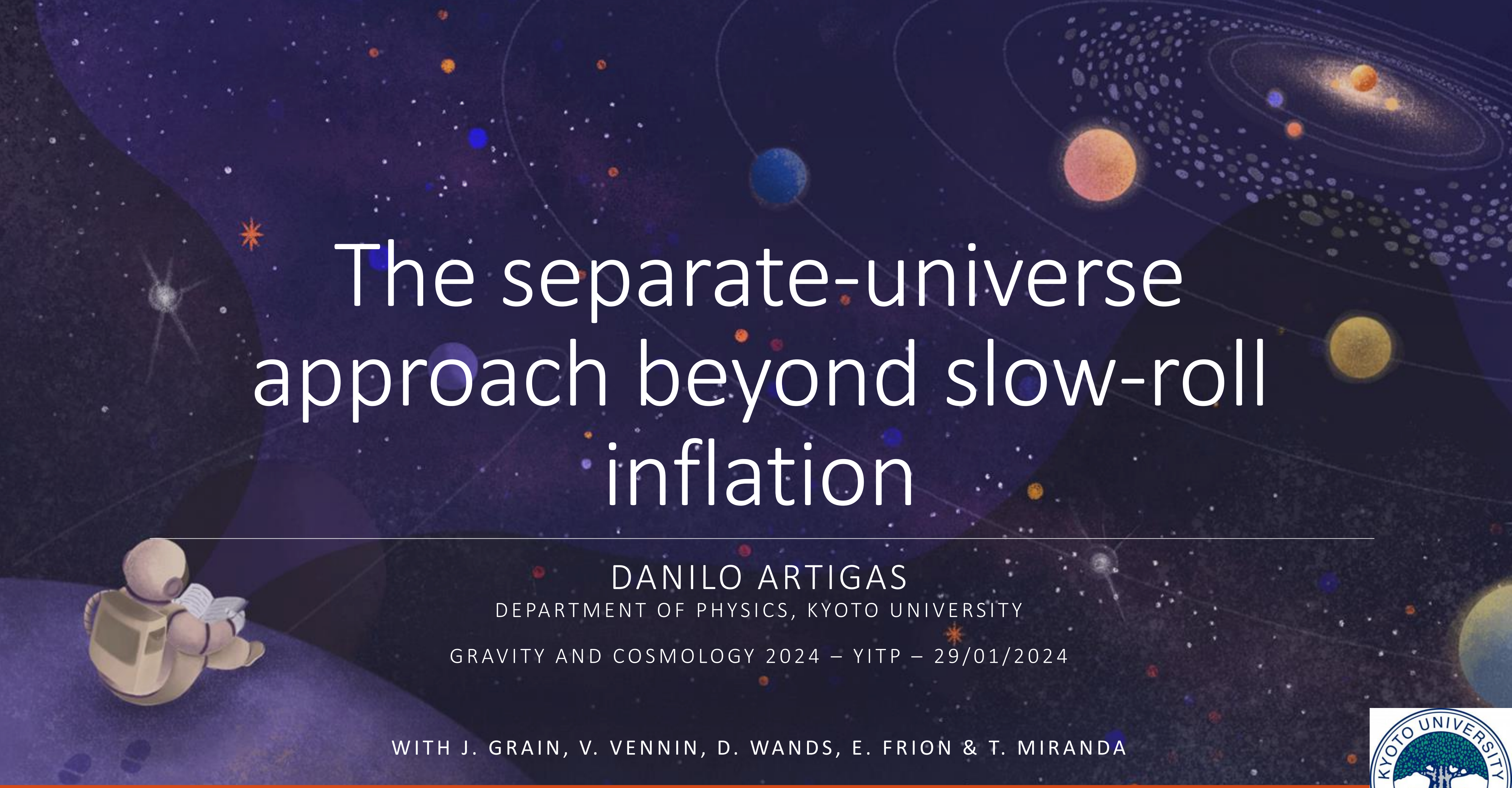

The separate-universe approach beyond slow-roll inflation

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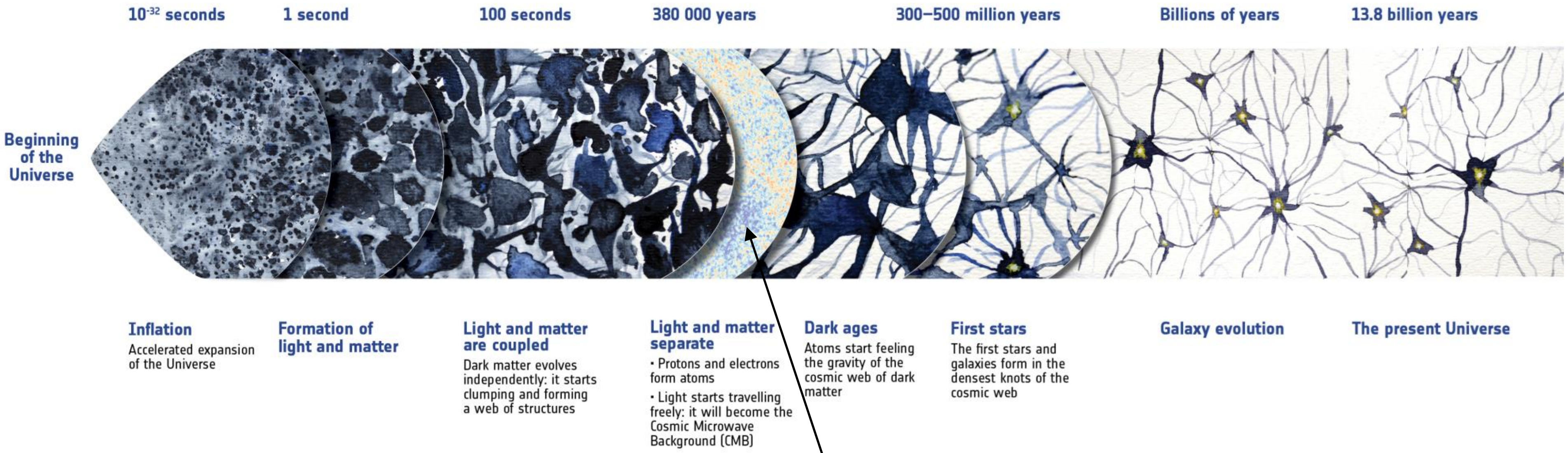
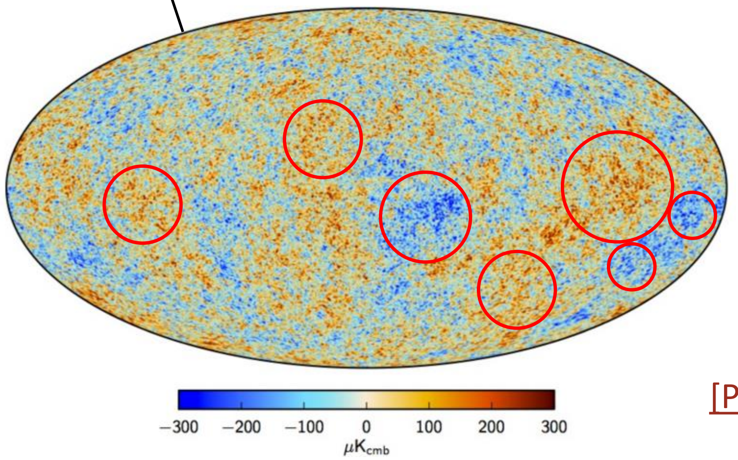
Credit: Sacha Berna

Introduction



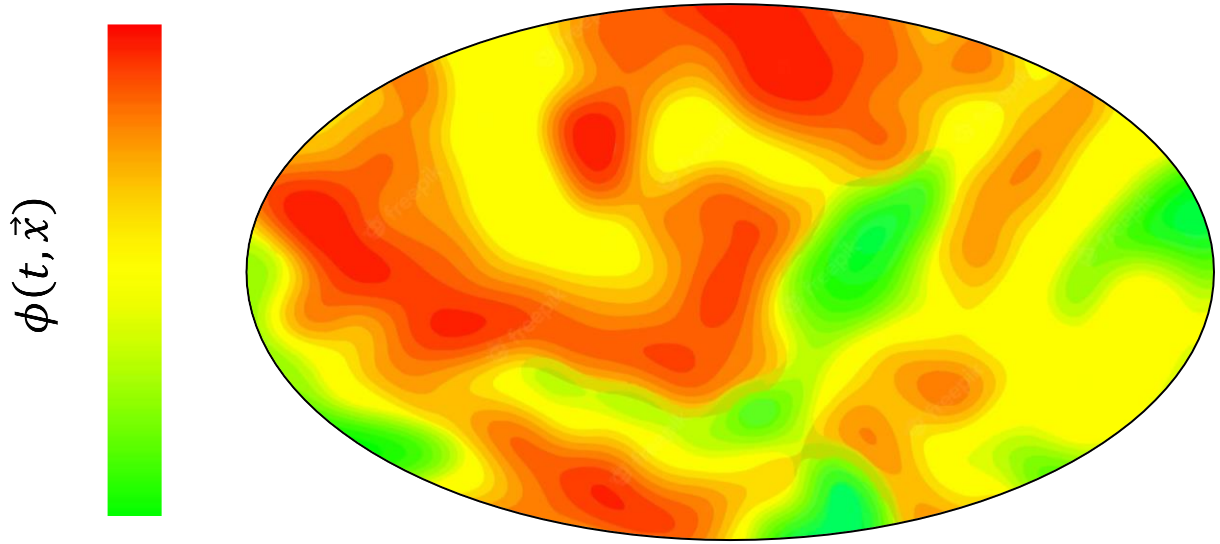
- **Inflation**

Quantum fluctuations seed large-scale inhomogeneities.



[Planck Collaboration (2016)]

Separate universe

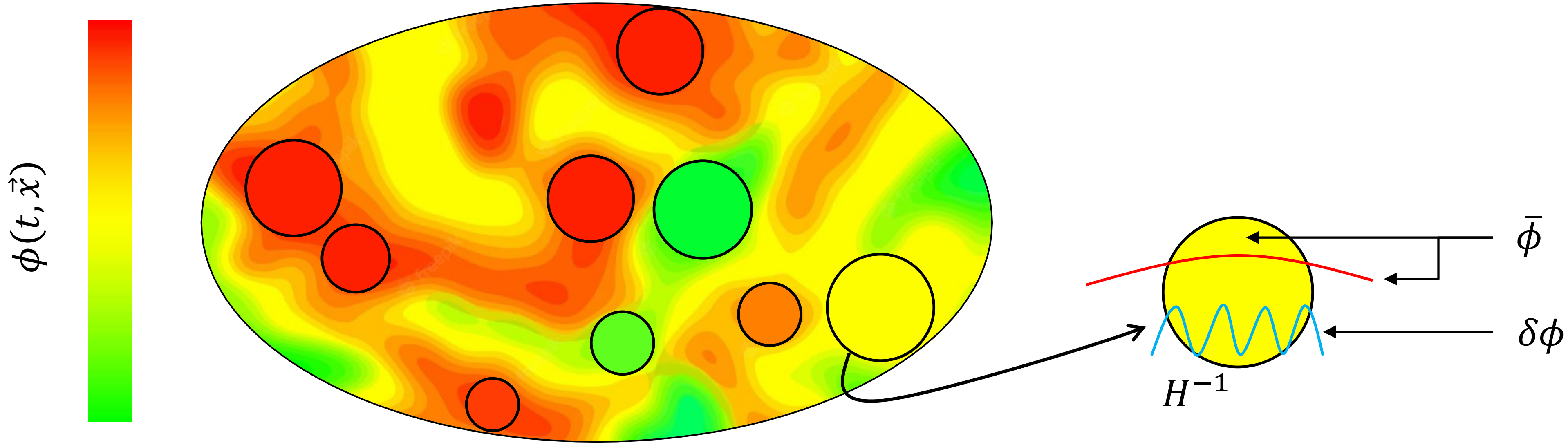


At the non-linear level:

Gradient expansion / $\delta\mathcal{N}$ formalism

[Starobinsky (1983)]
[Salopek, Bond (1990)]
[Sasaki, Stewart (1996)]
[Sasaki, Tanaka (1998)]
[Wands, Malik, Lyth, Liddle (2000)]

Separate universe



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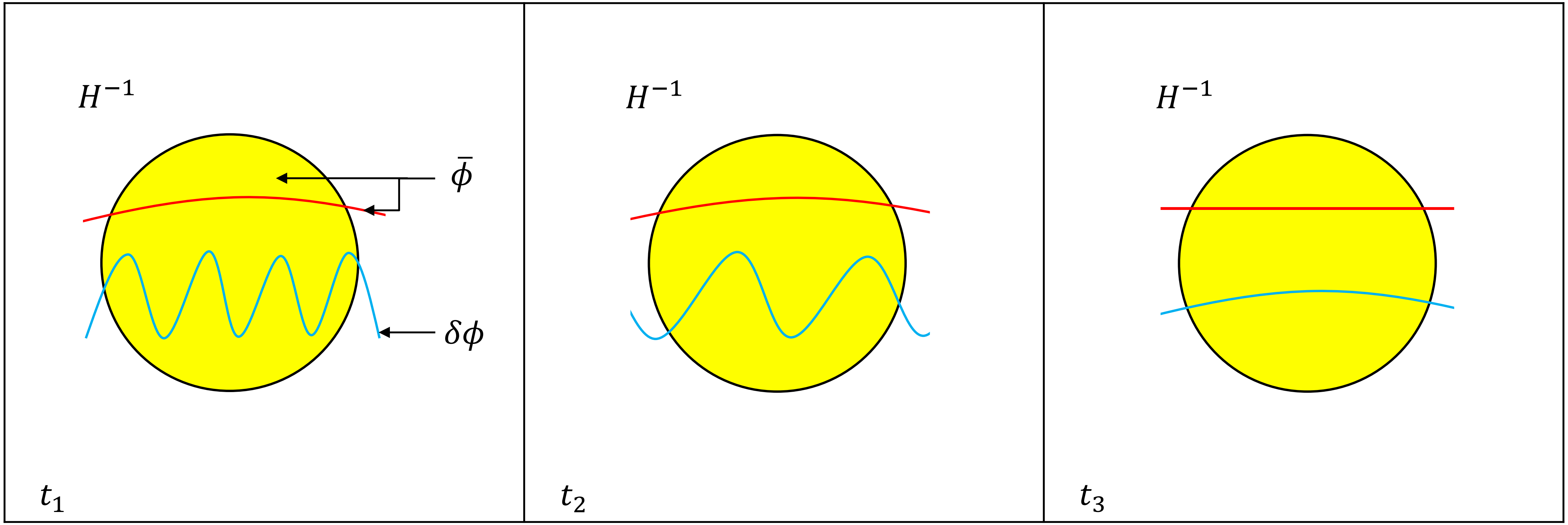
Stochastic inflation

$$\dot{\bar{\phi}} = \frac{N}{\sqrt{\gamma}}\dot{\bar{\pi}}_\phi + \xi_\phi$$

$$\dot{\bar{\pi}}_\phi = -N\sqrt{\gamma}\, V_{,\phi} + \xi_{\pi_\phi}$$

[Starobinsky (1982)]
[Nambu, Sasaki (1988)]
[Fujita et al. (2013)]
[Grain, Vennin (2017)]



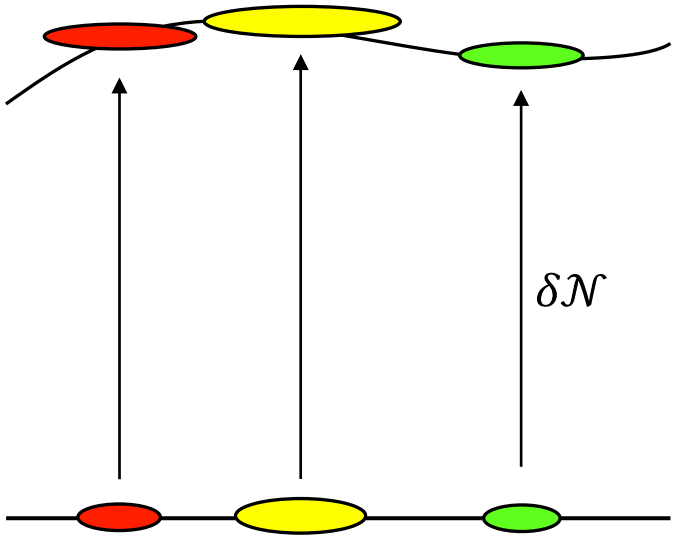
Assumptions:

- Quantum fluctuations experience a quantum-to-classical transition. [Micheli's talk] [Polarski, Starobinsky (1996)] [Lesgourgues, Polarski, Starobinsky (1997)] [Kiefer, Polarski, Starobinsky (1998)] [Martin, Vennin (2018)]
- The long-wavelength modes evolve as the background.
- Use of spatially-flat and of uniform-expansion gauges.

Gauges in stochastic inflation

$$\dot{\bar{\phi}} = \frac{N + \cancel{\delta N}}{\sqrt{\gamma + \cancel{\delta\gamma}}}\bar{\pi}_\phi + \xi_\phi + \mathrm{grad.}$$

$$\dot{\bar{\pi}}_\phi = -(N + \cancel{\delta N})\sqrt{\gamma + \cancel{\delta\gamma}}\, V_{,\phi} + \xi_{\pi_\phi} + \mathrm{grad.}$$



Uniform-expansion gauge

$\delta\mathcal{N}(t_{in}, t_f, \vec{x}) = \zeta(t_f, \vec{x})$

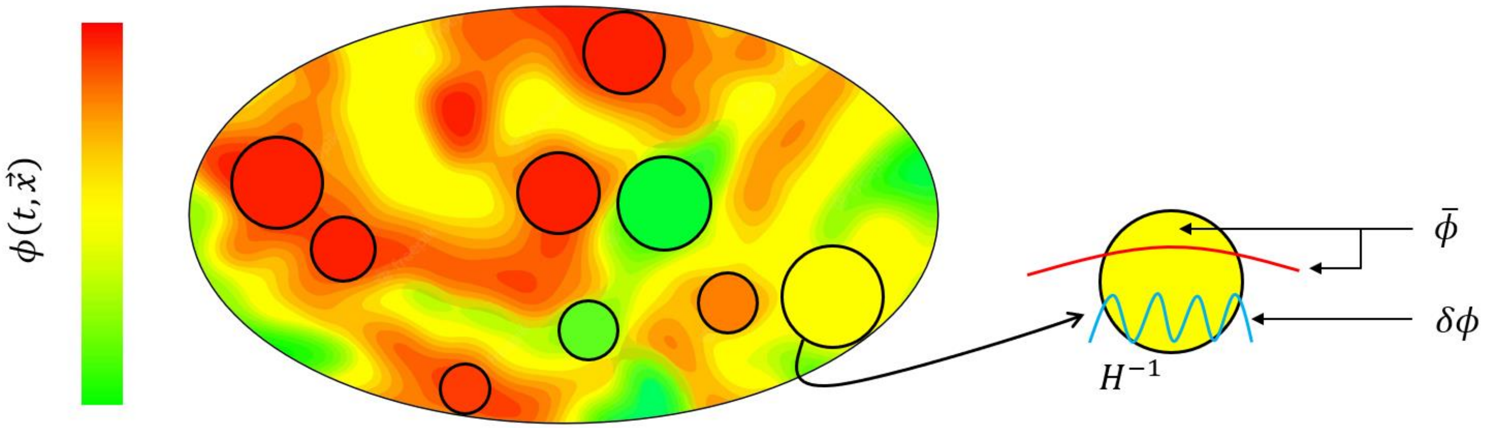
Spatially-flat gauge

(FLRW metric)

Can we safely neglect gradient terms with those gauges?

HOW CAN WE FORMALISE THE SEPARATE-UNIVERSE APPROACH BEYOND SLOW ROLL?

[Pattison, Vennin, Assadullahi, Wands (2019)]
[Miranda, Frion, Wands (2020)]
[Briaud, Vennin (2023)]
[Fanizza, Marozzi, Medeiros (2023)]



Long-wavelength modes evolution

Cosmological perturbation theory (CPT)	Separate universe (SU)
1. Perturb and expand $$\mathcal{H}_{GR} \to \mathcal{H}_{GR}^{(0)} + \delta\mathcal{H}_{GR}$$ 2. Set a homogeneous and isotropic background $$\to \mathcal{H}_{FLRW}^{(0)} + \delta\mathcal{H}_{GR}$$	1. Perturb and expand $$\mathcal{H}_{FLRW} \to \mathcal{H}_{FLRW}^{(0)} + \delta\mathcal{H}_{FLRW}$$ Perturbations evolve as the background.

Compare at large scales

The separate-universe approach is valid if:

- $\partial_i\partial_j = \mathcal{O}(k^2)$ negligible $(k \ll aH)$

$$k^2 \ll a^2 \left|\frac{\pi_\phi^2}{a^6} - V(\phi)\right|, \qquad k^2 \ll a^2 \left|\frac{\pi_\phi^2}{a^6} + \frac{V(\phi)}{2}\right|, \qquad k^2 \ll \left|V_{,\phi,\phi}\right|$$

[Jackson et al. (2023)]

- Anisotropic perturbations and shift vector negligible
- The gauge is carefully chosen

[**DA**, Grain, Vennin (2022)]

Gauges in SU

CPT	SU
$(\delta N, \delta N_1), (\delta\phi, \delta\pi_\phi), (\delta\gamma_1, \delta\pi_1), (\delta\gamma_2, \delta\pi_2)$ lapse & shift · scalar field · isotropic · anisotropic	$(\overline{\delta N}, \cancel{\delta N_1}), (\overline{\delta\phi}, \overline{\delta\pi_\phi}), (\overline{\delta\gamma_1}, \overline{\delta\pi_1}), (\cancel{\delta\gamma_2}, \cancel{\delta\pi_2})$ shift · anisotropic
Spatially-flat gauge $\delta\gamma_1 = \delta\gamma_2 = 0.$ $\delta\dot{\gamma}_2 = -2\sqrt{\frac{2}{3}}\, a^2\, k\, \delta N_1 + 4N\, a\, \delta\pi_2 = 0$ $\Longrightarrow \delta N = N \frac{\pi_\phi}{2\, a^3 H}\, \delta\phi$ ⟷	**Spatially-flat gauge** $\overline{\delta\gamma_1} = 0.$ $\overline{\delta N} = -\frac{N}{6\, H^2}\left(V_{,\phi}\, \overline{\delta\phi} + \frac{\pi_\phi}{a^6}\, \overline{\delta\pi_\phi}\right)$

[**DA**, Grain, Vennin (2022)]

Gauges in SU

Regarding the **uniform-expansion gauge**, the gauge

$$\delta N = \delta\gamma_1 = 0$$

doesn't fix all gauge degrees of freedom.

Can we have a gauge that fixes all gauge-degrees of freedom

$$\begin{cases} \delta X = 0 \\ \delta\gamma_1 = 0 \end{cases} \xRightarrow[E.O.M.]{} \begin{cases} \delta N = 0 \\ \delta N_1 \approx 0 \end{cases}$$

It is very unlikely…

[**DA**, Grain, Vennin (2023)]

The momentum constraint

$$D^{(1)} = \underbrace{\pi_\phi \delta\phi - \frac{1}{\sqrt{3}}\, a\, H\, \delta\gamma_1 - \frac{2\, a^2}{\sqrt{3}}\, \delta\pi_1}_{\overline{D^{(1)}} \neq 0} - 2\sqrt{\frac{2}{3}}\,(a\, H\, \delta\gamma_2 + a^2\, \delta\pi_2) = 0$$

[Salopek, Bond (1990)]
[Cruces, Germani, Prokopec (2018)]
[Rigopoulos, Wilkins (2021)]

In spatially-flat gauge in USR:

$$\overline{D^{(1)}} = \lim_{k\to 0}\left(-\frac{\pi_\phi}{2V_{,\phi}}\, \delta P_{nad}\right) \to \infty$$

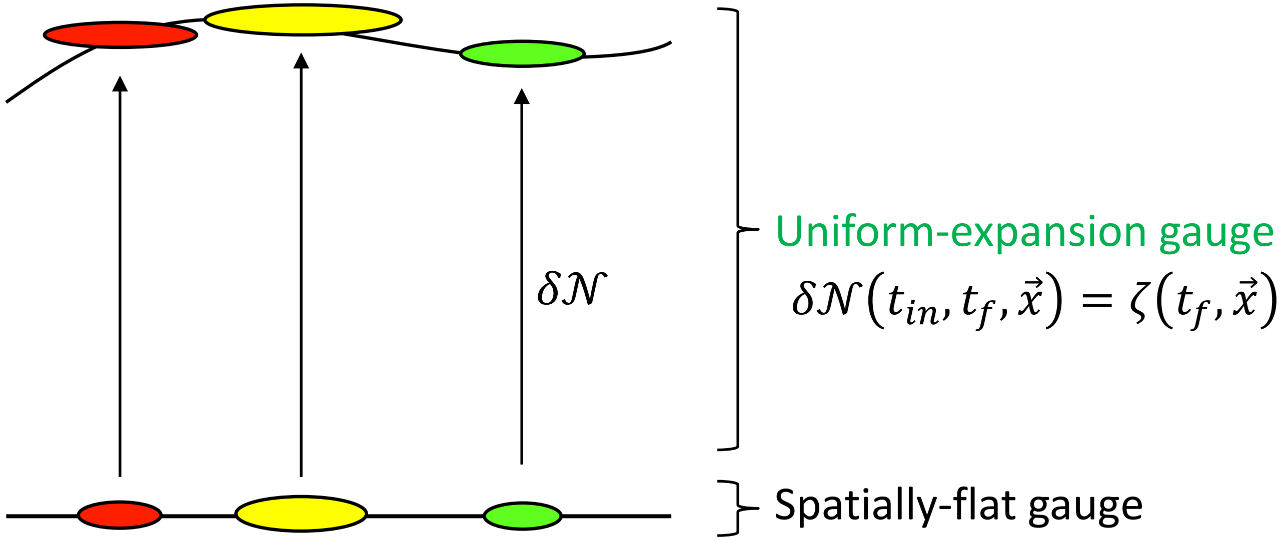
[**DA**, Frion, Miranda, Vennin, Wands (in prep.)]

One should not use the momentum constraint in separate universe beyond slow roll.

However, $\overline{\dot{D}^{(1)}} = 0$ at large scales.

Summary and future directions

- Apply the spatially-flat gauge in slow roll only. [**DA**, Grain, Vennin (2022)]
- The uniform-expansion gauge may be too restrictive. Apply the $\delta\mathcal{N}$ in another gauge. [**DA**, Grain, Vennin (2023)]
- Maybe there are more suitable gauges to consider ? [with Małkiewicz]



$$\dot{\phi} = \frac{N + \delta N}{\sqrt{\gamma + \cancel{\delta\gamma}}} \bar{\pi}_\phi + \xi_\phi + \text{grad.}$$

$$\dot{\bar{\pi}}_\phi = -(N + \delta N)\sqrt{\gamma + \cancel{\delta\gamma}}\, V_{,\phi} + \xi_{\pi_\phi} + \text{grad.}$$

Summary and future directions

- Anisotropic separate universe: captures vector and tensor perturbations.
  Should allow a broader class of gauges (e.g. spatially-flat gauge).

  [Tanaka, Urakawa (2021)]
  [Tanaka, Urakawa (2023)]
  [with Tanaka, Urakawa]

- Keep gradient terms at the SR-USR transition.

  [with Tanaka, Shi Pi]

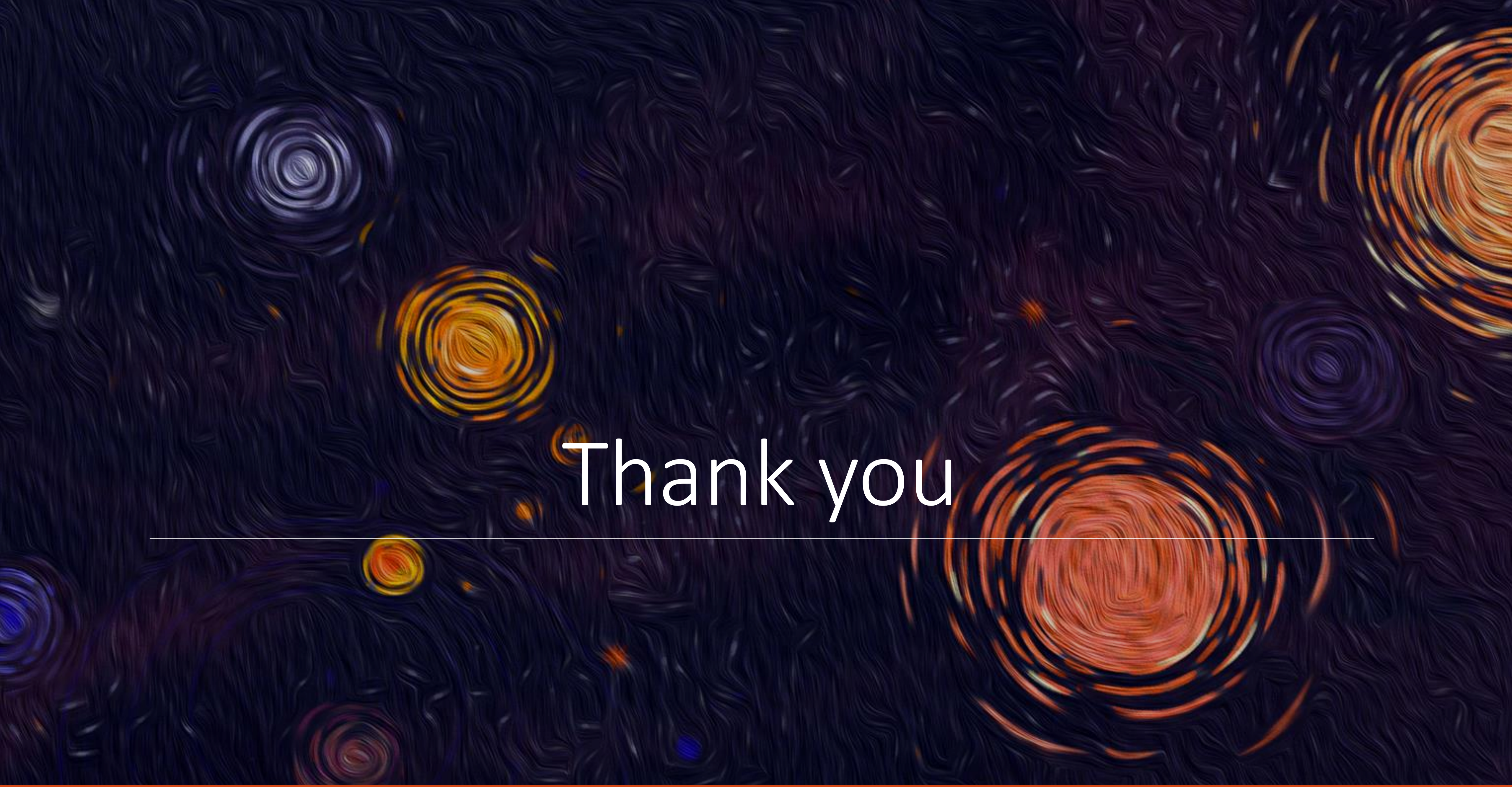
- USR with non-Bunch-Davies vacuum.
  The SR $\rightarrow$ USR transition can shift the vacuum at the onset of USR.

  [Jackson et al. (2023)]
  [**DA**, Frion, Miranda, Vennin, Wands (in prep.)]

- Application: stochastic formalism in a power-law collapse period.

  [with Tada, S. Yokoyama]

Thank you



Credit: Sacha Berna

Counting the degrees of freedom

Cosmological perturbation theory

8 variables

$$\delta N, \delta N_1$$
$$\delta\phi, \delta\gamma_1, \delta\gamma_2, \delta\pi_\phi, \delta\pi_1, \delta\pi_2$$

2 constraints

$$S^{(1)} = 0, D^{(1)} = 0$$

2 gauge conditions preserved over time

$$\delta\gamma_1 = 0, \delta\gamma_2 = 0$$
$$\delta\dot{\gamma}_1 = 0, \delta\dot{\gamma}_2 = 0$$

1 physical degree of freedom (2 variables).

Separate universe

5 variables

$$\delta N$$
$$\delta\phi, \delta\gamma_1, \delta\pi_\phi, \delta\pi_1$$

1 constraint

$$S^{(1)} = 0$$

1 gauge condition preserved over time

$$\delta\gamma_1 = 0$$
$$\delta\dot{\gamma}_1 = 0$$

1 physical degree of freedom (2 variables).

Use of the Hamiltonian formalism in SU

- Complementary picture.

- The stochastic noise comes from quantum fluctuations that is naturally formulated in the Hamiltonian framework.

- The initial vacuum state reduces to a phase-space parametrisation choice.

- Slow-roll inflation is an attractor, which removes the dependence on initial velocites $\delta\dot{\phi}_0$. Beyond slow roll, this is not the case anymore.

- In Lagrangian: n stochastic second-order differential equations.
- In Hamiltonian: $2n$ first-order differential equations.

Gauge-invariant separate universe

Cosmological perturbation theory (CPT)

$$\ddot{v} + \left(k^2 - \frac{\ddot{z}}{z}\right) v = 0$$

Separate universe (SU)

$$\ddot{v} + \left(k^2 - \frac{\ddot{z}}{z}\right) v = \sqrt{\frac{\epsilon_1}{2}}\epsilon_2 \, M_{Pl} H^2 \overline{D^{(1)}}$$

[**DA**, Grain, Vennin (2022)]

- And $\overline{D^{(1)}}$ is conserved at large scales
- In general $D^{(1)} = \overline{D^{(1)}} + D^{(1)}_{Aniso}$ and

$$\overline{D^{(1)}} = -\frac{\pi_\phi}{2V_{,\phi}} \, \delta P^{k\to 0}_{nad}$$

[**DA**, Frion, Miranda, Vennin, Wands (in prep.)]

Proportional to the comoving density perturbation.

- Even if $\overline{D^{(1)}} \neq 0$ during inflation, the SU approach may be useful to compute the curvature perturbation after inflation, when it coincides with the comoving density perturbation.