# Effective Theory of Warped Compactifications and the Implications for KKLT

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YKIS Feb 2022

# Motivation

- Is existence of string theory realizations of de Sitter space ~settled?
  - Notoriously difficult
  - After 20 years not entirely clear
  - Key question for string theory, and de Sitter space theory
- KKLT paradigm proposed construction
  - Questions because of "many moving parts"
  - No one has full Lagrangian
  - Hard to explicitly construct the 10d model of de Sitter space
- Yet exist compelling probe approximation and effective **4d** theory arguments

- One issue that others and I raised is that a relevant (and light) field ignored
  - Field is "**radion**" associated with throat
    - For explicit constructions was conifold deformation parameter
  - Was neglected in most earlier analyses
- Seemed ot lead to an instability for $\sqrt{g_s}M > \sim 7$ (in presence of antibrane)
    - Bena, Dudas, Grana, Luest, Blumenhagen, Klawer Schlechter independently found related instability
  - Could do effective 5d theory and identify origin and meaning of instability
- Destabilization for too small $g_s M^2$
  - M related to flux
  - leads to runaway of IR brane
  - **Independent of volume modulus metastability**

# New Work

- This talk (based on recent work with S Lust)
  - Effective potentials in warped compactifications more subtle
  - Need to take account of constraints
  - Significant change in IR of throat
  - Related to light KK modes in IR, even of the stabilized Kahler moduli

# Outline: New Work

- Turns out effective theories for warped compactifications much more subtle
- Need to impose metric constraints
- Low energy potential construction requires understanding full metric
- Qualitative change of potential behavior
- IN IR!

# Big Lesson

- Kahler moduli stabilized
- But in warped geometries their KK modes are still light
- Comparable in mass to conifold deformation parameter
- Runaway behavior goes away

# Outline

- Introduce KKLT: way of finding perturbative/manageable loophole
- Review
  - “conifold destabilization”
  - 5d EFT radion IS conifold deformation parameter
  - Seemed uplift destabilizes conifold (radion) if $M$ too small--Really a runaway radion
- Show why EFT must be modified
  - And how it resolves issue

# KKLT: Construction of de Sitter

- 10d Calabi-Yau /F-theory construction
  - Fluxes stabilize all complex structure moduli
  - But Kahler (volume) modulus $\sigma$ remains undetermined
- KKLT resolution
  - Step 1: Break no-scale structure with nonperturbative gauge contributions to stabilize Kahler modulus at large volume
    - Yields AdS4 as low-energy theory
- **Uplift energy**
  - Anti D3 brane; but in warped geometry (KS throat)
    - Suppresses uplift

# Warped Geometry (String Theory)

(Kachru, Polchinski, Verlinde)

- Cartoon: RS warped AdS throat glued onto CY
- CY acts as UV brane
- But Klebanov-Strassler AdS space
  - Constantly changing (increasing) AdS curvature
  - $AdS_5$ but with "running $N_{eff}$"
    - $N_{UV}$=MK; $N_{IR}$= M; hierarchy from $e^{-2 \Pi K/Mg_s}$
- Caps off at a critical length
- Conifold deformation region is "IR brane"
- KPV paper: 4d Mink space as low-energy EFT

# Potential Issue

- Slicing needs consistent UV and IR boundary conditions
  - Or additional space-dependent energy in bulk
- If all heavy fields integrated out (and ignored) you don't get a consistent geometry
- Need uplift energy to be present in UV
  - Otherwise you are assuing UV source already
    - No need for throat
  - Consistency requires means for transferring energy
- Whatever stabilizes system plays this role
- How to resolve?
- Clearly need a backreaction of some sort
  - Associated with stabilized geometry
  - Can absorb and transfer energy
- In 5d language, Goldberger-Wise Mechanism!

# Note: 10d

- Full construction too hard
  - Branes and antibranes
  - Many moduli fields
  - Which to keep?
- From 4d perspective perfectly fine
  - But is the underlying framework consistent and stable?
- We will grant ALL KKLT assumptions
  - String elements assumed possible
- Can construct 5d theory that has many of the essential features
  - Necessary preliminary for consistency of theory
  - Points to what we need to keep in 10d construction
- And show still a potential instability

# **5d** EFT

5d theory has all ingredients to probe consistency and stability of the gravity construction

I. Warping: 5d Klebanov-Strassler Geometry
   – $AdS_5$ but with "running $N_{eff}$"

II. Hierarchy from $e^{-2\Pi K/Mg_s}$

III. CY space serves as UV AdS boundary

IV. Conifold deformation ends space on IR boundary
   – (negative tension) Note has to be different AdS

V. Can construct full toy model
   Including stabilization mechanism
   Important but omitted

So let's study essential 5d ingredients: bent branes on boundary determining geometry and GW stabilization to allow for consistent geometry

# GW

- Guarantees radion moves so that
- Both junction conditions are satisfied
  - And entire bulk can be consistently sliced
- Notice radion is localized in IR
- Responding to mismatch in boundary conditions
- Clearly any stabilized geometry needs analog field
- GW bulk field, and radion

# GW Response to Perturbation

IR. To see more explicitly how the matching works, we add the IR boundary term

$$\mathcal{L} \supset \int d\phi e^{-4\sigma} \delta T \frac{\delta(\phi - \pi)}{r_c}. \tag{3.9}$$

The relevant equations of motion are the Einstein Equations that relate the metric to both the bulk and the brane energy. It will be important that we include the energy of the kinetic term of the GW field, since a shift in radion adjust the kinetic term in such a way that energy redistributes throughout the bulk. We use a general RS type metric

$$ds^2 = e^{-A(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2, \tag{3.10}$$

where to leading order $A = 2kr_c\phi$. We are interested in the deviation in the presence of a perturbing IR brane energy.

We use [56]

$$\left(\frac{\delta A}{\delta \phi}\right)^2 - k^2 r_c^2 - \frac{2}{3} k^2 r_c^2 \epsilon \Phi^2 - \frac{1}{6} \left(\frac{\delta \Phi}{\delta \phi}\right)^2 = 0 \tag{3.11}$$

and the junction condition at the IR brane is

$$\frac{\partial A(\phi)}{\partial \phi} = -kr_c \left(1 + \frac{1}{3} \delta T\right) \tag{3.12}$$

# Response

- Perturb radion slightly
- Leads to required shift in bulk energy

We need to include the change in bulk energy from the change in $\partial\Phi/\partial\phi$, which arises from the shift in $r_c$ in the presence of the perturbation. The dominant contribution to the change comes from the $A$ term in the GW solution above and is approximately

$$\delta\left(\frac{\partial\Phi(\pi)}{\partial\phi}\right) = (4+\epsilon)\epsilon v_v k\pi\delta r \tag{3.13}$$

whereas the initial dominant contribution to the derivative comes from the $B$ term and is approximately

$$\left(\frac{\partial\Phi(\pi)}{\partial\phi}\right) = -\epsilon v_v \tag{3.14}$$

which gives us the correct junction condition if

$$\delta r = \frac{(kr_c)^2 T}{\epsilon^2 k\pi v^2} \tag{3.15}$$

The important thing (here we are assuming the uplift yields Minkowski) is that the GW field take the form such that the uplift is that required in the UV as well. We have

$$\delta\left(\frac{\partial\Phi(0)}{\partial\phi}\right) = (4+\epsilon)\epsilon v_v k\pi\delta r e^{-4kr_c\pi}e^{-\epsilon k\pi r_c} \tag{3.16}$$

$$\left(\frac{\partial\Phi(0)}{\partial\phi}\right) = -\epsilon v_h$$

# Can identify GW field in 10d theory!

$$V = \lambda_1 \phi^4 + \lambda_2 \phi^{4-\epsilon}$$

From a dual perspective, you really have a running coupling
Explicit breaking conformal invariance from running
Spontaneous breaking at IR brane position

Can explicitly identify running in KKLT: Hedecker et al

in the IR. Ref. [19] identifies the GW field $H$ and argues its slowly varying potential in the radial direction is a result of the kinetic term for a field originating in the 10d theory from the flux of the NS 2-form potential $B_2$ on the $S^2$ cycle of the $T^{1,1}$. They explicitly construct a potential consistent with "running $N_{eff}$" and describe how with this field they can stabilize a geometry that consists of the CY region, a conifold region with constant warp factor, and the warped deform conifold. This is in the spirit of the dual interpretation of the GW mechanism,

# Can Identify Radion in KKLT!

First let's consider the "conifold instability"

- *S*: Conifold deformation parameter

$$\sum_{a=1}^{4} \omega_a^4 = S\,. \tag{3.10}$$

The deformation parameter $S$ is the complex structure modulus whose absolute value corresponds to the size of the 3-sphere at the tip of the cone.

$$\int_A \Omega_3 = S\ , \tag{3.11}$$

# Potential for *S*

The supersymmetric potential for this field induced by the Klebanov-Strassler geometry is

$$V_{KS} = \frac{\pi^{3/2}}{\kappa_{10}} \frac{g_s}{(Im\rho)^3} \left[ c \log \frac{\Lambda_0^3}{|S|} + c' \frac{g_s(\alpha' M)^2}{|S|^{4/3}} \right]^{-1} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2 , \tag{3.12}$$

where $g_s$ is the stabilized vev of the dilaton, $Im\rho = (\mathrm{Vol}_6)^{3/2}$, $c$ as we argue below is not relevant here (and is in any case suppressed in the small $S$ region), whereas the constant $c'$, multiplying the term coming solely from the warp factor, denotes an order one coefficient, whose approximate numerical value was determined in [46] to be $c' \approx 1.18$.

Add potential from antibrane:

The antibrane contributes a perturbation

$$V_{\overline{D3}} = \frac{\pi^{1/2}}{\kappa_{10}} \frac{1}{(Im\rho)^3} \frac{2^{1/3}}{I(\tau)} \frac{|S|^{4/3}}{g_s(\alpha' M)^2} . \tag{3.18}$$

We follow [56] and define $c'' = \frac{2^{1/3}}{I(0)} \approx 1.75$ . For $p$ anti-D3 branes the potential is multiplied by $p$, and this is taken care by simply replacing $c'' \to c'' p$.

# “Conifold” instability

The general form of the potential (we factor out $\lambda_1^2 \pi g_s / c'$) is

$$V = S^{4/3} \left( 1 + \epsilon \log \frac{S}{\Lambda_0^3} \right)^2 + \delta S^{4/3} \tag{3.28}$$

The barrier disappears when $\delta / \epsilon^2 = 9/16$.

We see that the perturbation from the antibrane (yielding the $\delta$ type perturbation above) yields the potential proportional to the above with $\delta = c'' c' g_s / \pi K^2$ and $|\epsilon| = M g_s / 2\pi K$. By writing it this way we keep $\epsilon$ and $\delta$ as small parameters. This gives precisely the stability condition found in [59], namely

$$\sqrt{g_s} M > M_{\min} \qquad \text{with} \qquad M_{\min} = \frac{8}{3} \sqrt{\pi c' c''} \approx 6.8 \sqrt{p}\,. \tag{3.29}$$

# S Potential

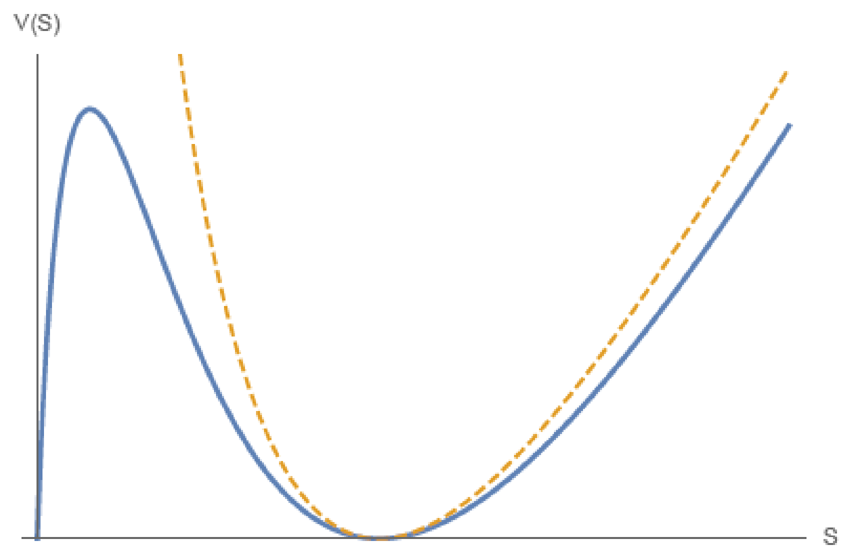



Figure 1: The potential $V_{KS}$ of [16] for the complex structure modulus $S$ of the Klebanov-Strassler throat given in (2.17). The solid blue line corresponds to the full potential, while the dotted orange line does shows the naïve potential that does not take into account the effects of warping ($c' = 0$). Both potentials have the same supersymmetric minimum but differ drastically at small $S$.

The potential (2.17) has a supersymmetric minimum, corresponding to $\partial_S W = 0$, which, for $S \ll \Lambda_0^3$, is at

$$s_{\mathrm{KS}} \simeq \Lambda_0^3 \exp\left(-\frac{2\pi K}{g_s M}\right) . \tag{2.19}$$

# With Uplift

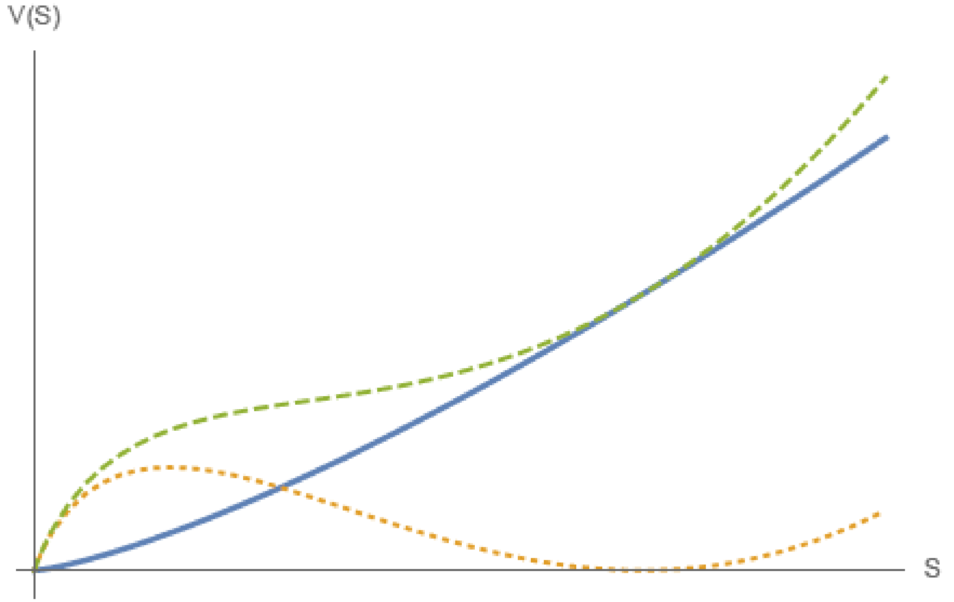



Figure 2: The contribution $V_{\overline{D3}}$ (solid blue line) of an $\overline{D3}$-brane placed in the Klebanov-Strassler throat to the potential for $S$. The two other lines represent the original potential $V_{KS}$ (dotted orange line) for the specific value $\sqrt{g_s}M = 6$ as well as the superposition $V_{KS} + V_{\overline{D3}}$ (dashed green line).

# Radion is the Conifold Deformation Parameter

- We identify based on its effect on metric
- And its potential
- $S \sim \Phi^3$ where $\Phi$ is the radion in GW potential
- This means checking for stability of RS type geometry is checking for  stability wrt conifold deformation parameter!
- Exactly what Bena, Dudas, Grana, Luest; Blumenhagen, Klaewer Shlechter did

# "Radion" Scalar: S~Φ³

We can also expand about $S_{KS}$ to find

$$V \approx S^{4/3} \lambda_2^2 (\frac{S - S_{KS}}{S_{KS}})^2$$

mass is not suppressed by $\epsilon$. When we use

$$m_S^2 \equiv \frac{1}{M_{pl}^2} G^{S\bar{S}} \partial_{\bar{S}} \partial_S V \Big|_{S=s_{\rm KS}} . \qquad (3.15)$$

we find the $S$ mass squared is suppressed by $1/g_s M^2$. In terms of the properly normalized field $\phi$ (see below), the mass squared scales (over the exponential suppression) as $1/(g_s M^2)^2$, which is how all KK masses associated with the IR region of the conifold throat would scale as well

Eq. 3.16 as,

$$\phi = \frac{3 M_{pl} \sqrt{c'}}{\pi^{1/2} \|\Omega\| V_w^{1/2}} \alpha' \sqrt{g_s} M S^{1/3} = \frac{3 \sqrt{g_s} M \sqrt{c'}}{8 \pi^4 \alpha' \|\Omega\|} S^{1/3} , \qquad (3.16)$$

where $c'' \approx 1.75$ so that the parameter $S$ is indeed related to $\phi^3$, and is the parameter determining the warping in the throat. This $\phi$ is precisely the radion of GW and has the correct potential to both determine the length of the throat and the warping in the IR as well as to respond to perturbations to generate a consistent geometry. The radion mass squared, as with the values of KK mass squared. is suppressed by a factor $1/(g_s M)^2$ in units of the confinement scale, where the confinement scale is suppressed relative to the warped string scale by $1/(\sqrt{g_s M})$.

# Notice form of potential: conifold ⟷ radion

The supersymmetric potential for this field induced by the Klebanov-Strassler geometry is

$$V_{KS} = \frac{\pi^{3/2}}{\kappa_{10}} \frac{g_s}{(Im\rho)^3} \left[ c \log \frac{\Lambda_0^3}{|S|} + c' \frac{g_s(\alpha' M)^2}{|S|^{4/3}} \right]^{-1} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2 , \qquad (3.12)$$

where $g_s$ is the stabilized vev of the dilaton, $Im\rho = (\mathrm{Vol}_6)^{3/2}$, $c$ as we argue below is not relevant here (and is in any case suppressed in the small $S$ region), whereas the constant $c'$, multiplying the term coming solely from the warp factor, denotes an order one coefficient, whose approximate numerical value was determined in [46] to be $c' \approx 1.18$.

The potential for the $S$ field is essentially the potential above that we had for a GW field, but takes a slightly different form than that above due to supersymmetry, namely

$$V = S^{4/3} \left( \lambda_1 - \lambda_2 \log \frac{\Lambda_0^3}{S} \right)^2 = \lambda_1^2 S^{4/3} \left( 1 - \frac{\lambda_2}{\lambda_1} \log \frac{\Lambda_0^3}{S} \right)^2 \approx \lambda_1^2 S^{4/3} \left( 5 - 6 \left( \frac{S}{\Lambda_0^3} \right)^\epsilon + 2 \left( \frac{S}{\Lambda_0^3} \right)^{2\epsilon} \right) \qquad (3.13)$$

where rewriting the potential in this GW form breaks down near the "IR brane" where $(2\pi M/K) \log S/\Lambda_0^3$ gets big. Really the original form is enough to see that we have weakly explicitly broken scale invariance. Here $\lambda_1 = K g_s$ and $\lambda_2 = (M/2\pi)$, and $\epsilon = \lambda_2/\lambda_1$. The minimum occurs at $S_{KS} = \Lambda_0^3 e^{-2\pi k/M g_s} = \Lambda_0^3 e^{-\lambda_1/\lambda_2} = \Lambda_0^3 e^{-1/\epsilon}$. Here the $S^{4/3}$ dependence comes from the Kahler potential whereas the remaining dependence is from the superpotential. The nonrernormalization theorems in the supersymmetric potential guarantee the full potential is always proportional to the leading order potential.

# Runaway radion if too big a perturbation

The general form of the potential (we factor out $\lambda_1^2 \pi g_s / c'$ is

$$V = S^{4/3}\left(1 + \epsilon \log \frac{S}{\Lambda_0^3}\right) + \delta S^{4/3} \tag{3.19}$$

The barrier disappears when $\delta/\epsilon^2 = 9/16$.

We see that the perturbation from the antibrane (yielding the $\delta$ type perturbation above) yields the potential proportional to the above with $\delta = c" c' / g_s \pi \lambda_1^2$ and $|\epsilon| = M g_s / 2\pi K$. By writing it this way we keep $\epsilon$ and $\delta$ as small parameters. This gives precisely the stability condition found in [56], namely

$$\sqrt{g_s} M > M_{\text{min}} \qquad \text{with} \qquad M_{\text{min}} = \frac{8}{3}\sqrt{\pi c' c''} \approx 6.8\sqrt{p}\,. \tag{3.20}$$

# Real potential instability

- Need largish $g_s M^2$
- But hard to satisfy
- Hierarchy problematic
  - $K/Mg_s \sim KM/M^2 g_s$
  - KM bounded in a given geometry
- Another problem
  - Cosmological phase transition for RS like geometries
    - Cremenelli, Nicolis, Rattazzi//Hassanain, March-Russell, Schellvinger
  - High temperature AdS/Schwarschild
  - Cosmological phase transition won't complete
  - Need to evolve to RS
  - Upper bound on $M^2 \sim 21$ for this geometry
- Caveat: We are assuming supergravity solution applies even for small M
  - However if it doesn't we still have to work out solution to have example

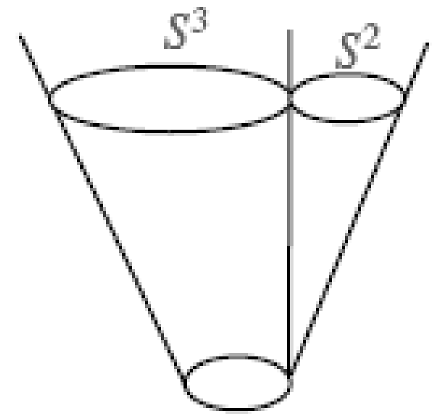
# Warped Conifold Potential

- Turns out the assumed S potential is not correct
  - In IR!
- Need to impose various constraints
- Let’s get a taste of how this wokrs

# A CLOSER LOOK AT THE CONIFOLD POTENTIAL

- deformed conifold: $$\sum_{i=1}^{4} z_i^2 = S$$

  $\rightarrow$ S = complex structure modulus



$S \sim vol(S^3)$

- Superpotential:

$$W \sim \int G_3 \wedge \Omega = \frac{M}{2\pi i} S\left(\log \frac{\Lambda_0^3}{S} + 1\right) + \frac{i}{g_s} KS$$

- Kähler potential requires knowledge of warp factor: Klebanov-Strassler solution:

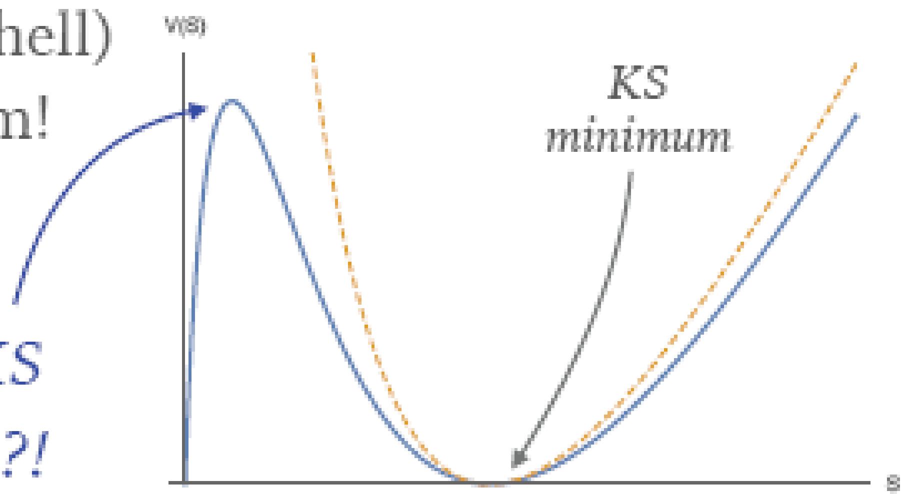
$$e^{-4A(\tau)} \sim \frac{g_s(\alpha' M)^2}{|S|^{\frac{4}{3}}} I(\tau)$$

# A CLOSER LOOK AT THE CONIFOLD POTENTIAL

- Kähler metric: [Douglas, Shelton, Torroba, '07, '08]

$$G_{S\bar{S}} = \partial_S \partial_{\bar{S}} K \sim \int e^{-4A} \chi_S \wedge \bar{\chi}_{\bar{S}} \approx e^{-4A(\tau=0)} \sim \frac{g_s(\alpha' M)^2}{|S|^{\frac{4}{3}}}$$

- Anti-brane instability:
  Requires knowledge of the (off-shell) potential away from the minimum!



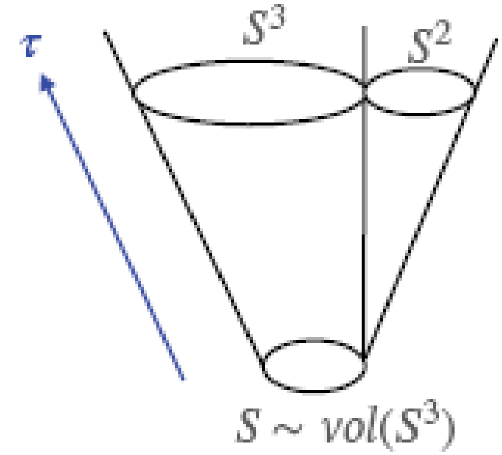

*can we use the KS warp factor here?!*

# COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- Deformed conifold in $\mathbb{C}^4$:

$$\sum_{i=1}^{4} z_i^2 = S$$

S = complex structure modulus




- metric on the deformed conifold:

$$ds^2_{DC} = \frac{S^{2/3}}{2} K(\tau) \left[ \frac{1}{3K^3(\tau)} \left[ d\tau^2 + (g^5)^2 \right] + \cosh^2\left(\frac{\tau}{2}\right) \left[ (g^3)^2 + (g^4)^2 \right] + \sinh^2\left(\frac{\tau}{2}\right) \left[ (g^1)^2 + (g^2)^2 \right] \right]$$

complex structure just a conformal factor?!

Answer: did not fix gauge (coordinates) yet!

# COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- First: understand gauge fixing without warping:

$$ds_{10}^2 = ds_4^2 + ds_{DC}^2$$

- Gauge fixing of Calabi-Yau deformations:

$$g_{ij} \to g_{ij} + \delta g_{ij}$$

[Candelas, de la Ossa '91]

$$\Rightarrow \qquad g^{ij}\delta g_{ij} = 0 \qquad \nabla^i \delta g_{ij} = 0$$

*(traceless)* *(harmonic)*

*(will get modified in the presence of warping!)* [Giddings, Maharana '05], [Shiu et al. '08], [Douglas, Torroba '08]

- Deformed conifold:

$$\delta g_{ij} = \partial_S g_{ij} \sim \frac{1}{S} g_{ij}$$

harmonic but not traceless!

With warp factor no longer traceless!

# COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- Add compensating diffeomorphism:

$$\delta g_{ij} = \partial_S g_{ij} + 2\nabla_{(i}\eta_{j)}$$

Ansatz:

$$\eta = \left(\eta^\tau(\tau), 0, 0, 0, 0, 0\right)$$

Solution:

$$\eta^\tau(\tau) = -\frac{1}{2S}\frac{\sinh(2\tau) - 2\tau}{\sinh^2\tau}$$

- Interpretation:

Replace $\tau$ with "new" $S$-dependent radial variable: $\tau \to T(\tau, S)$

$$\frac{dT}{dS} = \eta^\tau\left(T(\tau,S),S\right) \longrightarrow$$

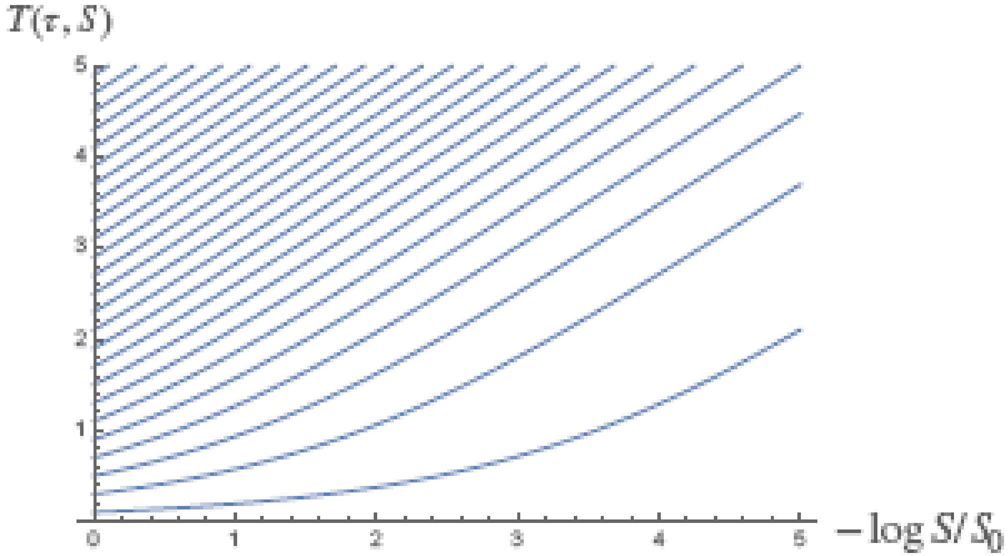
Analytic solution:

$$T(\tau,S) = F\left[F^{-1}(\tau) - \frac{1}{4}\log\frac{S}{S_0}\right]$$

with $F(x) = \frac{1}{2}\log\left[\sinh(2x) - 2x\right]$

# COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- The radial coordinate as a function of $S$:




- UV behavior ($\tau \to \infty$): $T(\tau, S) \to \tau - \log S/S_0$
- Compare with UV expansion of the metric:

$$ds^2_{DC}(\tau \to \infty) \to S^2 e^{2T/3}\left(\frac{1}{9}dT^2 + \frac{1}{6}ds^2_{T^{1,1}}\right) = S_0^2 e^{2\tau/3}\left(\frac{1}{9}d\tau^2 + \frac{1}{6}ds^2_{T^{1,1}}\right)$$

Deformation acts only in the IR!

# Deformtoins of warped geometries

The most general form of a background which preserves all isometries of a four-dimensional maximally-symmetric spacetime takes the form

$$ds_{10}^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) dy^m dy^n \,, \tag{4.1}$$

We need to consider both the variations of the warp factor $\delta A$ and the variations of the internal metric $\delta g_{mn}$. It was found in [9, 10] that these are not independent but related by

$$\delta A = -\frac{1}{4} g^{mn} \delta g_{mn} \,. \tag{4.3}$$

This can be understood as an extension of the traceless condition (2.3) to the warped case. The harmonic gauge condition (2.2) also needs to be modified accordingly and becomes

$$\nabla^m (e^{2A} \delta \pi_{mn}) = 0 \,, \tag{4.4}$$

# Solve using diffeomorpism

To find solutions $\delta A$ and $\delta g_{mn}$ satisfying these conditions, one can start from a deformation $\delta g^0_{mn}$ and add an infinitesimal diffeomorphism which acts as a compensating gauge transformation,

$$\delta g_{mn} = \delta g^0_{mn} + \nabla_m \eta_n + \nabla_n \eta_m \,. \tag{4.6}$$

With this ansatz the modified harmonic gauge condition (4.4) becomes a set of second order differential equations on $\eta_m$ which have to be solved to find $\delta g_{mn}$. Subsequently, one can use (4.3) to determine $\delta A$.

# Even More General

Here, $u^I(x^i)$ denotes a set of four-dimensional scalar fields, parametrizing moduli or also massive excitations of the background solution. It seems to be natural to introduce a similar $x^i$ dependence for the warp factor as $A\left[\tau, u^I(x^i)\right]$. However, as we will see this notation has to be taken with a grain of salt.

The general space-time dependent ansatz for the metric now reads

$$ds^2 = e^{2A\left(\tau, u^I(x)\right)} g_{\mu\nu}(x) dx^\mu dx^\nu + \left[e^{f\left(\tau, u^I(x)\right)} d\tau + K_\mu(\tau, x) dx^\mu\right]^2 , \qquad (8.2)$$

where $g_{ij}$ denotes the components of an a-priori undetermined four-dimensional metric and we also allow for possible off-diagonal components $K_i$.

$$3D_\tau^2 A + 6(D_\tau A)^2 + g_{ab} D_\tau \phi^a D_\tau \phi^b + 2V(\phi) - \frac{1}{4} e^{-2A} R^{(4)} = 0 .$$

Satisfy higher d eq of motion

above as the off-diagonal $E_{i5}$ component (8.8) of the five-dimensional Einstein equations. We recall that it takes the form

$$3D_I D_\tau A + 2g_{ab} D_I \phi^a D_\tau \phi^b = 0 , \qquad (8.15)$$

No off-diagonal

constraint arises from the traceless part of the five-dimensional Einstein equatio[n] vanishing four-dimensional momenta, $\partial_\mu u^I = 0$, and a symmetric background spa[ce] $R_{\mu\nu} = \frac{1}{4} g_{\mu\nu} R$, it reduces to

$$\left(\nabla_\mu \partial_\nu u^I - \tfrac{1}{4} g_{\mu\nu} \Box u^I\right) \left[2D_I A + D_I f\right] = 0 .$$

Traceless EE

The constraint therefore reads

$$2D_I A + D_I f = 0 .$$

# Solve

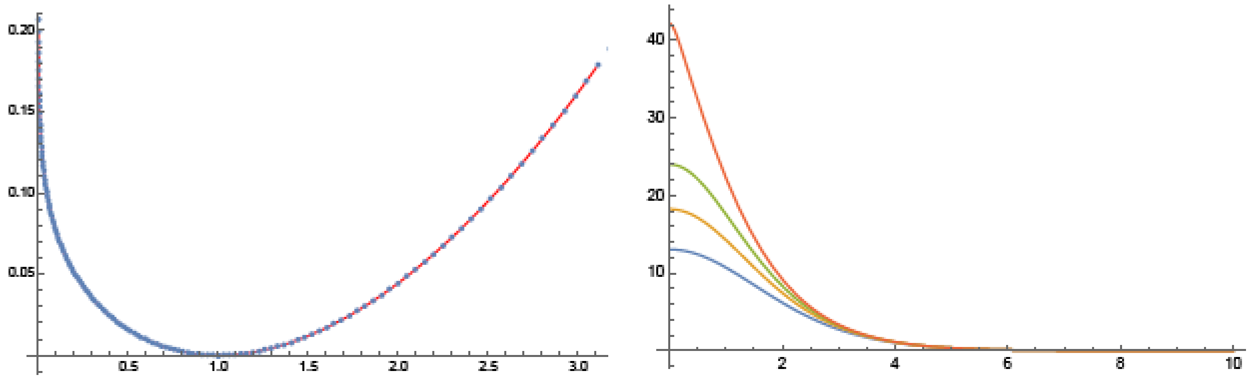



Figure 2: Left: The potential as a function of $S/S_0$. Right: The warp factor $e^{-4A(\tau)}$ for $S/S_0 = 2.0, 1.0, 0.5, 0.1$ Both plots are created using the differential constraints (8.15) and (8.17) but ignoring the Hamiltonian constrain (8.13).

# No Second Minimum

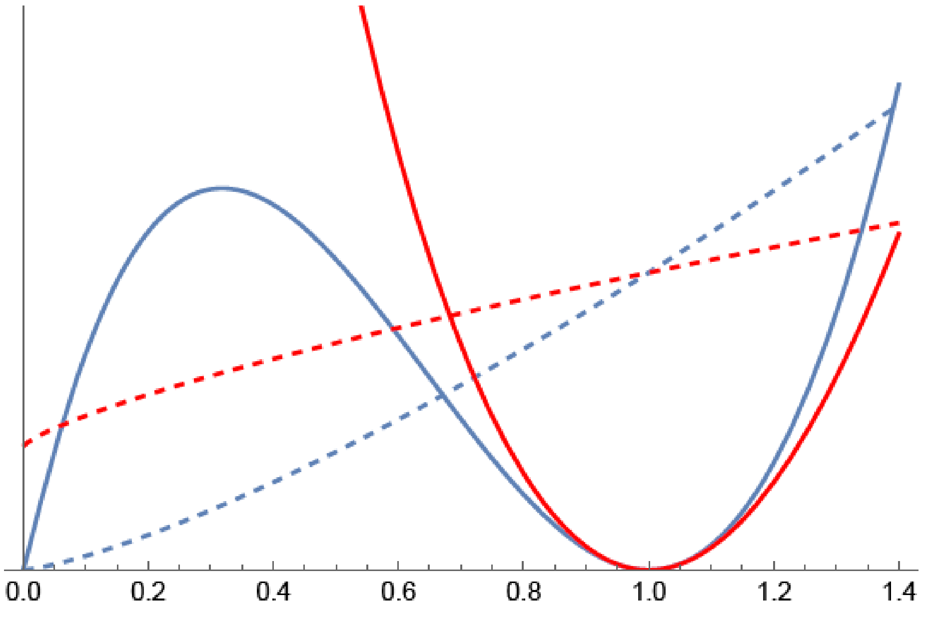



Figure 3: Comparison of the potential computed by [10] (blue) and our potential (red). The solid line is the potential for the conifold modulus $S$ and the dashed line the contribution from the antibrane. Their superposition is illustrated in Figure 4.

# Punchline

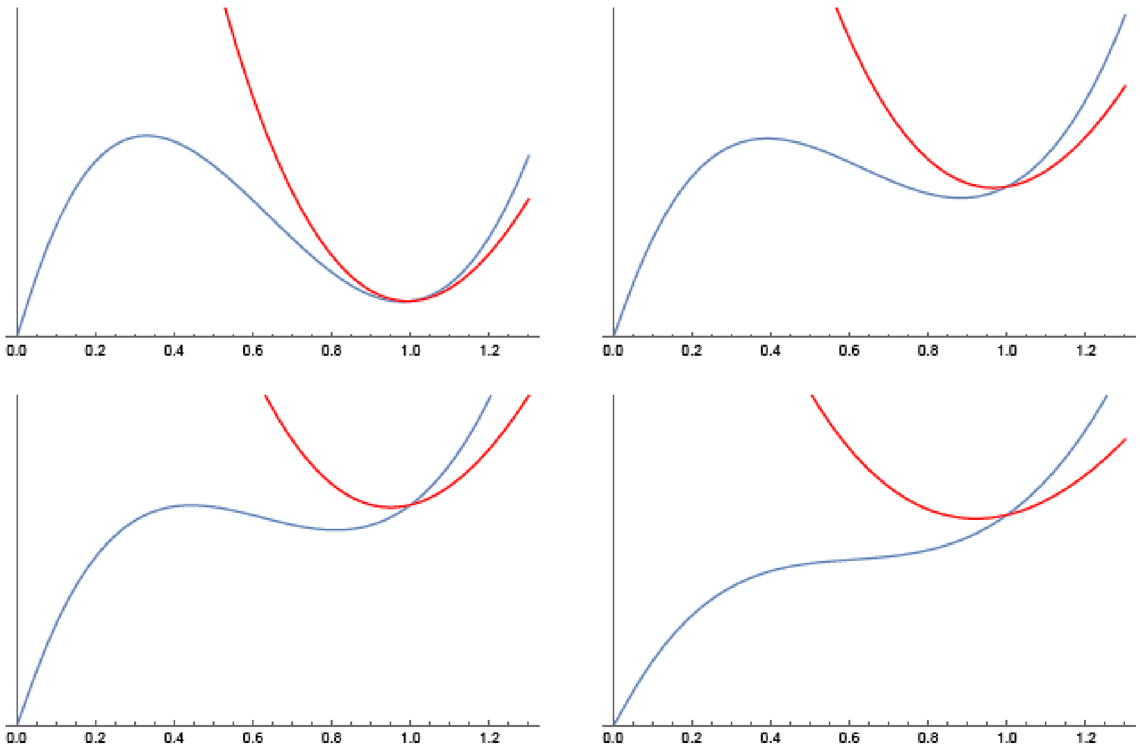



Figure 4: Superposition of the potential for $S$ and the antibrane potential for different values of $M^2$ (from large to small). The "old" potential is in blue, for small values of $M$ its minimum disappears. The red potential, which was computed using the constraint (8.15), always has a minimum, irrespective of the value of $M$.

# Conclude

- Low energy effective theory nontrivial in context of warped compactifications
- Solving Einstein's Equations consistently even off-shell leads to qualitative change in form of potential
- Here related to fact that warped compactificaton shape can change in the IR
- Not determined solely by the UV stabilization
- Essentially allows for KK modes of volume moduli
- Though not yet explicit in our formalism
- Resolves the mysterious and now-seen-to-be spurious instability
- Can have interesting consequences in the future