Stochastic tunneling in de Sitter Spacetime

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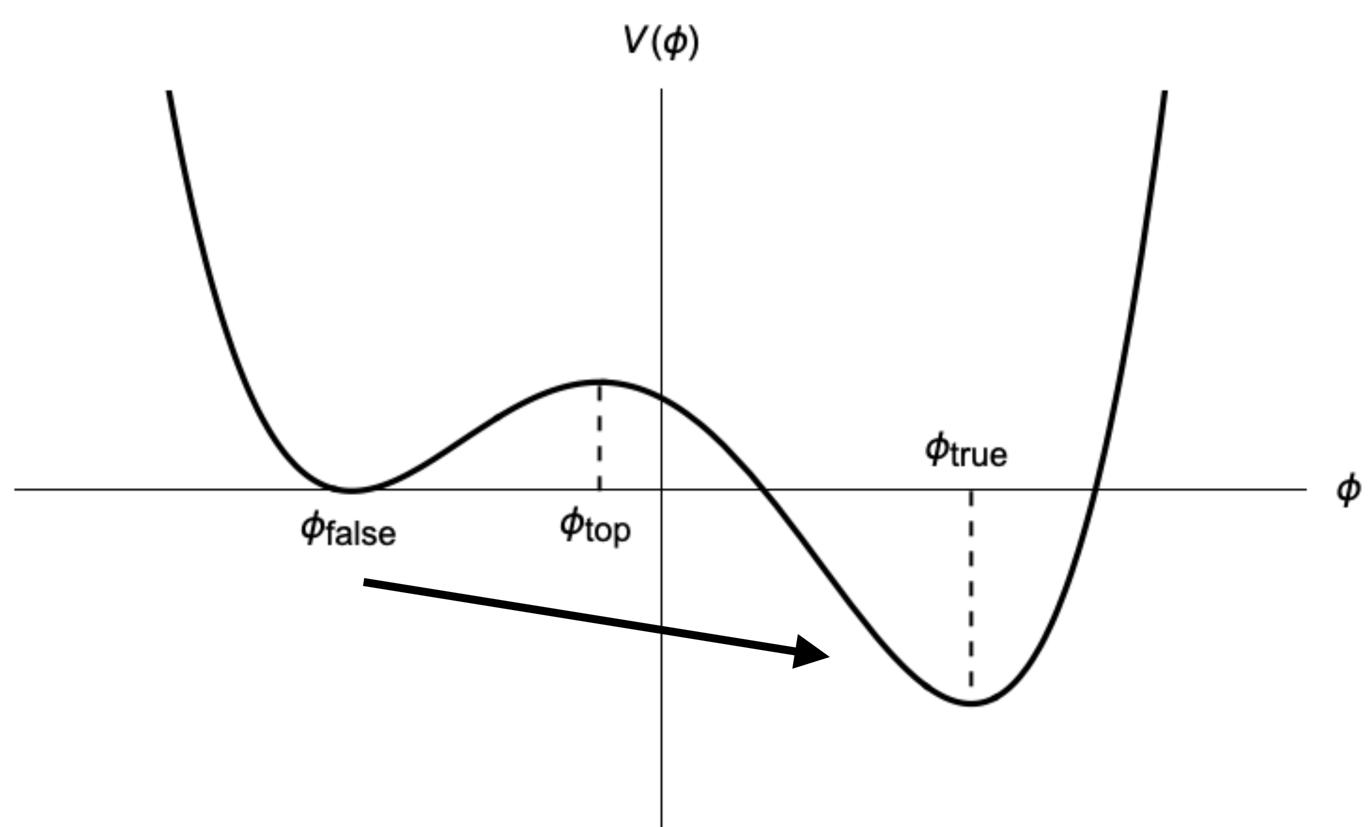
Tunneling in cosmology

Tunneling is interesting phenomenon in early universe!

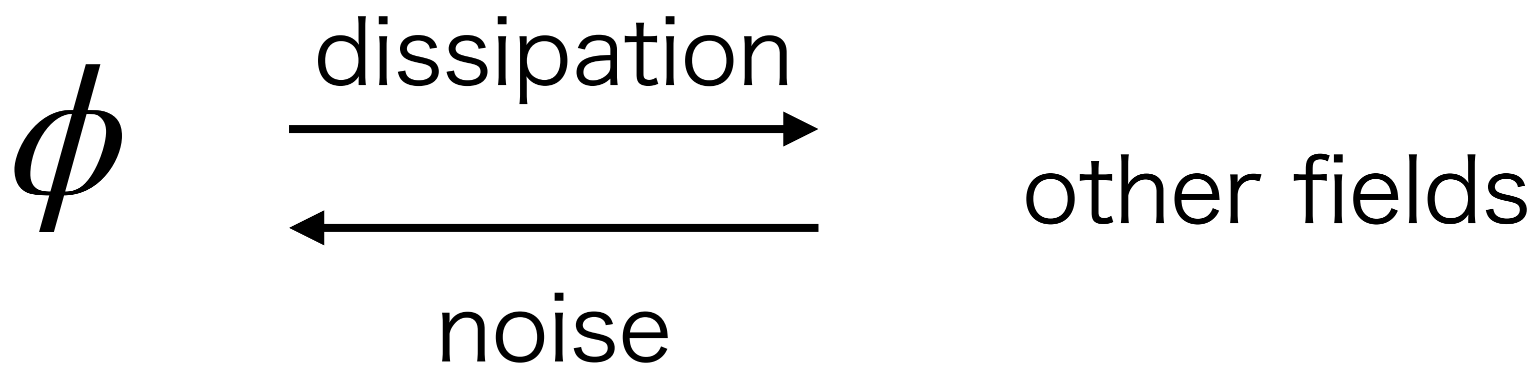
- Gravitational waves from bubbles collisions

  [Kosowsky, Turner, Watkins (1992)]
- Electro weak baryogenesis [Kuzmin, Rubakov, Shaposhnikov (1985)]
- Chain inflation [Freese, Spolyar (2005)]
- etc



Tunneling with dissipation and noise

- When a tunneling field coupled to other fields,



- Dissipation and noise effects to ϕ in real time evolution.

$\longrightarrow$ It is hard to include such effects in imaginary time scheme.

About our work

One way is that describing tunneling in real time

- In de Sitter spacetime, “Stochastic approach” is convenient way (we will see later).

- We construct “path integral” formulation which can be include dissipation and noise (future work)

- In this talk, I will concentrate on explaining the validity of our formalism.

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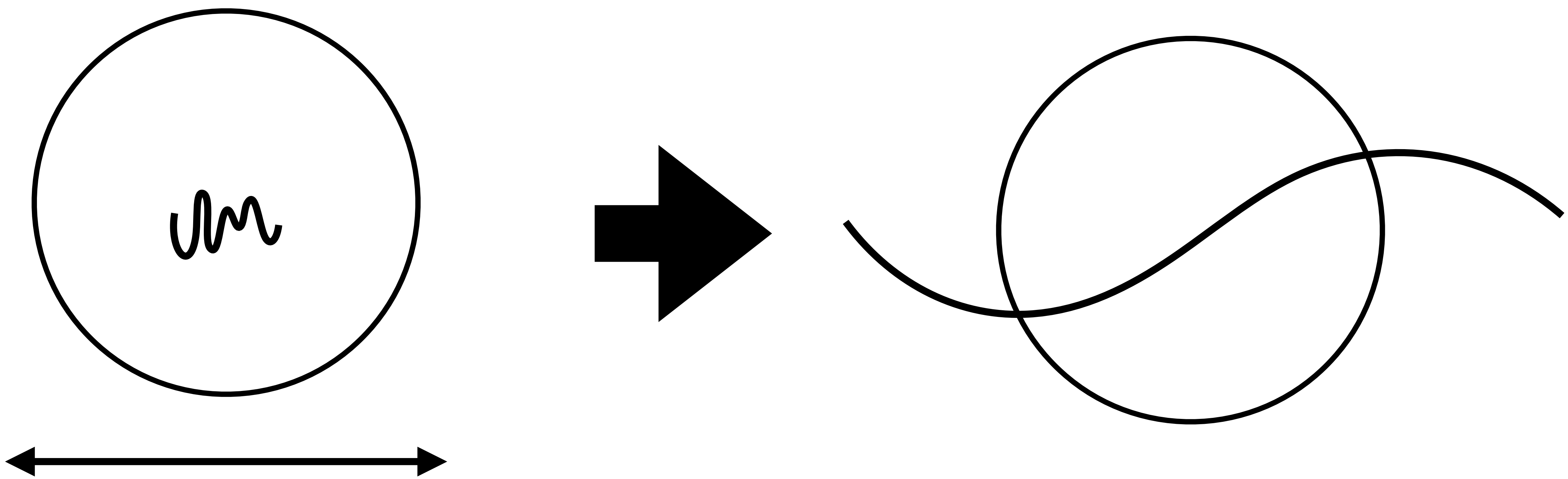
Idea of stochastic approach

In de Sitter spacetime, (quantum) short wavelength modes (UV) are stretched to (classical) long wavelength modes (IR).



$$\sim \frac{1}{\varepsilon H}$$

[A. A. Starobinsky and J. Yokoyama (1994)]

- We focus on the IR dynamics.
- The UV modes are contribute as a quantum noise.

Noise effects

- Focusing on IR dynamics $\phi = \phi_{IR} + \phi_{UV}$

- Langevin equation (Homogeneity + slow-roll)

$$\dot{\phi}_{IR} = -\frac{1}{3H}V'(\phi_{IR}) + \xi^{\phi}$$

ξ^{ϕ} : quantum noise

$$\langle \xi^{\phi}(t) \rangle = 0, \quad \langle \xi^{\phi}(t)\xi^{\phi}(t') \rangle = \frac{H^3}{4\pi^2}\delta(t-t')$$

- Quantum noise ξ^{ϕ} pushes up ϕ_{IR} to the top of potential barrier. [A. D. Linde (1992)]

Path integral representation

- Martin-Siggia-Rose-Jansenn-de Dominicis functional integral

$$p(\phi_{IR}, t \,|\, \phi'_{IR}, t') = \int_{\phi_{IR}(t')=\phi'_{IR}}^{\phi_{IR}(t)=\phi_{IR}} \mathscr{D}(\phi_{IR}, \Pi_\Delta) \, \exp\left[\int dt \, (\Pi_\Delta \dot{\phi}_{IR} - H(\phi_{IR}, \Pi_\Delta)) \right]$$

$$H(\phi_{IR}, \Pi_\Delta) := -\frac{V'(\phi_{IR})}{3H} \Pi_\Delta - \frac{H^3}{8\pi^2} \Pi_\Delta^2$$

$p(\phi_{IR}, t \,|\, \phi'_{IR}, t')$: transition probability

[P.C. Martin, E.D. Sigma, H.A. Rose (1973)],
[C. De Dominicis, E. Brezin, J. Zinn-Justin (1975)], [H.K. Janssen (1976)]

- Π_Δ : Auxiliary field which describes noise effects

- We estimate the probability by a tunneling configuration

Path integral representation (for full Langevin eq)

- Relaxing homogeneity and slow-roll,

$$p(\phi_{IR}(\mathbf{x}), t \,|\, \phi'_{IR}(\mathbf{x}), t') = \int_{\phi_{IR}(t',\mathbf{x})=\phi'_{IR}(\mathbf{x})}^{\phi_{IR}(t,\mathbf{x})=\phi_{IR}(\mathbf{x})} \mathscr{D}(\phi_{IR}, \Pi_{IR}, \phi_\Delta, \Pi_\Delta) \exp\left[\int d^4x \left(\Pi_\Delta \dot{\phi}_{IR} - \phi_\Delta \dot{\Pi}_{IR} - H(\phi_{IR}, \Pi_{IR}, \phi_\Delta, \Pi_\Delta)\right)\right]$$

$$H(\phi_{IR}, \Pi_{IR}, \phi_\Delta, \Pi_\Delta) = \frac{\Pi_{IR}\Pi_\Delta}{a^3} - (a\nabla^2\phi_{IR} - a^3 V'(\phi_{IR}))\phi_\Delta - \frac{1}{2}\sum_{\alpha,\beta}\int d^4x' X^\alpha_\Delta(x) G^{\alpha\beta}(x,x') X^\beta_\Delta(x')$$

$$G^{\alpha\beta}(x,x') := \langle \xi^\alpha(x)\xi^\beta(x')\rangle, \quad (\alpha,\beta = \phi, \Pi)$$

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Phase space for $H(\phi_{IR}, \Pi_\Delta)$

- We can visualize the existence of tunneling configurations on the "phase space" (ϕ_{IR}, Π_Δ). **[V. Elgart and A. Kamenev (2004)]**

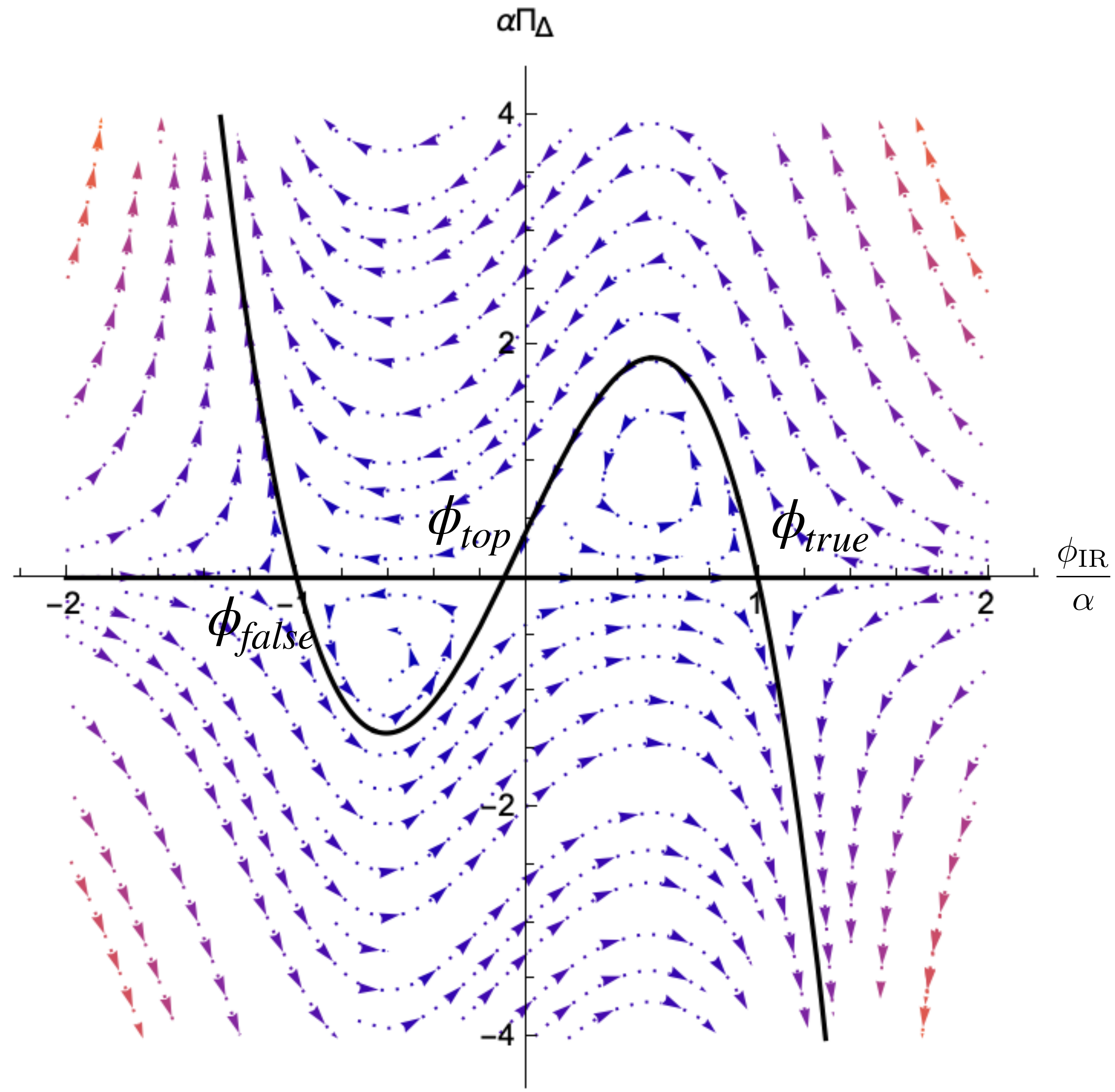
- Black lines : $H(\phi_{IR}, \Pi_\Delta) = 0$

- The EoM on the black lines

$$\dot{\phi}_{IR} = -\frac{V'(\phi_{IR})}{3H} \quad (\Pi_\Delta = 0)$$

$$\dot{\phi}_{IR} = +\frac{V'(\phi_{IR})}{3H} \quad (\Pi_\Delta = -\frac{8\pi^2}{3H^4}V'(\phi_{IR}))$$

- There is a flow starting from ϕ_{false} to ϕ_{true}



The flow on the phase space (ϕ_{IR}, Π_Δ)

Tunneling rate

- The action for this non-trivial configuration can be calculated analytically.

$$I = \int_{t'}^{t} dt \left[\Pi_\Delta \dot{\phi}_{IR} - H(\phi_{IR}, \Pi_\Delta)\right] = -\frac{8\pi^2}{3H^4}\int_{t'}^{t_*} dt\, \dot{\phi}_{IR} V'(\phi_{IR}) = -\frac{8\pi^2}{3H^4}\Delta V$$

ΔV : hight of potential barrier

- This completely matches with the Hawking-Moss instanton in Euclidean method! **[S.W. Hawking, I.G. Moss (1982)]**

- We can also reproduce the same result in the case of path integral with 1+3 dimensional action.

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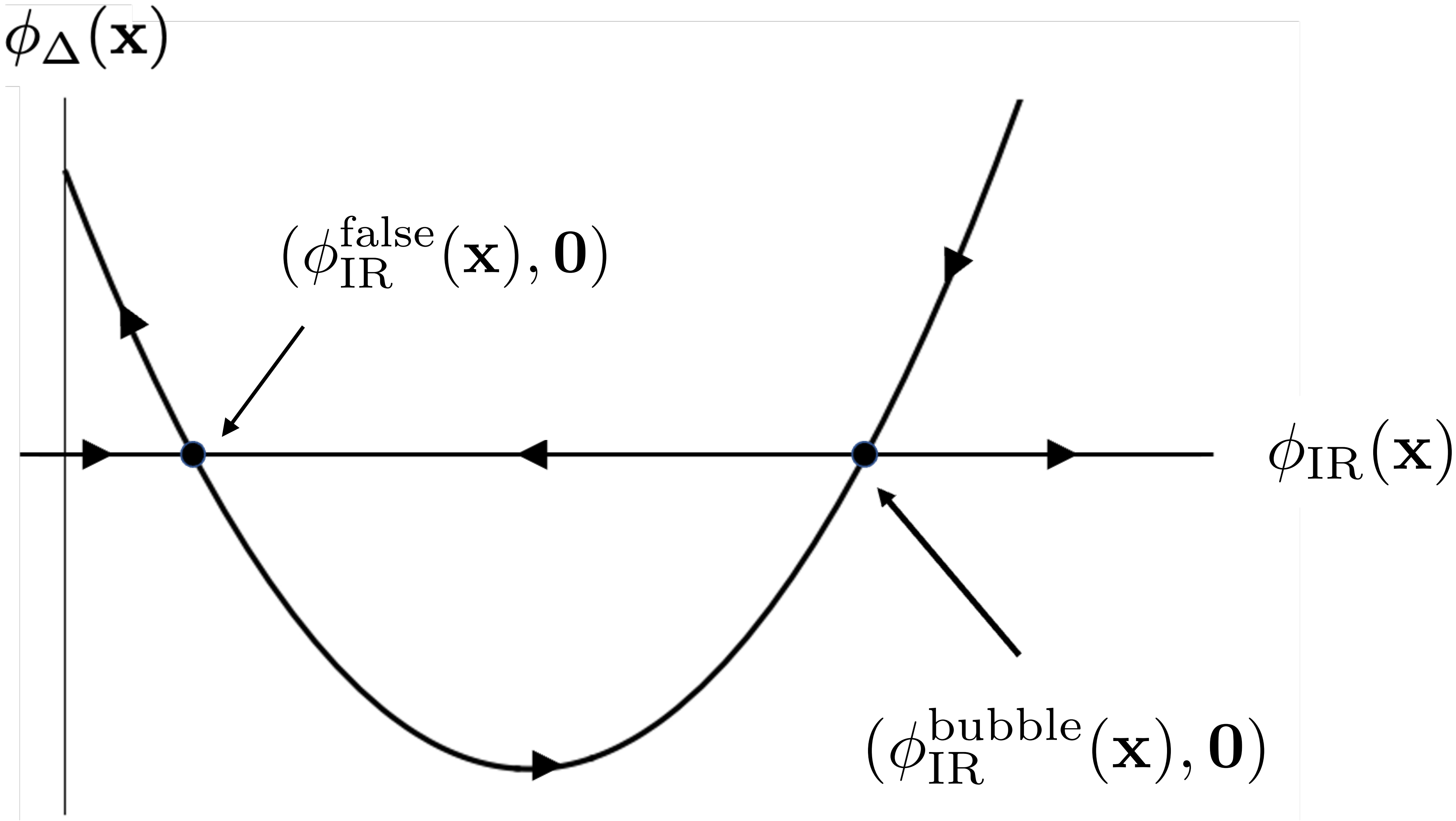
Is it possible to describe CDL?

- Schematic picture of flows in the "field" phase space $(\phi_{IR}(\mathbf{x}), \phi_\Delta(\mathbf{x}))$



CDL configuration

- The path integral ($\epsilon \gg 1$)

$$p(\phi_{IR}, t\,|\,\phi_{IR}, t') \simeq \int \mathscr{D}(\phi_{IR}, \phi_\Delta) \exp\left[\left[\int d^4x \left(a^3 \dot{\phi}_\Delta \dot{\phi}_{IR} - H_{CDL}(\phi_{IR}, \phi_\Delta) \right)\right]\right.$$

$$H_{CDL}(\phi_{IR}, \phi_\Delta) \sim -\left(a\nabla^2\phi_{IR} - a^3 V'(\phi_{IR})\right)\phi_\Delta - \frac{H^2\varepsilon}{6\pi} a^3 \phi_\Delta^2$$

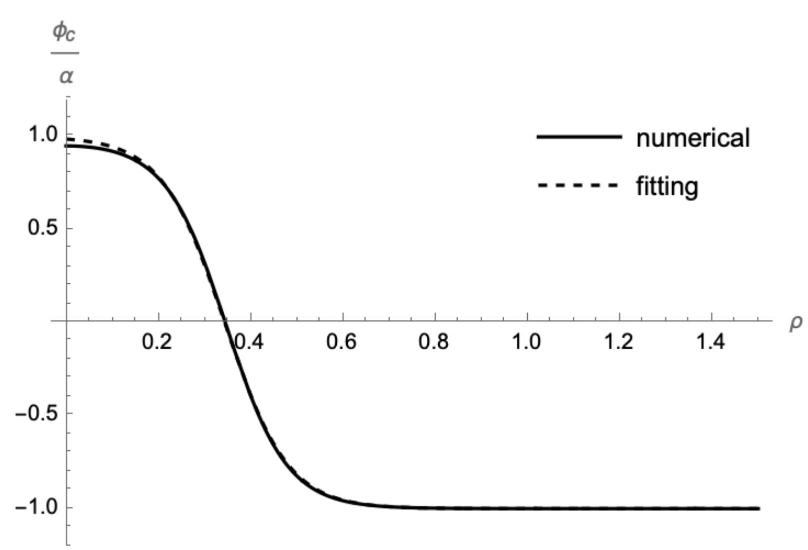
- The nontrivial equation on $H_{CDL}(\phi_{IR}, \phi_\Delta) = 0$ with $\phi_\Delta \neq 0$

$$\ddot{\phi}_{IR} + 3H\dot{\phi}_{IR} = V'(\phi_{IR}) - a^{-2}\nabla^2\phi_{IR}, \qquad (\phi_\Delta \neq 0)$$

- This EoM has Euclidean AdS symmetry.
- We can impose O(4) symmetry and solve.

$$\frac{\partial^2\phi}{\partial\rho^2} + \frac{3}{\tanh\rho}\frac{\partial\phi}{\partial\rho} = \frac{V'(\phi_{IR})}{H^2}$$

$$\lim_{\rho\to\infty} \phi_{IR} = \phi_{false}, \quad \left.\frac{d\phi_{IR}}{d\rho}\right|_{\rho=0} = 0$$



Tunneling rate

・The action for this configuration (numerically)

$$I \simeq \int_{\Sigma} d^4x \left[a^3 \dot{\phi}_{\Delta} \dot{\phi}_{IR} - H_{CDL}(\phi_{IR}, \phi_{\Delta}) \right] := -\frac{24\pi^2\alpha^2}{H^2\varepsilon} \tilde{I}(\mu, \bar{\rho})$$

α : potential parameter, ϵ : cutoff scale parameter

$\tilde{I}(\mu, \bar{\rho})$: dimensionless action for the non-trivial configuration

・When the potential is steep, the tunneling rate becomes larger than Hawking-Moss one.

$\rightarrow$ Consistent with the Euclidean method.

・The relation to the Euclidean method is not clear (future work).

Summary

- We introduce the path integral formalism for the stochastic approach and formulate the tunneling probability in real time.
- We find the configuration which reproduce the result of the HM instanton.
- We also find the Coleman-de Luccia like configuration and calculating the tunneling rate.

Future work

- Relation to Euclidean method.
- Relation to other real-time formalisms (ex: Lefschetz thimble method, Evolution of an initial fluctuation).
- Tunneling with dissipation and noise.

Thanks for your attention !

Back up

Interpretation of HM instanton

- How to interpret such a constant solution in Lorentzian spacetime?
- Thermal interpretation [A. R. Brown, E. J. Weinberg (2005)]

$$\Gamma \sim \exp\left[-\frac{8\pi^2}{3H^4}\left(V(\phi_{top}) - V(0)\right)\right] = \exp\left[-\underbrace{\left(\frac{H}{2\pi}\right)}_{\text{de Sitter temperature}}{}^{-1}\underbrace{\frac{4\pi}{3H^3}}_{\text{Habble volume}}\left(V(\phi_{top}) - V(\phi_{false})\right)\right]$$

But tunneling is non-equilibrium process!

Langevin equation

- Klein-Gordon eq (in de Sitter spacetime)

$$\partial_t^2\phi + 3H\partial_t\phi - \frac{1}{a^2}\nabla^2\phi + \frac{\partial V}{\partial\phi} = 0$$

- Split into UV parts and IR parts.

$$\phi = \phi_{IR} + \phi_{UV}$$
$$\Pi = \Pi_{IR} + \Pi_{UV}$$

- Mode expansion of the ϕ_{UV} and Π_{UV}

$$\phi_{UV} = \int \frac{d^3k}{(2\pi)^3}\theta(k-k_c)[a_{\boldsymbol{k}}u_k(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + a_{\boldsymbol{k}}^\dagger u_k^*(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}]$$

$$\pi_{UV} = \int \frac{d^3k}{(2\pi)^3}\theta(k-k_c)[a_{\boldsymbol{k}}\dot{u}_k(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + a_{\boldsymbol{k}}^\dagger \dot{u}_k^*(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}]$$

$k_c = \epsilon a(t)H$: cutoff scale

Langevin equation

- The mode function u_k satisfies

$$\partial_t^2 u_{\boldsymbol{k}} + 3H\partial_t u_{\boldsymbol{k}} + \frac{k^2}{a^2}u_{\boldsymbol{k}} + \frac{\partial^2 V(\phi_{IR})}{\partial \phi^2}u_{\boldsymbol{k}} = 0$$

※ We take $V(\phi_c + \phi_\Delta) \simeq V(\phi_c) + V'(\phi_c)\phi_\Delta$

- In this case, the Klein-Gordon eqs become

$$\begin{cases} \dot{\phi}_{IR} = a^{-3}\Pi_{IR} + \xi^\phi \\ \dot{\Pi}_{IR} = a\,\nabla^2\phi_{IR} - a^3\frac{\partial V}{\partial \phi} + \xi^\Pi \end{cases}$$

Langevin eqs.

where ξ^ϕ, ξ^π is quantum noise terms.

$$\xi^\phi = \dot{k}_c \int \frac{d^3k}{(2\pi)^3}\delta(k - k_c)[\hat{a}_{\boldsymbol{k}}\phi_k(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^\dagger \phi_k^*(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}]$$

$$\xi^\pi = \dot{k}_c \int \frac{d^3k}{(2\pi)^3}\delta(k - k_c)[\hat{a}_{\boldsymbol{k}}\dot{\phi}_k(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^\dagger \dot{\phi}_k^*(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}]$$

Statistical properties of the noises

- Bunch-Davies vacuum

$$u_k = \frac{H\eta}{\sqrt{2k}}\left(1\frac{1}{-ik\eta} - 1\right)e^{-ik\eta}$$

- Correlation functions of the noises

$$\langle 0 \,|\, \xi^{\alpha}(t, \mathrm{x}) \,|\, 0 \rangle = 0, \quad (\alpha, \beta = \phi, \Pi),$$

$$\langle 0 \,|\, \xi^{\alpha}(t, \mathrm{x}) \xi^{\beta}(t', \mathrm{x}') \,|\, 0 \rangle = \frac{1}{2\pi^2} \dot{k}_c(t) k_c(t)^2 \frac{\sin(k_c(t) r)}{k_c(t) r} g^{\alpha\beta}(t) \delta(t - t')$$

$$\begin{cases} g^{\phi\phi}(t) := |\, u_{k_c}(t) \,|^2 = \frac{1}{2Ha^3}\left(\varepsilon^{-3} + \varepsilon^{-1}\right) \\ g^{\Pi\Pi}(t) := a(t)^6 \,|\, \dot{u}_{k_c}(t) \,|^2 = \frac{Ha^3}{2}\varepsilon \\ g^{\phi\Pi}(t) = (g^{\Pi\phi})^* := a(t)^3 u_{k_c}(t) \dot{u}^*_{k_c}(t) = -\frac{1}{2}\varepsilon^{-1} + \frac{1}{2} \end{cases}$$

Reduced Langevin equation

- Assumption : $\varepsilon \ll 1$ + homogeneity + slow-roll

- Reduced Langevin equation (Effectively 0+1 dim)

$$\dot{\phi}_{IR} = -\frac{1}{3H}V'(\phi_{IR}) + \xi^{\phi}$$

$$\langle \xi^{\phi}(t) \rangle = 0, \quad \langle \xi^{\phi}(t)\xi^{\phi}(t') \rangle = \underbrace{\frac{H^3}{4\pi^2}\delta(t-t')}$$

↑ White noise

Fokker-Plank equation

- Fokker-Plank eq

$$\frac{\partial p(\phi, t)}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V'(\phi)}{3H}P(\phi, t)\right) + \frac{H^3}{8\pi^2}\frac{\partial^2 P(\phi, t)}{\partial \phi^2}$$

$p(\phi, t)$: Probability distribution for ϕ

- Most of works discussed the HM transition in this formalism. [A. D. Linde (1992), J. E. Camargo-Molina, A. Rajantie (2022), etc]

But

- It does not give the clear tunneling process.
- It is hard to include space dependence.
- The CDL tunneling cannot be described.

Derivation of MSRJD 1

- Let's consider the case for the reduced Langevin eq.

- Discretization

$$X_i := \phi_i - \phi_{i-1} + \Delta t\left(\frac{V'(\phi_i)}{3H} - \xi_i^\phi\right) = 0$$

- Expectation value

$$\langle \mathcal{O}[\phi_c(t)]\rangle_\xi = \int \mathcal{D}\phi_c\, \mathcal{O}[\phi_c]\langle \delta(\phi_c - \phi_c[\xi])\rangle_\xi = \int \mathcal{D}\phi_c\, \mathcal{O}[\phi_c]\left\langle \underbrace{\left|\frac{\delta X}{\delta \phi_c}\right|}_{=1} \delta(X)\right\rangle_\xi$$

$$= \int \mathcal{D}\phi_c\, \mathcal{O}[\phi_c]\left\langle \prod_i \delta\left(\phi_i - \phi_{i-1} + \Delta t\left(\frac{V'(\phi_i)}{3H} - \xi_i^\phi\right)\right)\right\rangle_\xi$$

- Fourier transformation of delta function

$$\langle \mathcal{O}[\phi_c(t)]\rangle_\xi = \int \mathcal{D}(\phi_c, \tilde{\phi})\, \mathcal{O}[\phi_c]\left\langle \exp\left[i\int \mathrm{d}t\tilde{\phi}\left(\dot{\phi}_c + \frac{V'(\phi_c)}{3H} - \xi^\phi\right)\right]\right\rangle_\xi$$

Derivation of MSRJD 2

- Averaging over the noise

$$\langle \mathcal{O}[\phi_c(t)] \rangle_\xi = \int \mathcal{D}(\phi_c, \tilde{\phi})\, \mathcal{O}[\phi_c] \exp\left[\int \mathrm{d}t \left(i\tilde{\phi}\left(\dot{\phi}_c + \frac{V'(\phi_c)}{3H} \right) - \frac{H^3}{8\pi^2}\tilde{\phi}^2 \right)\right]$$

- Putting $\mathcal{O}[\phi_c] = \delta(\phi_c - \phi_c(t))$

$$p(\phi_c, t \,|\, \phi_c', t') = \int_{\phi_c(t')=\phi_c'}^{\phi_c(t)=\phi_c} \mathcal{D}(\phi_c, \tilde{\phi})\ \exp\left[\int \mathrm{d}t \left(i\tilde{\phi}\left(\dot{\phi}_c + \frac{V'(\phi_c)}{3H} \right) - \frac{H^3}{8\pi^2}\tilde{\phi}^2 \right)\right]$$

- Finally, introducing new variables $\Pi_\Delta := i\tilde{\phi}$

$$p(\phi_c, t \,|\, \phi_c', t') = \int_{\phi_c(t')=\phi_c'}^{\phi_c(t)=\phi_c} \mathcal{D}(\phi_c, \Pi_\Delta)\ \exp\left[\int \mathrm{d}t (\Pi_\Delta \dot{\phi}_c - H(\phi_c, \Pi_\Delta))\right]$$

$$H(\phi_c, \Pi_\Delta) := -\frac{V'(\phi_c)}{3H}\Pi_\Delta - \frac{H^3}{8\pi^2}\Pi_\Delta^2$$

Correspondence between FP and MSRJD

- Hamiltonian in MSRJD

$$H(\phi_c, \Pi_\Delta) := -\frac{V'(\phi_c)}{3H}\Pi_\Delta - \frac{H^3}{8\pi^2}\Pi_\Delta^2$$

- "Schrödinger" equation

$$-\frac{\partial p(\phi_c, t)}{\partial t} = H(\phi_c, \Pi_\Delta)p(\phi_c, t)$$

- Replacing $\Pi_\Delta \to \partial/\partial\phi$, we obtain Fokker-Plank eq.

$$\frac{\partial p(\phi, t)}{\partial t} = \frac{\partial}{\partial\phi}\left(\frac{V'(\phi)}{3H}P(\phi, t)\right) + \frac{H^3}{8\pi^2}\frac{\partial^2 P(\phi, t)}{\partial\phi^2}$$