}\dot{\varphi} - a^3\dot{\theta}\dot{\phi}\,,\end{aligned}$$

Noether Symmetries in GNT cosmology

The point-like canonical Lagrangian is:

$$\begin{aligned}\mathcal{L} =& 6c_2 a^2 \dot{a}\dot{\phi} f_\phi(\phi,\varphi) + 6c_2 a^2 \dot{a}\dot{\varphi} f_\varphi(\phi,\varphi) - 6c_1 a\dot{a}^2 f(\phi,\varphi) - \\ &- 6a^2\dot{a}\dot{\zeta} + 6a\theta\dot{a}^2 + 6a\dot{a}^2 - a^3\dot{\zeta}\dot{\varphi} - a^3\dot{\theta}\dot{\phi}\,,\end{aligned}$$

The Noether condition

$$X^{[1]}L + L\frac{d\xi}{dt} = \frac{df}{dt}\,,$$

gives a system of 43 differential equations, 19 of which are independent.
This yields 7 different forms for the distortion function, i.e. 7 classes of theories that are invariant under point transformations.

Nonlocal-Curvature: $R + Rf(\Box^{-1}R)$

We set at the general action:

$$f(\phi, \varphi) = f(-\phi + \varphi) = f(\psi)\,,\ c_1 = -c_2 = -1\,,\ \theta = -\zeta\,,$$

and the Lagrangian reads:

$$\mathcal{L} = 6a\dot{a}^2\left(f(\psi) + \theta + 1\right) + 6a^2\dot{a}\left(f'(\psi)\dot{\psi} + \dot{\theta}\right) + a^3\dot{\theta}\dot{\psi}\,.$$

Nonlocal-Curvature: $R + Rf(\Box^{-1}R)$

We set at the general action:

$$f(\phi, \varphi) = f(-\phi + \varphi) = f(\psi)\,,\ c_1 = -c_2 = -1\,,\ \theta = -\zeta\,,$$

and the Lagrangian reads:

$$\mathcal{L} = 6a\dot{a}^2\left(f(\psi) + \theta + 1\right) + 6a^2\dot{a}\left(f'(\psi)\dot{\psi} + \dot{\theta}\right) + a^3\dot{\theta}\dot{\psi}\,.$$

From the Noether condition we get a system of 18 differential equations which yield:

$$X = (C_5 + C_4 t)\partial_t + \frac{1}{3}a(C_4 - C_1)\partial_a + (C_3 + C_1\theta)\partial_\theta - 2C_2\partial_\psi\,,$$

$$f(\psi) = \begin{cases} -1 + \frac{C_3}{C_1} + C_6 e^{-\frac{C_1}{C_2}\psi}\,, & C_1 \neq 0\,, \\ C_6 + \frac{C_3}{2C_2}\psi\,, & C_1 = 0\,. \end{cases}$$

Cosmological solutions for the exponential coupling

The Lagrangian becomes

$$\mathcal{L} = 6a\,(1+\theta)\,\dot{a}^2 + 3C_6 a e^{-\frac{C_1}{2C_2}\psi}\left(2\dot{a}^2 - \frac{C_1}{C_2}a\dot{a}\dot{\psi}\right) + 6a^2\dot{a}\dot{\theta} + a^3\dot{\theta}\dot{\psi}\,,$$

and from the Euler Lagrange equations we get:

de-Sitter solutions $a(t) = e^{H_0 t}$

$$\psi(t) = -4H_0 t + \psi_1\,,$$
$$\theta(t) = 3C_6 e^{\frac{\psi_1}{2} - 2H_0 t} - \frac{\theta_1}{3H_0}e^{-3H_0 t} - 1\,,$$
$$f(\psi) = C_6 e^{\psi/2}\,.$$

Power-law solutions $a(t) = a_0 t^p$

$$\psi(t) = \frac{6p(1-2p)}{3p-1}\ln(t)\,,$$
$$\theta(t) = \frac{C_6(3p-1)}{p-1}t^{-2p} - 1\,,$$
$$f(\psi) = C_6 e^{\frac{(1-3p)\psi}{3(1-2p)}}\,.$$

Nonlocal-Teleparallel: $-T + Tf(\Box^{-1}T)$

This action is derived by setting in the general one:

$$f(\phi,\varphi) = f(\phi)\,,\;\; c_1 = 1\,,\;\; c_2 = 0\,,\;\; \zeta = 0\,,$$

and the Lagrangian becomes: $\mathcal{L} = 6a(-f(\phi) + \theta + 1\dot{a}^2 - a^3\dot{\theta}\dot{\phi})$.

Nonlocal-Teleparallel: $-T + Tf(\Box^{-1}T)$

This action is derived by setting in the general one:

$$f(\phi,\varphi) = f(\phi)\,,\ c_1 = 1\,,\ c_2 = 0\,,\ \zeta = 0\,,$$

and the Lagrangian becomes: $\mathcal{L} = 6a(-f(\phi) + \theta + 1\dot{a}^2 - a^3\dot{\theta}\dot{\phi})$.
The Noether condition yields 16 differential equations and they give:

$$X = (C_4 + C_5 t)\partial_t - \frac{1}{3}(C_2 - C_4)a\partial_a + (C_3 + C_2\theta)\partial_\theta + C_1\partial_\phi\,,$$

$$f(\phi) = \begin{cases} C_7 e^{\frac{C_2\phi}{C_1}} - \frac{C_3}{C_2} + 1\,, & C_2 \neq 0\,, \\ C_7 + \frac{C_3}{C_1}\phi\,, & C_2 = 0\,. \end{cases}$$

Cosmological Solutions for the exponential coupling

The Lagrangian becomes

$$\mathcal{L} = -6a\dot{a}^2\left(C_7 e^{\frac{C_3\phi}{C_1}} - \theta - 1\right) - a^3\dot{\theta}\dot{\phi}\,,$$

and from the Euler Lagrange equations we get:

de-Sitter solutions $a(t) = e^{H_0 t}$

$$\phi(t) = -2H_0 t\,,$$

$$\theta(t) = e^{-3H_0 t}\left(-C_7(3H_0 t + 1) - \frac{\theta_1}{3H_0}\right) - 1\,,$$

$$f(\phi) = C_7 e^{-3H_0 t}\,.$$

Power-law solutions $a(t) = a_0 t^p$

$$\phi(t) = \frac{6p^2\ln(t - 3pt)}{1-3p}\,,$$

$$\theta(t) = C_7(1-3p)^{3-3p}t^{2-3p} + \frac{\theta_0 t^{1-3p}}{1-3p} - 1\,,$$

$$f(\phi) = C_7 e^{\frac{(9p^2-9p+2)\phi}{6p^2}}\,.$$

Conclusions

- We introduced a new theory of gravity, which we call Generalized Nonlocal Teleparallel theory, and from which we can derive many interesting and already known theories by fixing the coupling constants.

Conclusions

- We introduced a new theory of gravity, which we call Generalized Nonlocal Teleparallel theory, and from which we can derive many interesting and already known theories by fixing the coupling constants.
- By using Noether's theorem we constrained the functional form of the action and found that in most cases the distortion function is either exponential or linear.

Conclusions

- We introduced a new theory of gravity, which we call Generalized Nonlocal Teleparallel theory, and from which we can derive many interesting and already known theories by fixing the coupling constants.
- By using Noether's theorem we constrained the functional form of the action and found that in most cases the distortion function is either exponential or linear.
- For these selected models we found cosmological solutions, such as de-Sitter and power-law.


Thank you!