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RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program

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Quantum features in cosmological perturbations?

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A. Micheli and P. Peter, « *Quantum Cosmological Gravitational Waves?* » in Handbook of Quantum Gravity

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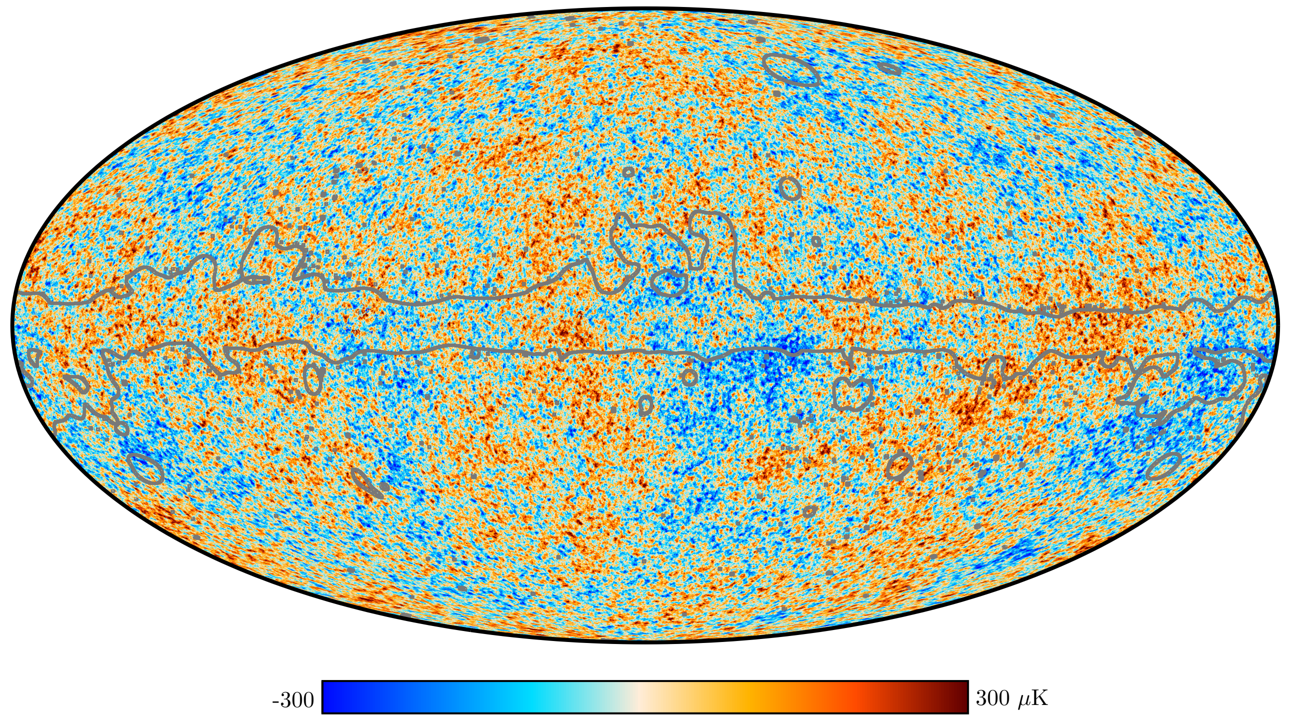
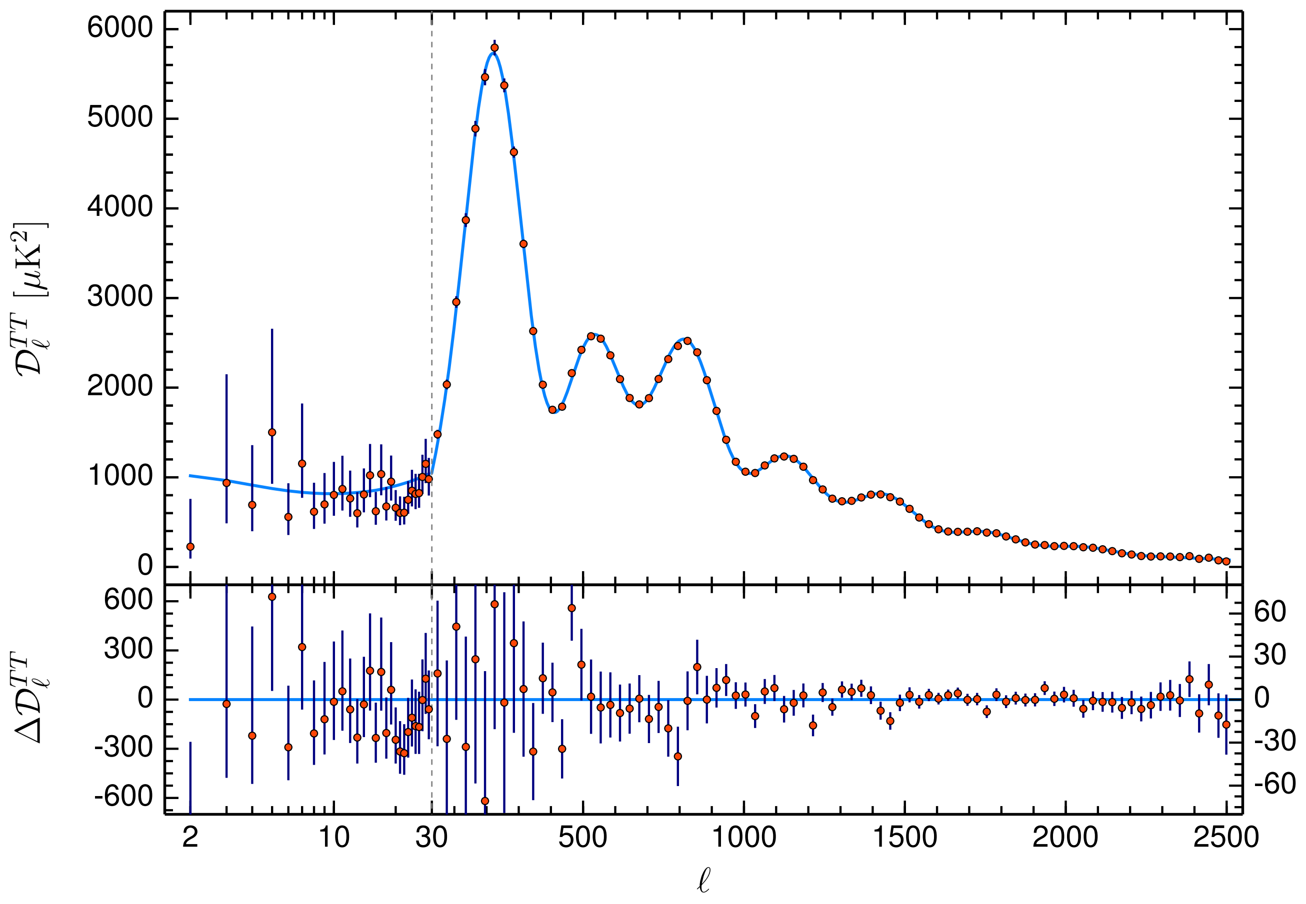
Context and motivations

Leading Scenario

Quantum fluctuations of matter and gravity during inflation gives initial conditions for structure formation in very good agreement with observation[1].





- Distribution of matter in the Universe is an (as of now, only) observational window on linearised quantum gravity!

Can we use it to probe this regime?

- In particular, a direct proof that inhomogeneities cannot have a classical origin would show that gravity can and should be quantised.

1. [arXiv:1807.06209 Planck Collaboration]

II - Naive quantum state of perturbations

Reasons to have hope

Quantum state cosmological perturbations

- Framework: GR and single field slow roll inflation, focus on scalar perturbations represented by Mukhanov-Sasaki $v(x)$, same can be done for gravitational waves[2,3].

- Dynamics at quadratic order: only mixing $\pm\boldsymbol{k}$ Fourier modes via

$$\hat{H}_{\boldsymbol{k},-\boldsymbol{k}} = \hat{\pi}_{\boldsymbol{k}}\hat{\pi}_{-\boldsymbol{k}} + k^2\hat{v}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}} + \frac{z'}{z}\left(\hat{\pi}_{\boldsymbol{k}}\hat{v}_{-\boldsymbol{k}} + \hat{v}_{\boldsymbol{k}}\hat{\pi}_{-\boldsymbol{k}}\right) \qquad \text{with} \qquad \hat{\pi}_{\boldsymbol{k}} = \hat{v}'_{\boldsymbol{k}} - \frac{z'}{z}\hat{v}_{\boldsymbol{k}},$$

$$[\hat{v}_{\boldsymbol{k}}, \hat{\pi}_{\boldsymbol{k}'}] = i\hbar\delta\left(\boldsymbol{k}+\boldsymbol{k}'\right), \quad z = M_{\mathrm{Pl}}a\sqrt{2\epsilon_1}$$

- Evolve as parametric oscillators: $\hat{v}''_{\boldsymbol{k}} + \left(k^2 - \frac{z''}{z}\right)\hat{v}_{\boldsymbol{k}} = 0$ *Initial amplitude?*

- Simplest assumption: vacuum initial state $\longrightarrow$ 2-mode squeezed vacuum (TMSV)

Fluctuations of quantities in a TMSV?

- Vacuum initial state Gaussian + quadratic hamiltonian: state remains Gaussian.

2. [PRD.42.3413 Grishchuk and Sidorov] 3. [arXiv:2211.00182 Micheli and Peter]

Fluctuations in a TMSV

- Gaussian state completely determined by covariance matrix γ_{ij} made of 2-point correlation functions of creation/annihilation operators $\hat{c}^{(\dagger)}_{\pm\boldsymbol{k}}$

- TMSV: - Homogeneity and isotropy[4]: $n_k = \langle \hat{c}^{\dagger}_{\pm\boldsymbol{k}} \hat{c}_{\pm\boldsymbol{k}} \rangle$ number of particles

  $c_k = \langle \hat{c}_{\boldsymbol{k}} \hat{c}_{-\boldsymbol{k}} \rangle$ pair correlation

  - Purity of the state $p_k = 1$ reduces it to two parameters: squeezing parameter r_k and angle φ_k : $n_k = \sinh^2(r_k)$; $c_k = -\sinh(2r_k)e^{i\varphi_k}/2$

Squeezing in inflation?

- Initially, uncorrelated vacuum fluctuations: $n_k = c_k = 0$

- After Hubble crossing, strong squeezing $r_k \approx N$ and correlations: $n_k \approx |c_k| \approx e^{2N}$

How quantum are they?

4. [arXiv:0505379 Campo and Parentani]

Quantum correlations

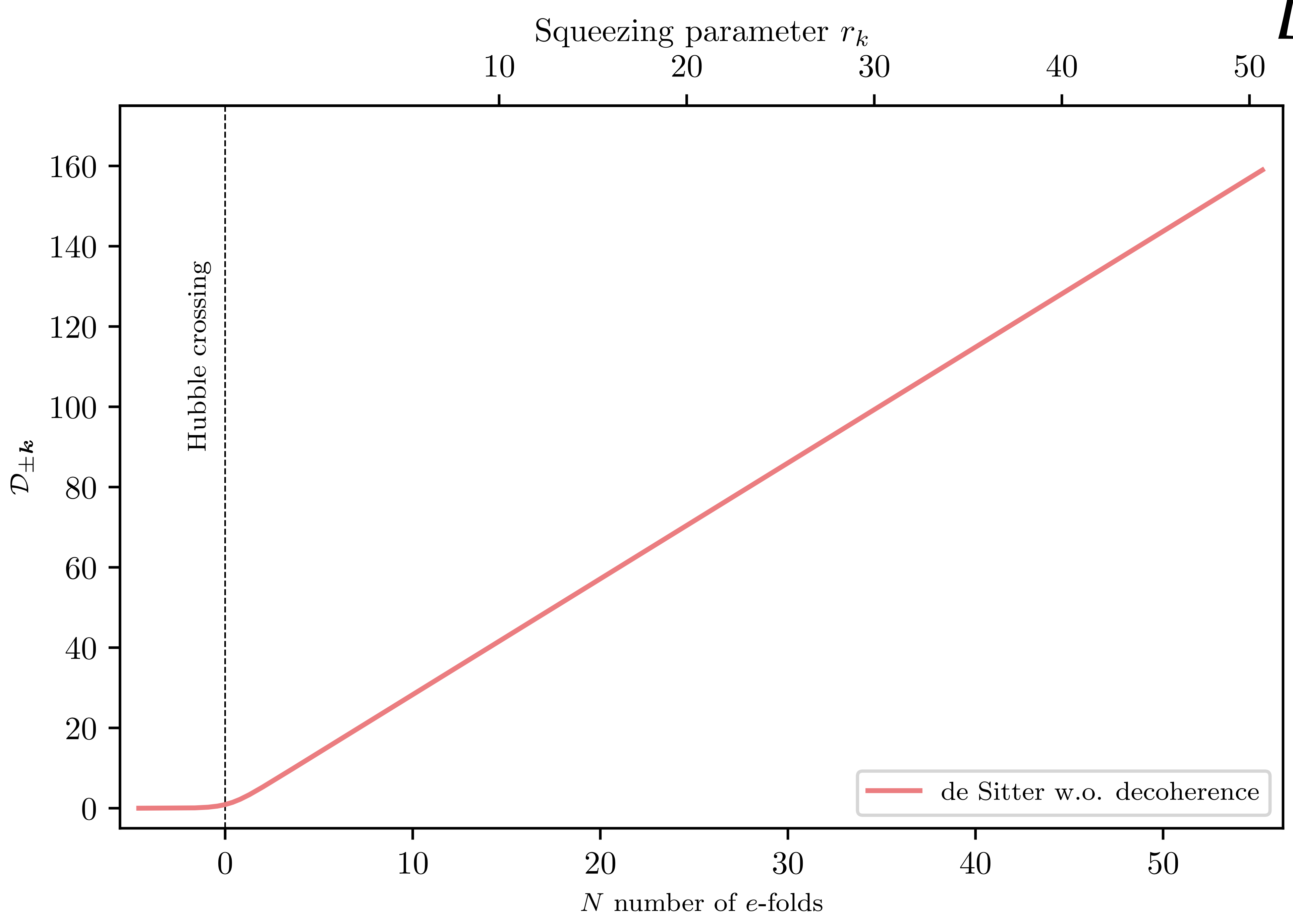
- Many measures of 'quantumness' of correlations between two systems $\mathcal{S}_1$ and $\mathcal{S}_2$ e.g. Bell inequalities.
- Consider Quantum Discord $\mathcal{D}(\mathcal{S}_1, \mathcal{S}_2)$ constructed such that

$\mathcal{S}_i$ described by classical probabilities

$$\mathcal{D}(\mathcal{S}_1, \mathcal{S}_2) = 0$$

Quantum setting $\mathcal{D}(\mathcal{S}_1, \mathcal{S}_2) \geq 0$

Discord of $\pm k$ pairs? Large[5]



Take Home Message 1

In the sense of most non-classicality criteria, two-mode squeezing generates strong quantum correlations between $\pm k$ modes[6].

Robustness?

Detectability?

5. [arXiv:1510.04038 Martin and Vennin]
6. [arXiv:2211.10114 Martin, Micheli, and Vennin]

III - Universe is not an ideal quantum optics experiment

The caveats

1) Robustness? - A simple model of decoherence

- Most systems $\mathcal{S}$ are not isolated. They interact and get correlated with their environment $\mathcal{E}$.
- Generically reduces purity of the state and tend to classicalise internal correlations $\longrightarrow$ Decoherence
- Minimal decoherence model for perturbations:

  - $\mathcal{S}$ = a pair of modes $\pm\boldsymbol{k}$
  - $\mathcal{E}$ = other pairs $\pm\boldsymbol{k}'$ of perturbations or modes of other fields
  - Take $\hat{\rho}_{\pm\boldsymbol{k}}$ to be a mixed 2-mode squeezed state[4,6] parametrised by r_k, φ_k and purity $0 \leq p_k \leq 1$.

- Can be dynamically realised by considering a linear coupling and deriving a Lindblad equation[7].

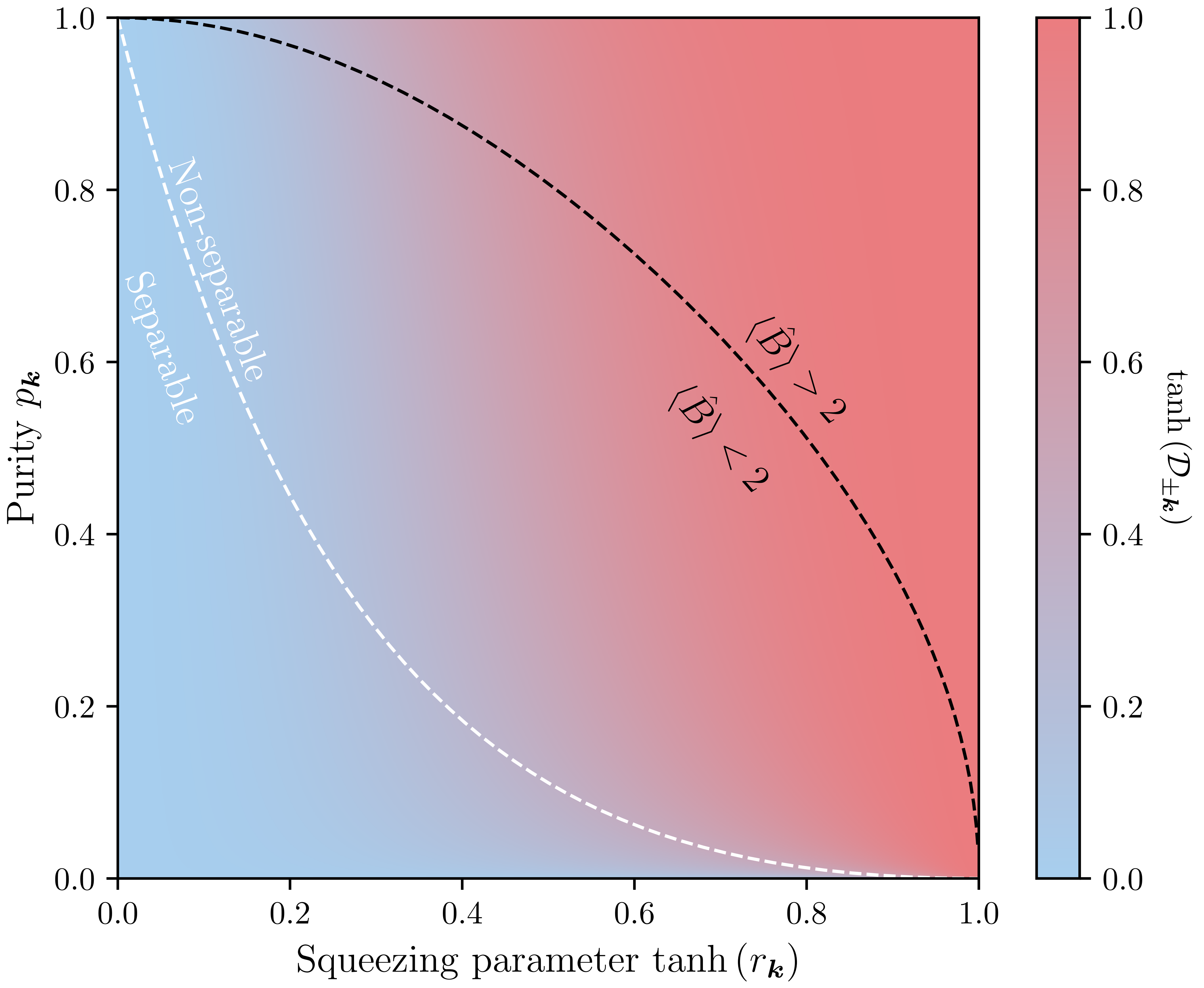
4. [arXiv:0505379 Campo and Parentani]

6. [arXiv:2211.10114 Martin, Micheli, and Vennin]

7. [arXiv:2112.05037 Martin, Micheli and Vennin]

1) Robustness? - Weakened correlations

- Discord in presence of decoherence[6,7]



Take Home Message 2

Quantum correlations can always be erased by sufficient decoherence but there is a competition between correlation build up and interaction erasing quantum features[4,6,7].

Left to answer: Where are we in this plot for the precise dynamics of inflation?

6. [arXiv:2211.10114 Martin, Micheli, and Vennin]
7. [arXiv:2112.05037 Martin, Micheli and Vennin]

4. [arXiv:0505379 Campo and Parentani]

2) Detectability?

- Measured operators fixed by cosmological dynamics: only $\hat{\zeta} \sim \hat{v}$ *Is it sufficient?* No
- For a generic operator $f\left(\hat{v}_{\pm\boldsymbol{k}}, \hat{\pi}_{\pm\boldsymbol{k}}\right)$ compare:

True quantum expectation values with TMSV

$$\left\langle f\left(\hat{v}_{\pm\boldsymbol{k}}, \hat{\pi}_{\pm\boldsymbol{k}}\right)\right\rangle$$

Stochastic average with a Gaussian probability $W\left(v_{\pm\boldsymbol{k}}, \pi_{\pm\boldsymbol{k}}\right)$ with same covariance as TMSV

$$\left\langle f\left(v_{\pm\boldsymbol{k}}, \pi_{\pm\boldsymbol{k}}\right)\right\rangle_{\mathrm{st.}} = \int \mathrm{d}v_{\pm\boldsymbol{k}}\, \mathrm{d}\pi_{\pm\boldsymbol{k}}\, f\left(v_{\pm\boldsymbol{k}}, \pi_{\pm\boldsymbol{k}}\right) W\left(v_{\pm\boldsymbol{k}}, \pi_{\pm\boldsymbol{k}}\right)$$

Take Home Message 2

Because squeezing is very large, for polynomial $f\left(\hat{v}, \hat{\pi}\right)$, we have $\left\langle f\left(\hat{v}, \hat{\pi}\right)\right\rangle \approx \left\langle f\left(v, \pi\right)\right\rangle_{\mathrm{st.}}$[8,3].

8. [arXiv:9303001 Albrecht et al.]
3. [arXiv:2211.00182 Micheli and Peter]

Conclusions and perspectives

- In this simple model, correlations are simple, perturbations exhibit quantum correlations, which might persist in presence of decoherence, but seems very challenging to measure them because requires complicated combinations of $\hat{v}$ and $\hat{\pi}$.

- To hope to find quantum signatures in the perturbations it seems necessary to consider richer situations beyond Gaussian level[9] or with more fields[10].

9. [arXiv:2001.09149 Green and Porto]
10. [arXiv:1508.01082 Maldacena]

Thank you for your attention!

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