From Laminar Flow to Wave Turbulence in Holographic Superfluid

Yu Tian (田雨)

University of Chinese Academy of Sciences
中国科学院大学

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with Shan-Quan Lan, Wen-Biao Liu, Hong Liu and Hongbao Zhang)

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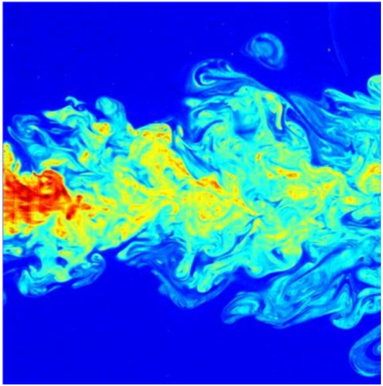
Turbulence: one of the most important scientific problems



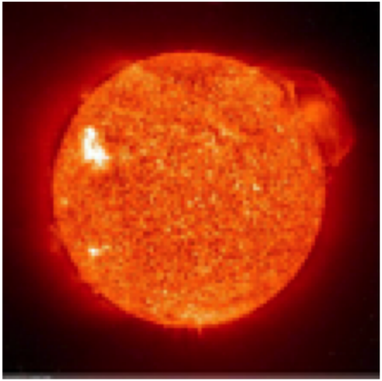


Figure: Turbulence in classical fluids

Turbulence: characteristics

- no consensus definition of turbulence
- spatially complex
- aperiodic in time
- spanning several orders of magnitude in spatial extent and temporal frequency
- chaotic
  sensitive to initial conditions (unpredictable)

Quantum turbulence

- turbulence in quantum fluids (superfluid Heliums, Bose-Einstein condensates, superconductors, etc)
- expected to be similar to classical turbulence at large scales
- expected to be distinct from classical turbulence at small scales set by the healing length, which is the characteristic size of local defects (basically vortices) of the order parameter

Vortex turbulence vs wave turbulence

- eddies in classical turbulence and vortices in quantum turbulence
  eddy (or vortex) as the fundamental characteristic of the traditional turbulence (vortex turbulence)
- wave turbulence: a type of turbulence different from the traditional one, where eddies (or vortices) exist but do not dominate the physics
- decomposition: vortex (incompressible) and wave (compressible)

$$\mathbf{v} = \mathbf{v}_i + \mathbf{v}_c$$

$$\nabla \cdot \mathbf{v}_i = 0, \qquad \nabla \times \mathbf{v}_c = 0$$

- vortex dominant vs wave dominant

Quantum wave turbulence: experiments and numerics

[Navon et al, Nature 539, 72 (2016)]

- onset of 3D turbulence in BEC by shaking
- numerical modeling using Gross-Pitaevskii equation

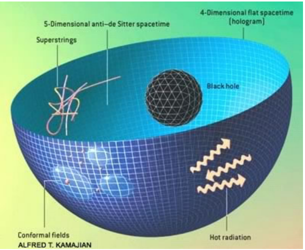
$$i\hbar\partial_t\varphi = \left(-\frac{\nabla^2}{2m} + V(t,\mathbf{x}) + g|\varphi|^2 - \mu\right)\varphi$$

  with excellent agreement with experimental measurements
- wave dominant (from numerics) isotropic steady turbulent state

Turbulence: open problems

- onset mechanism of (classical and quantum) turbulence
- control parameter of quantum turbulence (like the Reynolds number in classical turbulence)
- (non-)universal properties (Kolmogorov scaling law, Kolmogorov-Zakharov scaling law, energy cascade, etc)

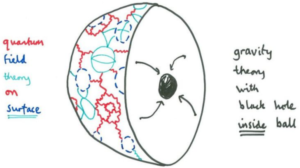
Why applied holography (AdS/CFT)?



- Quantum systems can be dually described by classical gravitational theories.
- Far-from-equilibrium dynamics as well as near-equilibrium transport processes can be easily realized.
- Dissipation at finite temperature is naturally included by putting a black hole in the bulk.

Facts about (applied) AdS/CFT

- Finite temperature field theory with finite chemical potential is dual to a charged black hole in the bulk AdS:



Temperature	$\longleftrightarrow$	Hawking temperature
Conserved charge	$\longleftrightarrow$	Charge
Chemical potential	$\longleftrightarrow$	Electric potential
Energy dissipation	$\longleftrightarrow$	Energy accretion

[YT, X.-N, Wu and H. Zhang, arXiv:1407.8273]

- Action of the simplest holographic superfluid model

  [Hartnoll, Herzog and Horowitz, arXiv:0803.3295]

$$I=\int_{\mathcal{M}} d^4x\sqrt{-g}\left(-\frac{1}{4}F_{AB}F^{AB}-|D\Psi|^2-m^2|\Psi|^2\right).$$

- Background metric

$$ds^2=\frac{L^2}{z^2}[-f(z)dt^2-2dtdz+dx^2+dy^2],\quad f(z)=1-\frac{z^3}{z_h^3}.$$

- Heat bath temperature

$$T=\frac{3}{4\pi z_h}.$$

- The hairless-hairy phase transition of the black hole occurs at the critical electric potential (chemical potential) $\mu_c = 4.06$.
- The above transition is interpreted as the normal-superfluid phase transition of the boundary system.

The laminar-turbulent transition by shaking (periodic driving)

- Shaking a holographic superfluid in a periodic box of length L with an appropriate frequency ω

$$u_x = A \sin \omega t$$

- Random initial perturbations
- The laminar-turbulent transition observed at the shaking amplitude $A = A_c$

- The case of laminar flow

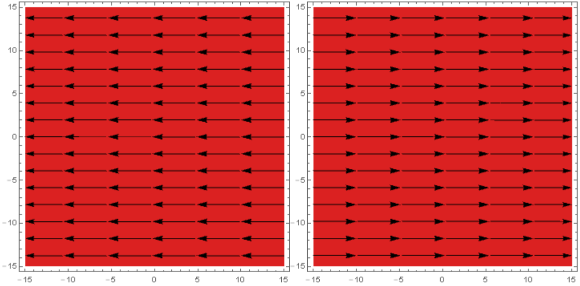


Figure: Superfluid velocity fields for the shaking amplitude $A < A_c$ at around the twentieth shaking cycles which changes direction and at the twentieth shaking cycles it is zero.

- The case of turbulent flow

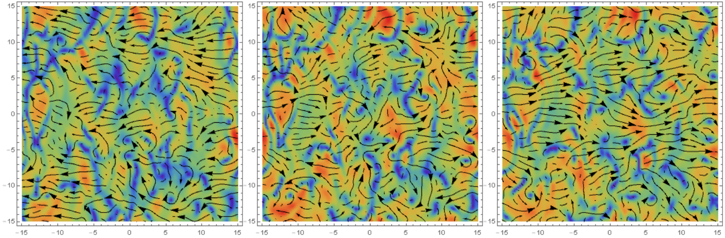


Figure: Superfluid velocity fields for the shaking amplitude $A > A_c$ before, at and after the twentieth shaking cycles where the total net velocity changes its direction. For the middle panel, the total net velocity is approximately zero.

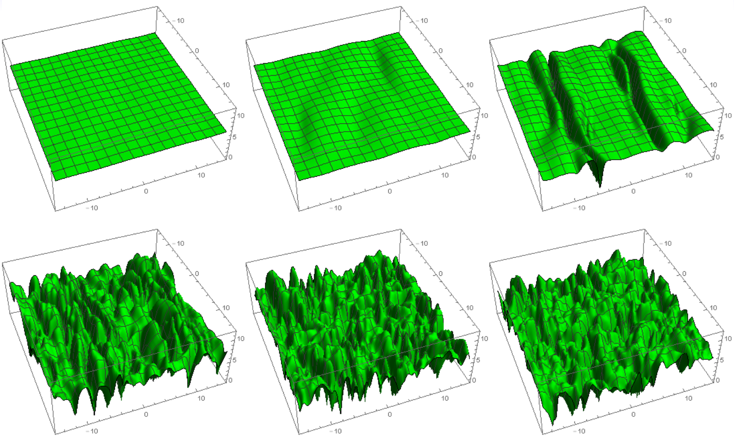


Figure: The configurations of $|\psi|$ after 1, 2, 3, 4, 5 and 25 shaking cycles for $A > A_c$.

- Characterization of the laminar-turbulent transition by the total kinetic energy

$$E_{\text{kin}}(t) = \int \frac{1}{2}\mathbf{u}^2|\psi|^2 d^2x$$

at integral shaking cycles

- Characterization of the laminar-turbulent transition by vortex formation

1. Motivation and introduction
2. Laminar-turbulent transition in holographic superfluids
3. **Wave turbulence in holographic superfluids**
4. Conclusion and discussion

Kinetic energy spectra and direct energy cascade

- Kinetic energy spectra:

$$E_{\text{kin}}(t) = \int_0^\infty \epsilon_{\text{kin}}(t,k)\,dk$$

- Direct energy cascade in holographic superfluids:

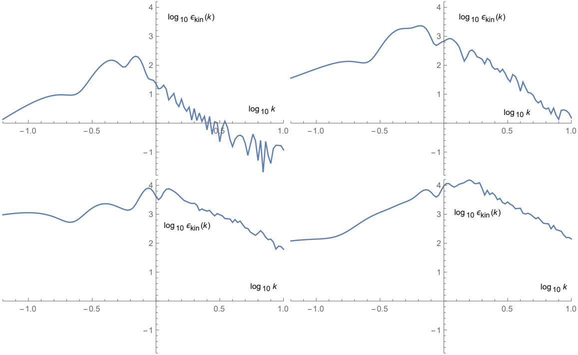


Figure: $\epsilon_{\text{kin}}(k)$ after 2, 3, 4 and 6 shaking cycles, respectively.

Kolmogorov $-5/3$ scaling law in vortex turbulence

- The turbulent dynamics is assumed to be characterized by the energy dissipation rate per unit mass ε at one end of an inertial range $k_- < k < k_+$.
- Dimensional analysis simply gives

$$\epsilon_{\rm kin}(k) = C\varepsilon^{2/3}k^{-5/3}$$

$$(\epsilon_{\rm kin} : [L^3 T^{-2}], \quad \varepsilon : [L^2 T^{-3}], \quad k : [L^{-1}])$$

  in the inertial range, where C is a dimensionless constant.
- Kolmogorov $-5/3$ scaling law and direct energy cascade in vortex turbulence (relaxation) of holographic superfluids [Chesler, Liu and Adams, 2012; Lan, YT and Zhang, 2016]

The scaling law in shaken holographic superfluids

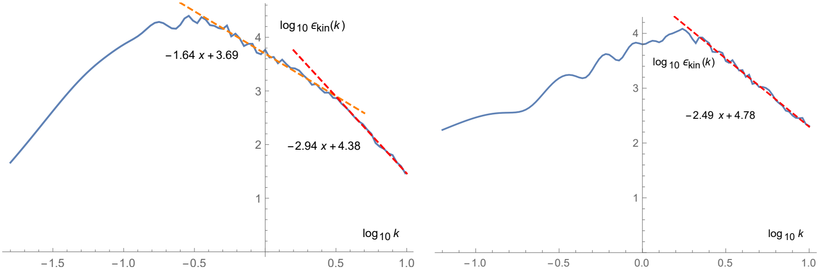


Figure: The kinetic energy spectra $\epsilon_{\mathrm{kin}}(k)$ for vortex relaxation (left) and shaking (right) in holographic superfluids. The left panel shows two scaling law, one the Kolmogorov $-5/3$ scaling law in the inertial range and the other the -3 scaling law characterizing free vortices. The right panel shows only one scaling law, which is different from $-5/3$ or -3. Wave turbulence?

Wave turbulence in holographic superfluids

- decomposition: vortex (incompressible) and wave (compressible)

$$\mathbf{u} = \mathbf{u}_i + \mathbf{u}_c$$

$$\nabla \cdot \mathbf{u}_i = 0, \qquad \nabla \times \mathbf{u}_c = 0$$

$$E_{i,c}(t) = \int \frac{1}{2}\mathbf{u}_{i,c}^2 |\psi|^2 d^2x = \int_0^\infty \epsilon_{i,c}(t,k)\, dk$$

$$\epsilon_{\text{kin}}(t,k) = \epsilon_i(t,k) + \epsilon_c(t,k)$$

- compressible to incompressible ratios:

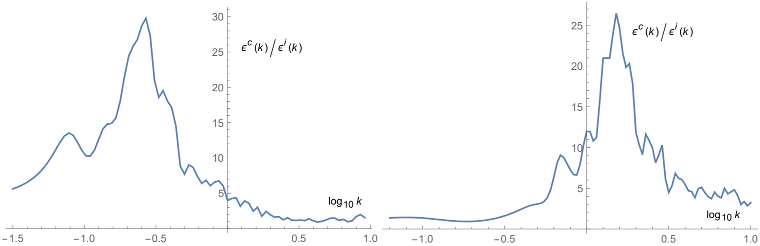


Figure: The typical ratios of compressible to incompressible energy spectra $\epsilon_c(k)/\epsilon_i(k)$ for vortex relaxation (left) and shaking (right). The left panel shows that waves mainly live at small k and vortices dominate the large k regime. The right panel is a telltale signature of wave turbulence.

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Conclusion

- The lamina-turbulent transition is observed by shaking 2D holographic superfluids.
- The turbulent state of the shaken holographic superfluids has a ~ -2.5 scaling law.
- The turbulent state of the shaken holographic superfluids is identified as a wave turbulence.

Discussion

- Onset mechanism of 2D quantum turbulence (in holographic superfluids)?
- Chaotic behavior from linear analysis (Lyapunov exponents)?
- More physical insights from the holographic point of view?

元宵节快乐！

Happy Lantern Festival!

Thanks for your attention!