NANOGrav Signal from Ultra Slow-Roll Inflation

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YITP long-term workshop

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Gravitational Waves: A New Window onto the Universe

A brief history of GW physics: past, present, future

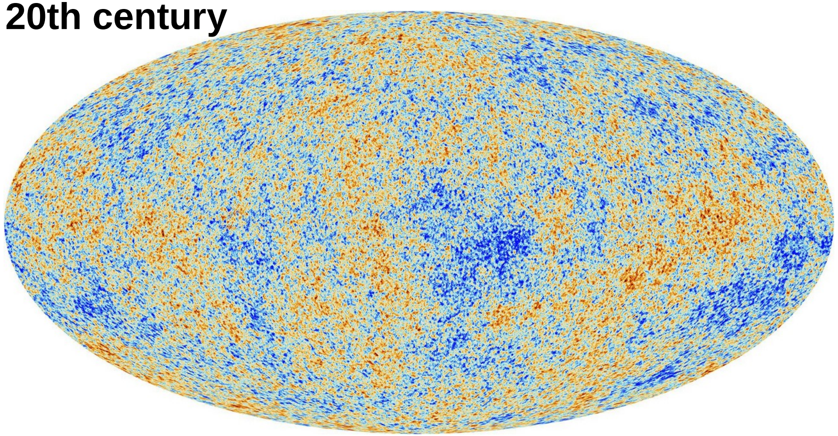
1916 Albert Einstein predicts GWs based on his theory of general relativity
2016 LIGO annouces first direct detection of a GW event (GW150914)
202x Next milestone: Detection of a stochastic GW background (GWB)

Big news on 29th june Compelling evidence for a GWB reported by several teams!

Cosmic Background of the 21th century

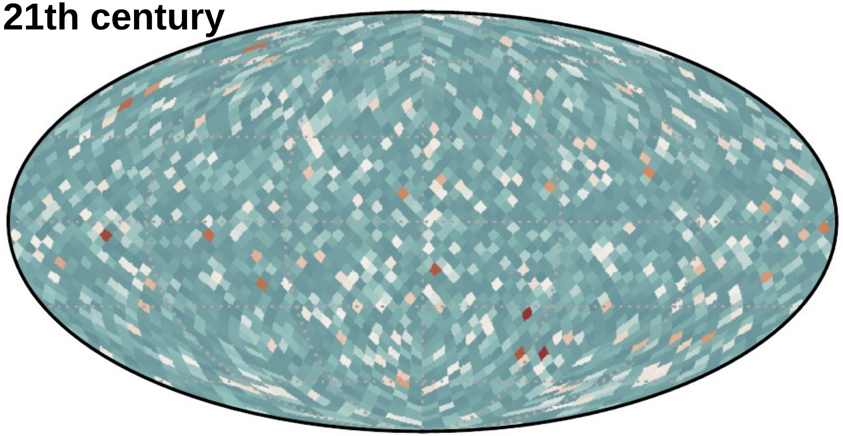
[PLANCK Collaboration]

20th century



[Sato-Polito, Kamionkowski: 2305.05690]

21th century



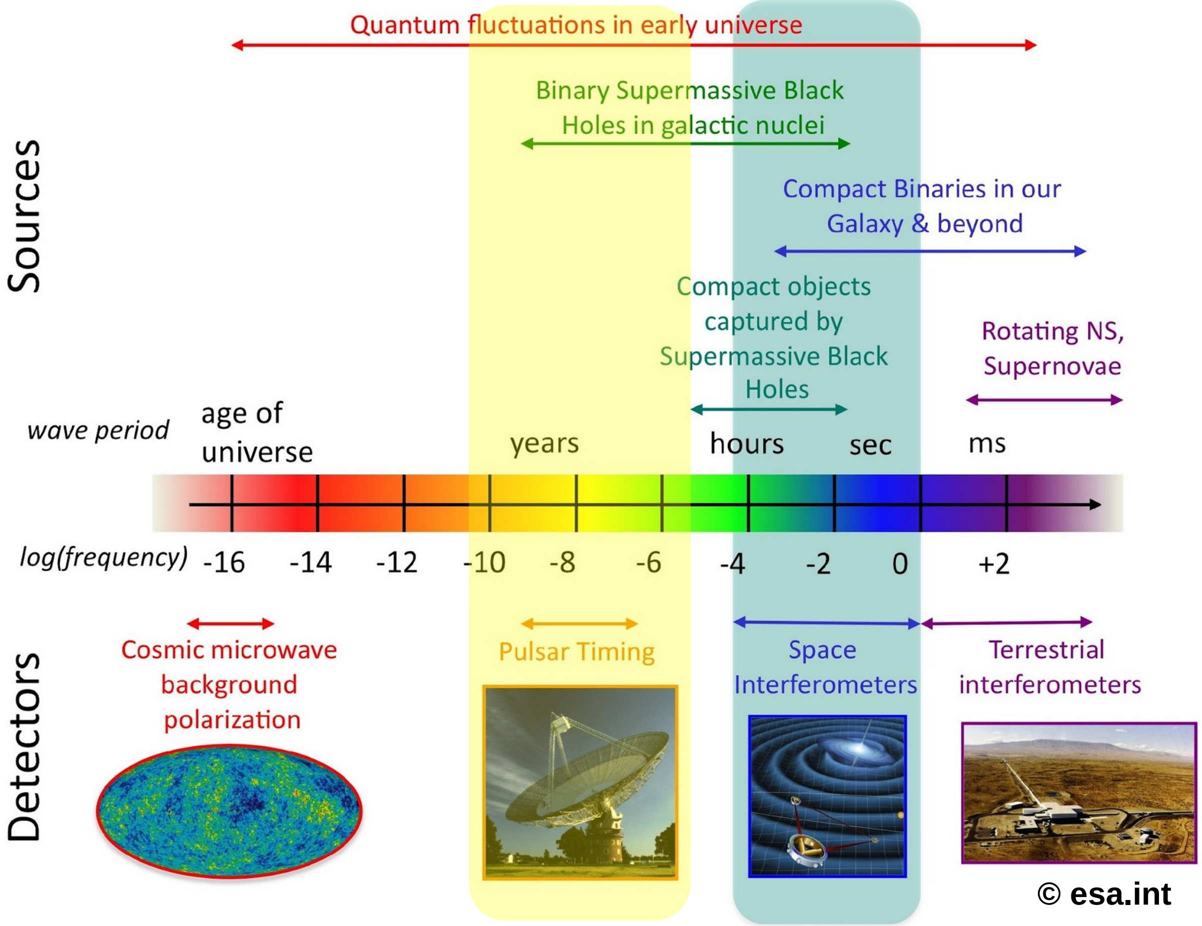
CMB: Cosmic microwave background

Relic photons from the early Universe

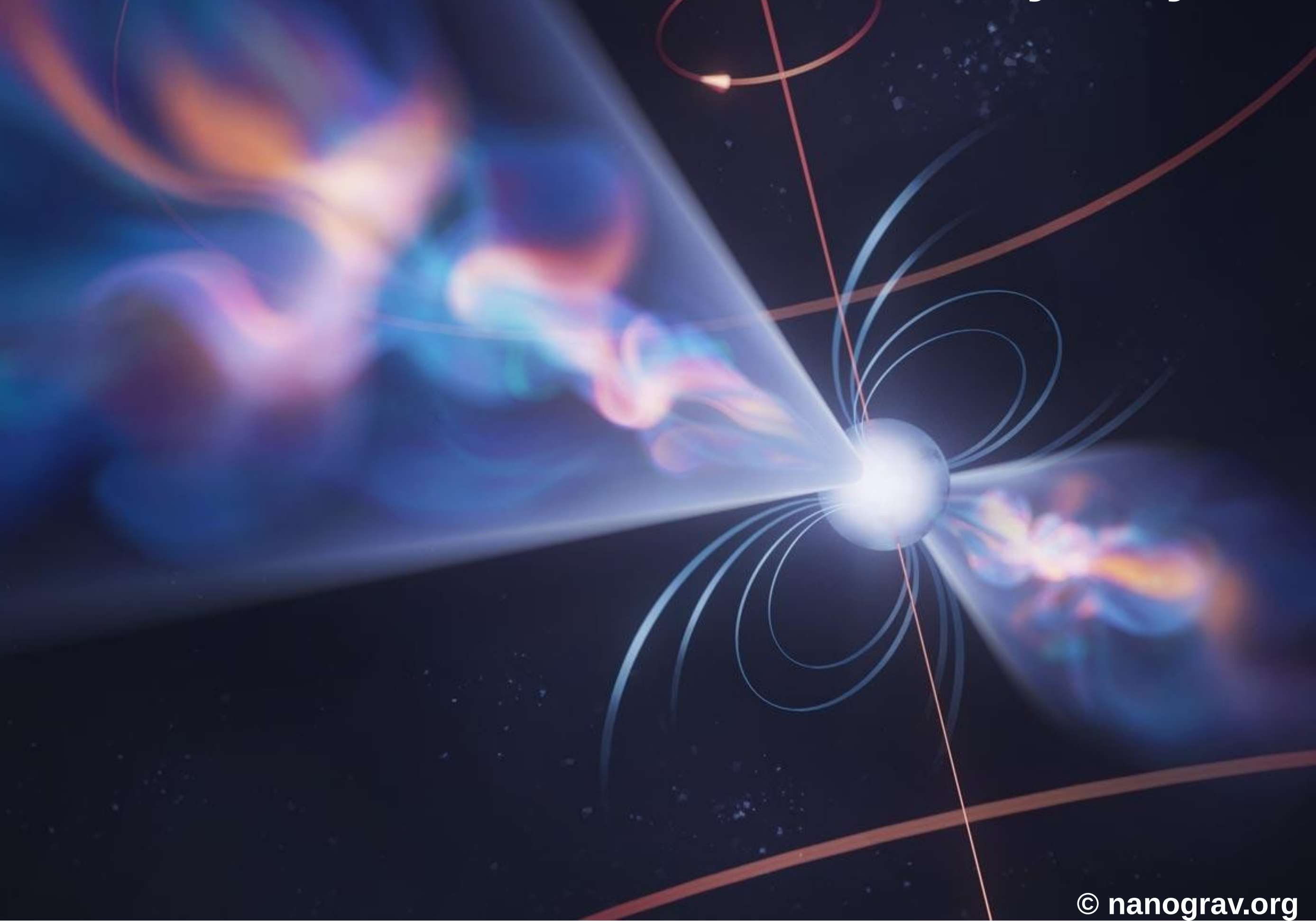
GWB: Gravitational-wave background

Relic gravitons from the early Universe
~ or ~ astrophysical signal

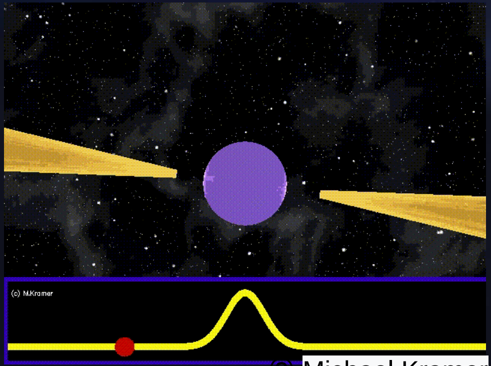
The Gravitational Wave Spectrum



Pulsars: cosmic clocks scattered across the Milky Way

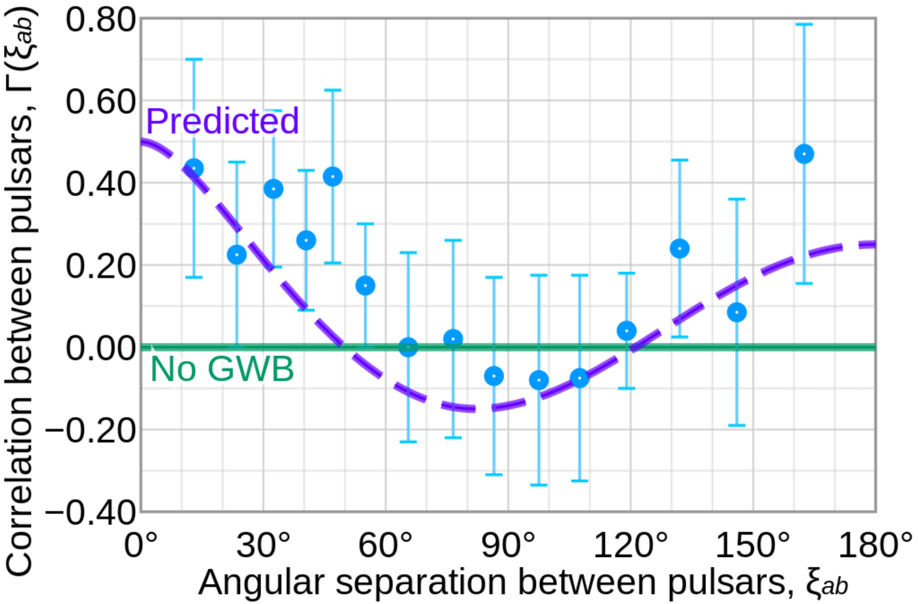


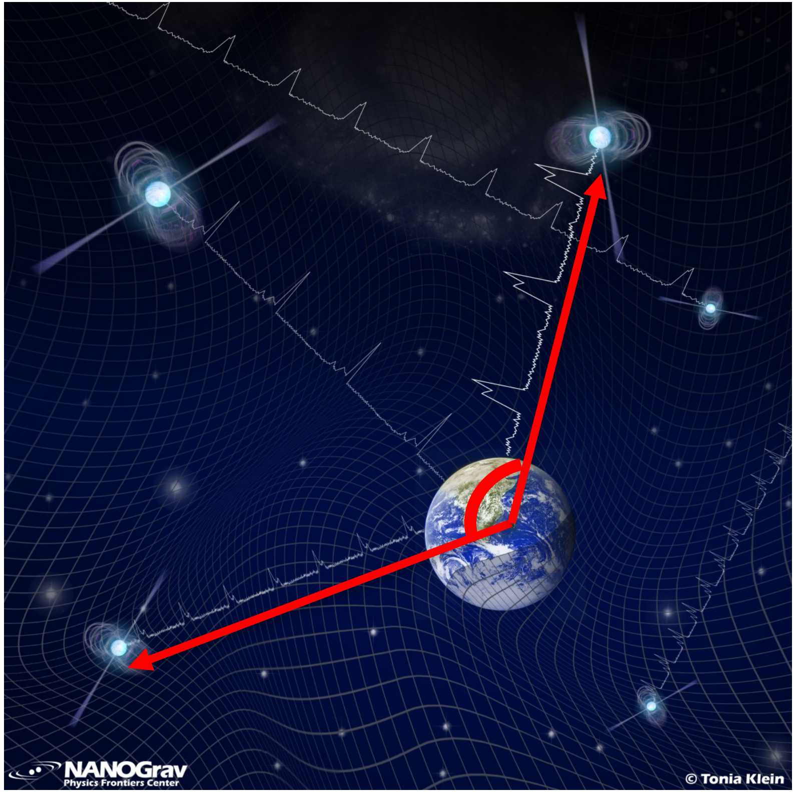
Cosmic lighthouse



PTAs are galaxy-sized GW detectors that allow us to search for nHz GWs

Hallmark signature in cross-correlation of timing residuals of pulsar pairs

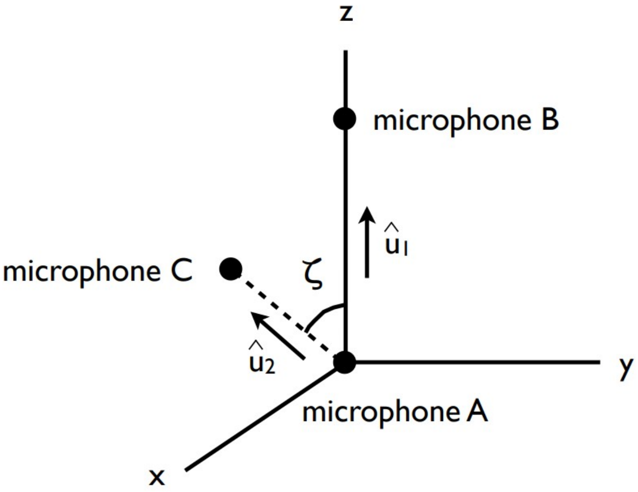




Quadrupolar correlations described by Hellings–Downs (HD) curve
[Hellings, Downs: Astrophys. J. 265 (1983) L39]

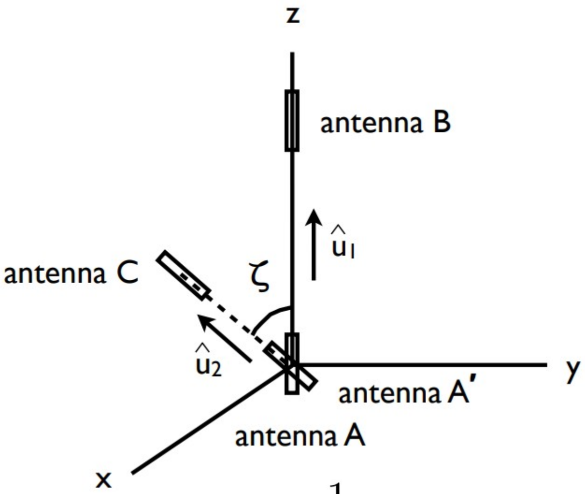
Hellings-Downs curve

Sounds Wave



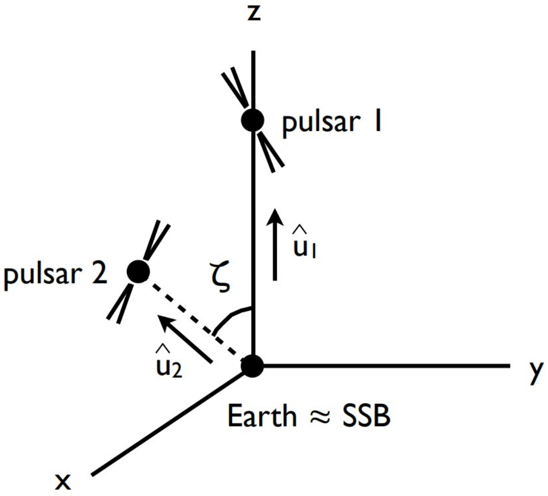
$$\chi(\zeta) \simeq \text{ Const.}$$

EM Wave



$$\chi(\zeta) \simeq \frac{1}{3}\cos\zeta$$

GW



$$\chi(\zeta) = \frac{1}{6} - \frac{1}{12}\left(\frac{1-\cos\zeta}{2}\right) + \frac{1}{2}\left(\frac{1-\cos\zeta}{2}\right)\ln\left(\frac{1-\cos\zeta}{2}\right)$$

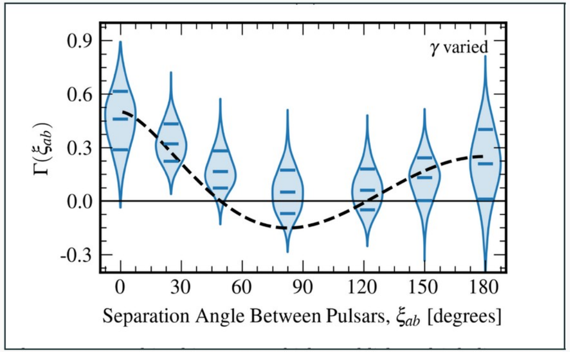
arXiv: 1412.1142

Lovell
Nançay
WSRT
SRT
Effelsberg
EPTA
GBT
Arecibo
VLA
CHIME
NANOGrav
FAST
CPTA
Parkes
PPTA
MeerKAT
MPTA
GMRT
InPTA
IPTA

PTArcade

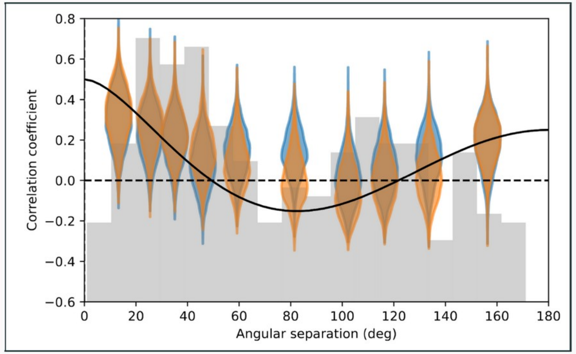
Compelling evidence for HD correlations

2306.16213: NANOGrav



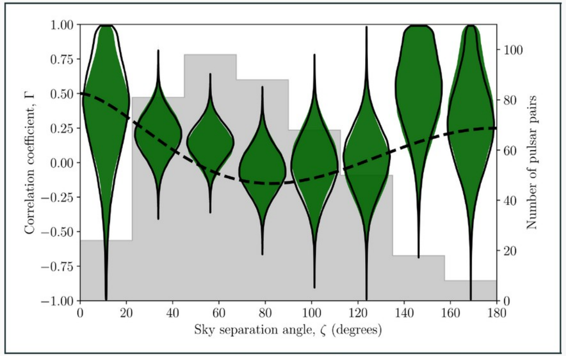
68 pulsars, 16 yr of data, HD at $\sim 3 \cdots 4\,\sigma$

2306.16214: EPTA+InPTA



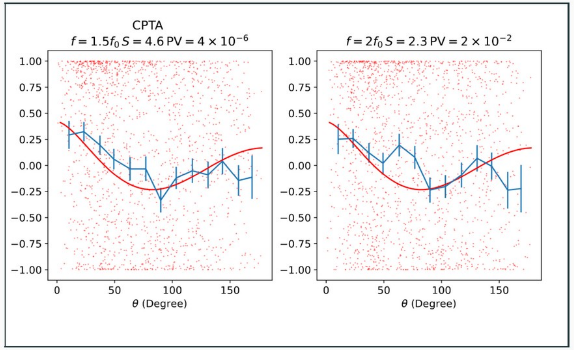
25 pulsars, 25 yr of data, HD at $\sim 3\,\sigma$

2306.16215: PPTA



32 pulsars, 18 yr of data, HD at $\sim 2\,\sigma$

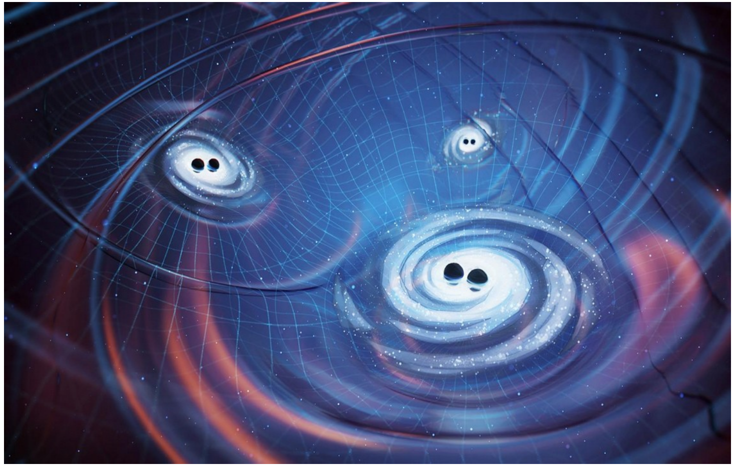
2306.16216: CPTA



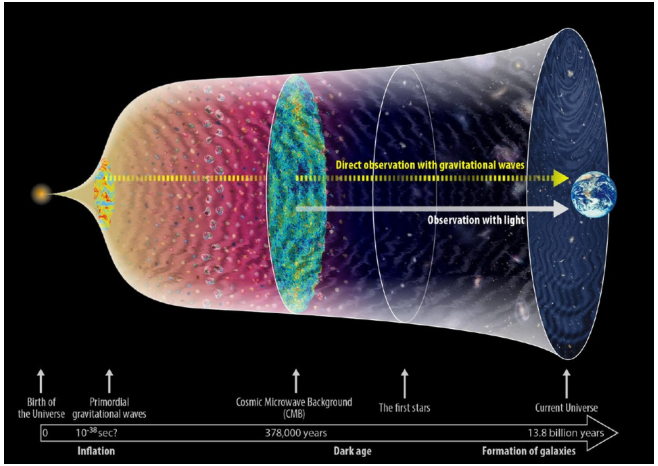
57 pulsars, 3.5 yr of data, HD at $\sim 4.6\,\sigma$

Interpretation: SMBHBs (realistic) or new physics (speculative)

Supermassive black-hole binaries



GWs from the Big Bang

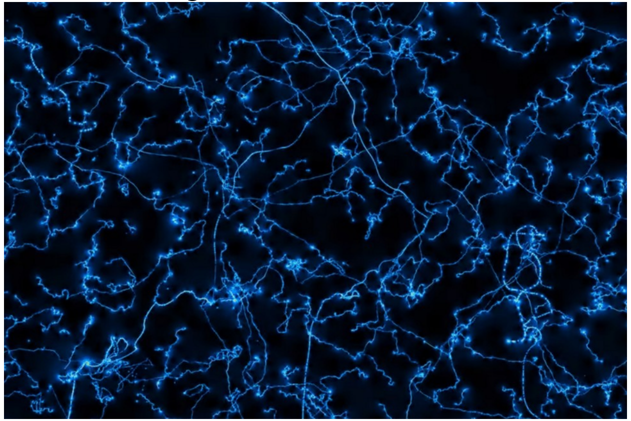


SMBHBs: No SMBHB mergers observed → data-driven field thanks to PTAs
New physics: Probe cosmology at early times, particle physics at high energies

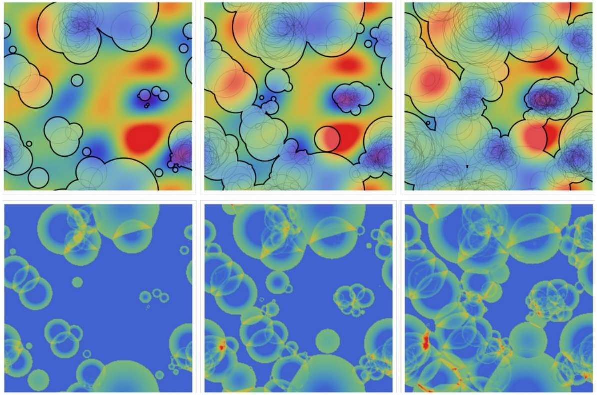
BSM scenarios: Inflationary gravitational waves, **scalar-induced gravitational waves**, cosmological phase transition, cosmic strings, domain walls, axions, and many more

New physics: many BSM models predicting a GWB from the Big Bang

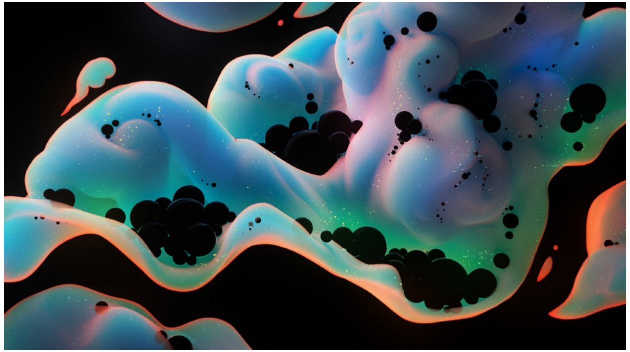
Cosmic defects
Cosmic strings, domain walls



Phase transition
Modified QCD transition, dark sector

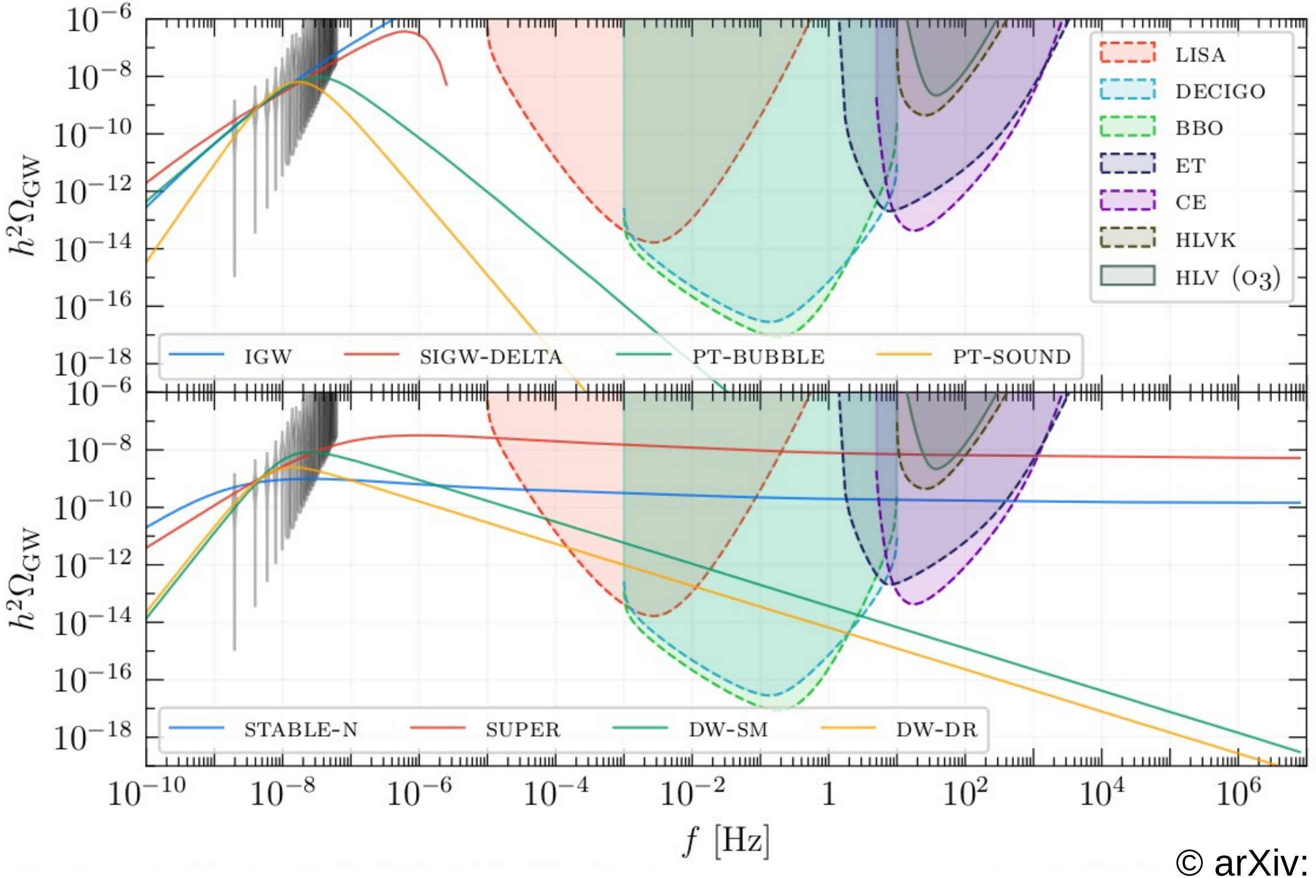


Inflation
Non-minimal blue-tilted models

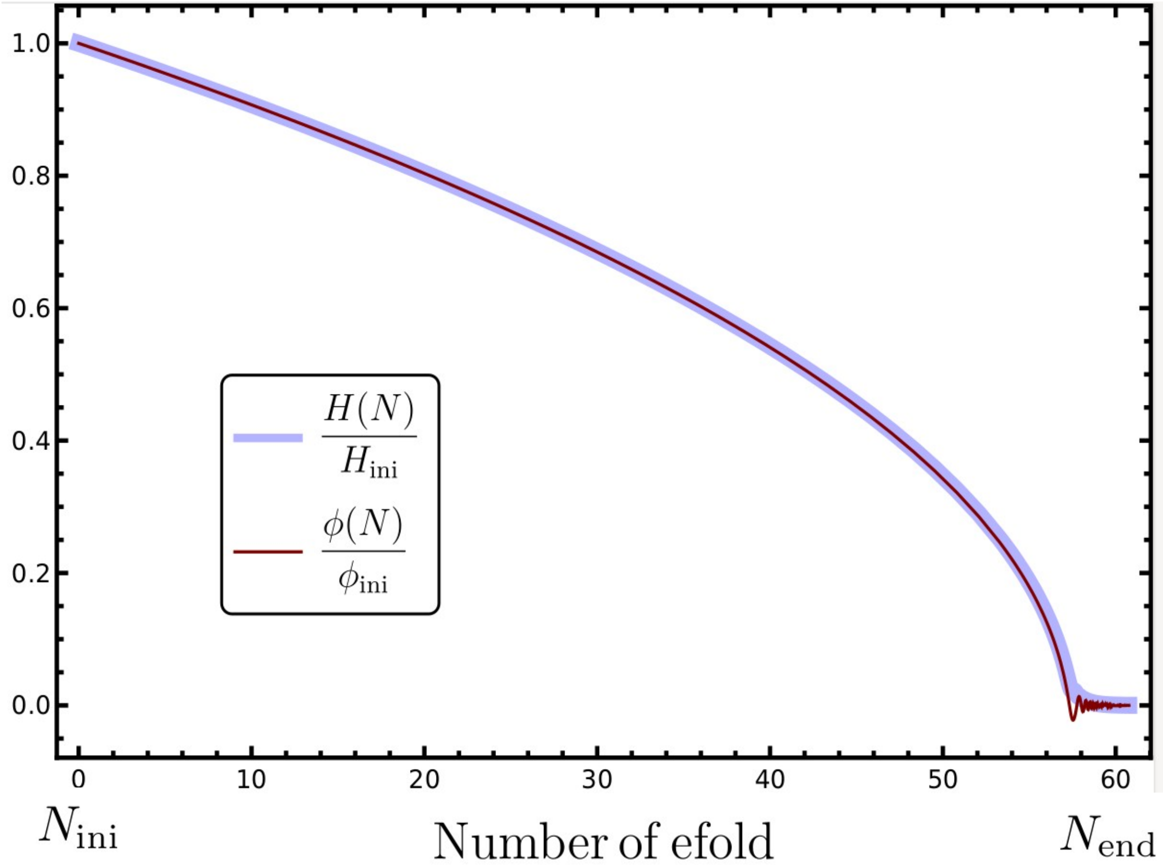


Scalar perturbations
Associated with primordial black holes
PBH dark matter, supermassive BHs

NANOGrav 15-year New-Physics Signals



Single (Slow-roll) Inflation



Action:

$$\mathcal{S} = \int \mathrm{d}^4x \, \sqrt{g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 \right]$$

Slow-roll Eqs.

$$3 M_{\rm Pl}^2 H^2 \simeq \frac{1}{2} m^2 \phi^2$$

$$3 H \dot{\phi} \simeq -m^2 \phi$$

$$N_{\rm end} \simeq \frac{1}{4} \left(\frac{\phi_{\rm ini}}{M_{\rm Pl}} \right)^2$$

Perturbations: Turn on Quantum Mechanics

$$\phi(t,\mathbf{x}) = \phi(t) + \delta\phi(t,\mathbf{x})$$

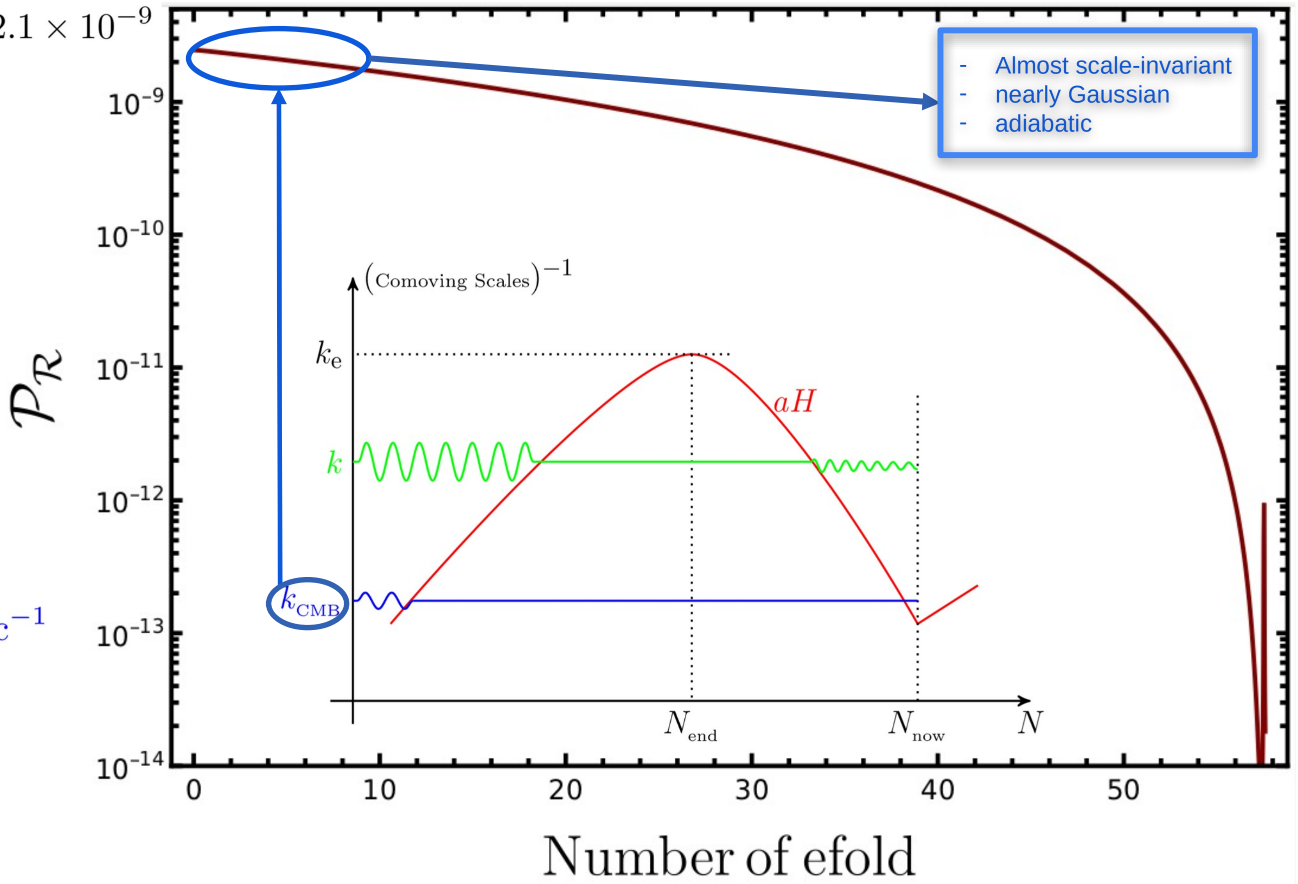
Spatially-flat gauge

$$\mathcal{R}(t,\mathbf{x}) = \frac{H(t)}{\dot{\phi}(t)}\delta\phi(t,\mathbf{x})$$

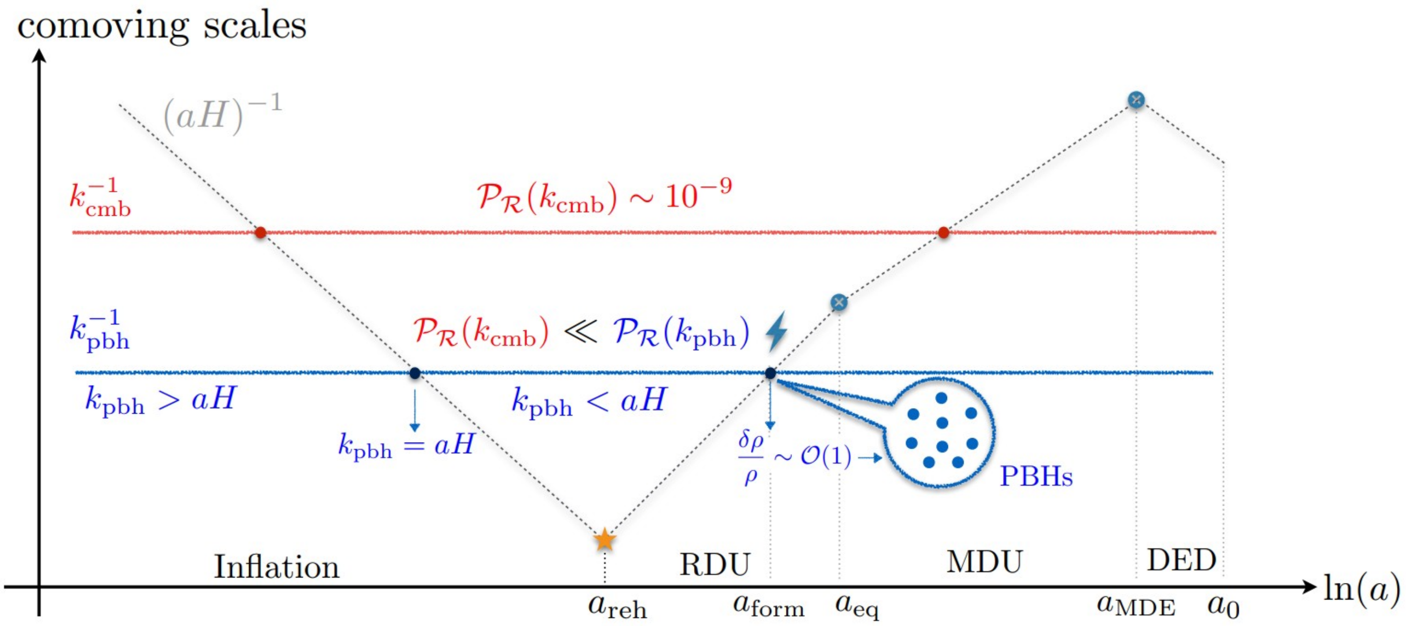
CMB Pivot Scale

$$k_{\rm CMB} \in [0.002, 0.05]\ {\rm Mpc}^{-1}$$

$$\mathcal{P}_{\rm COBE} \simeq 2.1 \times 10^{-9}$$



Modes exit the horizon during Inflation and re-enter during RD or MD era



Ozsoy&Tasinato (2301.03600)

Scalar-induced gravitational waves (SIGWs)

$$\mathrm{d}s^2 = -a^2\left[(1+2\Phi)\mathrm{d}\tau^2 + \left((1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$\Phi \simeq \Psi$$

$$h_{\mathbf{k}}^{\lambda\,\prime\prime}(\eta) + 2\mathcal{H}h_{\mathbf{k}}^{\lambda\,\prime}(\eta) + k^2 h_{\mathbf{k}}^{\lambda}(\eta) = 4S_{\mathbf{k}}^{\lambda}(\eta),$$

$$S_{\mathbf{k}}^{\lambda} = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\,\varepsilon_{ij}^{\lambda}(\hat{\mathbf{k}})\;q^i q^j\left[2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}\right)\left(\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}}\right)\right]$$

$$\Phi_{\mathbf{k}} = \frac{2}{3}\mathcal{T}(\mathbf{k}\tau)\mathcal{R}_{\mathbf{k}}\,.$$

integration kernel

$$\bar{\Omega}_{\mathrm{GW}}^{\mathrm{ind}}(f) = \int_0^{\infty}\mathrm{d}v\int_{|1-v|}^{1+v}\mathrm{d}u\,\mathcal{K}(u,v)\,\mathcal{P}_{\mathcal{R}}(uk)\,\mathcal{P}_{\mathcal{R}}(vk)$$

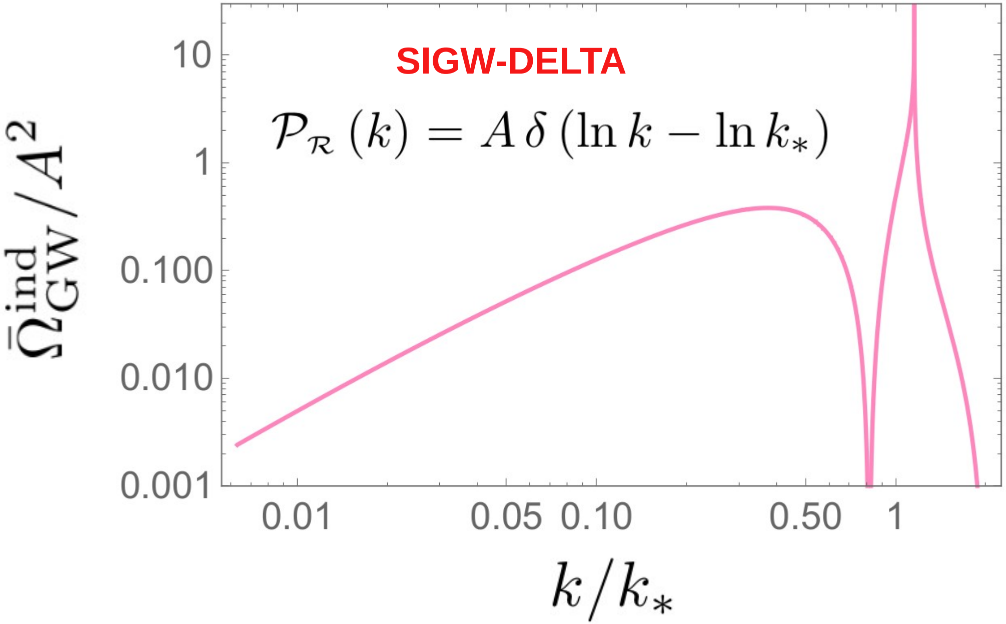
$$\Omega_{\mathrm{GW}}^{\mathrm{ind}}(f) = \Omega_{\mathrm{r}}\left(\frac{g_*(f)}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}(f)}\right)^{4/3}\bar{\Omega}_{\mathrm{GW}}^{\mathrm{ind}}(f)$$

Scalar-induced gravitational waves (SIGWs)

$$\mathcal{P}_h^{(\mathrm{ind})} \sim \int \mathrm{d}k \int \mathrm{d}k' \left[\int f(k,k',t)\mathrm{d}t\right]^2 \mathcal{P}_\mathcal{R}(k)\mathcal{P}_\mathcal{R}(k')$$

$\mathcal{R}$ → During **Radiation-dominated** era → **GWs**

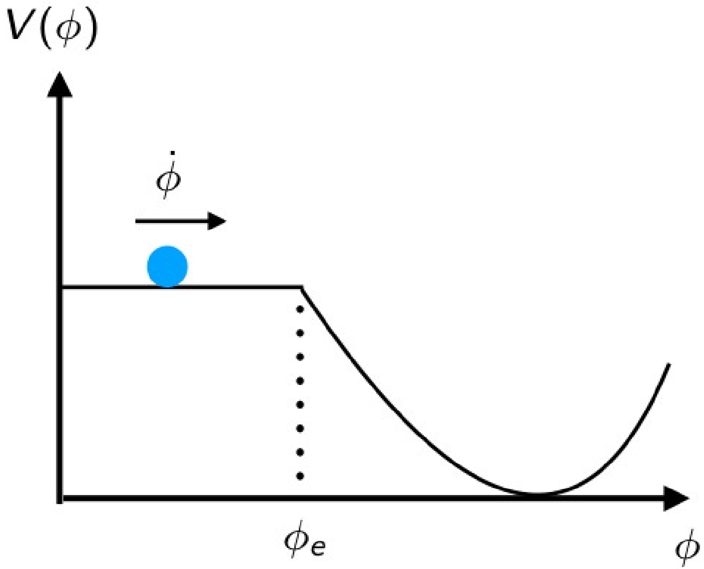
$f(k,k',t)$: oscillating function



Ultra-Slow-Roll (USR) model

USR inflation is a setup with a flat potential (Kinney 2006)

It was proposed as an example of single field model violating Maldacena's non-Gaussianity condition (M. H. Namjoo, H. F., M. Sasaki, 2012)



The background equations are given by

$$\ddot{\phi} + 3H\dot{\phi} = 0\,, \quad 3M_P^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V_0 \simeq V_0\,,$$

The setup is in a non-attractor phase so $N = N(\phi, \dot{\phi})$.

The setup: SR-USR-SR

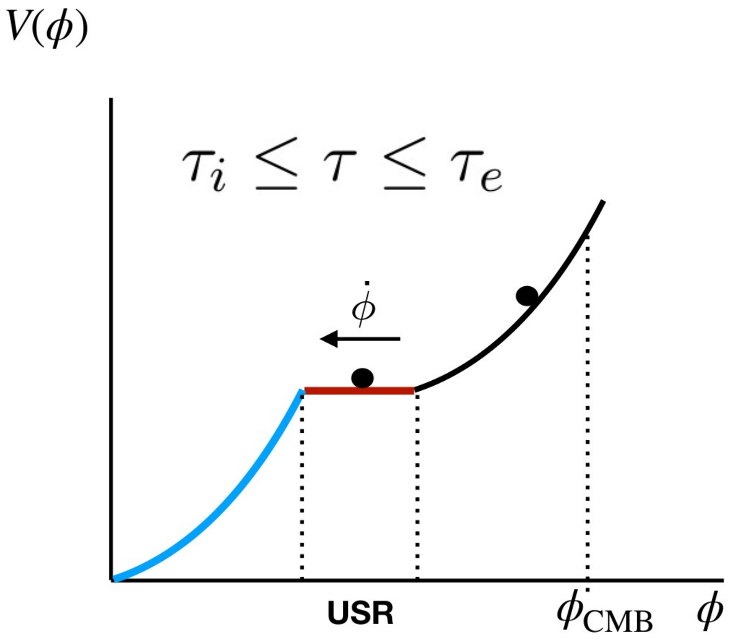
In collaboration with Hassan Firouzjahi
arXiv: 2307.03164

The setup is a three-phase model of inflation:

$SR \rightarrow$ USR $\rightarrow SR$

The CMB modes leave the horizon in first SR phase.

The USR modes experience growth: $\mathcal{R} \propto a(t)^3$



USR modes lead to PBHs formation and SIGWs during RD era!

The setup: SR-USR-SR

A key feature of the USR setup is that $\dot{\phi}$ falls off exponentially:

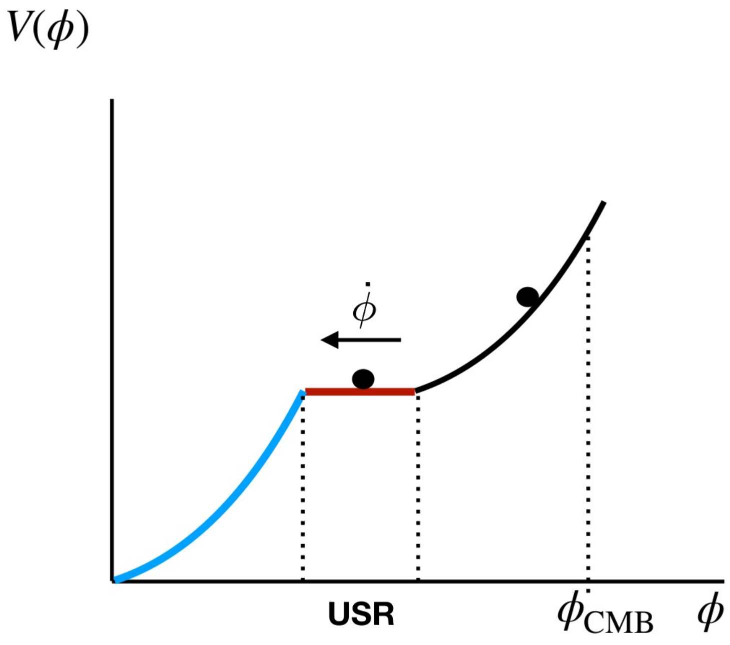
$$\dot{\phi} \propto a(t)^{-3} \longrightarrow \epsilon \equiv -\frac{\dot{H}}{H^2} \propto a(t)^{-6}$$

Here h measures the sharpness of the transition:

For a sharp transition $h \ll -1$.
For a mild transition $h \longrightarrow 0$.

As ϵ falls off exponentially, $\mathcal{R}$ grows exponentially:

$$\mathcal{R}_k = \frac{H}{M_P\sqrt{4\epsilon_i k^3}}\left(\frac{\tau_i}{\tau}\right)^3(1+ik\tau)e^{-ik\tau}$$



The setup: SR-USR-SR

$$(h, \Delta N) \text{ parameter space}$$

ΔN **: duration of the USR period**

$$\Delta N = \ln\left(\frac{\tau_i}{\tau_e}\right)$$

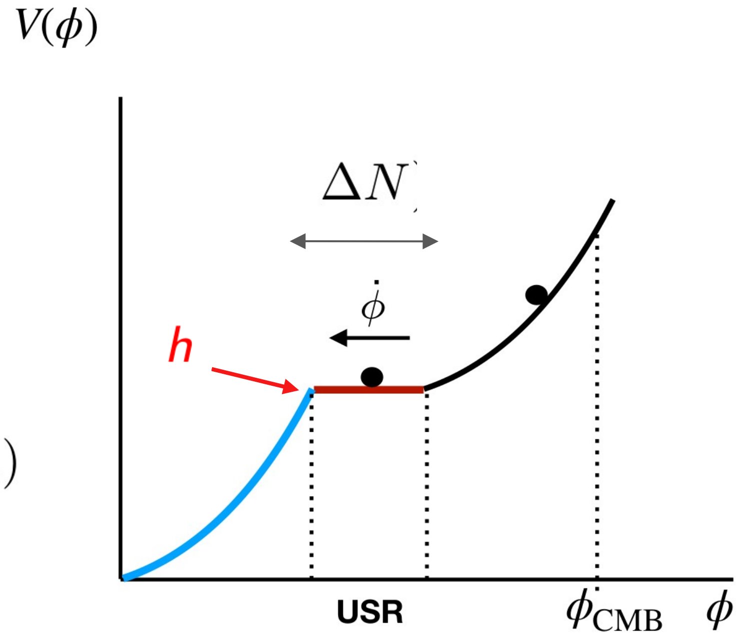
Here h measures the sharpness of the transition:

For a sharp transition $h \ll -1$.
For a mild transition $h \longrightarrow 0$.

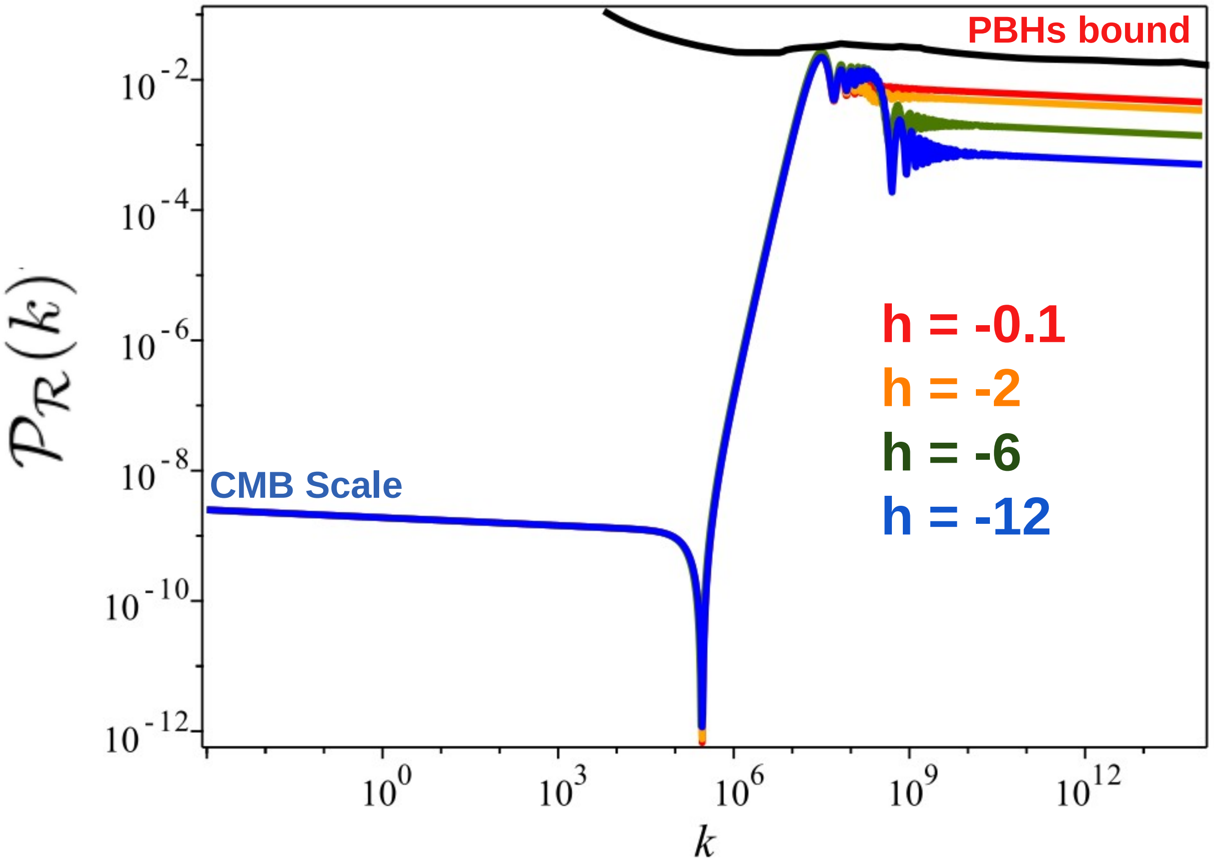
$$h \equiv -6\sqrt{\frac{\epsilon_V}{\epsilon_e}}$$

$$\mathcal{P}_{\mathcal{R}}(k, \tau = 0) \simeq \mathcal{P}_{\rm CMB}\; e^{6\Delta N} \left(\frac{h-6}{h}\right)^2 g(h, \tau_{\rm i}, \tau_{\rm e})$$

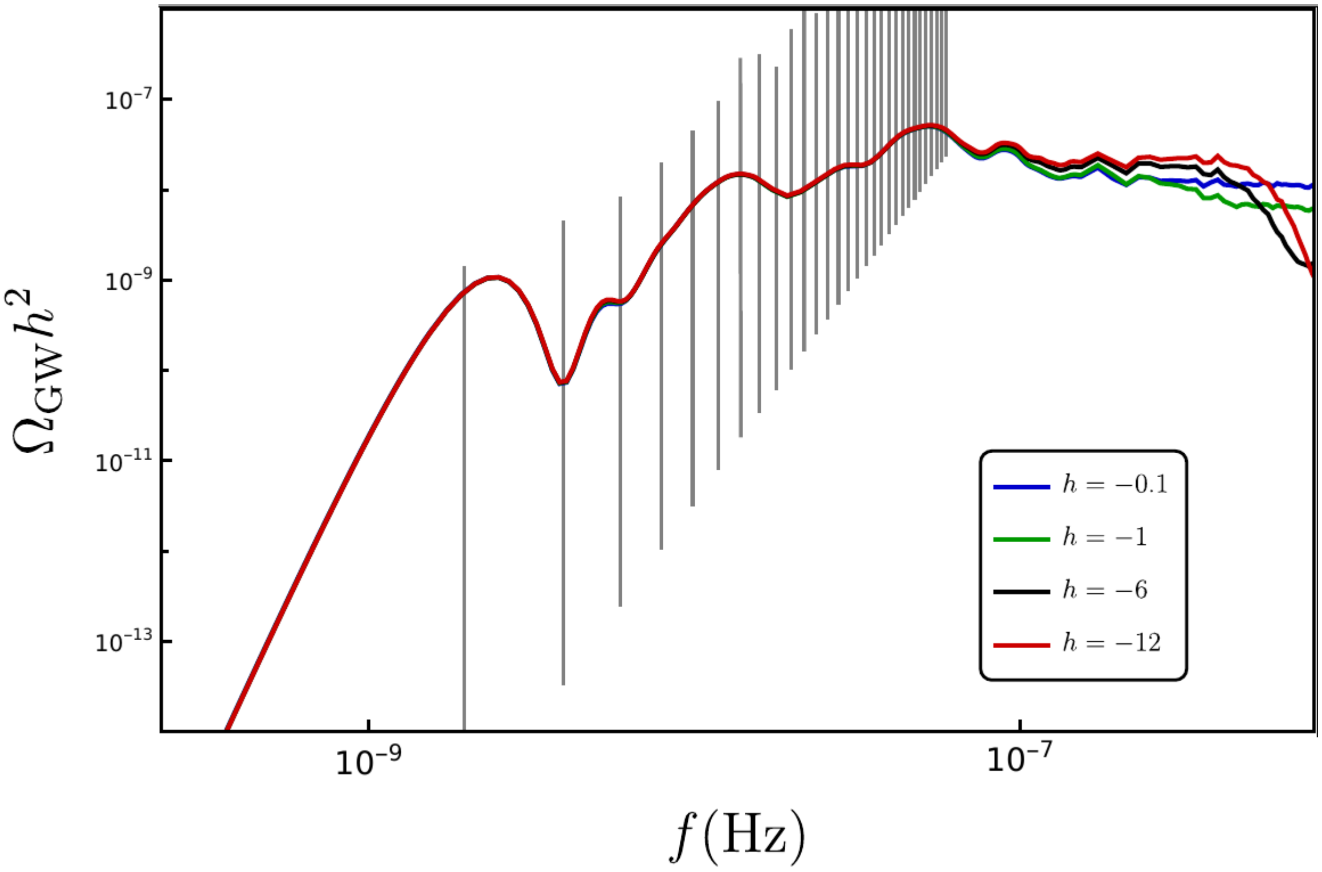
Local-type Non-G: $$f_{NL} = \frac{5h^2}{2(h-6)^2}$$



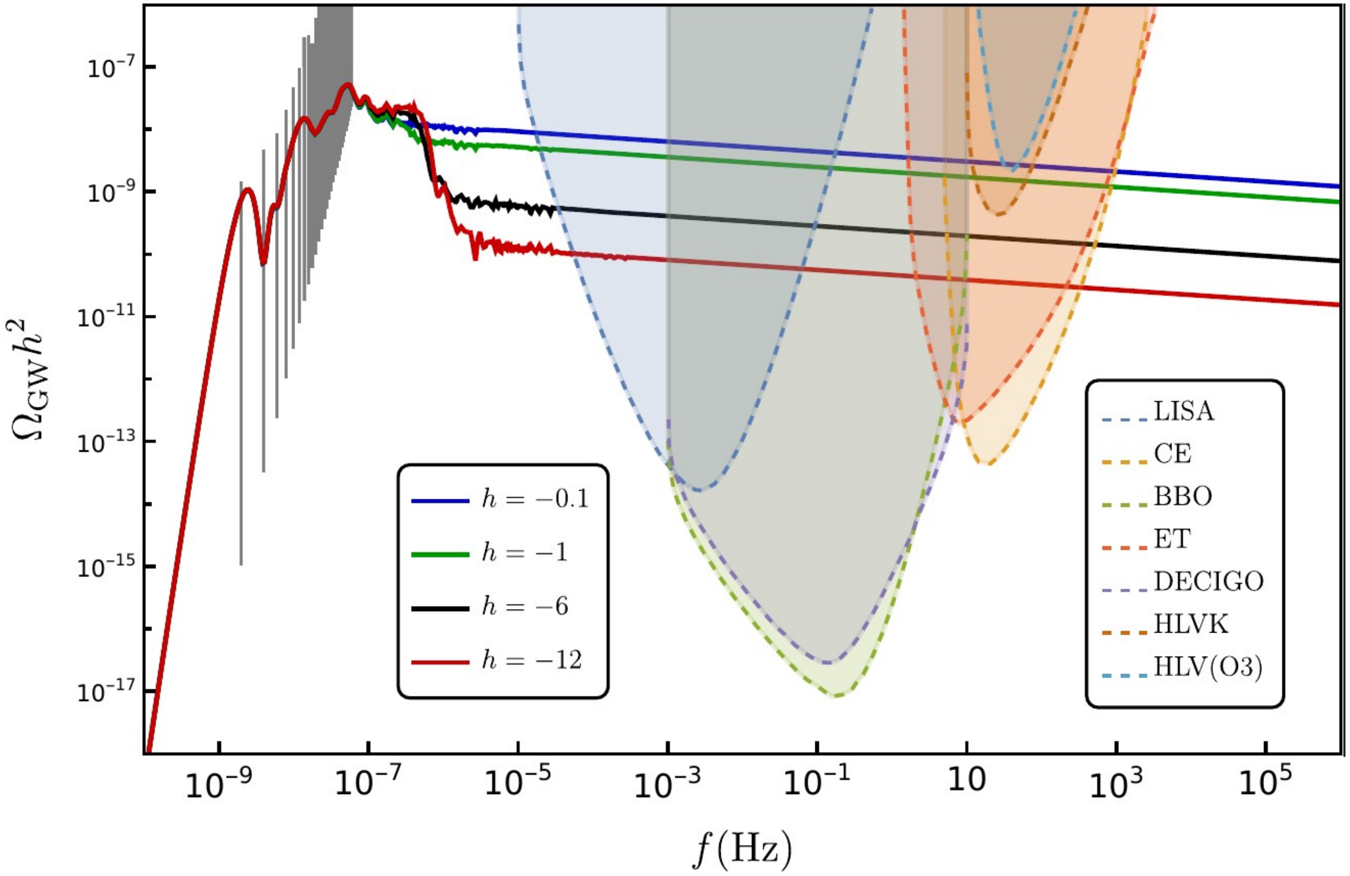
Enhanced Power spectrum



SIGW-USR: NanoGrav signal



SIGW-USR: future observations



Axion as Inflaton

Axion (inflaton): ϕ

$$\mathcal{L}_{\mathrm{matter}} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Chern-Simons interaction

$$A''_\lambda + \left(k^2 - 2\lambda\xi k a H\right)A_\lambda = 0$$

$$A^\lambda(\eta,k) = \frac{e^{\lambda\pi\xi/2}}{\sqrt{2k}}\,\mathrm{W}_{-i\lambda\xi,\frac{1}{2}}(2ik\eta)$$

$$\xi \equiv \frac{\alpha\, M_{\mathrm{Pl}}}{\sqrt{2}f}\mathrm{sgn}(\dot\phi)\sqrt{\epsilon_\phi}$$

Instability Parameter

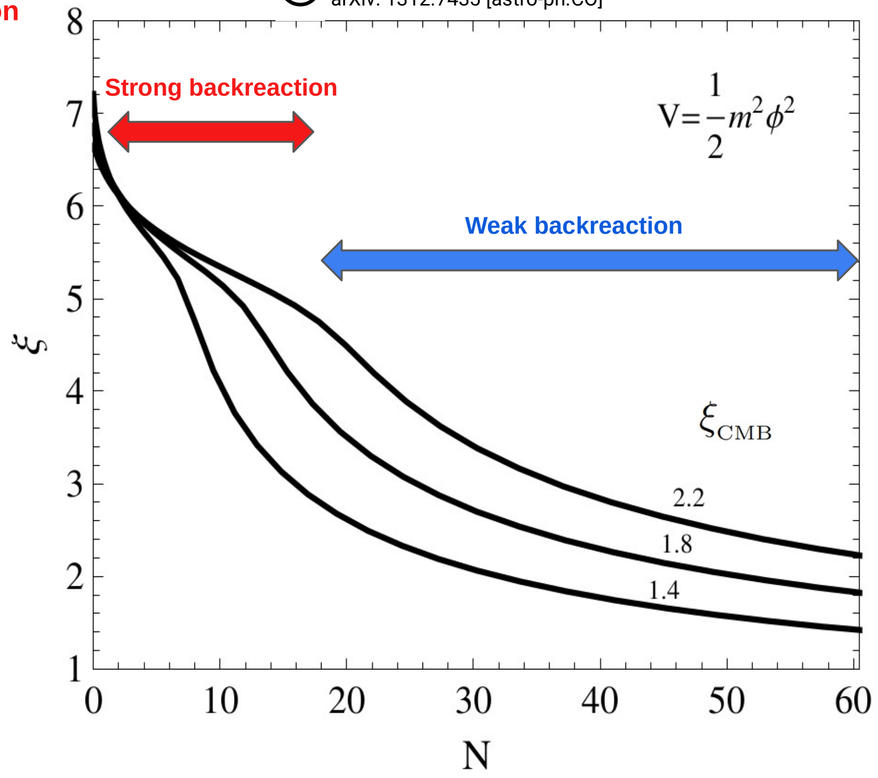
$$\epsilon_\phi \equiv \frac{\dot\phi^2}{2M_{\mathrm{Pl}}^2 H^2}$$

Tachyonic instability is experienced by modes

$$k < k_{\mathrm{cr}} \equiv 2|\lambda\xi|aH$$

Assuming $\xi > 0$:

$$A_+(k<k_{\mathrm{cr}}) \cong \frac{e^{\pi\xi - \sqrt{8\xi k/(\mathbf{a}H)}}}{\sqrt{2k}}\left(\frac{k}{2\xi\mathbf{a}H}\right)^{\frac{1}{4}} \propto e^{\pi\xi}$$



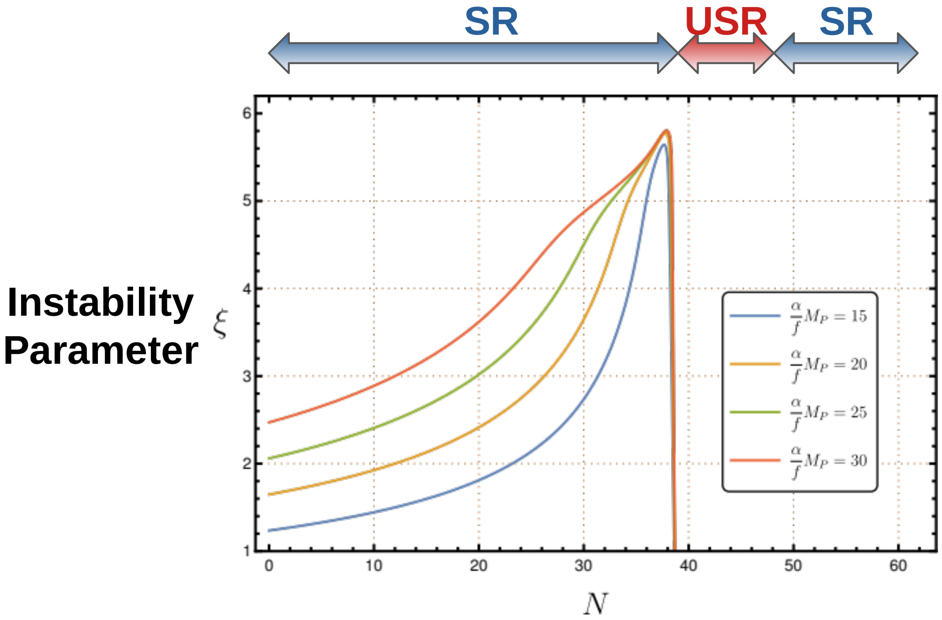
Axion-USR Model

In collaboration with Hassan Firouzjahi

$$S=\int d^4x\sqrt{-g}\Big[\frac{M_{\rm Pl}^2}{2}R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\alpha\phi}{4f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}\Big]$$

SR-USR-SR

The rolling field during the first SR phase contributes to instability parameter



Non-perturbative gauge field production:

$$A_k^{(-)} \propto e^{\pi\xi}$$

Axion-USR Model

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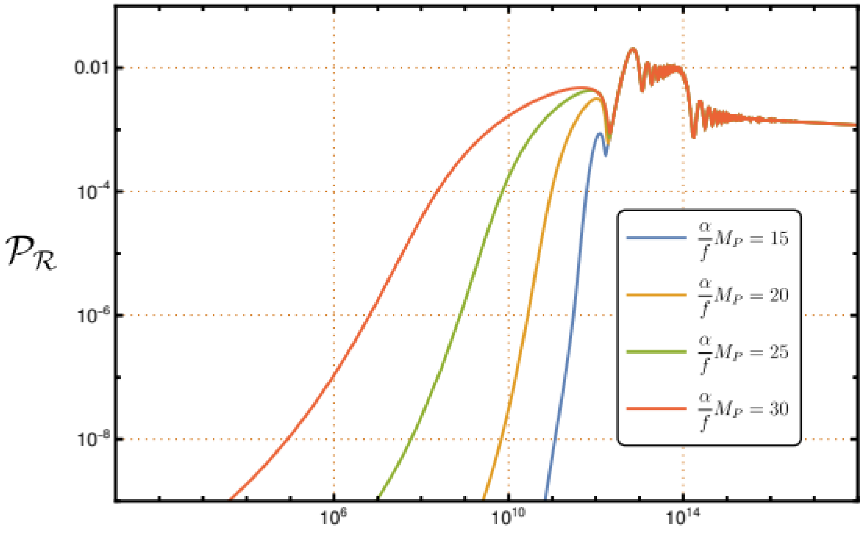
$$S = \int d^4x\sqrt{-g}\Big[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha\phi}{4f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}\Big]$$

$V(\phi)$ → SR-USR-SR → $\mathcal{R}_{\mathbf{k}}^{(\rm vac)}$

$\frac{\alpha\phi}{4f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$ → Chern-Simon term → tachyonic production of gauge field fluctuations during SR phase → inverse decay $\delta A + \delta A \to \delta\phi$ → $\mathcal{R}_{\mathbf{k}}^{(J)}$



$$\mathcal{P}_\mathcal{R}(\mathbf{k},\tau) = \mathcal{P}_\mathcal{R}^{(\rm vac)}(k,\tau) + \mathcal{P}_\mathcal{R}^{(J)}(\mathbf{k},\tau)$$

$$\mathcal{P}_\mathcal{R}(\mathbf{k},\tau=0) = \mathcal{P}_{\rm CMB}\left(\frac{\xi_{\rm CMB}}{\xi_i}\right)^2 e^{6\Delta N}\left(\frac{36}{h^2}\right)\left(\left|\alpha_k^{(3)} + \beta_k^{(3)}\right|^2 + \mathcal{P}_{\rm CMB}\, f_2(h, z_i, z_e, \xi_k)e^{4\pi\xi_k}\right)$$

Primordial GWs generations

$$\left(\partial_\tau^2 + k^2 - \frac{2}{\tau^2}\right)\hat{h}_k^\lambda(\tau) = \frac{-\mathbf{a}^3}{M_{\rm Pl}}\Pi_{ij}^\lambda(\mathbf{k}) \int \frac{d^3k\ e^{-i\mathbf{k}.\mathbf{x}}}{(2\pi)^{3/2}}\ [E_iE_j + B_iB_j]$$

transverse traceless projector Π_{ij}^λ

$$\hat{h}_\lambda = \hat{h}_\lambda^{(\rm vacuum)} + \hat{h}_\lambda^{(\rm source)}$$

Example

$$A_- = 0$$

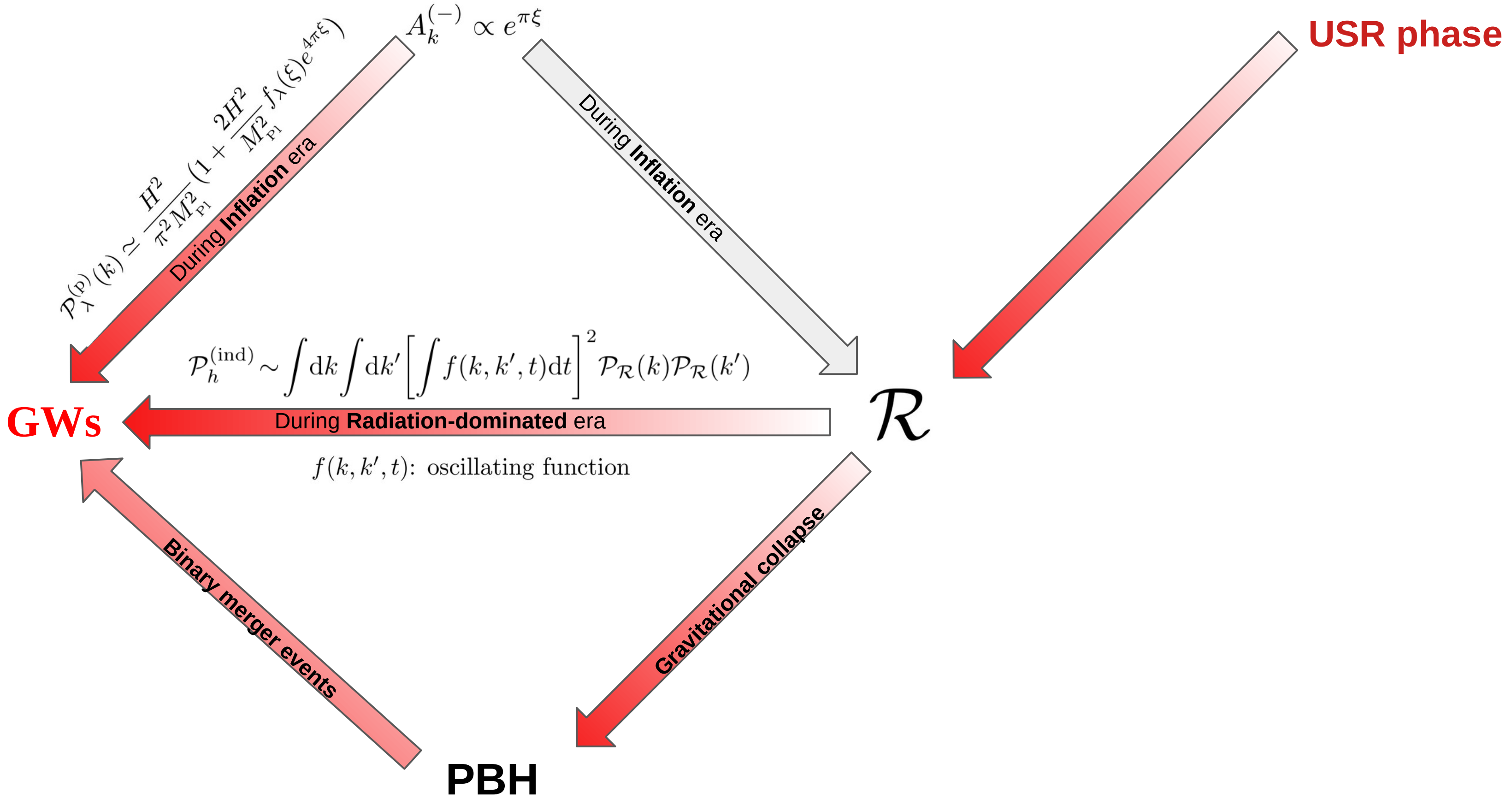
$$A_+(\tau, k) \simeq \frac{1}{\sqrt{2k}}\left(\frac{k}{2\,\xi\,aH}\right)^{1/4} e^{\pi\,\xi - 2\sqrt{2\xi\,k/aH}}$$

Chiral GWs

$$f_{\rm R}(\xi) \sim 10^{-7}/\xi^6$$
$$f_{\rm L}(\xi) \sim 10^{-9}/\xi^6$$

$$\mathcal{P}_\lambda^{(\rm p)}(k) \simeq \frac{H^2}{\pi^2 M_{\rm Pl}^2}\left(1 + \frac{2H^2}{M_{\rm Pl}^2} f_\lambda(\xi) e^{4\pi\xi}\right)$$

Gravitational Waves Production



Summary:

- Importance of stochastic GW Background
- The stochastic gravitational wave background (SGWB) detected recently by the pulsar timing arrays (PTAs) observations may have cosmological origins.
- We generated SIGW in nanoHertz frequency from:

  I- a three-phase model of inflation: Slow-Roll > USR > Slow-Roll

  II- Axion-USR model

Thank You for Your Attention!

Primordial Black Holes

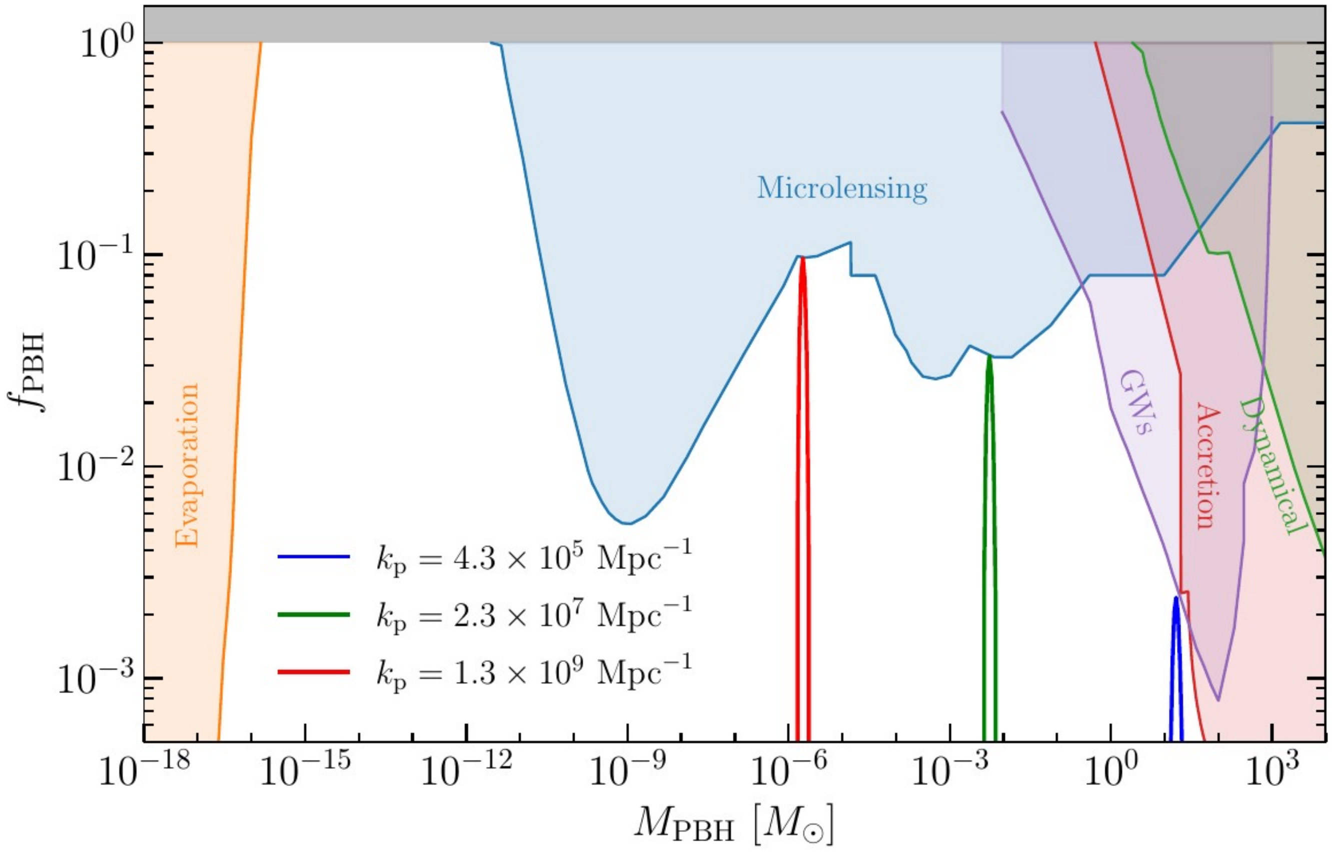
- **Black holes formed in the early Universe (soon after the Big Bang through a non-stellar way)**
  - Gravitational collapse of the overdense region of inhomogeneities During the radiation dominated era

$$\beta \simeq \int_{\mathcal{R}_c}^{\infty} f_{\mathcal{R}}(x)\ \mathrm{d}x \simeq \frac{1}{2}\mathrm{Erfc}\left(\frac{\mathcal{R}_c}{\sqrt{2\mathcal{P}_{\mathcal{R}}}}\right)$$

$$f_{\mathrm{PBH}}(M_{\mathrm{PBH}}) \simeq 2.7 \times 10^8 \left(\frac{M_{\mathrm{PBH}}}{M_{\odot}}\right)^{-\frac{1}{2}} \beta(M_{\mathrm{PBH}})$$

$$\frac{M_{\mathrm{PBH}}}{M_{\odot}} \simeq 30 \left(\frac{k_{\mathrm{p}}}{3.2 \times 10^5\ \mathrm{Mpc}^{-1}}\right)^{-2}$$

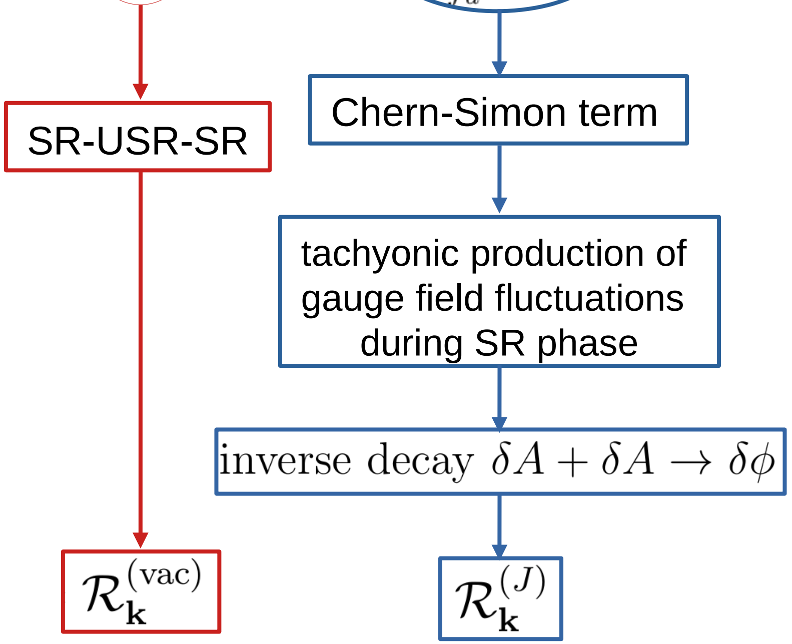
PBH abundance



Axion-USR Model

In collaboration with Hassan Firouzjahi

$$S = \int d^4x \sqrt{-g}\Big[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha\phi}{4f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}\Big]$$



$$\ddot{\phi} + 3H\dot{\phi} = -V_\phi + \langle J_{\rm em}\rangle\,,$$

$$3M_P^2H^2 - V(\phi) = \frac{1}{2}\dot{\phi}^2 + \langle\rho_{\rm em}\rangle\,,$$

$$\vec{A}'' - \nabla^2\vec{A} - \frac{\alpha}{f_a}\phi'\,\vec{\nabla}\times\vec{A} = 0$$

$$J_{\rm em} \equiv \frac{\alpha}{f_a}\vec{E}\cdot\vec{B}$$

$$\rho_{\rm em} \equiv \frac{1}{2}(\vec{E}^2 + \vec{B}^2)$$

$$\mathcal{R}_{\mathbf{k}} = \mathcal{R}_{\mathbf{k}}^{(\rm vac)} + \mathcal{R}_{\mathbf{k}}^{(J)}$$