Model Independent Constraints from Eikonal Scattering

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Kyoto, Mar. 2, 2018

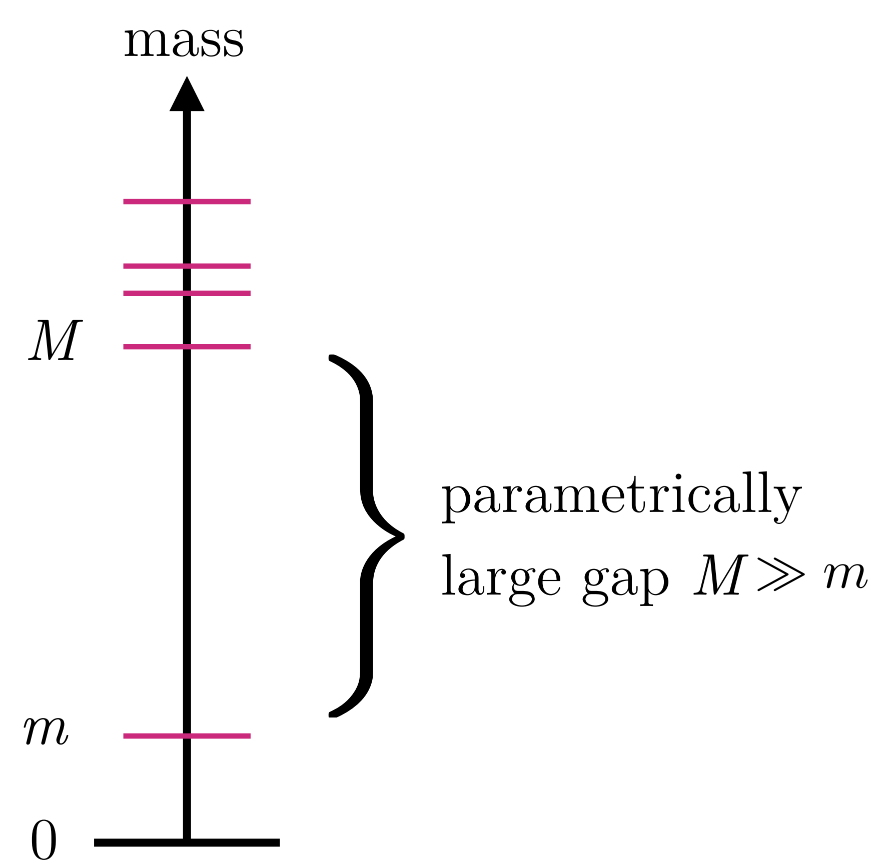
arxiv: 1708.05716 w/ Austin Joyce and Rachel Rosen

arxiv: arXiv:1712.10020 w/ James Bonifacio, Austin Joyce and Rachel Rosen

arxiv: 1803:xxxxx w/ James Bonifacio

Isolated massive spinning particles?

Is it possible to have a theory with a spectrum like this:



Spin 0, 1/2: Yes (pseudo Goldstones)

Spin 1, 3/2: Yes (spontaneously broken weakly coupled gauge theory/SUGRA)

Spin $\geq$ 2: ?

Common lore says No: a massive higher spin always comes with more states at parametrically the same mass

Isolated massive spinning particles?

Examples:

<u>Kaluza Klein theory:</u>

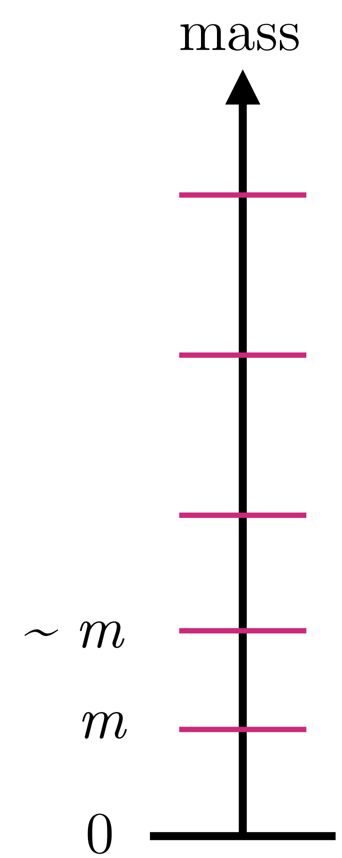
towers of spin ≤ 2 $\quad m^2 \sim \lambda_{\text{laplacian}}$

<u>Confining gauge theory</u>

towers of all spins $\quad m^2 \sim \Lambda^2_{\text{QCD}}$

<u>String theory</u>

towers of all spins $\quad m^2 \sim \dfrac{1}{\alpha'}$



no large gap $\gg m$

Isolated massive spinning particles?

Can there be "elementary" particles with spin ≥ 2 ?

Are there Hadrons with Compton wavelength $\gg$ intrinsic size ?

Could the graviton have a small Hubble-scale mass?

IR modification scale

$$V(r) \sim \frac{1}{r} e^{-mr}, \quad m \sim H$$

Isolated massive spinning particles?

If such isolated massive particles are possible, there must exist an *effective field theory* (EFT) for them with a cutoff parametrically larger than the mass:

$$\Lambda \gg m$$

If such an EFT doesn't exist: problem solved

If it does exist: must figure out if it can be UV completed

approach: look for such EFTs and find obstructions to UV completion.

Best possible EFT

What is the highest possible strong coupling scale in an EFT of a single massive particle?

e.g. Spin-2: Einstein-Hilbert + generic potential $\Lambda_5 \sim \left(M_P m^4\right)^{1/5}$

dRGT theory $\Lambda_3 \sim \left(M_P m^2\right)^{1/3}$

Equivalently: what is the softest possible UV behavior of the tree amplitude?

Einstein-Hilbert + generic potential $\mathcal{A}_4 \sim E^{10}$

dRGT theory $\mathcal{A}_4 \sim E^6$

Best possible EFT

KH, James Bonifacio (to appear)

Generic EFT:

$$\begin{aligned}\mathcal{L} \sim (\partial h)^2 &+ h^2 \\ &+h^3 + \partial^2 h^3 + \partial^4 h^3 + \cdots \\ &+h^4 + \partial^2 h^4 + \partial^4 h^4 + \cdots \\ &\vdots\end{aligned}$$

Field redefinitions $\rightarrow$ put fields on shell: transverse, traceless, $\Box \rightarrow -m^2$

Classify all on-shell cubic and quartic vertices

cubic vertices

Polarization tensors:

$$\epsilon_{\mu_1 \ldots \mu_s} \longrightarrow z_{\mu_1} z_{\mu_2} \ldots z_{\mu_s} \quad , \qquad z^2 = 0$$

No on-shell non-trivial functions of momenta:

$$p_1^\mu + p_2^\mu + p_3^\mu = 0 \qquad \Rightarrow \qquad p_1 \cdot p_2 = \frac{1}{2}\left(m_1^2 + m_2^2 - m_3^2\right) \ , \ \text{etc.}$$

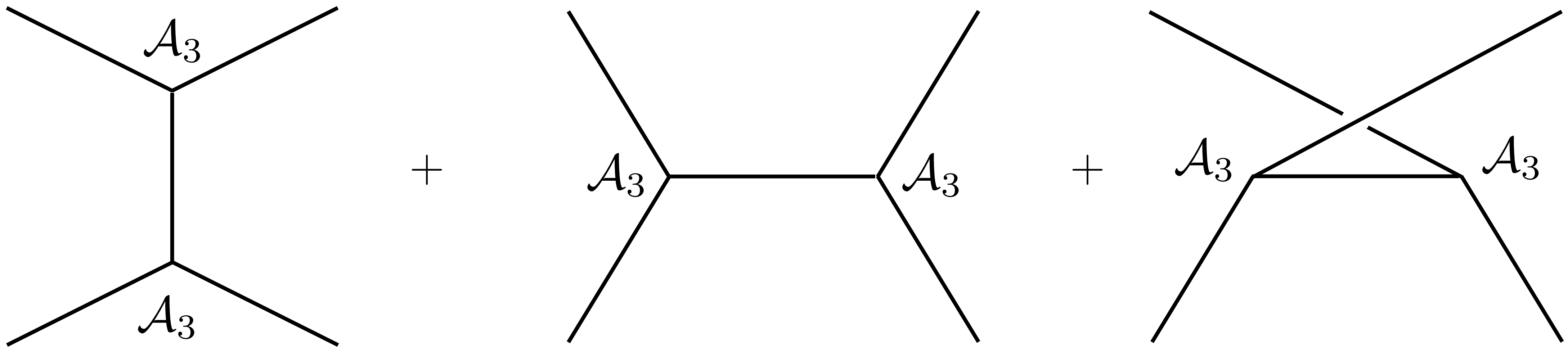
$$\mathcal{A}_3 \sim \ z_{12}^{n_{12}} z_{13}^{n_{13}} z_{23}^{n_{23}} zp_{12}^{m_{12}} zp_{23}^{m_{23}} zp_{31}^{m_{31}}$$

$$\begin{aligned} n_{12} + n_{13} + m_{12} &= s_1, \\ n_{12} + n_{23} + m_{23} &= s_2, \\ n_{13} + n_{23} + m_{31} &= s_3. \end{aligned}$$

Finite number of solutions. $\rightarrow$ On-shell cubic amplitudes nailed down by Lorentz invariance.

Best possible scaling

Build the exchange diagrams:



Finite number of cubic vertices $\rightarrow$ finite number of exchange diagrams $\rightarrow$ bounded growth with energy

$$\mathcal{A}_{\text{exchange}} \sim E^{\#}$$

Best possible scaling

KH, James Bonifacio (to appear)

Classify all analytic quartic amplitudes (contact terms):

2 independent invariants made of momenta (2 Mandelstams)

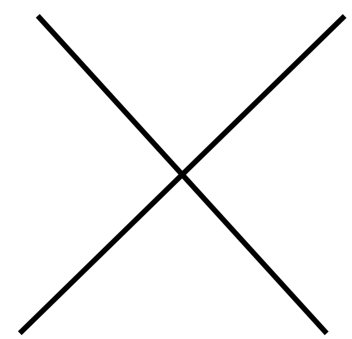
$$p_{12}^{k_{12}} p_{13}^{k_{13}} z_{12}^{n_{12}} z_{13}^{n_{13}} z_{14}^{n_{14}} z_{23}^{n_{23}} z_{24}^{n_{24}} z_{34}^{n_{34}} zp_{13}^{m_{13}} zp_{14}^{m_{14}} zp_{21}^{m_{21}} zp_{24}^{m_{24}} zp_{31}^{m_{31}} zp_{32}^{m_{32}} zp_{42}^{m_{42}} zp_{43}^{m_{43}}$$

unconstrained

$$\begin{aligned}
n_{12} + n_{13} + n_{14} + m_{13} + m_{14} &= s_1, \\
n_{12} + n_{23} + n_{24} + m_{21} + m_{24} &= s_2, \\
n_{13} + n_{23} + n_{34} + m_{31} + m_{32} &= s_3, \\
n_{14} + n_{24} + n_{34} + m_{42} + m_{43} &= s_4.
\end{aligned}$$

This is the contact diagram:

$$\mathcal{A}_{\text{contact}} \sim$$

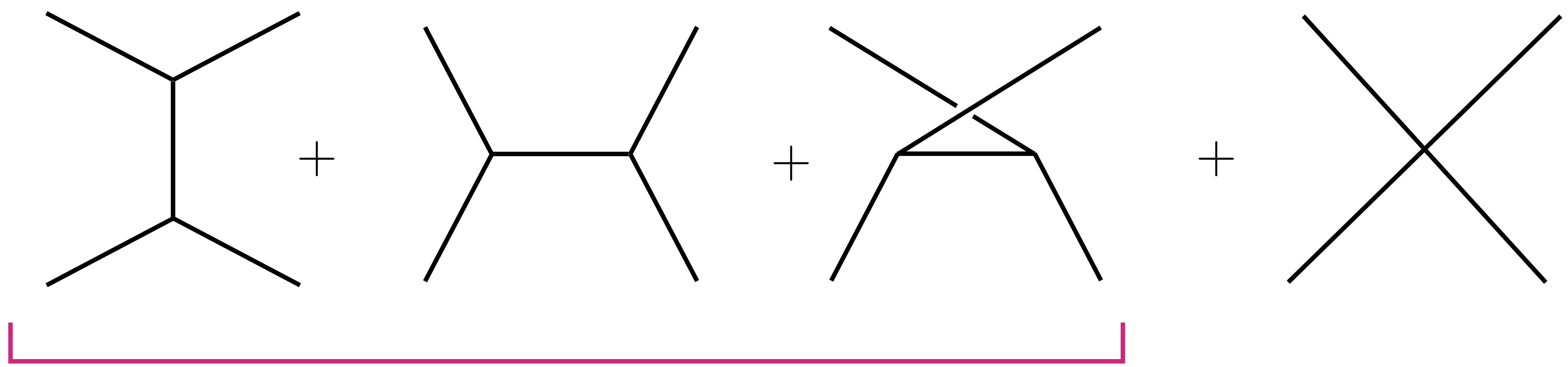


Best possible scaling

KH, James Bonifacio (to appear)

Try to cancel off highest energy scaling of exchange diagrams, working down:

$$\mathcal{A}_4 = \mathcal{A}_{\text{exchange}} + \mathcal{A}_{\text{contact}}$$



$\sim E^{\#}$

Best possible scalings

KH, James Bonifacio (to appear)

Best scaling for spin-1: E^4 $\Lambda_2 \sim (M_P m)^{1/2}$

allow additional scalar $\rightarrow$ can achieve E^0 $\rightarrow$ Higgs mechanism

Best scaling for spin-2: E^6 $\Lambda_3 \sim \left(M_P m^2\right)^{1/3}$

allow additional scalar+vector $\rightarrow$ no simple gravitational Higgs mechanism

Christensen, Stefanus (2014)
Nima Arkani-Hamed, Huang, Huang (2017)

Conjecture for higher spins:

$$\mathcal{A}_4 \sim \begin{cases} E^{3s} & s \text{ even}, \\ E^{3s+1} & s \text{ odd}. \end{cases}$$

$$\Lambda_{\text{max}} = \begin{cases} \Lambda_{\frac{3s}{2}} & s \text{ even}, \\ \Lambda_{\frac{3s+1}{2}} & s \text{ odd}. \end{cases}$$

$$\Lambda_n \equiv (M_p m^{n-1})^{1/n}$$

Best EFTs for spin-2

Spin-2 Result: Best possible scaling is E^6

Only theories that achieve this are dRGT theory and pseudo-linear

DRGT theory: de Rham, Gabadadze, Tolley (2011)

$$\frac{M_P^{D-2}}{2}\int d^Dx\,|e|R[e] - m^2\sum_n a_n \int \epsilon_{A_1\cdots A_D} e^{A_1}\wedge\cdots\wedge e^{A_n}\wedge 1^{A_{n+1}}\wedge\cdots\wedge 1^{A_n}$$

2 parameters

Pseudo-linear theory: KH: 1305.7227

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

$$+\lambda_3\frac{m^2}{M_p}\ \eta^{\mu_1\nu_1\cdots\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}$$

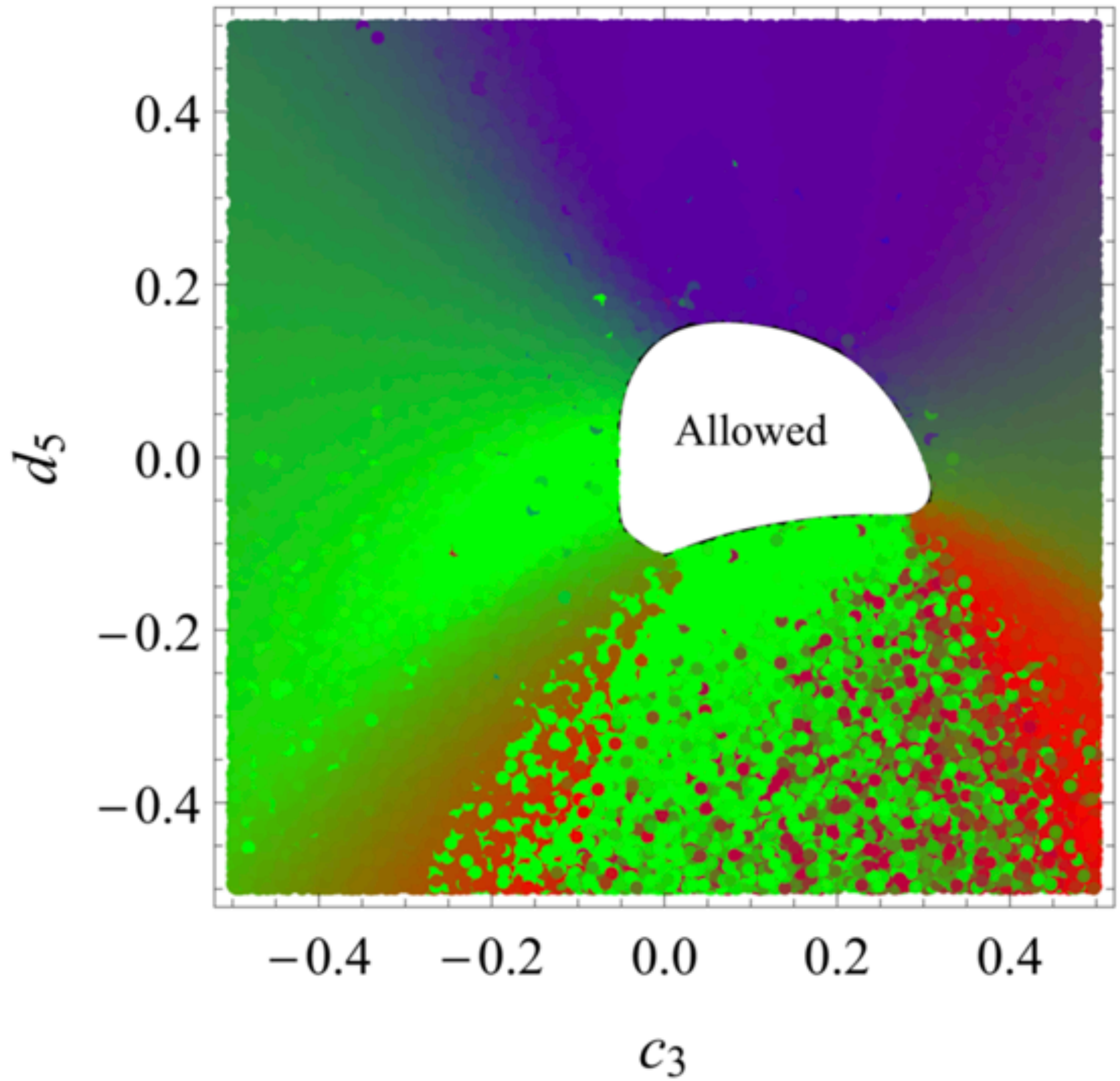
$$+\lambda_4\frac{m^2}{M_p^2}\ \eta^{\mu_1\nu_1\cdots\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

$$+\frac{1}{M_p}\ \eta^{\mu_1\nu_1\cdots\mu_4\nu_4}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\,h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

Also 2 parameters

Constraints from forward dispersion relations: dRGT theory allowed island

Cheung, Remmen (2016)



Constraints from forward dispersion relations: Pseudo-linear not allowed

James Bonifacio, KH, Rachel Rosen (1607.06084)

$$\mathcal{L}_{\rm int} = \frac{1}{M_p}\lambda_1\mathcal{L}_{2,3} + \frac{m^2}{M_p}\lambda_3\mathcal{L}_{0,3} + \frac{m^2}{M_p^2}\lambda_4\mathcal{L}_{0,4}$$

$\mathcal{L}_{2,3}$: $12\,\delta^{[\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_3}\delta^{\mu_4]}_{\nu_4}\,(\partial_{\mu_1}\partial^{\nu_1}h_{\mu_2}^{\ \nu_2})\,h_{\mu_3}^{\ \nu_3}\,h_{\mu_4}^{\ \nu_4}$

$\mathcal{L}_{0,3}$: $\frac{1}{6}\left([h]^3 - 3[h][h^2] + 2[h^3]\right)$

$\mathcal{L}_{0,4}$: $\frac{1}{24}\left([h]^4 - 6[h]^2[h^2] + 3[h^2]^2 + 8[h][h^3] - 6[h^4]\right)$

Forward amplitude: $\mathcal{A}_{\rm forward} \sim \dfrac{E^4}{M_P^2 m^2}$

$$f(TTTT)_- = \frac{\lambda_1^2}{m^2 M_p^2}$$

$$f(VVVV)_+ = -\frac{15\lambda_1^2 + 13\lambda_1\lambda_3 + 5\lambda_3^2}{12\,m^2 M_p^2}$$

$$f(SSSS) = -\frac{5\lambda_1^2 + 6\lambda_1\lambda_3 + \lambda_3^2 + 2\lambda_4}{9\,m^2 M_p^2}$$

$$\vdots$$

⇒ No allowed region

Constraints from eikonal scattering

Another traditional constraint on EFTs:
Superluminality of small fluctuations on non-trivial Lorentz-violating backgrounds (e.g. Velo-Zwanziger problem)

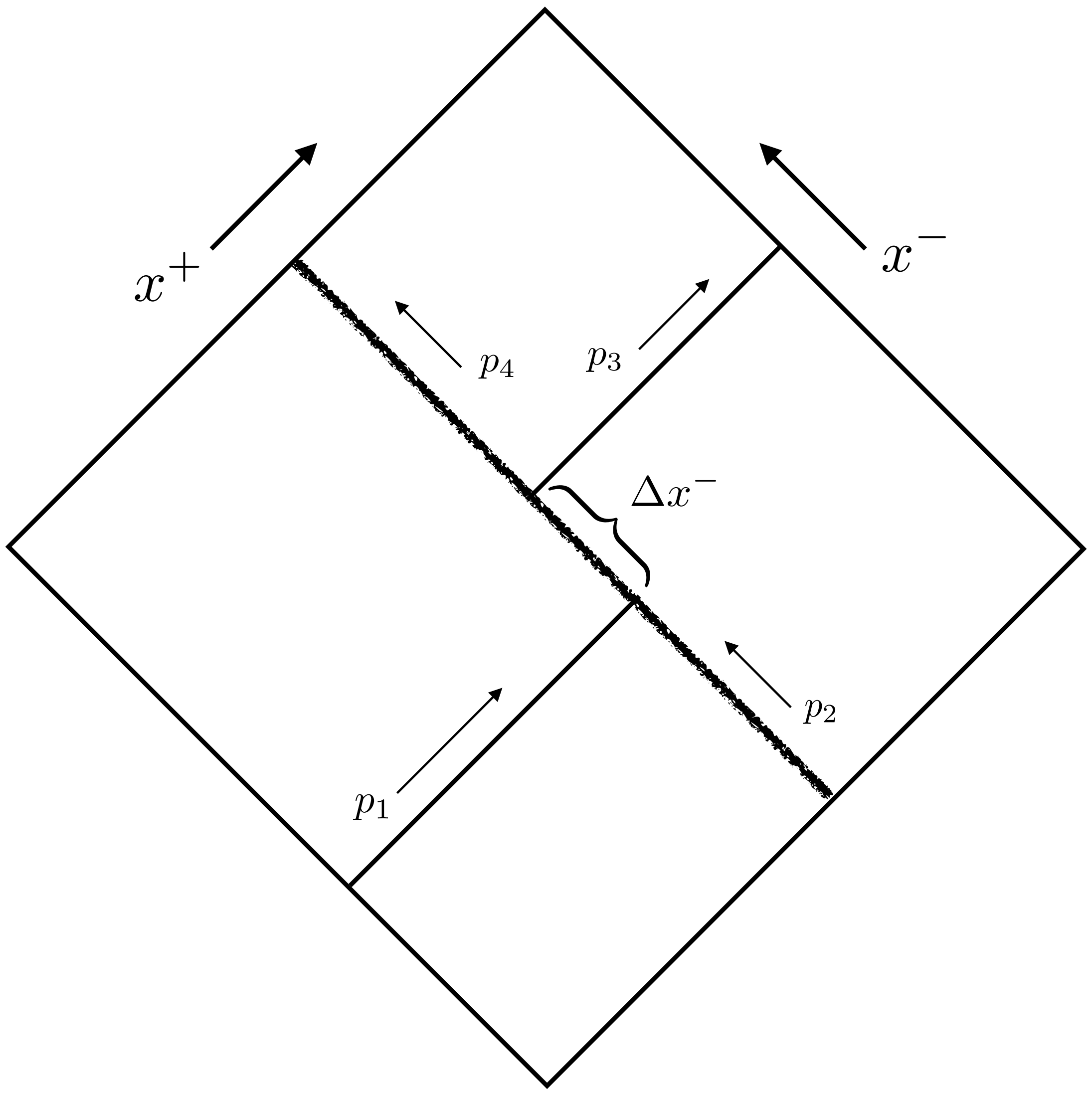
Less problematic: superluminality in the S-matrix

Camanho, Edelstein, Maldacena, Zhiboedov (2016)

Eikonal scattering:

high-energy, fixed impact parameter:

$$s/t \to \infty$$



Eikonal kinematics (massive)

KH, Austin Joyce, Rachel A. Rosen (1708.05716)

Light-cone coordinates:

large momenta → p^+ ; small momentum transfer → q^i

$$p_1^\mu = \left(\frac{1}{2p^+}\left(\frac{\vec{q}^{\,2}}{4} + m_A^2\right), p^+, \frac{q^i}{2}\right), \qquad p_3^\mu = \left(\frac{1}{2p^+}\left(\frac{\vec{q}^{\,2}}{4} + m_A^2\right), p^+, -\frac{q^i}{2}\right),$$

$$p_2^\mu = \left(p^-, \frac{1}{2p^-}\left(\frac{\vec{q}^{\,2}}{4} + m_B^2\right), -\frac{q^i}{2}\right), \qquad p_4^\mu = \left(p^-, \frac{1}{2p^-}\left(\frac{\vec{q}^{\,2}}{4} + m_B^2\right), \frac{q^i}{2}\right).$$

$$\epsilon_T^\mu(p_1) = \left(\frac{\vec{q}\cdot\vec{e}_1}{2p^+}, 0, e_1^i\right), \qquad \epsilon_L^\mu(p_1) = \left(\frac{1}{2m_A p^+}\left(\frac{\vec{q}^{\,2}}{4} - m_A^2\right), \frac{p^+}{m_A}, \frac{q^i}{2m_A}\right),$$

$$\epsilon_T^\mu(p_2) = \left(0, -\frac{\vec{q}\cdot\vec{e}_2}{2p^-}, e_2^i\right), \qquad \epsilon_L^\mu(p_2) = \left(\frac{p^-}{m_B}, \frac{1}{2m_B p^-}\left(\frac{\vec{q}^{\,2}}{4} - m_B^2\right), -\frac{q^i}{2m_B}\right),$$

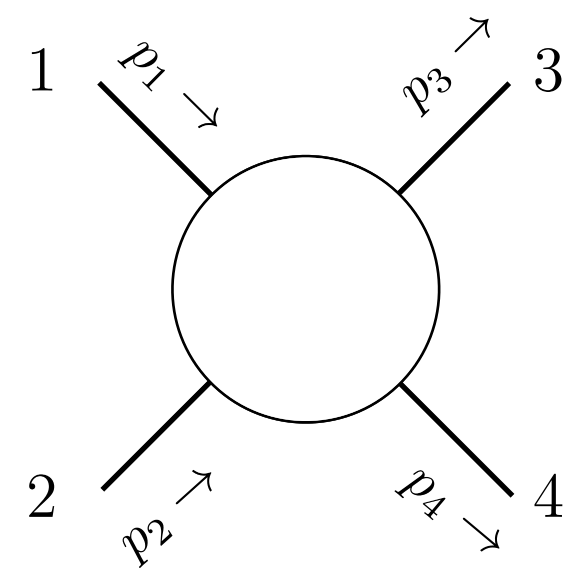
$$\epsilon_T^\mu(p_3) = \left(-\frac{\vec{q}\cdot\vec{e}_3}{2p^+}, 0, e_3^i\right), \qquad \epsilon_L^\mu(p_3) = \left(\frac{1}{2m_A p^+}\left(\frac{\vec{q}^{\,2}}{4} - m_A^2\right), \frac{p^+}{m_A}, -\frac{q^i}{2m_A}\right),$$

$$\epsilon_T^\mu(p_4) = \left(0, \frac{\vec{q}\cdot\vec{e}_4}{2p^-}, e_4^i\right), \qquad \epsilon_L^\mu(p_4) = \left(\frac{p^-}{m_B}, \frac{1}{2m_B p^-}\left(\frac{\vec{q}^{\,2}}{4} - m_B^2\right), \frac{q^i}{2m_B}\right).$$

$$\epsilon_T^{\mu\nu}(p_a) = \epsilon_T^\mu(p_a)\epsilon_T^\nu(p_a),$$

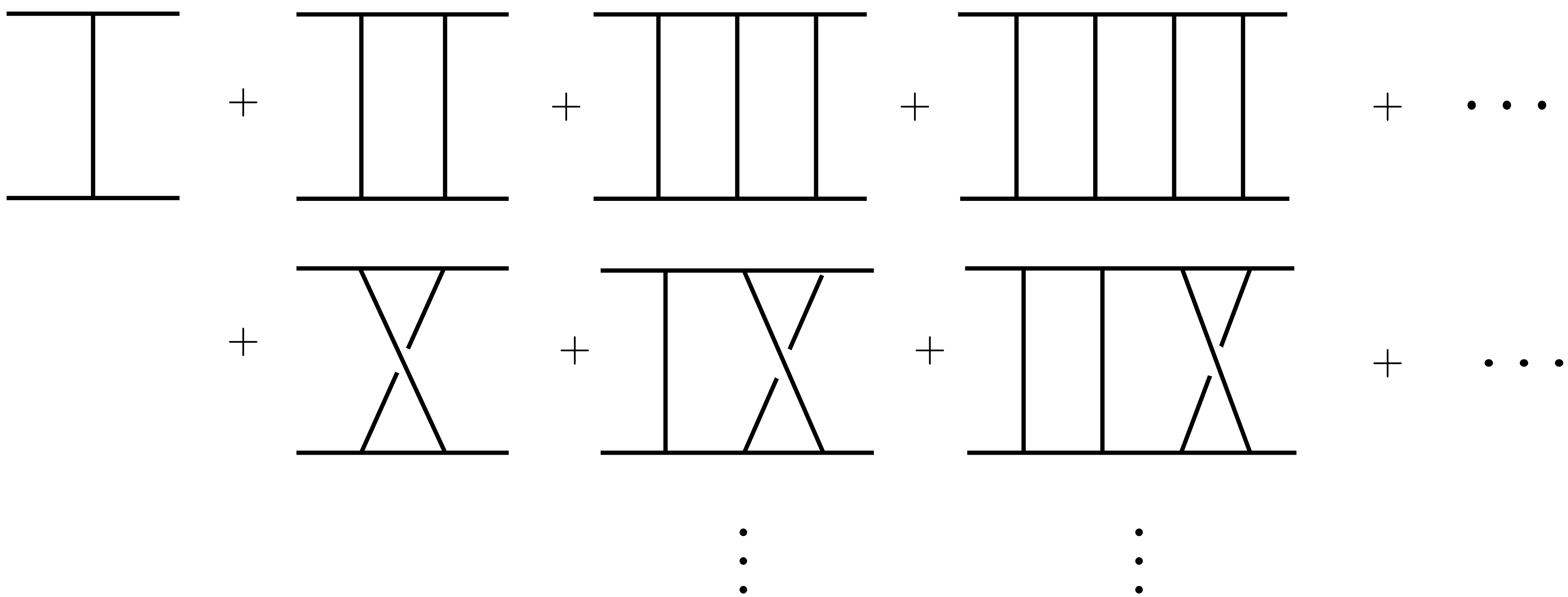
$$\epsilon_V^{\mu\nu}(p_a) = \frac{i}{\sqrt{2}}\left(\epsilon_T^\mu(p_a)\epsilon_L^\nu(p_a) + \epsilon_L^\mu(p_a)\epsilon_T^\nu(p_a)\right),$$

$$\epsilon_S^{\mu\nu}(p_a) = \sqrt{\frac{D-1}{D-2}}\left[\epsilon_L^\mu(p_a)\epsilon_L^\nu(p_a) - \frac{1}{D-1}\left(\eta^{\mu\nu} - \frac{1}{p_a^2}p_a^\mu p_a^\nu\right)\right].$$

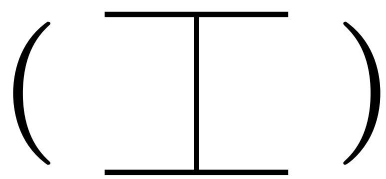


Eikonal limit

$\mathcal{A}_{\text{eikonal}} =$



$= \; e^{(\;\;\;)}$



$$i\mathcal{A}_{\text{eikonal}} = 4p^-p^+ \int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{q}} \left(e^{i\delta(\mathbf{b})} - 1\right) \quad , \qquad \delta(\mathbf{b}) = \frac{1}{4p^-p^+} \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\, e^{-i\mathbf{b}\cdot\mathbf{q}} \mathcal{A}_0(\mathbf{q})$$

Time delay: $\quad \Delta x^- = \dfrac{1}{p^-}\delta$

Eikonal

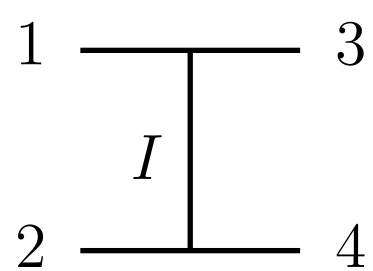
Camanho, Edelstein, Maldacena, Zhiboedov (2016)

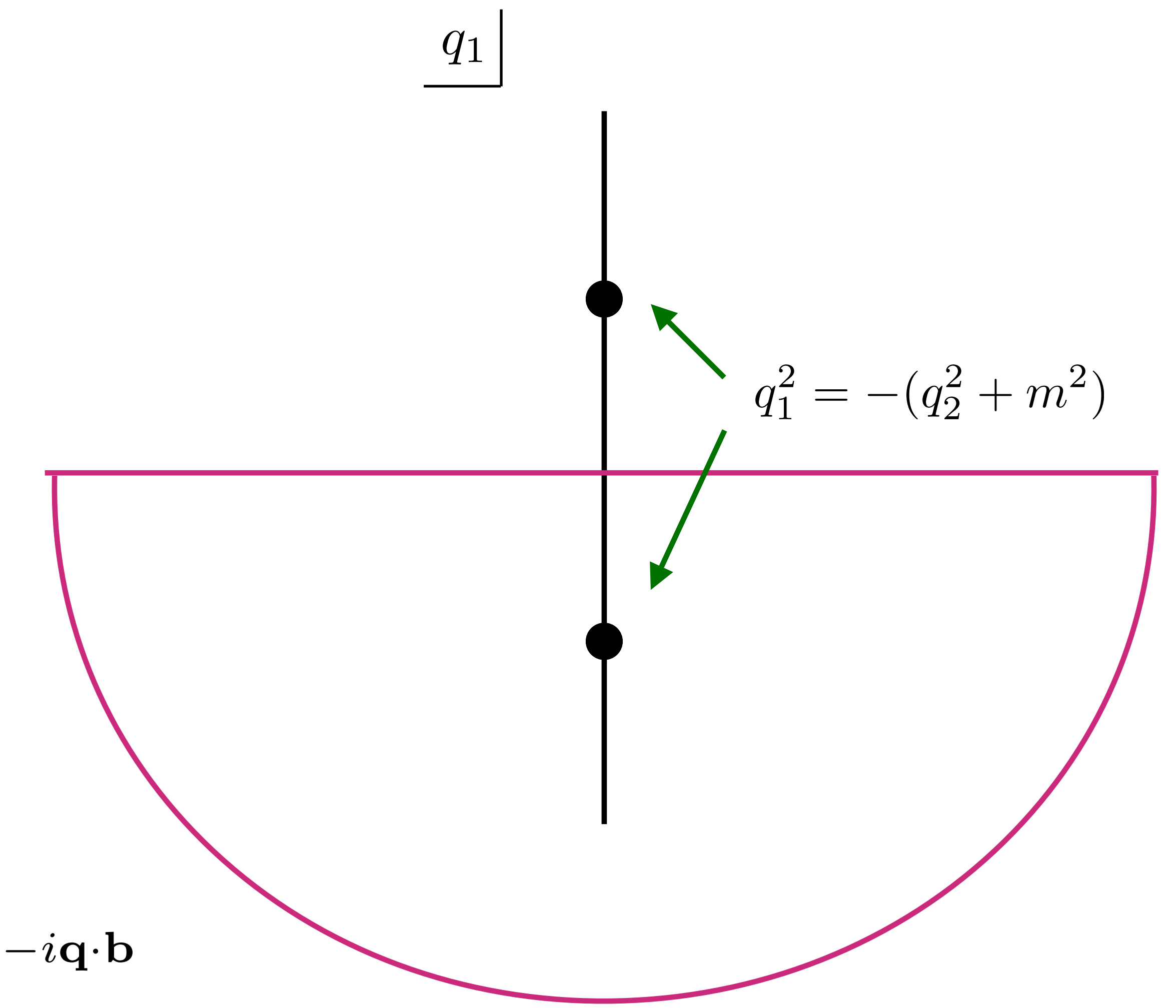
Eikonal phase depends only on *on-shell* three point amplitudes:

$$\delta(\mathbf{b}) = \frac{1}{4p^- p^+} \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\, e^{-i\mathbf{b}\cdot\mathbf{q}} \mathcal{A}_0(\mathbf{q})$$

$$\mathcal{A}_0(\mathbf{q}) \sim \frac{1}{\mathbf{q}^2 + m^2}$$

$$\text{res } \mathcal{A}_0 \sim \sum_I \mathcal{A}_3^{13I} \mathcal{A}_3^{I24}$$





$$\delta(s,b) = \frac{\sum_I \mathcal{A}_3^{13I}(i\partial_{\mathbf{b}})\mathcal{A}_3^{I24}(i\partial_{\mathbf{b}})}{2s} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{e^{-i\mathbf{q}\cdot\mathbf{b}}}{\mathbf{q}^2 + m^2}$$

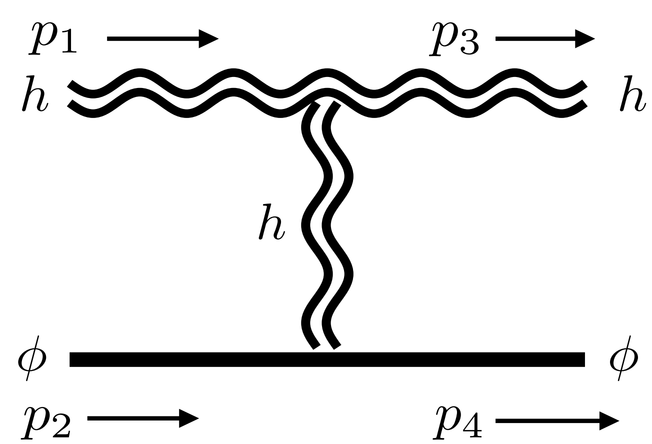
$$= \frac{\sum_I \mathcal{A}_3^{13I}(i\partial_{\mathbf{b}})\mathcal{A}_3^{I24}(i\partial_{\mathbf{b}})}{2s} \left[\frac{1}{2\pi} K_0(mb)\right]$$

Cubic massive spin-2 vertices

$\mathcal{A}_1$	$z_1 \cdot z_2 \;\; z_2 \cdot z_3 \;\; z_3 \cdot z_1$	$h_{\mu\nu}^3$
$\mathcal{A}_2$	$(p_1 \cdot z_3 \;\; z_1 \cdot z_2 + p_3 \cdot z_2 \;\; z_1 \cdot z_3 + p_2 \cdot z_1 \;\; z_2 \cdot z_3)^2$	$\sqrt{-g}R\big\|_{(3)}$
$\mathcal{A}_3$	$(p_1 \cdot z_3)^2\,(z_1 \cdot z_2)^2 + (p_3 \cdot z_2)^2\,(z_1 \cdot z_3)^2 + (p_2 \cdot z_1)^2\,(z_2 \cdot z_3)^2$	$\delta^{[\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_3}\delta^{\mu_4]}_{\nu_4}\partial_{\mu_1}\partial^{\nu_1}h_{\mu_2}^{\;\;\nu_2}h_{\mu_3}^{\;\;\nu_3}h_{\mu_4}^{\;\;\nu_4}$
$\mathcal{A}_4$	$p_1 \cdot z_3 \; p_2 \cdot z_1 \; p_3 \cdot z_2 \;\; (p_1 \cdot z_3 \; z_1 \cdot z_2 + p_3 \cdot z_2 \; z_1 \cdot z_3 + p_2 \cdot z_1 \; z_2 \cdot z_3)$	$\sqrt{-g}\left(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2\right)\Big\|_{(3)}$
$\mathcal{A}_5$	$(p_1 \cdot z_3)^2\,(p_2 \cdot z_1)^2\,(p_3 \cdot z_2)^2$	$\sqrt{-g}\;R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}\Big\|_{(3)}$

D=4: no $\mathcal{A}_4$, 2 additional parity violating amplitudes

Massive spin-2 eikonal



$$
\begin{array}{c} \\ \frac{1}{s}\delta^{\lambda,\lambda'} = \end{array}
\begin{array}{c} \\ T \\ T \\ V \\ V \\ S \end{array}
\begin{array}{ccccc}
T & T & V & V & S \\
\left(\begin{array}{ccccc}
\frac{48(b^2 m^2+6)K_1(bm)\alpha_5+K_0(bm)(b^2(\alpha_2-\alpha_3+18\alpha_5)m^3+144 b\alpha_5 m)}{2b^3 m^2 Mp^2 \pi} & 0 & \frac{K_1(bm)(b^2m^2(\alpha_3-24\alpha_5)-96\alpha_5)-48bmK_0(bm)\alpha_5}{4b^2m^2Mp^2\pi} & 0 & \frac{K_2(bm)(\alpha_3-18\alpha_5)}{2\sqrt{3}Mp^2\pi} \\
0 & \frac{K_0(bm)(b^3m^3(\alpha_2-\alpha_3+6\alpha_5)-144bm\alpha_5)-48(b^2m^2+6)K_1(bm)\alpha_5}{2b^2m^3Mp^2\pi} & 0 & \frac{bmK_1(bm)\alpha_3+48K_2(bm)\alpha_5}{4bmMp^2\pi} & 0 \\
\frac{K_1(bm)(b^2m^2(\alpha_3-24\alpha_5)-96\alpha_5)-48bmK_0(bm)\alpha_5}{4b^2m^2Mp^2\pi} & 0 & \frac{K_0(bm)(\alpha_1-2\alpha_2+24\alpha_5)-\frac{K_1(bm)(\alpha_3-24\alpha_5)}{bm}}{4Mp^2\pi} & 0 & \frac{K_1(bm)(2\alpha_1-5\alpha_2+\alpha_3+24\alpha_5)}{4\sqrt{3}Mp^2\pi} \\
0 & \frac{bmK_1(bm)\alpha_3+48K_2(bm)\alpha_5}{4bmMp^2\pi} & 0 & \frac{K_0(bm)(\alpha_1-2\alpha_2+\alpha_3)+\frac{K_1(bm)(\alpha_3-24\alpha_5)}{bm}}{4Mp^2\pi} & 0 \\
\frac{K_2(bm)(\alpha_3-18\alpha_5)}{2\sqrt{3}Mp^2\pi} & 0 & \frac{K_1(bm)(2\alpha_1-6\alpha_2+\alpha_3-24\alpha_5)}{4\sqrt{3}Mp^2\pi} & 0 & \frac{K_0(bm)(2\alpha_1-5\alpha_2-2(\alpha_3+6\alpha_5))}{4Mp^2\pi}
\end{array}\right)
\end{array}
$$

$\alpha_1 \quad \leftrightarrow \quad h^3_{\mu\nu}$

$\alpha_2 \quad \leftrightarrow \quad$ Einstein-Hilbert

$\alpha_3 \quad \leftrightarrow \quad$ Pseudo-linear

$\alpha_4 \quad \leftrightarrow \quad$ Gauss-Bonnet

$\alpha_5 \quad \leftrightarrow \quad$ Riemann^3

Massive spin-2 eikonal

diagonalize in powers of $1/b$

$$\frac{1}{s}\delta^{\lambda,\lambda'} \to \begin{pmatrix} \frac{144\,\alpha_5}{b^4\, m^4\, \mathrm{Mp}^2\, \pi} & 0 & 0 & 0 & 0 \\ 0 & -\frac{144\,\alpha_5}{b^4\, m^4\, \mathrm{Mp}^2\, \pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \alpha_5 = 0$$

$$\frac{1}{s}\delta^{\lambda,\lambda'} \to \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\alpha_3}{\sqrt{3}\, b^2\, m^2\, \mathrm{Mp}^2\, \pi} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\alpha_3}{4\, b^2\, m^2\, \mathrm{Mp}^2\, \pi} & 0 & 0 \\ 0 & 0 & 0 & \frac{\alpha_3}{4\, b^2\, m^2\, \mathrm{Mp}^2\, \pi} & 0 \\ \frac{\alpha_3}{\sqrt{3}\, b^2\, m^2\, \mathrm{Mp}^2\, \pi} & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[\text{eigenvalues}]{} \left\{-\frac{\alpha_3}{\sqrt{3}\,\pi\, b^2\, m^2\, \mathrm{Mp}^2}, \frac{\alpha_3}{\sqrt{3}\,\pi\, b^2\, m^2\, \mathrm{Mp}^2}, -\frac{\alpha_3}{4\,\pi\, b^2\, m^2\, \mathrm{Mp}^2}, \frac{\alpha_3}{4\,\pi\, b^2\, m^2\, \mathrm{Mp}^2}, 0\right\} \qquad \alpha_3 = 0$$

$$\frac{1}{s}\delta^{\lambda,\lambda'} \to \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\alpha_1 - 3\,\alpha_2}{2\sqrt{3}\; b\, m\, \mathrm{Mp}^2\, \pi} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_1 - 3\,\alpha_2}{2\sqrt{3}\; b\, m\, \mathrm{Mp}^2\, \pi} & 0 & 0 \end{pmatrix} \xrightarrow[\text{eigenvalues}]{} \left\{-\frac{\alpha_1 - 3\,\alpha_2}{2\sqrt{3}\;\pi\, b\, m\, \mathrm{Mp}^2}, \frac{\alpha_1 - 3\,\alpha_2}{2\sqrt{3}\;\pi\, b\, m\, \mathrm{Mp}^2}, 0, 0, 0\right\} \qquad \alpha_1 = 3\alpha_2$$

$$\frac{1}{s}\delta^{\lambda,\lambda'} \to \begin{pmatrix} \frac{K_0(b\,m)\,\alpha_2}{2\,\mathrm{Mp}^2\,\pi} & 0 & 0 & 0 & 0 \\ 0 & \frac{K_0(b\,m)\,\alpha_2}{2\,\mathrm{Mp}^2\,\pi} & 0 & 0 & 0 \\ 0 & 0 & \frac{K_0(b\,m)\,\alpha_2}{4\,\mathrm{Mp}^2\,\pi} & 0 & 0 \\ 0 & 0 & 0 & \frac{K_0(b\,m)\,\alpha_2}{4\,\mathrm{Mp}^2\,\pi} & 0 \\ 0 & 0 & 0 & 0 & \frac{K_0(b\,m)\,\alpha_2}{4\,\mathrm{Mp}^2\,\pi} \end{pmatrix}$$

non-negative

Massive spin-2 eikonal constraints

Allowed cubic vertex: $\mathcal{L}_3 \propto \frac{1}{2M_{\rm Pl}} R^{(3)}_{\rm EH} + \frac{m^2}{2M_{\rm Pl}} h^3_{\mu\nu}$

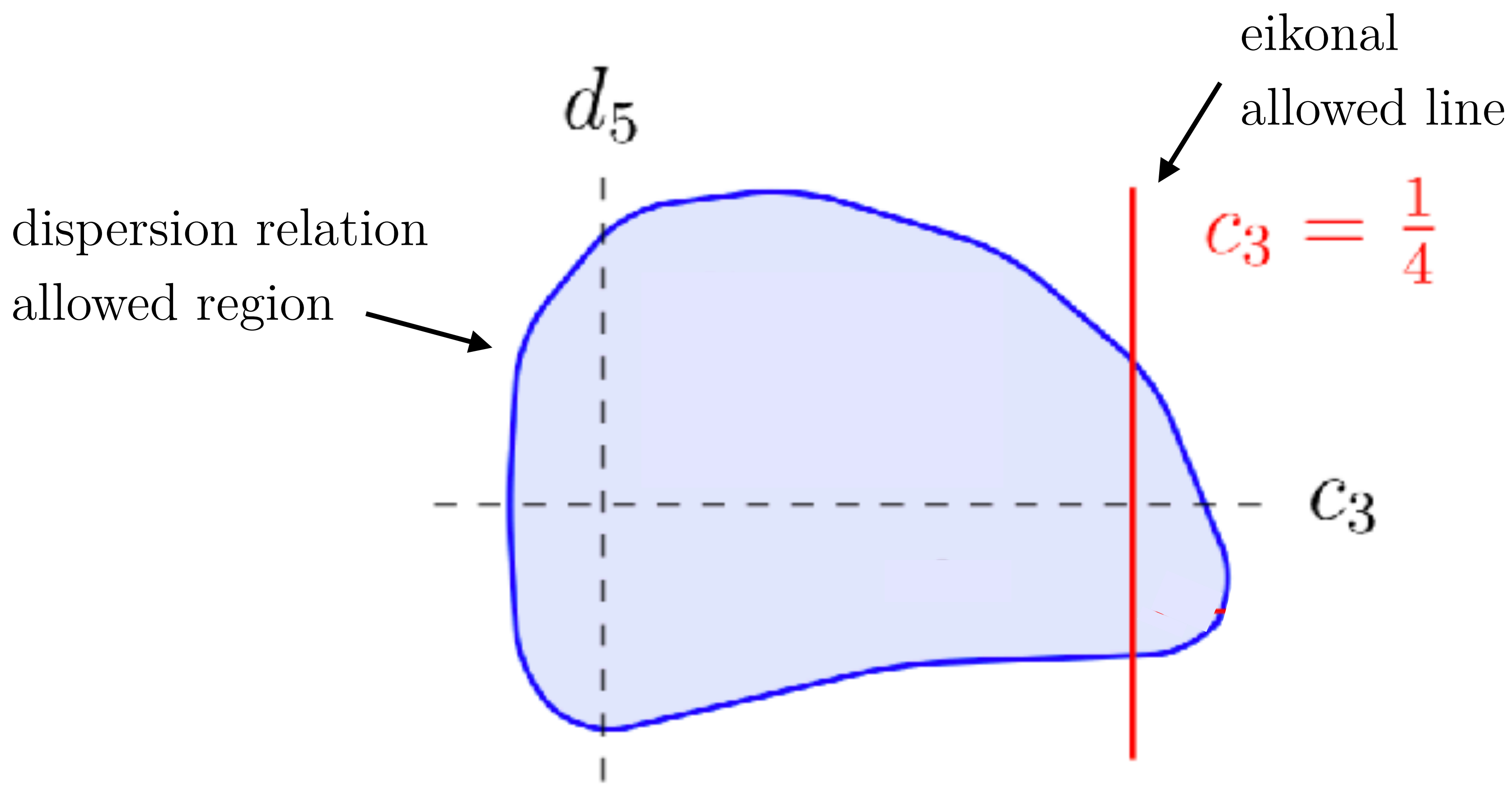
Vertex not of this form $\rightarrow$ new physics at m

KK reduction of Einstein-Hilbert

Conjecture: massive time delay avoided in KK theory by using this cubic vertex, not by cancellations among the KK tower

Massive spin-2 eikonal constraints

Constraints on dRGT theory:



Massless higher spins

KH, Austin Joyce, Rachel Rosen (1712.10021)

Cubic vertices:

Spin-1

$$\mathcal{A}_{\rm YM} \sim (p_1\cdot z_3)(z_1\cdot z_2)+(p_3\cdot z_2)(z_1\cdot z_3)+(p_2\cdot z_1)(z_2\cdot z_3)$$

$$\mathcal{A}_{F^3} \sim (p_1\cdot z_3)(p_2\cdot z_1)(p_3\cdot z_2)$$

Spin-2

Einstein-Hilbert $\sim (\mathcal{A}_{\rm YM})^2$

Gauss-Bonnet $\sim$ ~~$(\mathcal{A}_{\rm YM})(\mathcal{A}_{F^3})$~~ vanishes in D=4

$(\text{Riemann})^3 \sim (\mathcal{A}_{F^3})^2$

Spin-s

$(\mathcal{A}_{\rm YM})^s$

~~$(\mathcal{A}_{\rm YM})^{s-1}(\mathcal{A}_{F^3})$~~

~~$(\mathcal{A}_{\rm YM})^{s-2}(\mathcal{A}_{F^3})^2$~~

~~$\vdots$~~

~~$(\mathcal{A}_{\rm YM})(\mathcal{A}_{F^3})^{s-1}$~~

vanish in D=4

$(\text{linear curvature})^3 \sim (\mathcal{A}_{F^3})^s$

Massless higher spins

KH, Austin Joyce, Rachel Rosen (1712.10021)

Spin-s vertices:	$(\mathcal{A}_{\rm YM})^s$	$(\mathcal{A}_{F^3})^s$
gauge symmetry:	deforms	does not deform (linear)
Consistency/locality at quartic order (4 particle test) Benincasa, Cachazo (2007)	✗	✓
Eikonal constraints	✓	✗

$$\delta(b) = \pm\alpha^2 \frac{s^{s-1}}{2^{3s+2}\pi} \frac{(8s-2)!!}{b^{4s}}.$$

Conclusions

• Eikonal scattering and dispersion relations can provide useful model independent constraints on massive theories.

• An isolated massive spin-2 is not completely ruled out.

• Going beyond leading interactions: dispersion relations beyond the forward limit, subleading corrections to the Eikonal approximation may provide more information.

• May be useful as part of a bootstrap to solve large N QCD.