"Massive Spin 2's + Time Dependent Black Holes"

linear massive spin-2

$$\mathcal{L} = h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\rho}h_{\alpha\rho} - \frac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) + \frac{1}{M_p}h_{\mu\nu}T^{\mu\nu}$$

$\dot{M}$

$$V(r) = -\underbrace{\frac{4}{3}}_{\text{vDVZ}}\frac{1}{8\pi M_p^2}\frac{M}{r}e^{-mr}$$

Vainshtein 1972

nonlinear theory

$\dot{M}$ GR   classical nonlinear $r_V = \left(\frac{GM}{m^4}\right)^{1/5}$ classical linear

- non-linear theory of massive spin-2 contains vainshtein mech
- and avoids BD ghost

dRGT

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab} \qquad e^a = e_\mu{}^a dx^\mu$$
$$f_{\mu\nu} = \bar{e}_\mu{}^a \bar{e}_\nu{}^b \eta_{ab} \qquad \bar{e}^a = \bar{e}_\mu{}^a dx^\mu$$

$\swarrow \sqrt{g}R(g)$ $\qquad\qquad$ $\sim \sqrt{-g}$

$$S = \frac{M_p^2}{2}\int R^{ab}\wedge e^c\wedge e^d\,\epsilon_{abcd} - m^2\Big(\beta_0\, e^a\wedge e^b\wedge e^c\wedge e^d\,\epsilon_{abcd} + \beta_1\, e^a\wedge e^b\wedge e^c\wedge \bar{e}^d\,\epsilon_{abcd} + \beta_2\, e^a\wedge e^b\wedge \bar{e}^c\wedge \bar{e}^d\,\epsilon_{abcd} + \beta_3\, e^a\wedge \bar{e}^b\wedge \bar{e}^c\wedge \bar{e}^d\,\epsilon_{abcd}\Big)$$

- $(\beta_0 + 3\beta_1 + 3\beta_2 + \beta_3)m^2 = \Lambda$
- $\beta_1 + 2\beta_2 + \beta_3 = 1$

$M_p, \Lambda, m^2, \underline{\underline{\beta_2, \beta_3}}$

Black Hole solutions:

$$G_{\mu\nu} + m^2 \underset{(g,f)}{M_{\mu\nu}} = 8\pi G\, \underset{\uparrow\,\delta}{T_{\mu\nu}}$$

ansatz $\quad (\tau, \rho, \theta, \phi)$

$$ds_g^2 = -A_{00}^2(\rho)d\tau^2 + 2A_{01}(\rho)d\tau d\rho + A_{11}(\rho)^2 d\rho^2 + A_{22}^2(\rho)\rho^2 d\Omega^2$$
$$ds_f^2 = -d\tau^2 + d\rho^2 + \rho^2 d\Omega^2$$

- $G_{01} + m^2 M_{01} \sim m^2 \underbrace{A_{01}(\rho)}_{\mathrm{II}} \overbrace{\left(\beta_1 A_{22}(\rho)^2 + 2\beta_2 A_{22}(\rho) + \beta_3\right)}^{\mathrm{I}} = 0$

$\mathcal{O}(m^2)$

I : $ds_g^2 = -V(r)\,dt^2 + \frac{1}{V(r)}\,dr^2 + r^2 d\Omega^2$

$ds_f^2 = -C_0^2 dt^2 + C_0\sqrt{U(r)}\,dt\,dr + (C_1^2 - U(r))\,dr^2 + C_1^2 r^2 d\Omega^2$

- $V(r) = 1 - r_s/r - \frac{\Lambda}{3} r^2$
- $U(r) = \left(\frac{C_0^2}{V(r)^2} - \frac{C_1^2}{V(r)}\right)(1 - V(r))$

$\tau(t,r) = C_0 t - \int dr \sqrt{U(r)}$

$\rho(t,r) = C_1 r$

- $\beta_1 + 2C_1\beta_2 + C_1^2\beta_3 = 0$

$m^2(\beta_0 + 3C_1\beta_1 + 3C_1^2\beta_2 + C_1^3\beta_3) - \Lambda$

$\hookrightarrow$ infinitely strongly coupled

II $ds_g^2 = -B_0^2(r)\,dt^2 + B_1(r)^2 dr^2 + r^2 d\Omega^2$

$ds_f^2 = -dt^2 + \rho'(r)^2 dr^2 + \rho(r)^2 d\Omega^2$

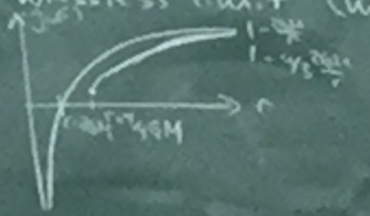
$$\left.\begin{array}{l} t = \tau \\ r(\rho) = \rho A_{22}(\rho) \end{array}\right\} B_0(r),\ B_1(r),\ \rho(r)$$

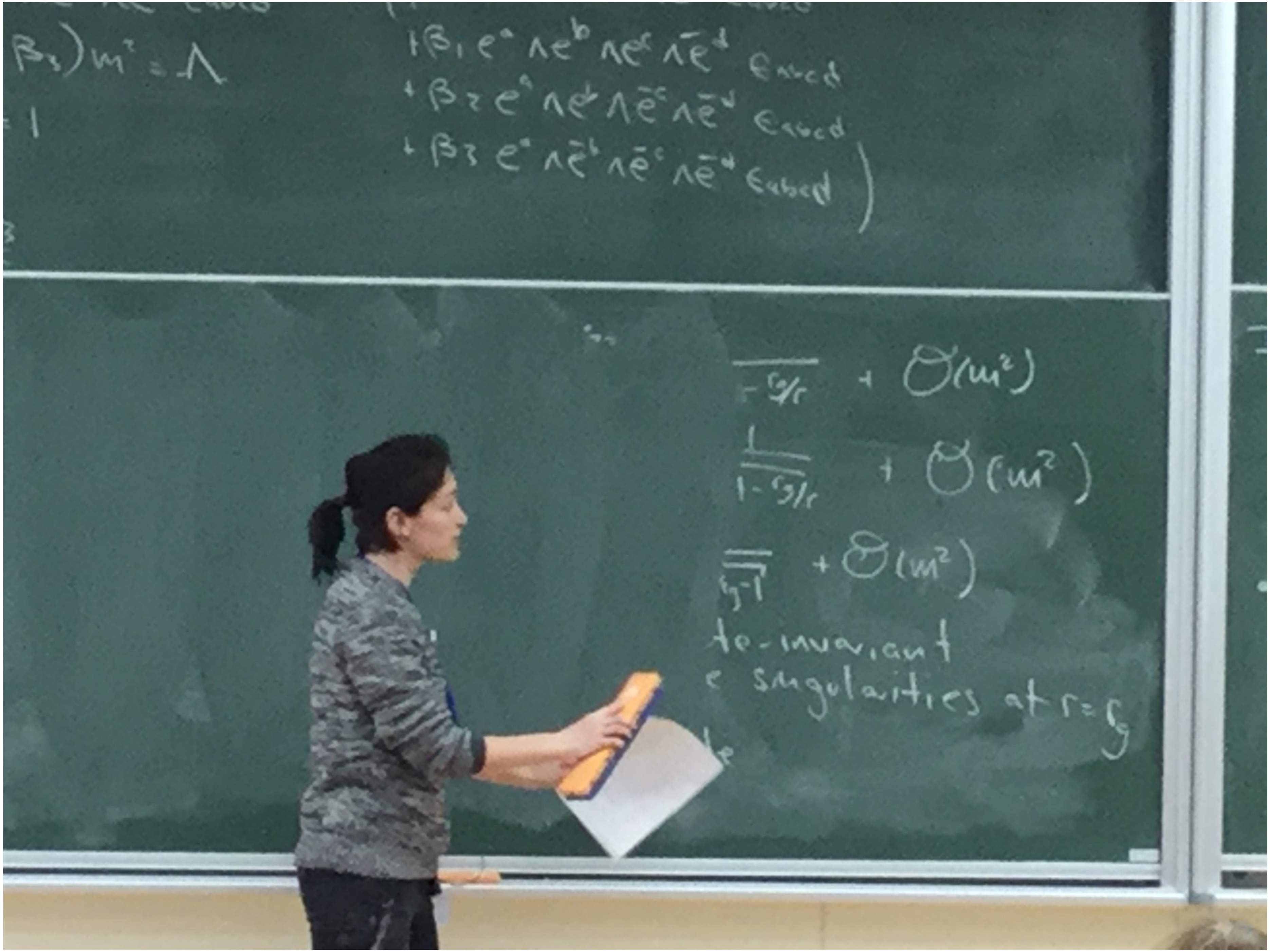
- minimal model : $\beta_1 \neq 0$ $\beta_2 = 0$, $\beta_3 = 0$

Solve exactly in massless limit $(m \to 0)$

$r \to \frac{9}{4} GM$

$-g_{00}(r) \to \left(\frac{r}{r_s}\right)^6$





β3) m² = Λ
+ β1 e^a ∧ e^b ∧ e^c ∧ ē^d ε_abcd
+ β2 e^a ∧ e^b ∧ ē^c ∧ ē^d ε_abcd
+ β3 e^a ∧ ē^b ∧ ē^c ∧ ē^d ε_abcd)
+ O(m²)
1 / (1 - r_g/r) + O(m²)
+ O(m²)
te-invariant
e singularities at r = r_g

Want: · asymptotically FP solutions
· smooth $m \to 0$ limit

$\beta_3 = 0$

assume: $m \to 0$ limit

(t,r) $\quad ds_g^2 = -(1 - r_g/r)\,dt^2 + \frac{1}{1 - r_g/r}\,dr^2 + r^2 d\Omega^2$

(τ,ρ) $\quad ds_f^2 = -d\tau^2 + d\rho^2 + \rho^2 d\Omega^2 = -[\dot\tau^2 - \dot\rho^2]\,dt^2 + 2[\dot\rho\rho' - \dot\tau\tau']\,dt\,dr + [\rho'^2 - \tau'^2]\,dr^2 + \rho(t,r)^2 d\Omega^2$

$\tau = \tau(t,r)$

$\rho = \rho(t,r)$

$$\hookrightarrow \left.\begin{aligned} G_{\mu\nu} + m^2 M_{\mu\nu} &= 0 \\ m^2 \nabla_\mu M^\mu{}_\nu &= 0 \end{aligned}\right\} \begin{aligned} \tau(t,r) &\simeq r_g \frac{\beta_1}{\beta_2}\sqrt{3}\,\sinh\left(\frac{t}{2r_g}\right)\left(\frac{r}{r_g} - 1\right)^{1/2} \\ \rho(t,r) &\simeq r_g \frac{\beta_1}{\beta_2}\sqrt{3}\,\cosh\left(\frac{t}{2r_g}\right)\left(\frac{r}{r_g} - 1\right)^{1/2} \end{aligned}$$

· check finiteness of $M^\mu{}_\mu$ at horizon ✓

· $G_{\mu\nu} + m^2 M_{\mu\nu} = 0$ $\qquad m^2 \quad \sinh\left(\frac{t}{2r_g}\right)$

as $m \to 0$ recover Schwarzschild

finite m, no timelike killing vector

· $ds^2 = e^{-2\Phi(t,r)}\left(1 - \frac{2GM(t,r)}{r}\right)dt^2 + \frac{1}{1 - \frac{2GM(t,r)}{r}}\,dr^2 + r^2 d\Omega^2$

apparent horizon: $2GM(t, r_H) = r_H \implies r_H(t) = r_g$