Multiple inflation with variable number of slow-roll inflatons

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Inflation

The inflationary hypothesis :

Some part of the world which includes all its presently observable part was as much symmetric as possible during some period in the past - both with respect to the geometrical background and to the state of all quantum fields (no particles).

Non-universal (due to the specific initial condition) explanation of the cosmological arrow to time - chaos, entropy (in some not well defined sense) can only grow after inflation.

Still this state is an intermediate attractor for a set of pre-inflationary initial conditions with a non-zero measure. Also it is not a unique one, there exists a class of such states leading to the same observable predictions.

Successive realization of this idea is based on the two more detailed and independent assumptions.

1. Existence of a metastable quasi-de Sitter stage in our remote past which preceded the hot Big Bang. During it, the expansion of the Universe was accelerated and close to the exponential one, $|\dot{H}| \ll H^2$.

2. The origin of all inhomogeneities in the present Universe is the effect of gravitational creation of pairs of particles - antiparticles and field fluctuations during inflation from the adiabatic vacuum (no-particle) state for Fourier modes covering all observable range of scales (and possibly somewhat beyond).

Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\mathcal{R}(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \; g_{,l}^{(a)}\, e_m^{l(a)} = 0, \; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

$\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

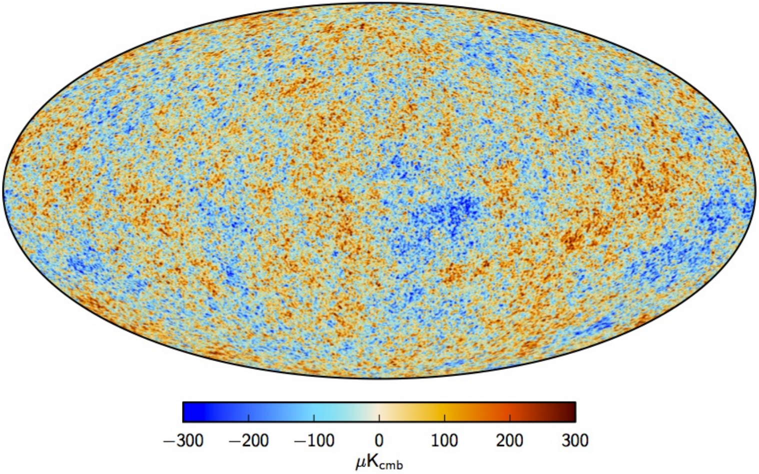
$$n_s(k) - 1 \equiv \frac{d \ln P_{\mathcal{R}}(k)}{d \ln k}, \quad r(k) \equiv \frac{P_g}{P_{\mathcal{R}}}$$

Existence of constant modes

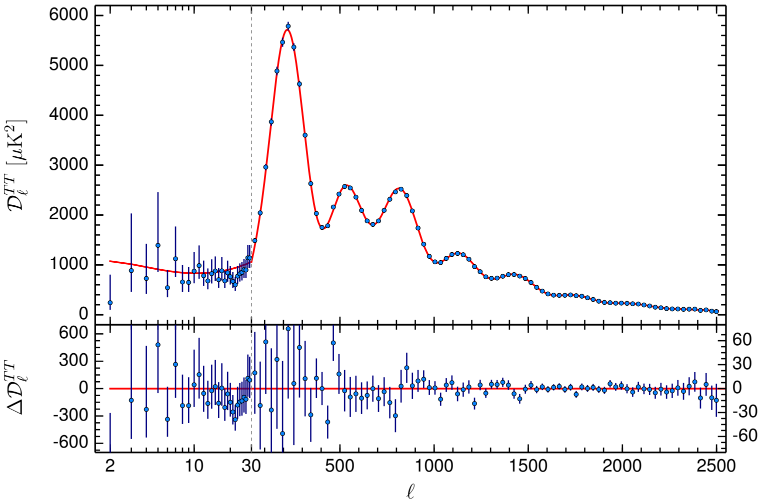
For FLRW models filled by ideal fluids, it was known already to Lifshitz (1946). For a wide class of modified scalar-tensor gravity theories, it was proved in A. A. Starobinsky, S. Tsujikawa and J. Yokoyama, Nucl. Phys. B 610, 383 (2001). However, their existence is much more general. From the mathematical point of view, constant modes appear simply due to the existence of non-degenerate solutions of the same gravity models in the isotropic and spatially flat FLRW space-time. By construction, these solutions always have 3 non-physical (gauge) arbitrary constants of integration due to the possibility of arbitrary and independent rescaling of all spatial coordinates. Making these constants slightly inhomogeneous converts them to the leading terms of physical constant modes (one scalar and two tensor ones). Moreover, it straightforwardly follows from this that these constants (now functions of spatial coordinates) need not be small, they can be arbitrarily large: $a^2(t)\delta_\alpha^\beta \to a^2(t)c_\alpha^\beta(\mathbf{r})$.

CMB temperature anisotropy

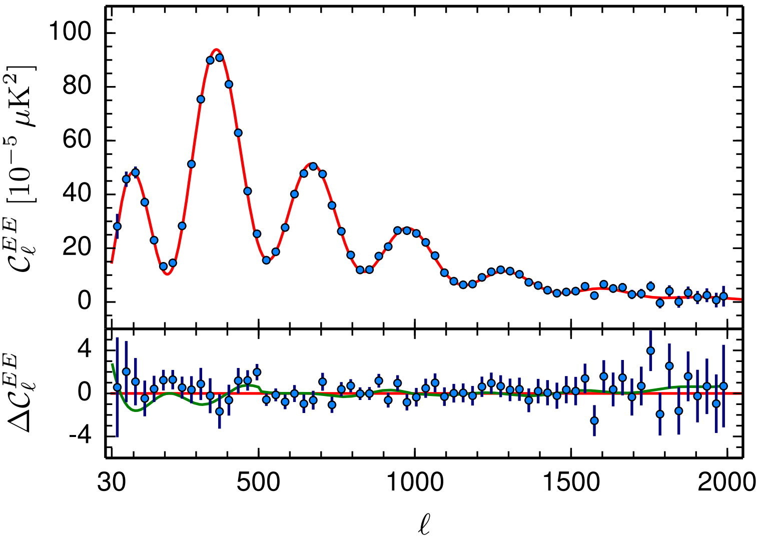
Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



CMB temperature anisotropy multipoles



CMB E-mode polarization multipoles



New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) >= \int \frac{P_{\mathcal{R}}(k)}{k}\, dk, \;\; P_{\mathcal{R}}(k) = (2.10 \pm 0.03)\cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.004$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s) N_H \sim 2$).

The most recent upper limit on r

BICEP/Keck Collaboration: P. A. R. Ade et al., Phys. Rev. Lett. 127, 151301 (2021); arXiv:2110.00483:

$r_{\,0.05} < 0.036$ at the 95% C.L.

For comparison, in the chaotic inflationary model $V(\varphi) \propto |\varphi|^n$, $r = \frac{4n}{N}$, $1 - n_s = \frac{n+2}{2N}$. The r upper bound gives $n \lesssim 0.5$ for $N_{0.05} = (55 - 60)$, but then $1 - n_s \le 0.023$. Thus, this model is disfavoured by observational data.

The simplest models producing the observed scalar slope

Arithmetical classification of inflationary models - by the number of free parameters of these models fixed by observations only.

The 3 simplest models having

$n_s - 1 = -\frac{2}{N}, \; r = \frac{12}{N^2} = 3(n_s - 1)^2$ are one-parametric.

1. The $R + R^2$ inflationary model.
2. The Higgs inflationary model.
3. The combined Higgs-R^2 model.

1. The $R + R^2$ model (Starobinsky 1980):

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \quad f(R) = R + \frac{R^2}{6M^2}, \quad M_{\rm Pl}^2 = G^{-1}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{\rm Pl} \approx 3.1 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \;\; r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.004, \;\; n_t = -\frac{r}{8}$$

$$N = \ln \frac{k_f}{k} = \ln \frac{T_\gamma}{k} - \mathcal{O}(10), \quad H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

2. The same prediction from the scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0, \;\; |\xi| \gg 1$ (Spokoiny 1984), including the Higgs inflationary model (Bezrukov & Shaposhnikov 2008).

Possible microscopic origins of this phenomenological model.

1. Follow the purely geometrical approach and consider it as the specific case of the fourth order gravity in 4D

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + (\text{small rad. corr.})$$

for which $A \gg 1,\ A \gg |B|$. Approximate scale (dilaton) invariance and absence of ghosts in the curvature regime $A^{-2} \ll (RR)/M_P^4 \ll B^{-2}$.

One-loop quantum-gravitational corrections are small (their imaginary parts are just the predicted spectra of scalar and tensor perturbations), non-local and qualitatively have the same structure modulo logarithmic dependence on curvature.

2. Another, completely different way:

consider the $R + R^2$ model as an approximate description of GR + a non-minimally coupled scalar field with a large negative coupling ξ ($\xi_{conf} = \frac{1}{6}$) in the gravity sector:

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{\xi R \phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \ .$$

Geometrization of the scalar:

for a generic family of solutions during inflation, the scalar kinetic term can be neglected, so

$$\xi R \phi = -V'(\phi) + \mathcal{O}(|\xi|^{-1}) \ .$$

No conformal transformation, we remain in the the physical (Jordan) frame!

These solutions are the same as for $f(R)$ gravity with

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \; f(R) = R - \frac{\xi R \phi^2(R)}{2} - V(\phi(R)).$$

For $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$, this just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$.

The same theorem is valid for a multi-component scalar field.

More generally, R^2 inflation (with an arbitrary n_s, r) serves as an intermediate dynamical attractor for a large class of scalar-tensor gravity models.

Inflation in the mixed R^2-Higgs model

The third model producing the same predictions for n_s and r as functions of N.

M. He, A. A. Starobinsky and J. Yokoyama, JCAP 1805, 064 (2018).

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right) - \frac{\xi R \chi^2}{2} + \frac{1}{2}\chi_{,\mu}\chi^{,\mu} - \frac{\lambda\chi^4}{4}, \ \ \xi < 0, \ \ |\xi| \gg 1$$

Can be conformally transformed to GR with two interacting scalar fields in the Einstein frame. The effective two scalar field potential for the dual model:

$$U = e^{-2\alpha\phi}\left(\frac{\lambda}{4}\chi^4 + \frac{M^2}{2\alpha^2}\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)^2\right)$$

$$\alpha = \sqrt{\frac{2}{3}}\,\kappa, \ \ \ \kappa^2 = 8\pi G, \ \ \ R = 3M^2\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)$$

One-field inflation in the attractor regime

In the attractor regime during inflation:

$$\alpha\phi \gg 1, \quad \chi^2 \approx \frac{|\xi| R}{\lambda}, \quad e^{\alpha\phi} \approx \chi^2 \left(|\xi|\kappa^2 + \frac{\lambda}{3|\xi| M^2} \right)$$

that directly follows from the geometrization of the Higgs boson in the physical (Jordan) frame. Thus, we return to the $f(R) = R + \frac{R^2}{6M^2}$ model with the renormalized scalaron mass $M \to \tilde{M}$:

$$\frac{1}{\tilde{M}^2} = \frac{1}{M^2} + \frac{3\xi^2\kappa^2}{\lambda}$$

Double-field inflation reduces to the single $(R + R^2)$ one for the most of trajectories in the phase space. For $\lambda = 0.01$,

$$|\xi| \leq \xi_c \approx 4400$$

Creation and heating of matter after inflation

These 3 models have different behaviour after the end of inflation depending on assumptions about scalaron coupling to usual matter or Higgs non-minimal coupling to gravity.
In the case of $R + R^2$ model with the scalaron decay into minimally coupled scalars and the longitudinal mode of vector bosons with $m \ll M$,

$$\Gamma = \frac{M^3}{192\, M_{\rm Pl}^2}, \quad N(k) \approx N_H + \ln \frac{a_0 H_0}{k} - \frac{5}{6} \ln \frac{M_{\rm Pl}}{M}$$

that gives $N(k/a_0 = 0.05\,\mathrm{Mpc}^{-1}) \approx 51$. For the Higgs and the mixed R^2-Higgs models $N(0.05\,\mathrm{Mpc}^{-1}) \approx 55$, the increase is mainly due to the large Higgs non-minimal coupling.
However, many interesting non-perturbative effects may occur in the mixed R^2-Higgs case, e.g. tachyonic preheating (M. He, R. Jinno, K. Kamada, A. A. Starobinsky and J. Yokoyama, JCAP 2101 (2021) 066 [arXiv:2007.10369]). See also M. He, JCAP 2105 (2021) 021 [arXiv:2010.11717].

Perspectives of future discoveries

- Primordial gravitational waves from inflation: r.
  $r \lesssim 8(1 - n_s) \approx 0.3$ (confirmed!) but may be much less. However, under reasonable assumptions one may expect that $r \gtrsim (n_s - 1)^2 \approx 10^{-3}$. The target prediction in the simplest (one-parametric) models is
  $r = 3(n_s - 1)^2 \approx 0.004$.
- A more precise measurement of $n_s - 1 \Longrightarrow$ duration of transition from inflation to the radiation dominated stage $\Longrightarrow$ information on inflaton (scalaron) couplings to known elementary particles at super-high energies $E \lesssim 10^{13}\ \mathrm{Gev}$.
- Local non-smooth features in the scalar power spectrum at cosmological scales (?).
- Local enhancement of the power spectrum at small scales leading to a significant amount of primordial black holes (?).

Generating peaks and troughs in the primordial scalar spectrum

To obtain large peaks and troughs in $P_{\mathcal{R}}$, temporal breaking of the slow-roll approximation during inflation is needed. The simplest way: fast break in the first derivative of the inflaton potential $V(\phi)$ (A. A. Starobinsky, JETP Lett. 55, 489 (1992)). Leads to a step in $P_{\mathcal{R}}$ with superimposed oscillations. To obtain a peak, two such features with opposite signs, or a fast break in the $V(\phi)$ itself are needed (so that an inflection point appears in between). However, it is not sufficient to have an inflection point only, it should be combined with a strong breaking of the slow-roll conditions.

Let $V(\phi) = V_0 + A_+\phi\,\theta(\phi - \phi_0) + A_-\phi\,\theta(\phi_0 - \phi)$ for ϕ close to ϕ_0. Then

$$\dot{\phi} = -\frac{A_+}{3H_0}\,\theta(-t) - \frac{A_- + (A_+ - A_-)e^{-3H_0 t}}{3H_0}\,\theta(t)$$

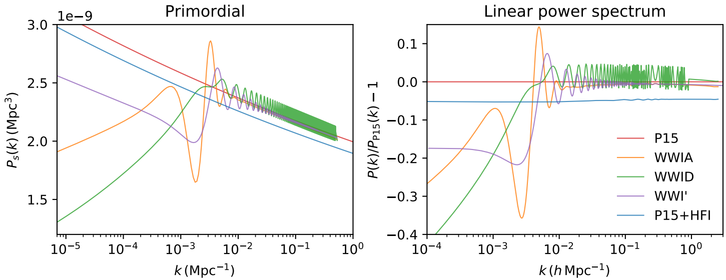
The slow-roll spectrum $P_{\mathcal{R}}$ is modulated by the multiplier

$$D^2 = 1 - 3\left(\frac{A_-}{A_+} - 1\right)\left[\left(1 - \frac{1}{y^2}\right)\sin 2y + \frac{2}{y}\cos 2y\right] +$$

$$\frac{9}{2}\left(\frac{A_-}{A_+} - 1\right)^2 \frac{1}{y^2}\left(1 + \frac{1}{y^2}\right) \times$$

$$\left[1 + \frac{1}{y^2} + \left(1 - \frac{1}{y^2}\right)\cos 2y - \frac{2}{y}\sin 2y\right],$$

$$y = \frac{k}{k_0},\;\; D(0) = \frac{A_-}{A_+},\;\; D(\infty) = 1$$



Primordial
1e−9
3.0
2.5
2.0
1.5
$P_s(k)\,(\mathrm{Mpc}^3)$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$k\,(\mathrm{Mpc}^{-1})$
Linear power spectrum
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$P(k)/P_{\mathrm{P15}}(k) - 1$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$k\,(h\,\mathrm{Mpc}^{-1})$
P15
WWIA
WWID
WWI'
P15+HFI

Non-scale-free features at cosmological scales

The most recent analysis of this type of spectra with power suppression at large scales (D. K. Hazra, D. Paoletti, I. Debono, A. Shafieloo, G. F. Smoot, A. A. Starobinsky, JCAP 2112 (2021) 038; arXiv:2107.09460) using the CMB temperature and polarization data from the Planck 2018 data release shows marginal (68% C.L.) preference of suppression from the large scale temperature angular power spectrum. However, the large-scale E-mode likelihood does not support this suppression and in the combined data the preference towards the suppression becomes negligible. For models with oscillatory features along with the suppression, unbinned data from the recently released CamSpec 12.5 likelihood was used which updates Planck 2018 results. Comparison of the Bayesian evidences of the feature models with their baseline slow-roll inflaton potentials showed that the latter are moderately preferred against potentials with features.

Perturbations in multiple slow-roll inflation

The constant mode of $\mathcal{R}$ always exists in the super-Hubble regime, but it is not tangent to the trajectory of a background solution in the field space generically.

The simplest case: minimally coupled to gravity scalar fields with $V = \sum_n V_n(\phi_n)$.
A. A. Starobinsky, JETP Lett. 42, 152 (1985).
The number of e-folds:

$$N = -8\pi G \sum_n \int^{\phi_n} \frac{V_n(\tilde{\phi}_n)}{dV_n/d\tilde{\phi}_n}\, d\tilde{\phi}_n \,,$$

where only slow-roll inflatons are taken into account in the sum.

The scalar perturbation spectrum (δN formalism):

$$\mathcal{R}(\mathbf{r}) = \left(\frac{\delta N}{\delta \phi_n}\right)_b \delta\phi_n(\mathbf{r}),$$

where the subscript b means the background quantity.
In the Fourier space:

$$\mathcal{R}(\mathbf{k}) = 4G \sum_n \left(\frac{HV_n}{dV_n/d\phi_n}\right)_k c_n(\mathbf{k}),$$

where $c_n(\mathbf{k})$ are independent Gaussian variables with zero average and unit dispersion, and the subscript b means that the background quantity is taken at the moment of the mode Hubble radius crossing $k = aH$ during inflation.
Generalization to an arbitrary $V(\phi_n)$: M. Sasaki, E. D. Stewart, Prog. Theor. Phys. 95 (1996) 71; astro-ph/9507001.

PBHs and small-scale GWs in two-field models of inflation

The simplest one-parameter inflationary models do not predict PBHs, at least whose existing at present with $M > 10^{15}$ g.
Previously known ways to obtain a large peak in the primordial power spectrum of scalar adiabatic perturbations at small scales:
1. A local feature in the inflaton potential $V(\phi)$ (a rapid change of its slope or its amplitude, an inflection point with a large $V'''(\phi)$).
2. A rapid turn of the inflaton trajectory in the field space in the case of many-field models of inflation.
3. Phase transitions leading to large isocurvature perturbations which transform to adiabatic ones afterwards.
Generically, the dimensionality of the internal space of slow-roll inflatons changes with time during multiple inflation.

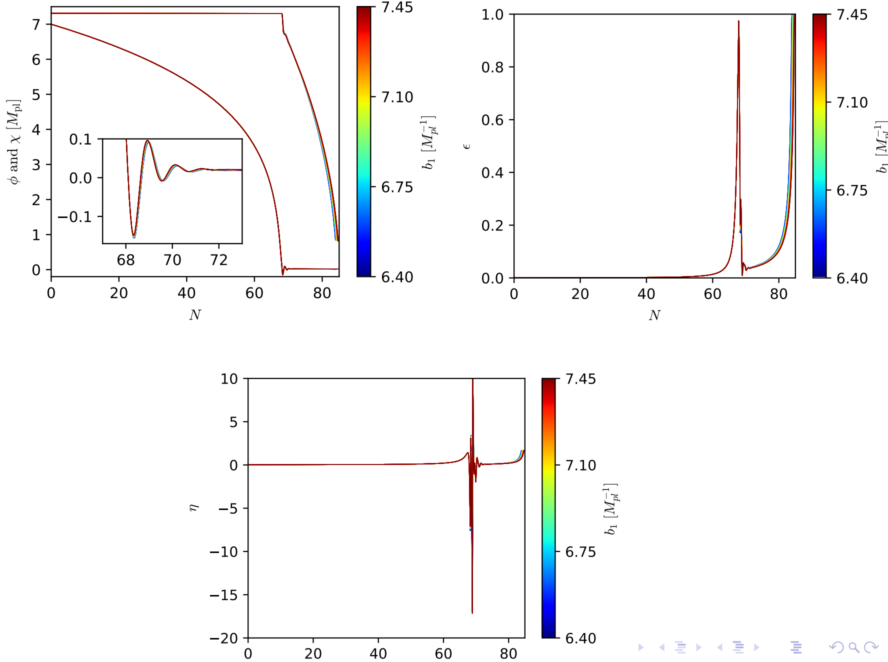
Two-field inflation with large kinetic coupling

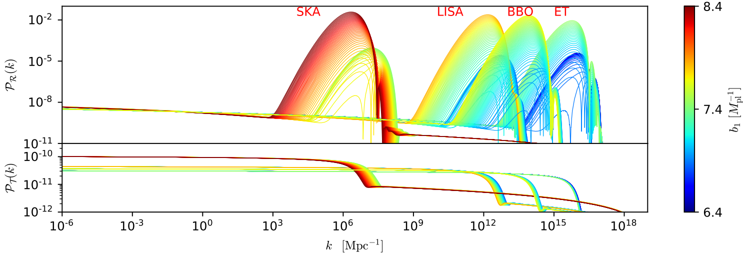
A novel mechanism: a two-field inflation with different inflaton effective masses (that leads to two stages of inflation) and a large non-standard kinetic coupling of the heavier field.
M. Braglia, D. K. Hazra, F. Fimelli, G. F. Smoot, L. Sriramkumar, A. A. Starobinsky, JCAP 2008 (2020) 001.

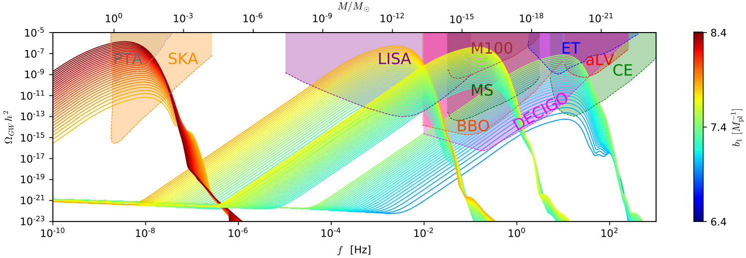
$$S(\phi,\chi) = \int d^4x \sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{f(\phi)}{2}(\partial\chi)^2 - V(\phi,\chi)\right]$$

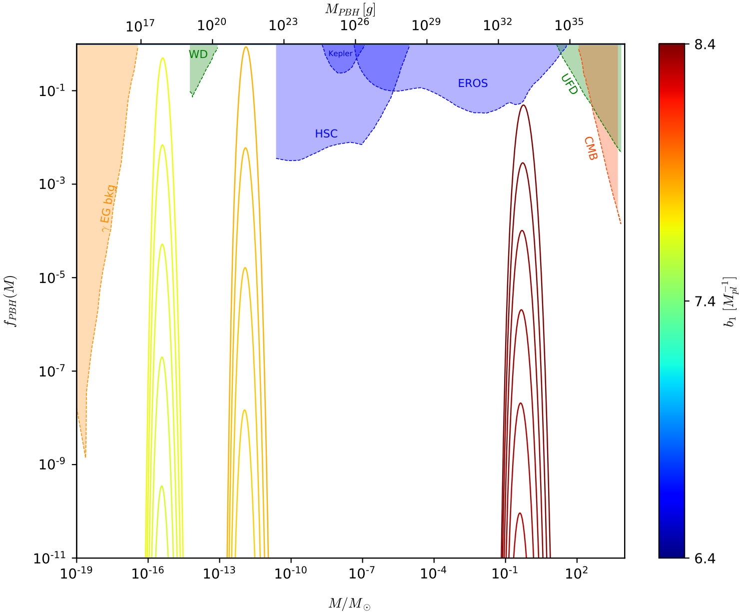
$$V(\phi,\,\chi) = V_0\frac{\phi^2}{\phi_0^2+\phi^2} + \frac{m_\chi^2}{2}\chi^2$$

Large kinetic coupling: $f(\phi) = \exp(b\phi),\ bM_{Pl} \gg 1$. The peak in the spectrum arises when the heavier field goes out of the slow-roll regime. It can lead to the formation of PBHs with a wide range of masses and to the generation of stochastic background of primordial gravitational waves produced by second order scalar perturbations.


ϕ and χ $[M_{\rm pl}]$
N
b_1 $[M_{pl}^{-1}]$
7.45
7.10
6.75
6.40
0.1
0.0
−0.1
68
70
72
ϵ
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
η
10
5
−5
−10
−15
−20


SKA
LISA
BBO
ET
$\mathcal{P}_\mathcal{R}(k)$
$\mathcal{P}_\mathcal{T}(k)$
k [Mpc^{-1}]
b_1 [$M_{\rm pl}^{-1}$]
8.4
7.4
6.4


$M/M_\odot$
PTA
SKA
LISA
M100
ET
aLV
CE
MS
BBO
DECIGO
$\Omega_{GW}h^2$
f [Hz]
b_1 [$M_{\rm pl}^{-1}$]
8.4
7.4
6.4



$M_{PBH}\,[g]$
10^{17}
10^{20}
10^{23}
10^{26}
10^{29}
10^{32}
10^{35}
WD
Kepler
EROS
HSC
UFD
CMB
γ EG bkg
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
$f_{PBH}(M)$
10^{-19}
10^{-16}
10^{-13}
10^{-10}
10^{-7}
10^{-4}
10^{-1}
10^{2}
$M/M_\odot$
8.4
7.4
6.4
$b_1\,[M_{pl}^{-1}]$

Conclusions

- The 3 simplest inflationary models without features in primordial power spectra of perturbations which have $n_s - 1 = -\frac{2}{N}$ ($R + R^2$, Higgs and the combined Higgs-R^2) are one-parametric and predict $r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.004$. However, actual values of $N(k)$ are slightly different for them.
- In two-field inflationary models with a large non-standard kinetic coupling of a heavier inflaton field, it is possible to produce a large peak at small scales in the primordial power spectrum of scalar adiabatic perturbations leading to the formation of PBHs and to the peak in the stochastic background of primordial gravitational waves produced by second order scalar perturbations. However, large peaks in the spectra require a large value of some of model parameters.