No-hair theorems and hairy black holes

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Parametrizations vs. theories

Advantages of parametrizations:

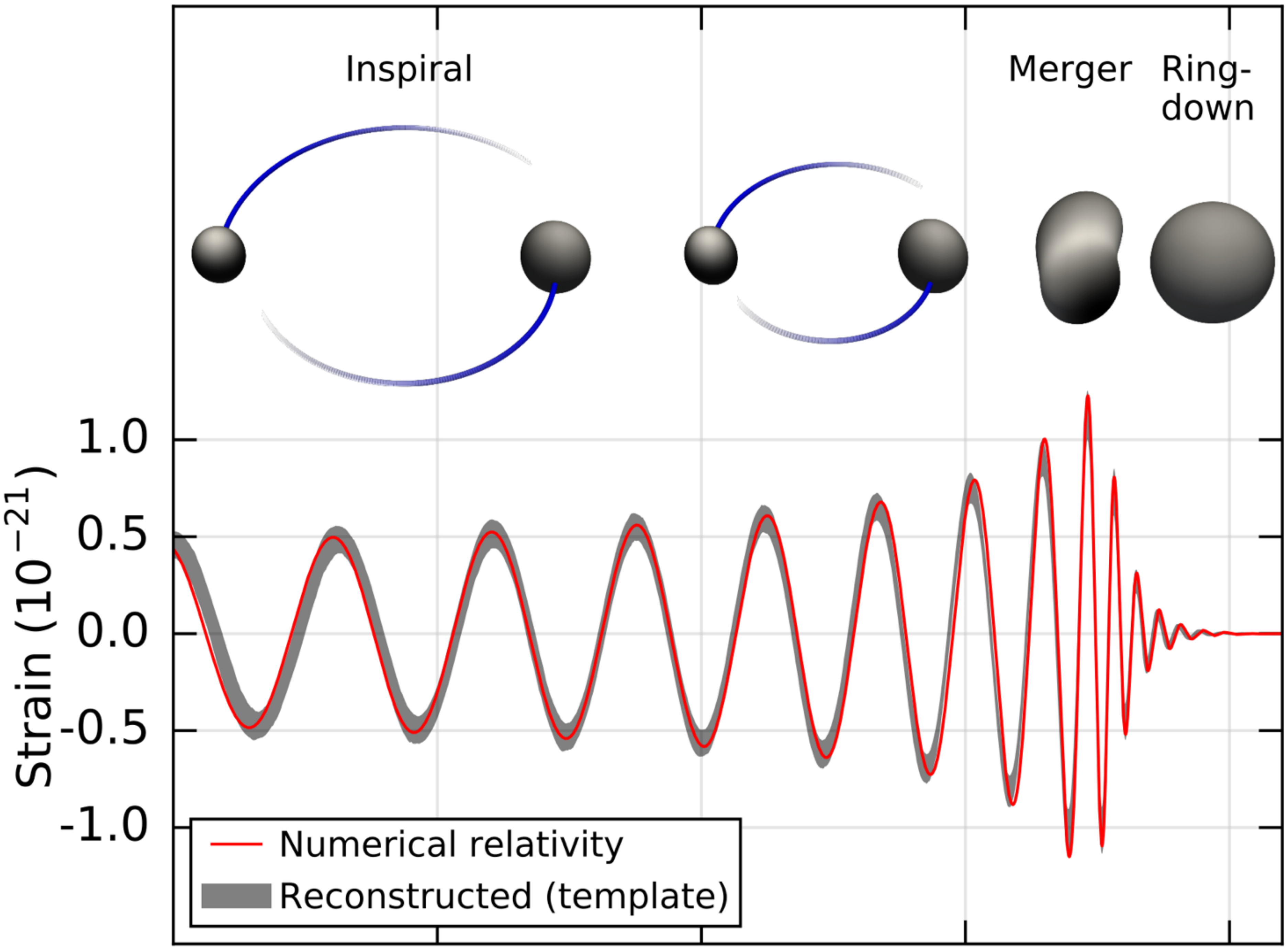
- We do not need to know the theory!

Disadvantages of parametrizations:

- They only get us half way there - they need interpretation in terms of a theory
- They give us a false sense of achievement - constraints can be meaningless or not independent
- They have limited range of validity

We need theory-specific tests as well!

Waveform



taken from B. P. Abbott et al. (LIGO -Virgo) Phys. Rev. Lett. 116, 061102 (2016)

Extracting new physics

Step-by-step guide for your favourite candidate:

- Study compact objects and determine their properties
- Model the inspiral (post-Newtonian); model the ringdown (perturbation theory)
- Do full-blown numerics to get the merger (requires initial value formulation!)

Scalar fields in BH spacetimes

The equation $$\Box\phi = 0$$

admits only the trivial solution in a BH spacetime that is

- stationary, as the endpoint of collapse
- asymptotically flat, i.e. isolated

S.W. Hawking, Comm. Math. Phys. 25, 152 (1972).

The same is true for the equation

$$\Box\phi = U'(\phi)$$

with the additional assumption of local stability

$$U''(\phi_0) > 0$$

T. P. S. and V. Faraoni, Phys. Rev. Lett. 108, 081103 (2012)

Scalar-tensor theory

Jordan frame action:

$$S_{\rm st} = \int d^4x\sqrt{-g}\Big(\varphi R - \frac{\omega(\varphi)}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - V(\varphi) + L_m(g_{\mu\nu},\psi)\Big)$$

Redefinitions:

$$\hat{g}_{\mu\nu} = \varphi g_{\mu\nu} = A^2(\phi)g_{\mu\nu} \qquad 4\sqrt{\pi}\varphi d\phi = \sqrt{2\omega(\varphi)+3}\,d\varphi$$

Einstein frame action:

$$S_{\rm st} = \int d^4x\sqrt{-\hat{g}}\Big(\frac{\hat{R}}{16\pi} - \frac{1}{2}\hat{g}^{\nu\mu}\partial_\nu\phi\partial_\mu\phi - U(\phi)\Big) + S_m(g_{\mu\nu},\psi)$$

No difference from GR?

Actually there is...

- Perturbations are different!

  E. Barausse and T.P.S., Phys. Rev. Lett. 101, 099001 (2008).

- They even lead to new effects, e.g. superradiance

  A. Arvanitaki and S. Dubovksy, Phys. Rev. D 83, 044026 (2011)
  R. Brito, V. Cardoso and P. Pani, Lect.Notes Phys. 906, 1 (2015)

- In general, relaxing the symmetries of the scalar can lead to "hairy" solutions.

  C. A. R. Herdeiro and E. Radu, Phys. Rev. Lett. 112, 221101 (2014).

- Cosmic evolution or matter could also lead to scalar "hair"

  T. Jacobson, Phys. Rev. Lett. 83, 2699 (1999);
  M. W. Horbatsch and C. P. Burgess, JCAP 1205, 010 (2012).
  V. Cardoso, I. P. Carucci, P. Pani and T. P. S., Phys. Rev. Lett. 111, 111101 (2013)

Generalized Galileons

G. W. Horndeski, Int. J. Theor. Phys. 10, 363 (1974)
C. Deffayet et al., Phys. Rev. D 80, 064015 (2009)

One can actually have terms in the action with more than 2 derivatives and still have second order equations:

$$\delta((\partial\phi)^2\Box\phi) = 2[(\partial^\mu\partial^\nu\phi)(\partial_\mu\partial_\nu\phi) - (\Box\phi)^2]\delta\phi$$

- Inspired by galileons: scalars that enjoy galilean symmetry

A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D 79, 064036 (2009)

- It includes well-know terms, such as

$$(\nabla_\mu\phi)(\nabla_\nu\phi)G^{\mu\nu} \qquad \phi\left(R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right)$$

Known "hairy" solutions! For example, for the coupling

$$e^\phi(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda})$$

P. Kanti et al., Phys. Rev. D 54, 5049 (1996).

Generalized Galileons

No-hair for **shift-symmetric** generalised galileons

$$\nabla_\mu J^\mu = 0$$

Assumptions: L. Hui, A. Nicolis, Phys. Rev. Lett. 110, 241104 (2013).

- Staticity and spherical symmetry
- Asymptotic flatness
- J^2 must be finite on the horizon
- Restrictions on the dependence of J^μ on ϕ

Straightforward generalisation to slowly-rotating solutions

T.P.S. and S.-Y. Zhou, Phys. Rev. Lett. 112, 251102 (2014);
Phys. Rev. D 90, 124063 (2014).

Hairy black holes with (linearly) time-dependent hair exist

E. Babichev and C. Charmousis, JHEP 1408, 106 (2014)

A simple exception

T.P.S. and S.-Y. Zhou, Phys. Rev. Lett. 112, 251102 (2014);
Phys. Rev. D 90, 124063 (2014).

Consider the action

$$S = \frac{m_P^2}{8\pi} \int d^4x \sqrt{-g} \left(\frac{R}{2} - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \alpha \phi \mathcal{G} \right)$$

The corresponding scalar equation is

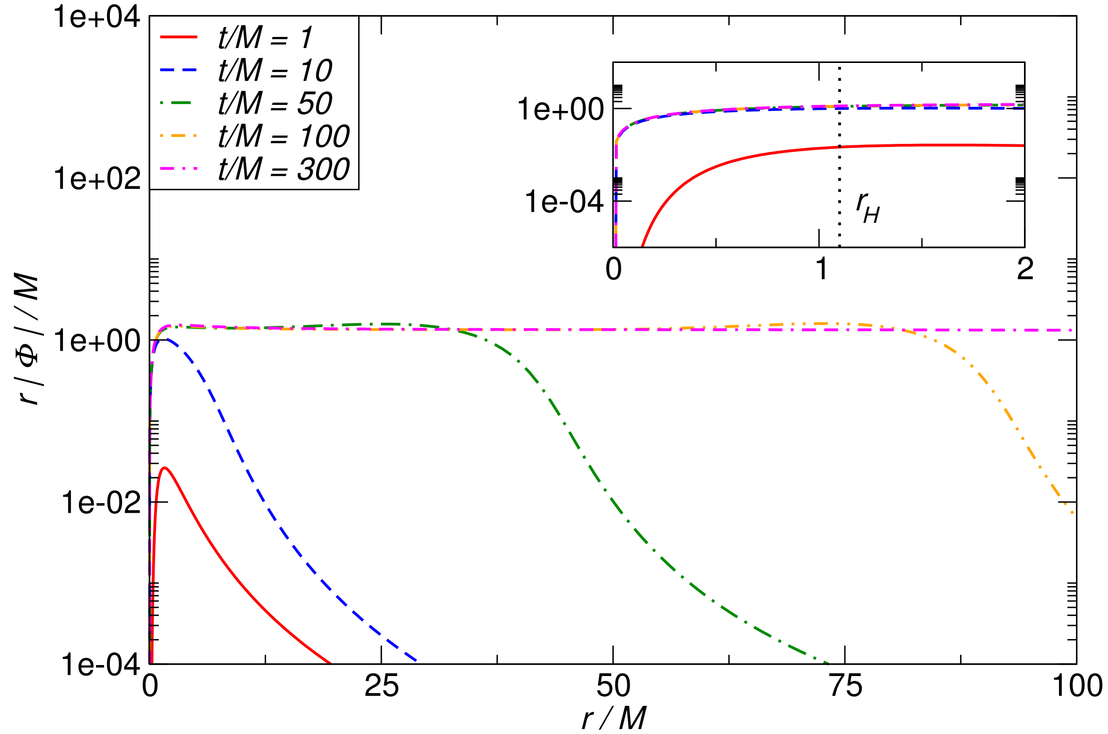
$$\Box \phi + \alpha \mathcal{G} = 0$$

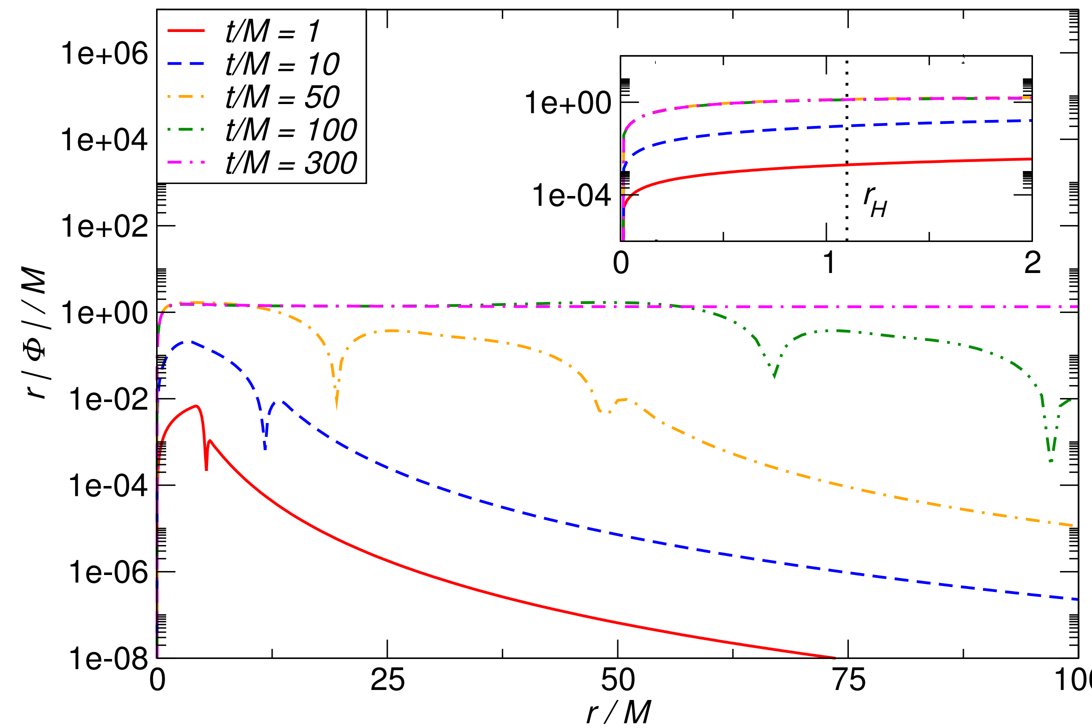
The Gauss-Bonnet term does not vanish in BH spacetimes!

Perturbative solution in Schwarzschild spacetime

$$\phi' = -\frac{2\alpha(r^2 + 2Mr + 4M^2)}{Mr^4}$$

Dynamical formation of hair





- First evidence that hair form from collapse

R. Benkel, T.P.S. and H. Witek, Phys. Rev. D 94 (R), 121503 (2016); Class. Quant. Grav. 34, 064001 (2017)

- Stars have zero scalar “charge”

N. Yunes and L. C. Stein, Phys. Rev. D 83, 104002 (2011).

Black hole scalarization

H. O. Silva, J. Sakstein, L. Gualtieri, T.P.S, and E. Berti, arXiv:1711.02080 [gr-qc]

No-hair theorem for the action

$$S = \frac{m_P^2}{8\pi} \int d^4x \sqrt{-g} \left(\frac{R}{2} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + f(\phi) \mathcal{G} \right)$$

provided that $f'(\phi_0) = 0 \,, \quad f''(\phi_0)\mathcal{G} < 0$

That is, for the equation

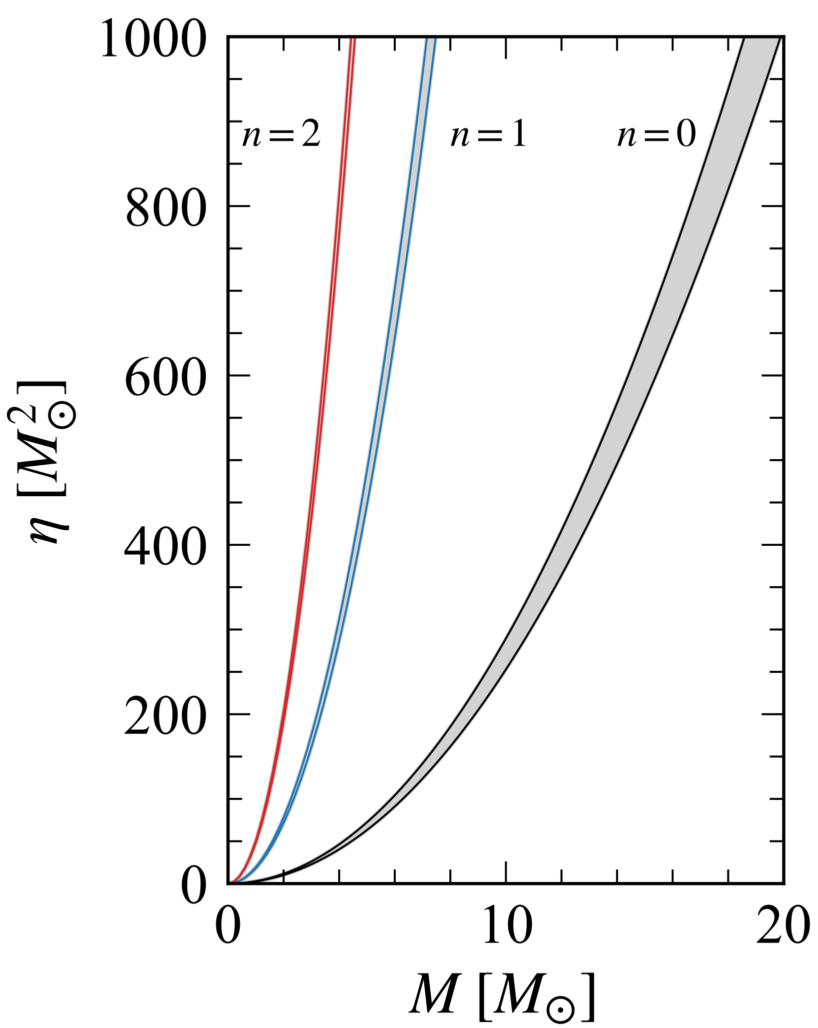
$$\Box \phi = -f'(\phi)\mathcal{G}$$

trivial solutions are unique if admissible, if the effective mass is positive

- But if it is negative then there can be"scalarization"!

Black hole scalarization

H. O. Silva, J. Sakstein, L. Gualtieri, T.P.S, and E. Berti, arXiv:1711.02080 [gr-qc]



Simplest but generic model:

$$f = \frac{\eta}{8}\phi^2$$

- Scalarization in bands! (Non-perturbative)
- It works for neutron stars too! (work underway)

See also: D. D. Doneva and S. S. Yazadjiev, arXiv:1711.0187 [gr-qc]

Conclusions

- Studying stationary compact objects beyond GR is the starting point of producing waveforms
- No-hair theorems act as uniqueness theorems for certain classes of theories
- …but they also help us identify interesting models
- Exciting phenomenology waits to be tested!
- Major obstacle: lack of predictions