Analytic Formulae for Inflationary Correlators with Dynamical Mass

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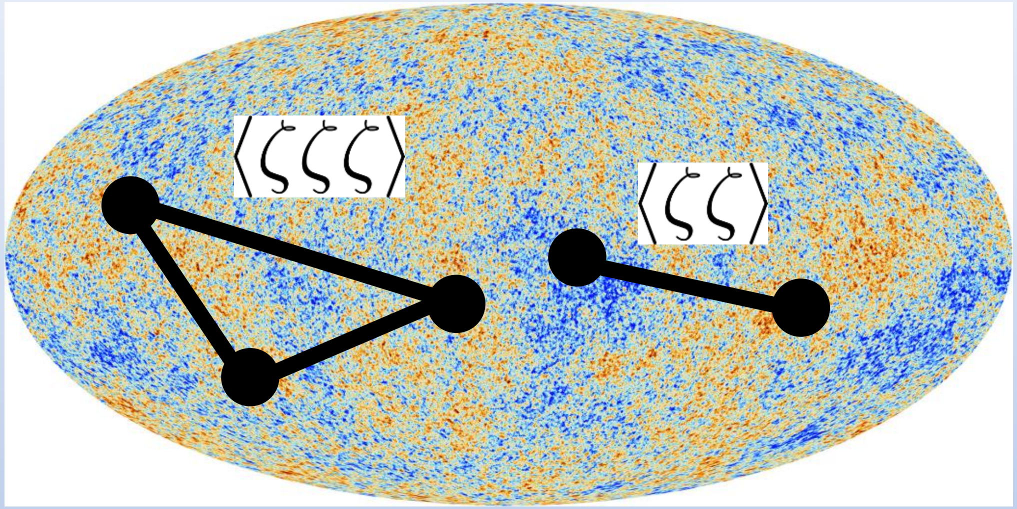
Observables for Inflationary Cosmology



2pt. correlation function (power spectrum)

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle_{\text{inf. end}} = (2\pi)^3 \delta^3(\boldsymbol{k}+\boldsymbol{k}') \frac{2\pi^2}{k^3} P_\zeta$$

$$P_\zeta \simeq \frac{H^2}{8\pi^2 \epsilon} \left(\frac{k}{k_*} \right)^{n_s - 1} , \quad n_s \simeq 0.965 , \quad \frac{dn_s}{d\log k} \simeq 0.002$$

[Planck ’18]

➡ Degeneracy of inflation models

3pt. correlation function (bispectrum) ⟵ Not yet observed in sufficient accuracy

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle_{\text{inf. end}} = (2\pi)^7 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \frac{P_{\zeta *}^2}{(k_1 k_2 k_3)^2} S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3} \right)$$

[Chen and Wang ’09]

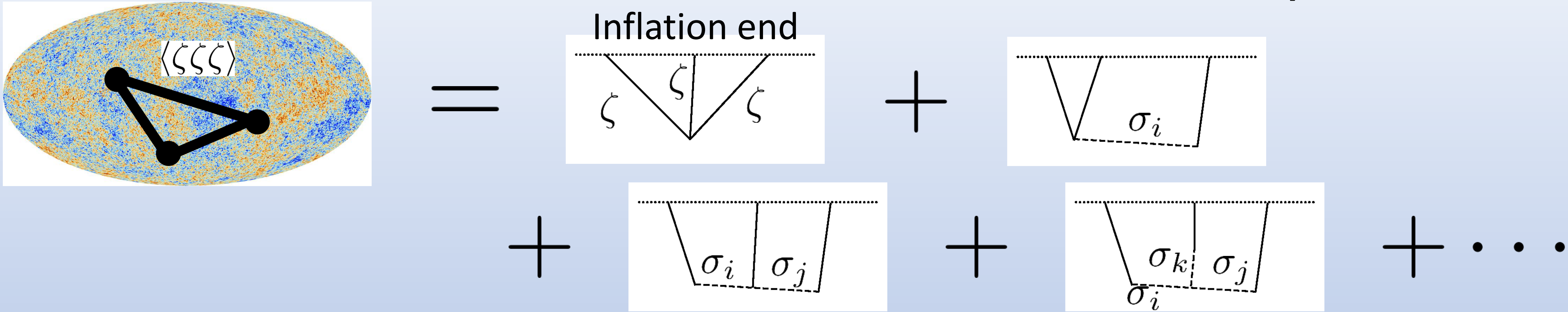
S : dim. less, model dependent shape function

$\langle\zeta\zeta\zeta\rangle$: effects of interactions in perturb. theory of QFT

➡ Probe for inflation models and BSM physics

Cosmological Collider Project

[Chen and Wang ’09, Noumi et al. ’12, Arkani-Hamed and Maldacena, ’15, Lee et al. ’16 etc.]

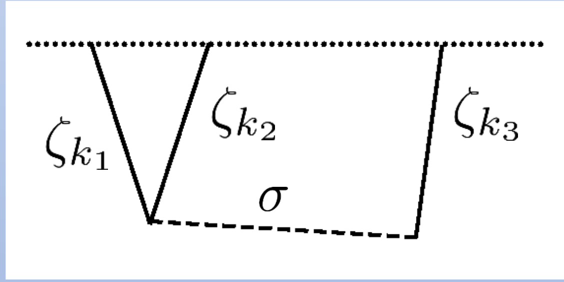


Signals for massive particles

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_1}{k_3} + \delta\right)$$

$k_3 \ll k_1 \simeq k_2$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$



Mass: wavelength of the shape function

Dictionary for BSM particles in high energy scale $\rho_{\rm inf}^{1/4} \lesssim 10^{15}\ \mathrm{GeV}$

Supersymmetry, gauge symmetry, CP violation, swampland, …

[Baumann and Green ’12] [Maru and Okawa ’21] [Liu et al. ’21] [Reece et al. ’22]

Q. Distinction of Interactions like Colliders on Ground?

Some Directions:

- Phase of oscillation [Qin and Xianyu '22]

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_{\rm L}}{k_{\rm S}} + \underline{\underline{\delta}}\right)$$

→ Expected to be uniquely determined by

$$\mu,\ \frac{k_{\rm L}}{k_{\rm S}},\ {\rm spin},\ {\rm diagram}$$

- Non-unity sound speed in EFT [Jazayeri et al. '22, Jazayeri and Renaux-Petel '23]

A peak in not-so-squeezed region $\frac{k_{\rm L}}{k_{\rm S}} \sim c_s$ (sound horizon crossing)

- Beyond scale invariant approx. (our work)

Scale dependence ⟷ De Sitter sym. breaking ⟷ Non-der. ints.

Derivative ints.: $f(\partial_\mu\phi, \sigma, \partial_\mu\sigma)$

- respect shift sym. of ϕ (de Sitter)
- EFT, SUGRA, etc.

Non-derivative ints.: $f(\phi, \sigma, \partial_\mu\sigma)$

- break shift sym. (slow-roll effects)
- Higgs, axion, extra dim., etc.

$\phi\bar{\psi}\psi \quad \phi F^{\mu\nu}\tilde{F}_{\mu\nu} \quad e^{\alpha\phi/M_{\rm Pl}}\sigma^2$

Demonstration of Scale Dependence (our calculation)

Approximated / numerical results: [Wang '19, Reece et al. '22]

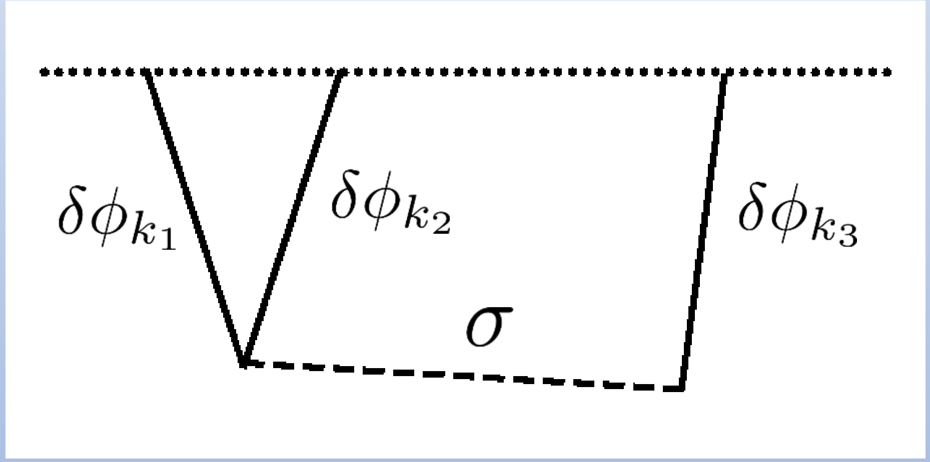
Action for Inflaton ϕ + massive scalar spectator σ

$$S = \int dx^4 \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_0^2\sigma^2 - M_{\rm pl} y \phi \sigma^2 + \mathcal{L}_{\rm diag}\right]$$

Sym. breaking interaction

Interactions for the diagram

$$\mathcal{L}_{\rm diag} \supset c_2(-\tau)^{-3}\sigma\delta\phi' + c_3(-\tau)^{-2}\sigma(\delta\phi')^2$$



Time dependent mass (excursion of inflaton)

$$m_{\rm eff}^2 = m_0^2 + 2yM_{\rm pl}\phi_0$$

$$\phi_0' = \frac{\sqrt{2\epsilon}M_{\rm pl}}{\tau}$$

Slow-roll approx. $\phi_0 = \sqrt{2\epsilon}M_{\rm pl}\log\frac{\tau}{\tau_0}$

➡ Linear approx. $\phi_0(\tau) \simeq \phi_{*0} - \sqrt{2\epsilon}M_{\rm pl}\left(1 - \frac{\tau}{\tau_*}\right)$

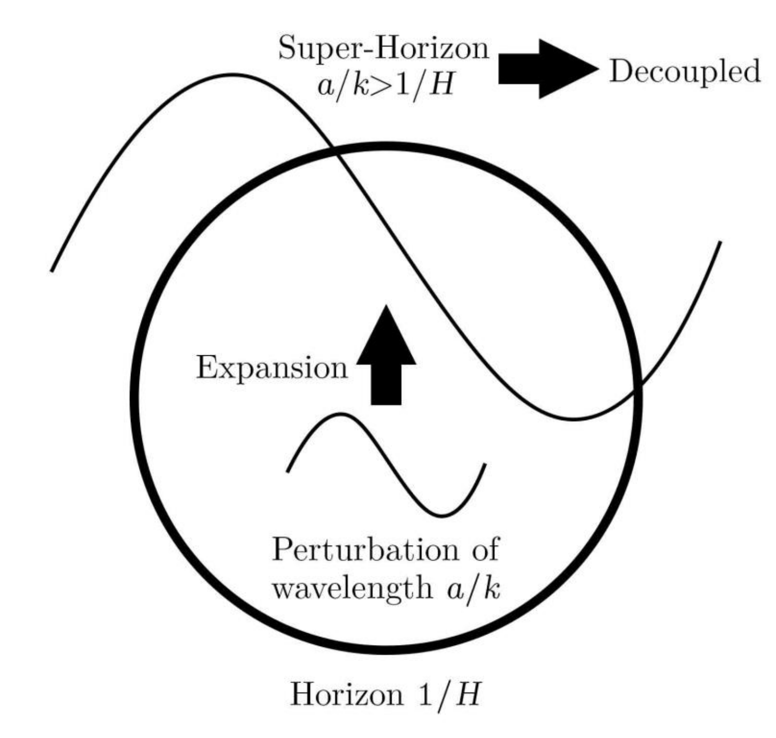
Initial condition $\phi_{*0} \simeq \sqrt{2\epsilon}M_{\rm pl}\log\frac{\tau_*}{\tau_0}$

Additional scale τ_0, τ_*

Effects of Time-Dependent Mass

Evo. of perturb. ⬅ **Time of horizon crossing** ⬌ **Scales**

$k\tau = -1$



Constant mass: Scale invariant $S(k_1/k_3, k_2/k_3)$

Time dependent mass: Scale dependent

Different mass for each horizon crossing scale

➡ Dependence on values of scales itself

Fixing additional scales

Two additional scales τ_0, τ_*

$$\phi_0(\tau) \simeq \phi_{*0} - \sqrt{2\epsilon}M_{\rm pl}\left(1 - \frac{\tau}{\tau_*}\right)$$

$$\phi_{*0} \simeq \sqrt{2\epsilon}M_{\rm pl}\log\frac{\tau_*}{\tau_0}$$

$$\begin{cases} k_i\tau_* = -1 & \text{(expansion at horizon crossing)} \\ \phi_{*0} \simeq \sqrt{2\epsilon}M_{\rm pl}\log\left(\frac{k_0}{k_*}\right) \sim \mathcal{O}\left(N_{\rm CMB}\sqrt{2\epsilon}M_{\rm pl}\right) & \text{at the largest scale of CMB} \end{cases}$$

τ_0, τ_* ⬌ k_0, k_*

$k_* \sim 10^{-4}\ {\rm Mpc}^{-1}$

Method: Cosmological bootstrap [Series of papers by Baumann, Lee, Pimentel et al. ’18, ’20, ’21, Pajer et al. ’20, Qin and Xianyu ’22, ’23 etc.]

Analytical Results

□ : scale dependence

□ : same as const. mass signal

Bispectrum

$$S = \sum_{a,b=\pm}\left[\frac{k_1 k_2}{k_3^2}\mathcal{U}_{ab}^{0,-2}\left(\frac{2k_3}{k_{123}},\frac{k_3}{k_0}\right)+\frac{k_2 k_3}{k_1^2}\mathcal{U}_{ab}^{0,-2}\left(\frac{2k_1}{k_{123}},\frac{k_1}{k_0}\right)+\frac{k_3 k_1}{k_2^2}\mathcal{U}_{ab}^{0,-2}\left(\frac{2k_2}{k_{123}},\frac{k_2}{k_0}\right)\right]$$

where

$$\mathcal{U}_{\pm\pm}^{p_1 p_2}(u,v) = D_1(p_1,p_2,\mu_v,\gamma)u^{5+p_{12}}{}_3F_2\left[\begin{matrix}1, 3+p_2\mp i\gamma, 5+p_{12}\\ \frac{7}{2}+p_2-i\mu_v, \frac{7}{2}+p_2+i\mu_v\end{matrix}\middle| u\right]$$
$$\mp D_2(p_1,p_2,\mu_v,\gamma)u^{5/2+p_1\pm i\mu_v}{}_2\mathcal{F}_1\left[\begin{matrix}p_1+\frac{5}{2}\pm i\mu_v, \frac{1}{2}\pm i\mu_v\mp i\gamma\\ 1\pm 2i\mu_v\end{matrix}\middle| u\right]+(\mu_v\to-\mu_v)$$

$$\mathcal{U}_{\pm\mp}^{p_1 p_2}(u,v) = C(p_1,p_2,\mu_v,\gamma)u^{5/2+p_1\pm i\mu_v}{}_2\mathcal{F}_1\left[\begin{matrix}p_1+\frac{5}{2}\pm i\mu_v, \frac{1}{2}\pm i\mu_v\mp i\gamma\\ 1\pm 2i\mu_v\end{matrix}\middle| u\right]+(\mu_v\to-\mu_v)$$

$$k_{123}=k_1+k_2+k_3\ ,\qquad \gamma=\pm\frac{y\sqrt{2\epsilon}M_{\rm pl}^2}{H^2}\ ,\qquad \mu_v^2=\frac{1}{H^2}\left(m_0^2+2y\sqrt{2\epsilon}M_{\rm pl}^2\log v\mp 2y\sqrt{2\epsilon}M_{\rm pl}^2\right)-\frac{9}{4}$$

from evaluation at horizon crossing

Observational Signals

cf. const. mass

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_3}{k_1}\right) \qquad \mu = \sqrt{\left(\frac{m_0}{H}\right)^2 - \frac{9}{4}}$$

Scale dependence : mass of short mode at the time of horizon crossing

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_3}{k_1}\right)} \cos\left[\mu\left(v\frac{k_3}{k_1}\right) \log \frac{k_3}{k_1}\right] \quad \text{in} \quad k_3 \ll k_1 \simeq k_2$$

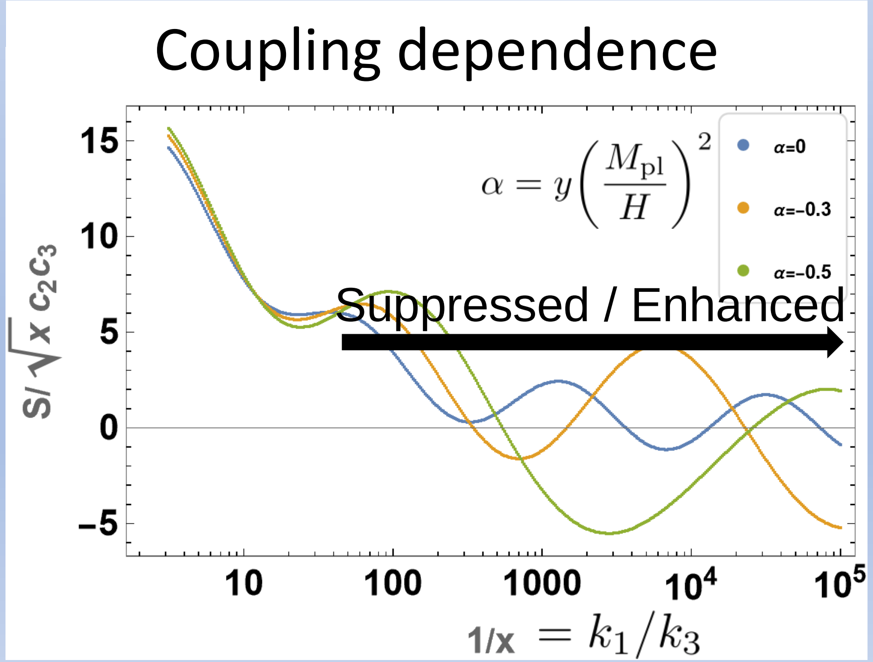
$$v = k_1/k_0 = 10^4 k_1, \qquad \mu^2 = \frac{1}{H^2}\left(m_0^2 + 2y\sqrt{2\epsilon}M_{\rm pl}^2 \log\left(v\frac{k_3}{k_1}\right) \mp 2y\sqrt{2\epsilon}M_{\rm pl}^2\right) - \frac{9}{4}$$

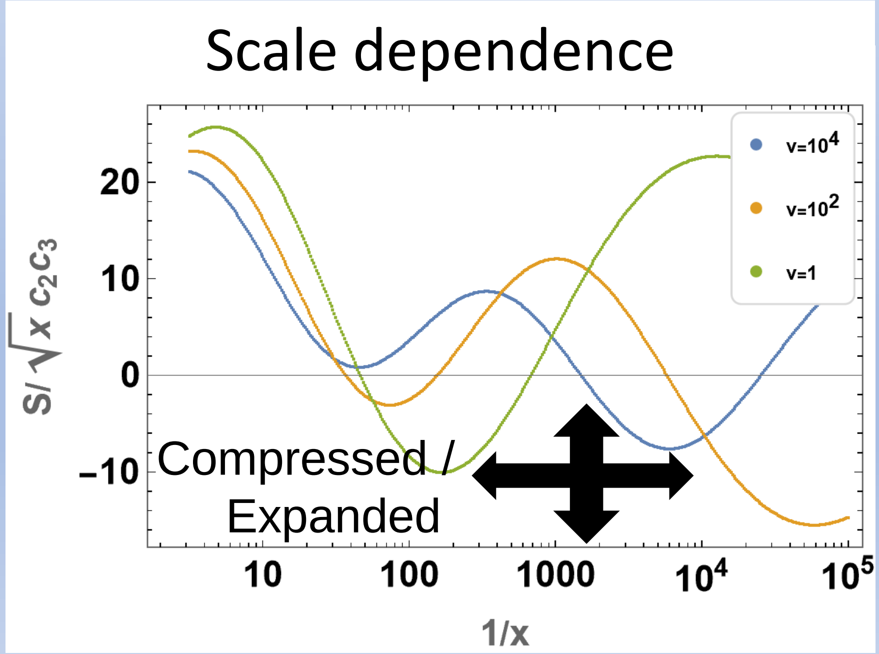
$$(vk_3/k_1 = k_3/k_0)$$

$$\Delta\phi \sim N\sqrt{\epsilon}M_{\rm pl}$$

[Lyth '96]

Not slow-roll suppressed thanks to the hierarchy $M_{\rm pl}/H \gtrsim 10^5$ in case of non-der. ints.





Probing Ints. 1: Der. vs Non-Der. Ints.

Scale dependence : mass at horizon crossing

(1) Non-derivative coupling, e.g., $\frac{\alpha}{M_{\rm pl}^{n-2}}\phi^n\sigma^2$

$$\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \alpha\left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

→ Large scale dependence

(2) Derivative coupling, e.g., $\frac{\beta}{M_{\rm pl}^{n(m+1)-2}}(\partial^m\phi)^n\sigma^2$ nm : even

$$\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \beta\left(\frac{H}{M_{\rm pl}}\right)^{nm-2} \epsilon^{nm-n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

Stronger suppression because of slowroll $\partial_t^m\phi \sim \epsilon^{m-1/2}H^m M_{\rm pl}$

(same order as the signal)

➡ Large scale dependence ⇔ Non-derivative coupling

Probing Ints. 2: Among Non-Der. Ints.

Boltzmann suppression of the signal $S \sim \left(\frac{k_3}{k_1}\right)^{1/2} \underline{\underline{e^{-\pi\mu}}} \cos\left(\mu \log \frac{k_3}{k_1}\right)$

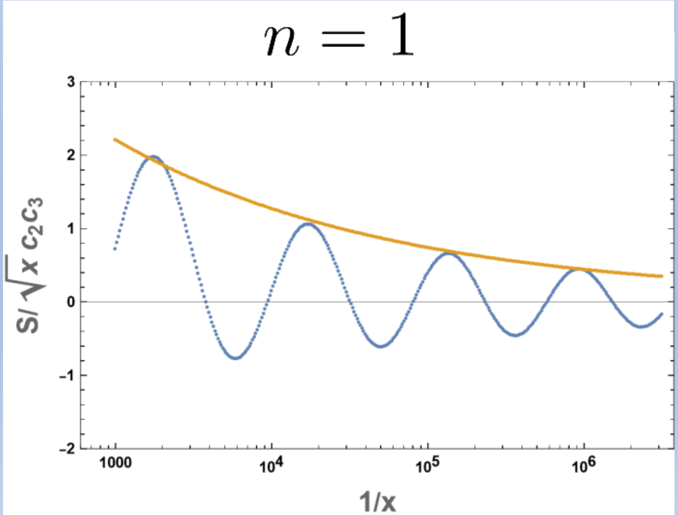
E.g., power function $\frac{\alpha}{\Lambda^{n-2}}\phi^n\sigma^2$ ➡ $\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \alpha\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$

$$\Rightarrow \quad e^{-\pi\mu} \sim \exp\left[-\pi\sqrt{\alpha\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2}\underline{\underline{\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n}}}\,\right]$$

Determination of n from the suppression

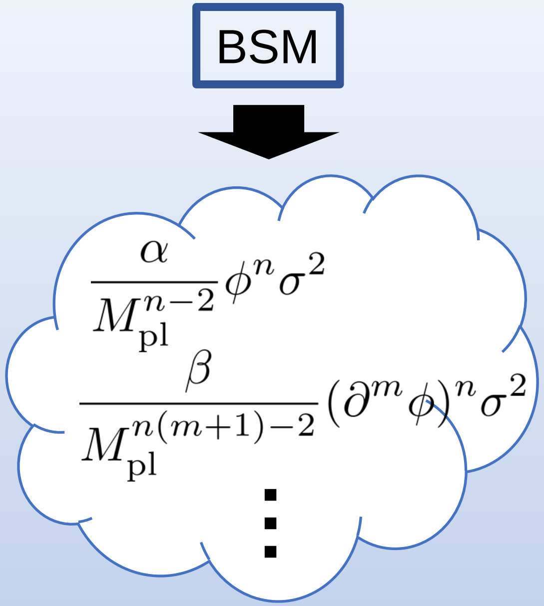
More generally, $\mathcal{L}_{\rm int} = g(\phi)\sigma^2$

$$\Rightarrow \quad e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{g\left(\underline{\underline{M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_3}{k_1}\right)}}\right)}\,\right]$$



Suppression / enhancement rate is uniquely characterized by $g(\phi)$

Summary



Cosmological collider project: $k_3 \ll k_1 \simeq k_2$

- Dictionary for particles $S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu}\cos\left(\mu\log\frac{k_3}{k_1}\right)$
- Scale dependence: types of interactions

Signals: horizon crossing (e.g., $\mu \to \mu(vk_3/k_1)$)

Distinguishing ints. by scale dependence in $\Delta m_{\rm eff}$:

O Derivative vs. Non-derivative interactions

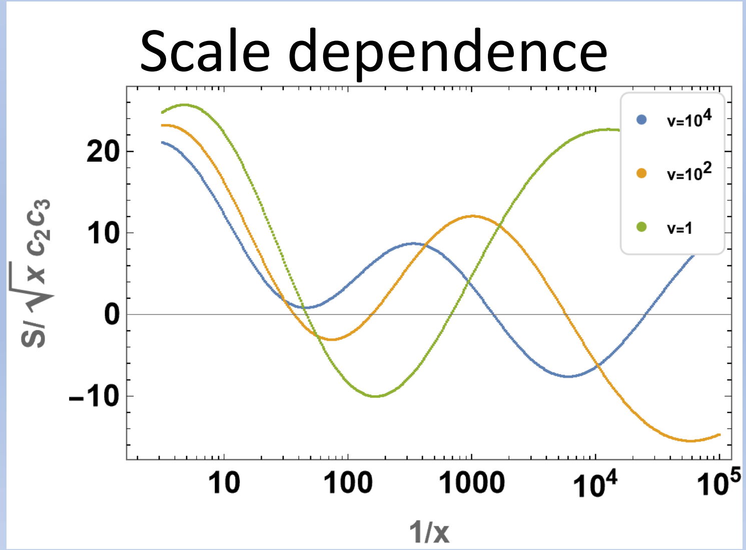
Derivative ints. Non-derivative ints.

$$\left(\frac{H}{M_{\rm pl}}\right)^{nm-2}\epsilon^{nm-n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n << \left(\frac{M_{\rm pl}}{H}\right)^2\epsilon^{n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

Observably large thanks to $M_{\rm pl}/H \gtrsim 10^5$

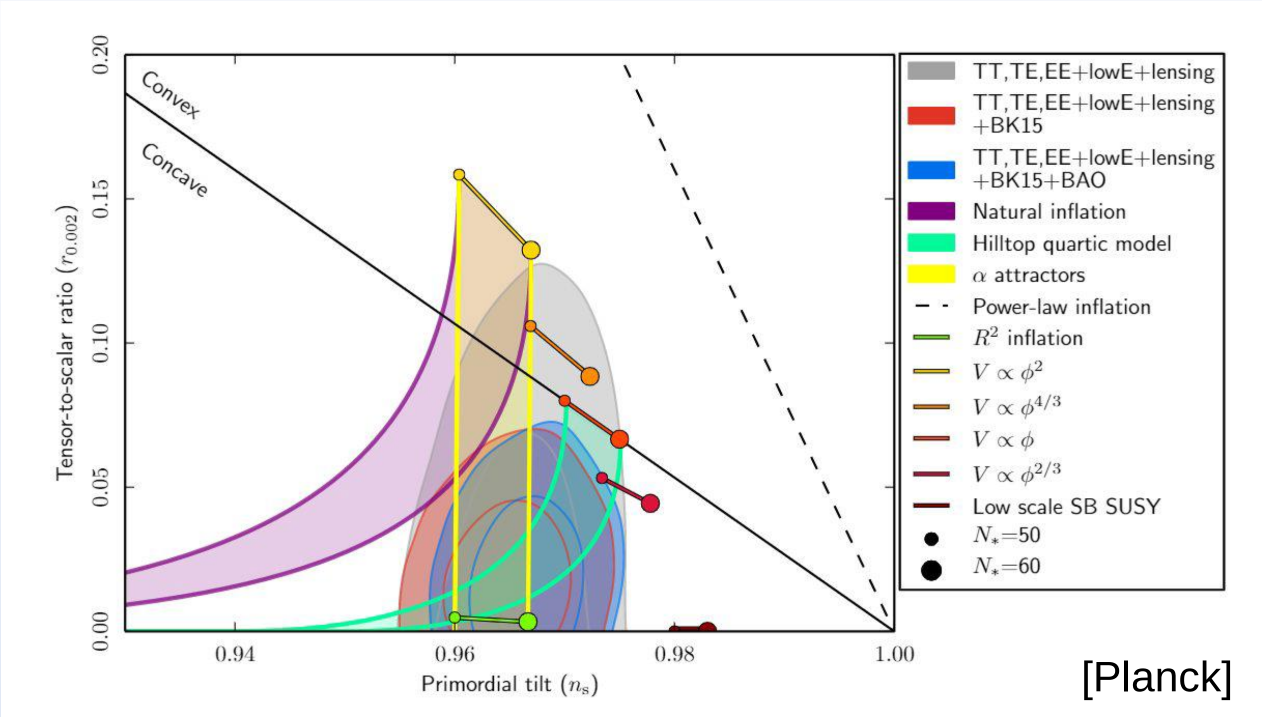
O Determining a non-der int. $g(\phi)\sigma^2$

$$e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{g\left(M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_3}{k_1}\right)\right)}\right]$$



Appendices

Planck 2018



Linear perturbations:

$$P_\zeta \simeq 2 \times 10^{-9}, \quad n_s \simeq 0.0965$$

Tensor: not yet detected

$$r = \frac{P_\gamma}{P_\zeta} < 0.056$$

Isocurvature perturbation: not detected

➡ Single field inflation is preferred.

Non-Gaussianities:

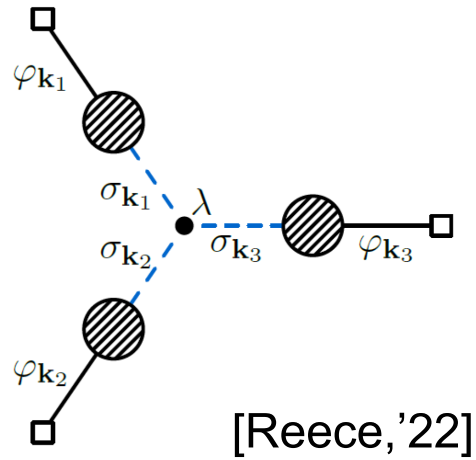
Squeezed: $f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$, Equilateral: $f_{\rm NL}^{\rm equil} = -26 + 47$

Form factor: insufficient resolution

Future experiment: 21 cm line ➡ resolution $\mathcal{O}(10^{-2})$

Non-derivative Ints., Numerical

[Reece et al. '22]



[Reece,'22]

Setup

Effective mass of the heavy particle: $m_{\rm eff} = e^{\alpha\phi/M_{\rm pl}} m_0^2$

Oscillatory feature

Wavelength } amplified or dumped
Amplitude

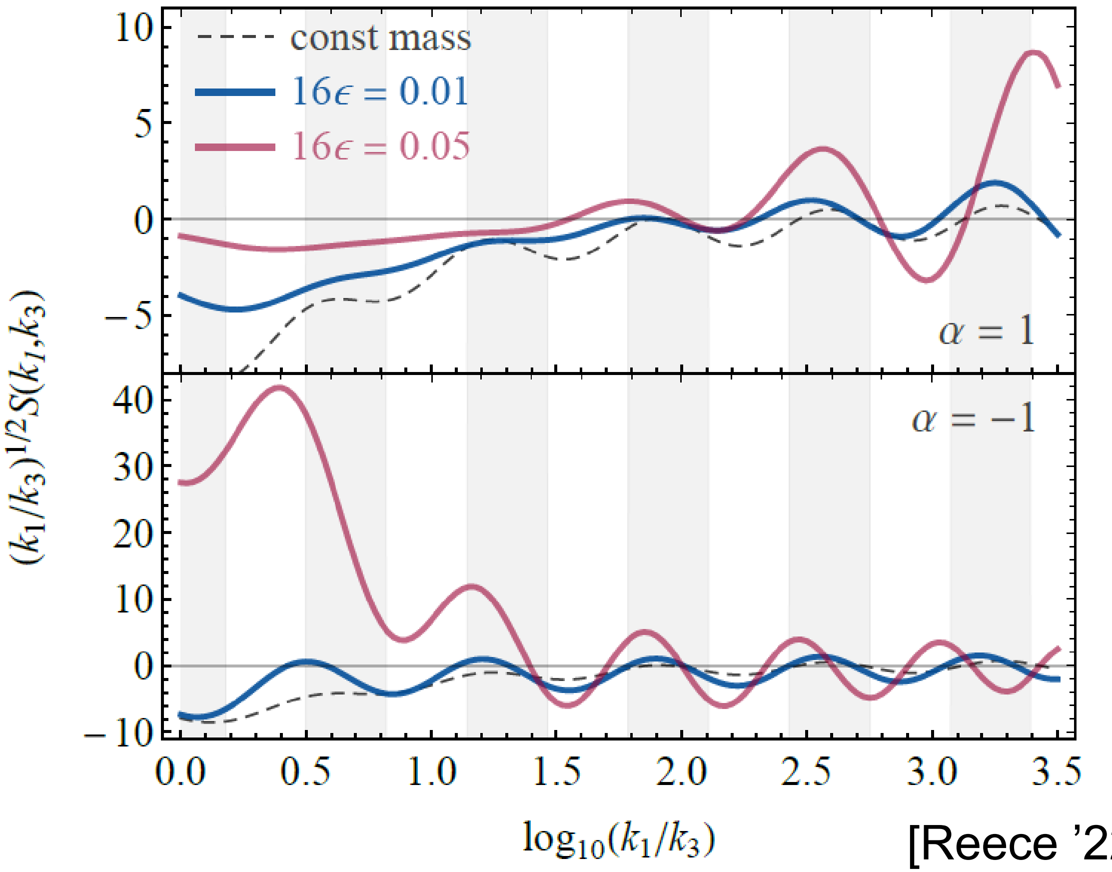
Things not clear in numerical work:

-physical interpretation

-model dependence

-scale dependence

Breaking down of de Sitter



[Reece '22]

Mode Functions of the Heavy Field

Mode expansion

$$\sigma(x) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left(v_k(\tau)a_{\boldsymbol{k}} + v_k^*(\tau)a_{-\boldsymbol{k}}^\dagger\right)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \quad ,$$

$$[a_{\boldsymbol{k}}, a_{\boldsymbol{k}'}^\dagger] = (2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}')$$

v_k : Mode function

Equation of motion for σ

$$v_k'' - \frac{2}{\tau}v_k' + \left(k^2 + \frac{m_{\rm eff}^2}{H^2\tau^2}\right)v_k = 0 \quad ,$$

$$m_{\rm eff}^2 = m_0^2 + 2yM_{\rm pl}^2\left[\frac{\phi_{*0}}{M_{\rm pl}} \mp \sqrt{2\epsilon}\left(1-\frac{\tau}{\tau_*}\right)\right]$$

Mode functions for σ (Bunch-Davies vacuum)

$$v_k = \frac{e^{\pi\gamma/2}}{\sqrt{2k}}(-H\tau)W_{-i\gamma,i\mu}(2ik\tau)$$

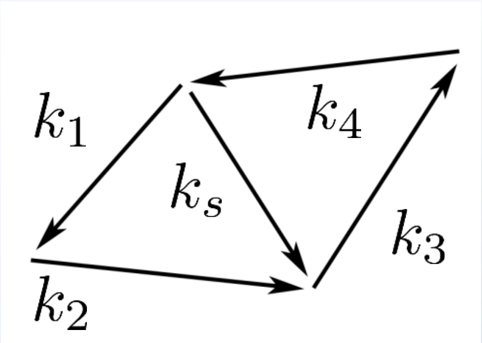
$$\mu^2 = \frac{1}{H^2}\left(m_0^2 + 2yM_{\rm pl}\phi_{*0} \mp 2y\sqrt{2\epsilon}M_{\rm pl}^2\right) - \frac{9}{4}$$

$$\gamma = \pm\frac{y\sqrt{2\epsilon}M_{\rm pl}^2}{H^2}$$

$y \to 0$: const. mass mode function

$$v_k = e^{-\pi\mu/2}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_{i\mu}^{(1)}(-k\tau)$$

Soft Limit to Obtain Bispectrum



Bispectrum: $\langle \zeta^3 \rangle \propto \frac{c_2 c_3}{8 k_1 k_2 k_3^4} \lim_{k_4 \to 0} \sum_{\mathrm{a,b}=\pm} \mathcal{I}_{\mathrm{ab}}^{0,-2} + (k_3 \to k_1, k_2)$

$$u_1 = \frac{2k_s}{k_{12} + k_s} \to \frac{2k_3}{k_{123}} \qquad u_2 = \frac{2k_s}{k_{34} + k_s} \to 1$$

$$\mathcal{I}_{ab}^{p1,p2} = \sum_{c,d=\pm} A_{ab|cd} \mathcal{V}_{a|c}^{p_1}(u_1) \mathcal{V}_{b|d}^{p_2}(u_2) + \mathcal{G}_{ab}^{p_1 p_2}(u_1, u_2)$$

$$\mathcal{V}_{a|c}^{p}(u) = iac 2^{iac\mu} \pi \operatorname{csch}(2\pi\mu) \left(\frac{u}{2}\right)^{5/2+p+iac\mu} {}_2\mathcal{F}_1 \left[\begin{matrix} \frac{5}{2} + p + iac\mu, \frac{1}{2} - ia\gamma + iac\mu \\ 1 + 2iac\mu \end{matrix} \middle| u \right]$$

$$\mathcal{G}_{ab}^{p_1 p_2} = \delta_{ab} \frac{H^2 e^{-a \frac{\pi}{2} i p_{12}} \Gamma(p_{12} + 5)}{2^{p_{12}+5}} \sum_{n=0}^{\infty} u_1^{n+p_{12}+5} \left(1 - \frac{1}{u_2}\right)^n \left(\begin{matrix} n + p_{12} + 4 \\ n \end{matrix} \right)$$
$$\times \frac{1}{\mu^2 + \left(\frac{5}{2} + n + p_2\right)^2} {}_3F_2 \left[\begin{matrix} 1, 3 + n + p_2 - ia\gamma, 5 + n + p_{12} \\ \frac{7}{2} + n + p_2 - i\mu, \frac{7}{2} + n + p_2 + i\mu \end{matrix} \middle| u_1 \right]$$

Infinite sum. in $\mathcal{G}_{ab}^{p_1 p_2}$

Only $n = 0$ contributes

➡ resolved

Divergences in $\mathcal{V}_{a|c}^{p}$

$${}_2\mathcal{F}_1 \left[\begin{matrix} \frac{5}{2} + p + iac\mu, \frac{1}{2} - ia\gamma + iac\mu \\ 1 + 2iac\mu \end{matrix} \middle| u \right] \xrightarrow[u \to 1-0]{} (1-u)^{-2+ia\gamma} \Gamma(-2 + ia\gamma) + \cdots \longrightarrow \infty$$

is canceled after the summation $\sum_{c,d=\pm} A_{ab|cd} \mathcal{V}_{a|c}^{p_1}(u_1) \mathcal{V}_{b|d}^{p_2}(u_2)$

➡ Final expression: next slide

Equilateral limit $k_1 = k_2 = k_3$

$v = k_1/k_0$ dependence

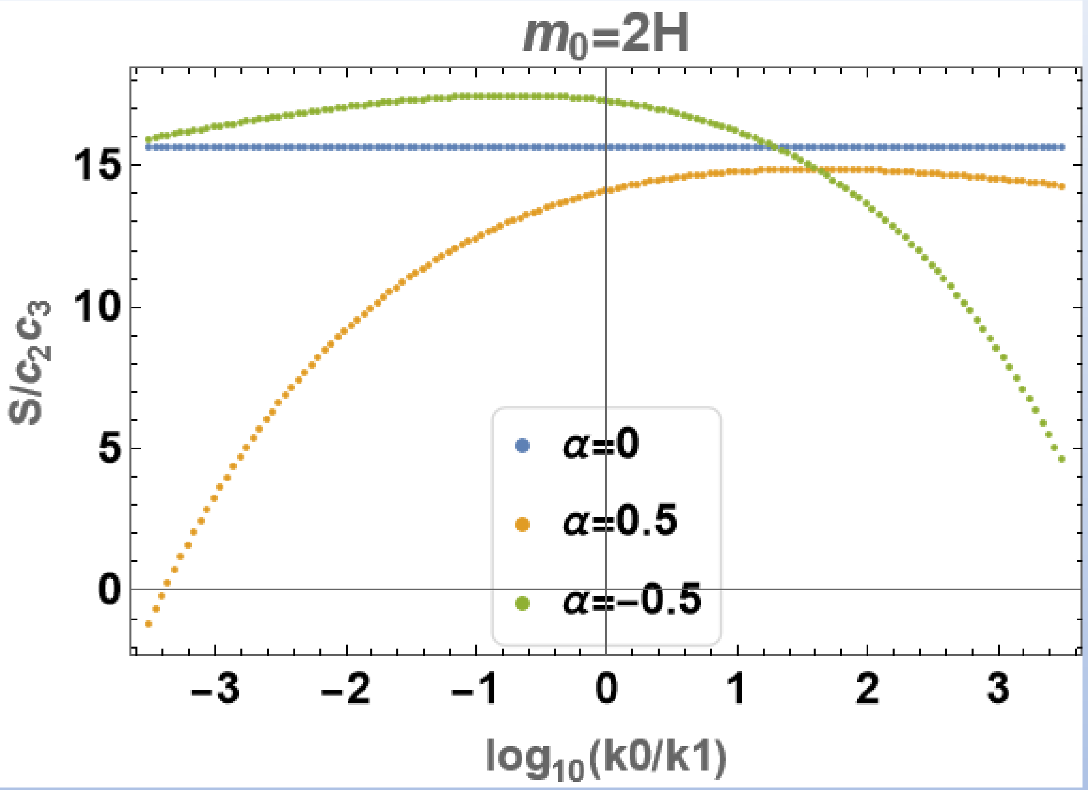
$$\frac{\partial S}{\partial v} = f(m_0)\frac{\sqrt{\epsilon}\alpha}{v} + \mathcal{O}(\epsilon)$$

The same scale dependence as

the general single field inflation [Chen, '07]

(Consistent to EFT description

integrating out heavy field)



Amplitude

$$S_{\rm eq}(\approx f_{\rm NL}^{\rm eq}) \sim c_2 c_3 \mathcal{O}(10)$$

$c_2 c_3$: dim. less ➡ $\mathcal{O}(1)$? $\mathcal{O}(\epsilon)$?