YITP long-term workshop

Gravity and Cosmology 2024

Cosmological Correlators Through the Looking Glass

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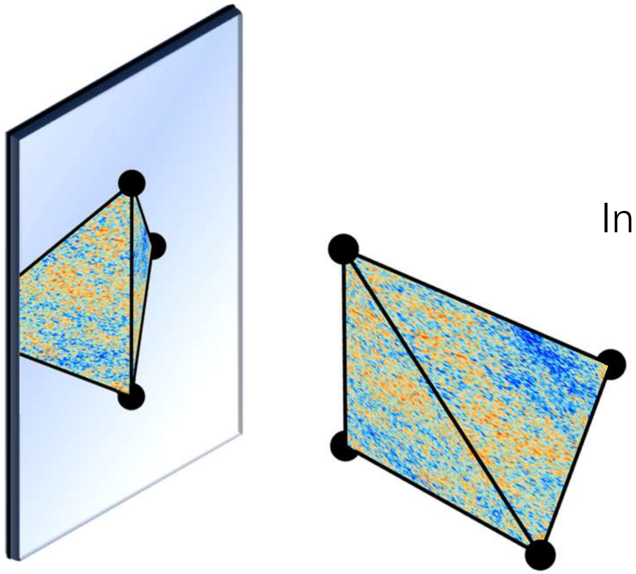


In collaboration with: David Stefanyszyn and Xi Tong

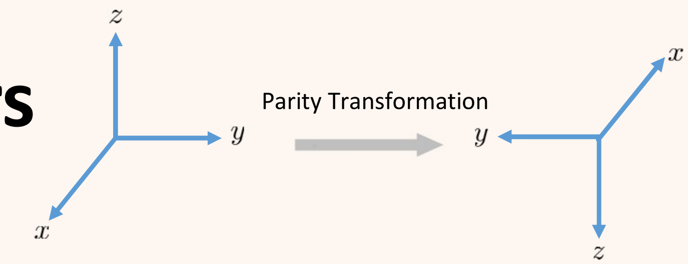
Based on 2309.07769

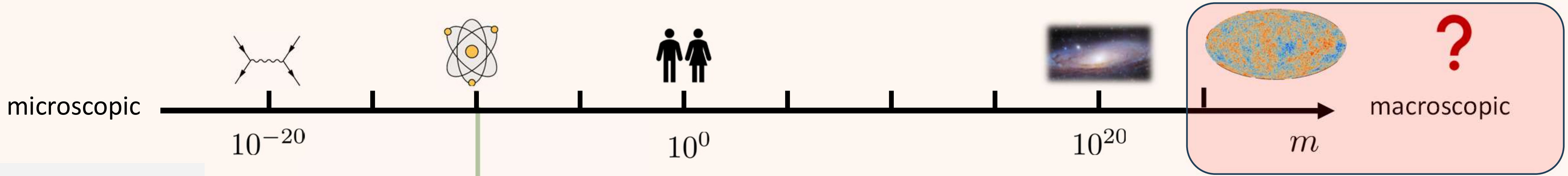
Institute for Basic Science

Outline

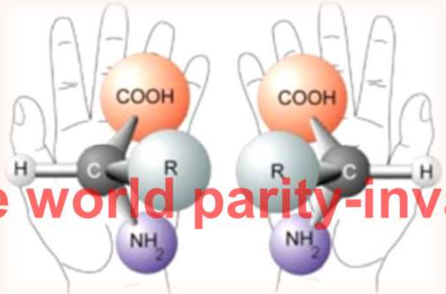
- ➢ Introduction
    - Parity in Correlators
- ➢ Exact result for Parity Odd (PO) Correlator
    - Parity-odd Factorization Theorem
- ➢ Application
    - Trispectrum from exchanging spin-1 and spin-2 particles
- ➢ Conclusion

Test Parity From Cosmological correlators



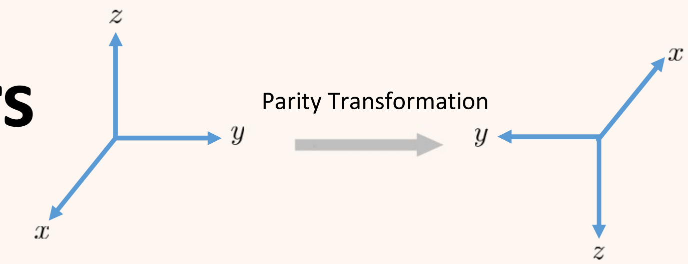


Parity Violation



Is the world parity-invariant on cosmological scales?

Test Parity From Cosmological correlators

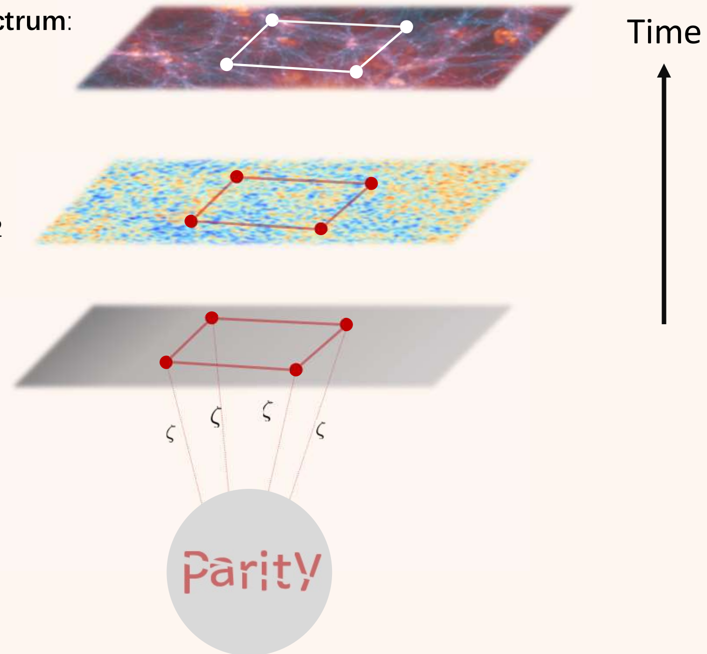


Test Parity From Cosmological correlators

LSS Parity-odd trispectrum:
Hou et al. 2022
Philcox 2022

CMB birefringence:
Minami, Komatsu 2020
Diego-Palazuelos et al. 2022

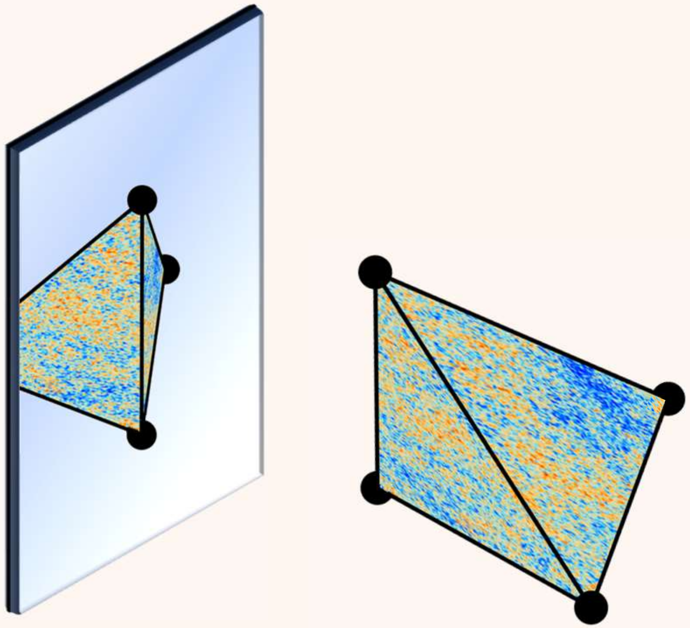
End of inflation



Fluctuations in the CMB and LSS, are believed to be seeded by ***microscopic*** quantum fluctuations during inflation

- It is natural to expect at least some level of parity violation in the primordial universe !

Cosmological correlators in the Mirror



$$\text{(Parity odd)} \sim i\,\mathrm{Im}\,\langle \zeta^n \rangle, \quad n \geq 4$$

- Imaginary Part

In Position Space:
$\zeta(\vec{x}) = \zeta^\dagger(\vec{x})$

→

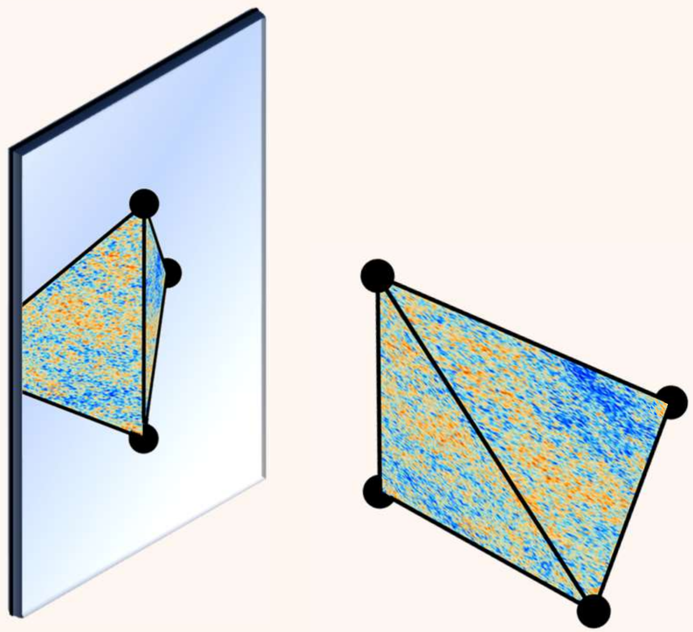
In Momentum Space:
$\zeta(\vec{k}) = \zeta^\dagger(-\vec{k})$

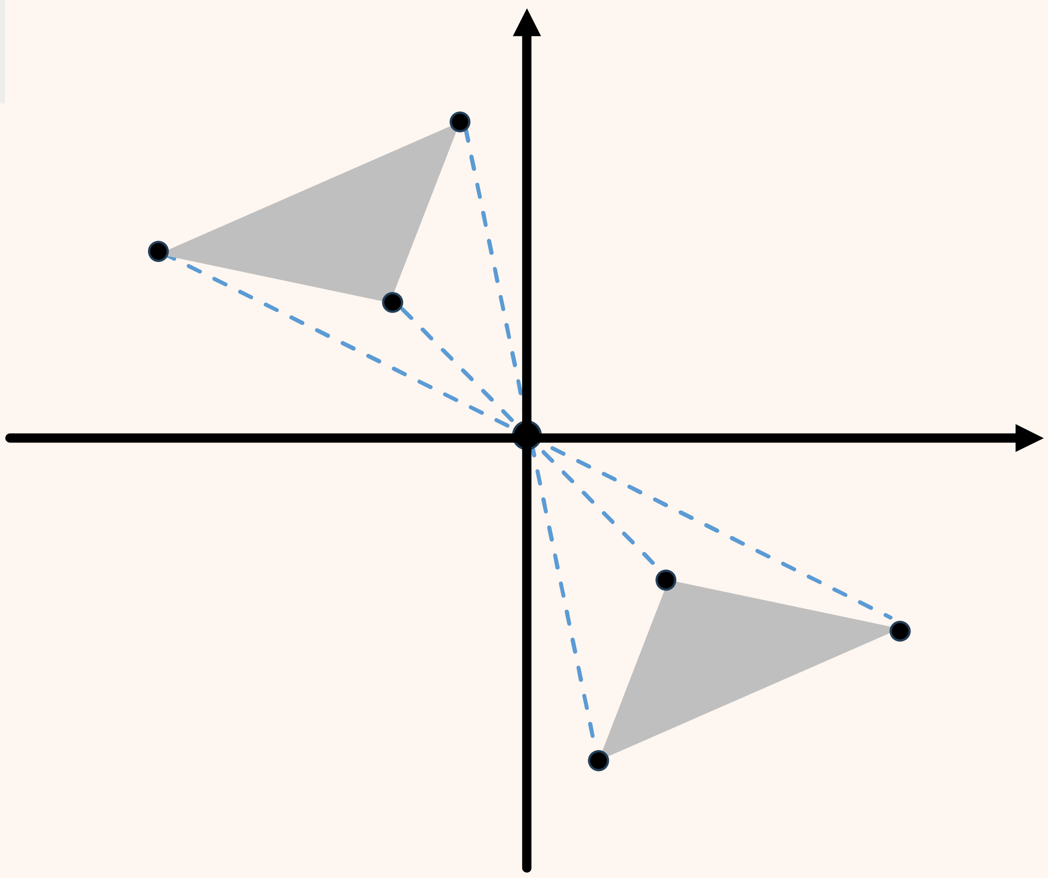
$$\langle \zeta^4(\vec{k}_{\mathrm{a}}) \rangle^{\mathrm{PO}} \equiv \frac{\langle \zeta^4(\vec{k}_{\mathrm{a}}) \rangle - \langle \zeta^4(-\vec{k}_{\mathrm{a}}) \rangle}{2} = i\,\mathrm{Im}\langle \zeta^4 \rangle$$

Cosmological correlators in the Mirror

$$\text{(Parity odd)} \sim i\,\mathrm{Im}\,\langle \zeta^n \rangle, \quad n \geq 4$$

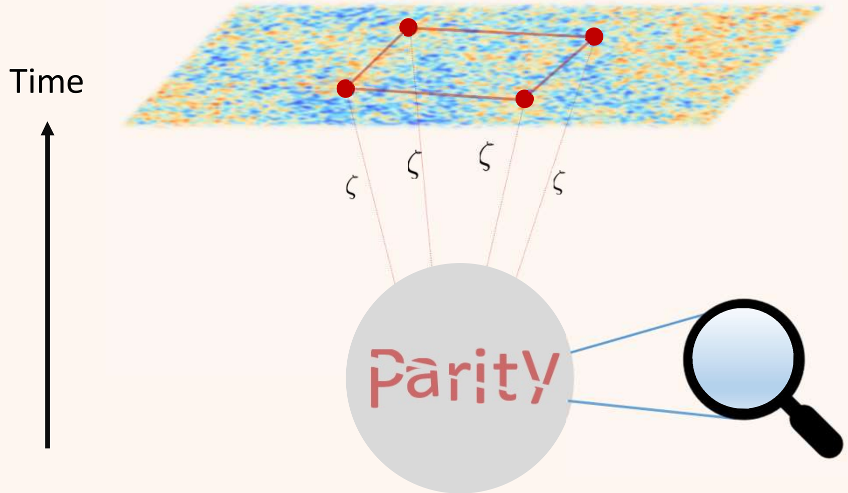
- At least 4-pts correlators (Trispectrum)





For 3pt functions: Parity Transformation = Translation + Rotation

PO in the primordial scalar trispectrum



How to generate parity violation during inflation?

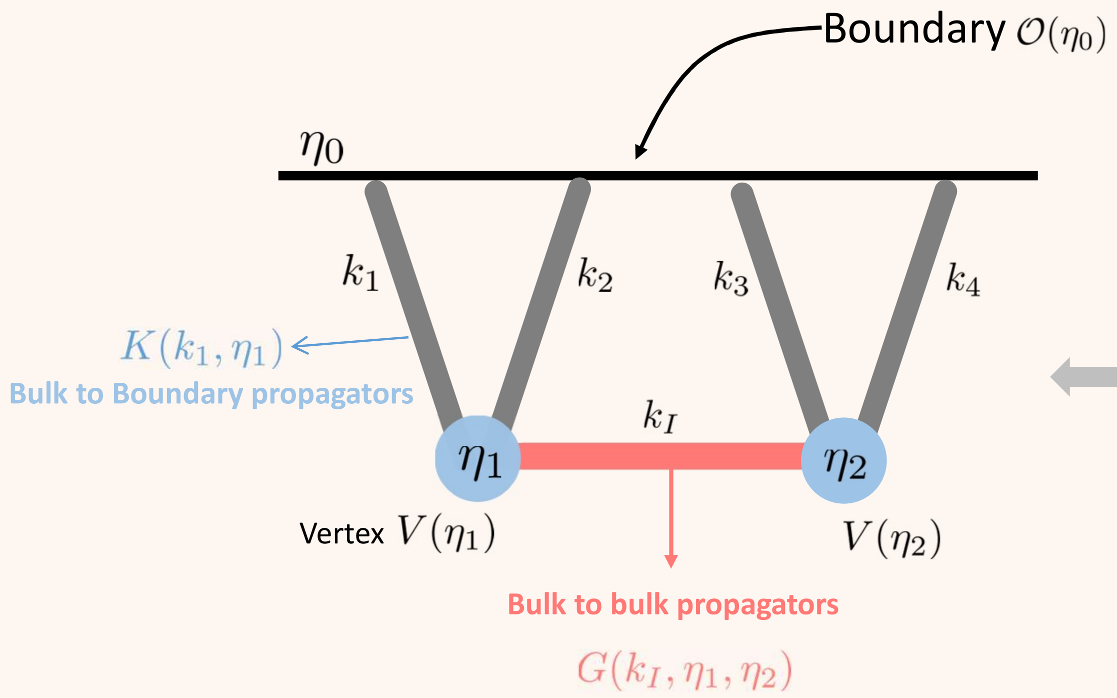
Not easy!

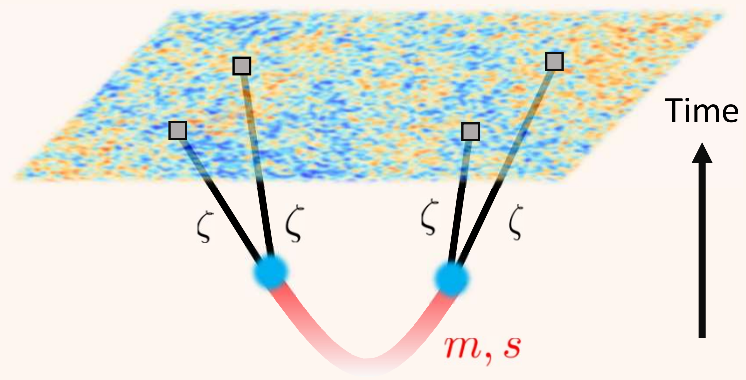
No-go theorem: [Liu et al., 2019] [Cabass et al., 2022]

Parity odd is absent in single-field inflation at *tree level*, under the assumption of a *Bunch-Davies vacuum* and *scale invariance* ***+ Locality***

[Jazayeri, Renaux-Petel, Tong, Werth and **YZ**, 2023]

Analytical Calculation of cosmological correlators





$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle' = \int d\eta_1 \int d\eta_2 V(\eta_1)V(\eta_2) G(k_I,\eta_1,\eta_2) K(k_1,\eta_1)K(k_2,\eta_1)K(k_3,\eta_2)K(k_4,\eta_2)$$

Too difficult to solve analytically!

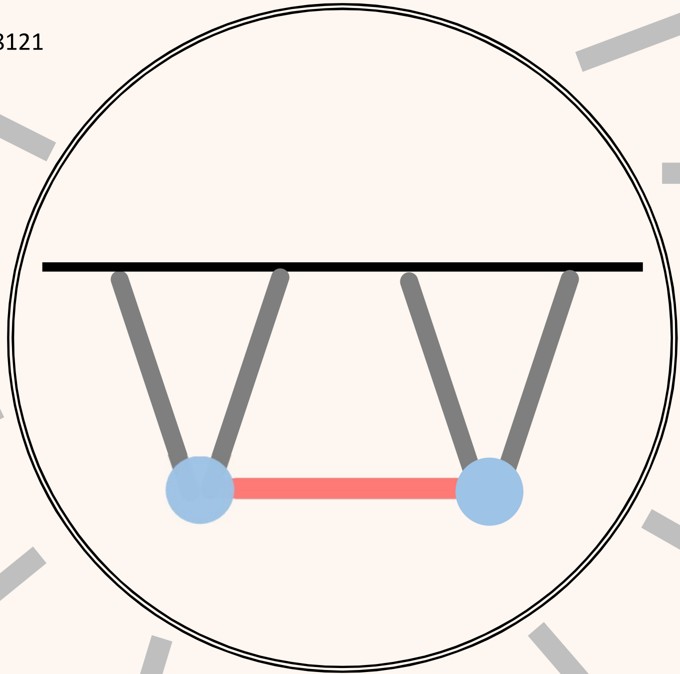
(1) Nested Time integral

$$G \sim \theta(\eta_1-\eta_2)v_k(\eta_1)v_k^*(\eta_2)+\theta(\eta_2-\eta_1)v_k^*(\eta_1)v_k(\eta_2)$$

(2) Mode functions are complicate

$$v_k \sim H_{i\mu} \text{ or } W_{i\kappa,i\mu}$$

Recent Developments



dS Cosmological bootstrap

Arkani-Hamed, Baumann, Lee, Pimentel 1811.00024
Baumann, Pueyo, Joyce, Lee, Pimentel 1910.14051 2203.08121

Boostless bootstrap

Pimentel, Wang, 2205.00013
Jazayeri, Renaux-Petel 2205.10340
……

Unitarity cutting rules

Goodhew, Jazayeri, Lee, Pajer, 2205.00013
Melville, Pajer 2103.09832
Baumann et al., 2021
……

Causality cutting rules

Tong, Wang, Zhu 2112.03448
Agui-Salcedo, Melville 2308.00680

Bulk locality

Jazayeri, Pajer,Stefanyszyn 2103.08649
……..

AdS-inspired Mellin method

Sleight, Taronna, 1907.01143, 2007.09993, 2106.00366

Partial Mellin-Barnes method

Qin,Xianyu 2205.01692,2208.13790,
2211.03810,2304.13295
…….

Cosmological polytope

Arkani-Hanmed, Benincasa, Postnikov 1709.02813
Benincasa 1909.02517
……..

Kinematic flow

Arkani-Hanmed, Baumann,Himan,Joyce,Lee,Pimentel
2312.05300 ……..

Cosmo flow

Pinol, Renaux-Petel, Werth
2302.00655,2312.06559

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle' = \mathrm{Re}\langle \zeta^4 \rangle + i\, \mathrm{Im}\langle \zeta^4 \rangle$$

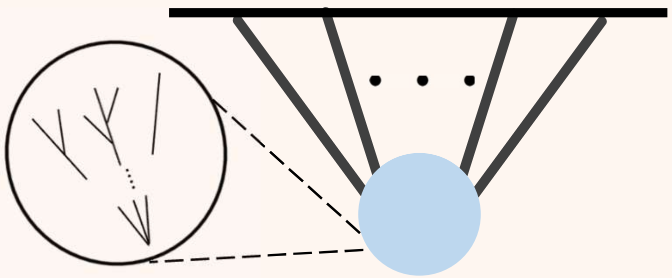
Too difficult to solve analytically!

Parity-odd part

Parity violation is hidden in the imaginary part,
we don't need to include all components!

For Parity-odd correlators, we can find exact results easily

General diagram



Parity odd interactions

$$\mathcal{L}_N^{\mathrm{PO}} = \lambda\, a^{1-2m-n} \epsilon^{(3)} \partial_i^{3+2m} \partial_\eta^n \zeta^M \sigma^{N-M}$$

Scale invariance $a = -\frac{1}{H\eta}$

Then the integral of the general diagram is given schematically as:

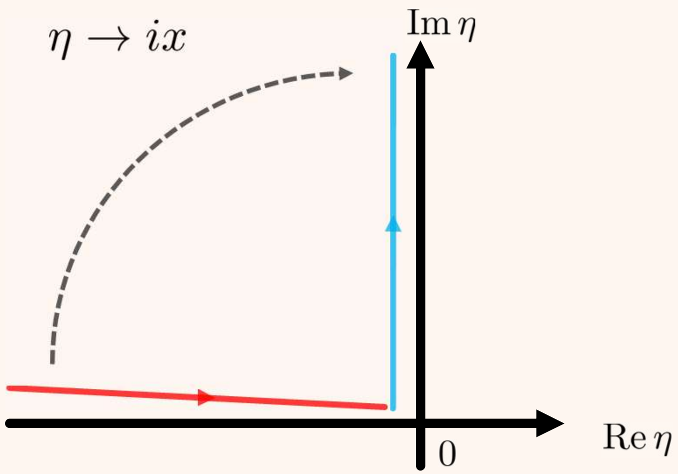
$$\mathcal{I} = \int_{-\infty(1-i\epsilon)}^{0} \left[\prod_{v=1}^{V} d\eta_v\, i\lambda_v\, D_v\right] \left[\prod_{e=1}^{n} K_e\right] \left[\prod_{e'=1}^{I} G_{e'}\right]$$

Vertex

Bulk-Boundary

Bulk-Bulk

Wick Rotation



$$\mathcal{I} = \int_{-\infty(1-i\epsilon)}^{0} \left[\prod_{v=1}^{V} d\eta_v \, i\lambda_v \, D_v\right] \left[\prod_{e=1}^{n} K_e\right] \left[\prod_{e'=1}^{I} G_{e'}\right]$$

$$\mathcal{L}_N = \lambda \, a^{1-2m-n} \epsilon^{(3)} \partial_i^{3+2m} \partial_\eta^n \zeta^M \sigma^{N-M}$$

Bulk to boundary propagators:
(Curvature field ζ)

BD vacuum

$$K = \frac{H^2}{2k^3}(1 - ik\eta)e^{ik\eta} \longrightarrow \frac{H^2}{2k^3}(1 + kx)e^{-kx} \in \mathbb{R}$$

Vertex:

Scale invariant + Unitarity

$$D \sim \epsilon^{(3)} \eta^{n+2m-1} (ik)^{3+2m} \partial_\eta^n \longrightarrow i^{n+2m-1} i^{3+2m} i^n \in \mathbb{R}$$

Bulk to bulk propagators:

$$G \sim \theta(\eta_1 - \eta_2) v_k(\eta_1) v_k^*(\eta_2) + \theta(\eta_2 - \eta_1) v_k^*(\eta_1) v_k(\eta_2)$$

$v_k \sim$ *Hankel, Whittaker* $\notin \mathbb{R}$

Only need the imaginary part !

Separation of bulk-bulk propagators

$$G \sim \theta(\eta_1 - \eta_2) v_k(\eta_1) v_k^*(\eta_2) + \theta(\eta_2 - \eta_1) v_k^*(\eta_1) v_k(\eta_2)$$

$$G = C + F$$

$$C(\eta_1, \eta_2, k) = \theta(\eta_1 - \eta_2) v(\eta_1, k) v^*(\eta_2, k) + (\eta_1 \leftrightarrow \eta_2)$$
$$+\mathcal{A}\, v^*(\eta_1, k) v^*(\eta_2, k) \qquad \in \mathbb{R}$$

$$F(\eta_1, \eta_2, k) = -\mathcal{A}\, v^*(\eta_1, k) v^*(\eta_2, k)$$

For the simplest case: $\mathcal{A} \sim i \cosh \pi\mu$ $\quad \in \mathbb{C}$

All contribution of Parity-odd

- Why must the separation exist?

$$\left(\eta_1^2 \frac{\partial^2}{\partial \eta_1^2} - 2\eta_1 \frac{\partial}{\partial \eta_1} + c_{h,S}^2 \eta_1^2 k^2 + \frac{m^2}{H^2}\right) G_\sigma^{(h)}(\eta_1, \eta_2, k) = -iH^2 \eta_1^2 \eta_2^2 \delta(\eta_1 - \eta_2)$$

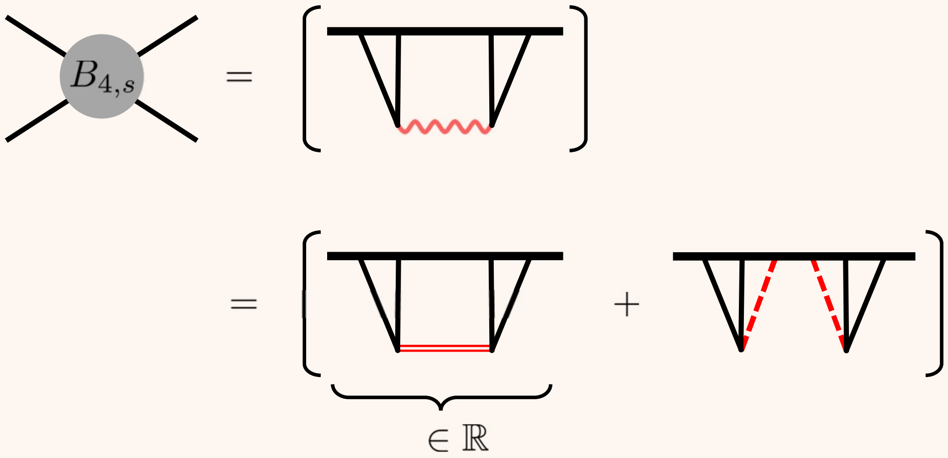
After Wick rotation, real coefficients differential equation

→ $\exists$ Real solution

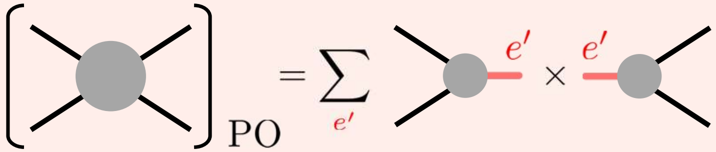
Separation of bulk-bulk propagators

Theorem 4.3. (Parity-odd factorisation) *The parity-odd part of any tree-level correlator of massless scalar fields is factorised and admits a Taylor expansion around* $k_T = 0$*, in theories containing an arbitrary number of fields of any mass, spin, coupling, sound speed and chemical potential, under the assumptions of locality, unitarity, scale invariance, IR convergence and a Bunch-Davies vacuum.*

For Trispectrum:



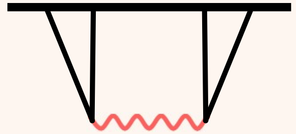
Factorization:



$$B_4^{\rm PO} = \text{constants} \times \text{kinematics} \times {}_2F_1(L) \times {}_2F_1(R)$$

(***Any*** spin, mass, etc.)

Example I: Exchange Massive spin-1



$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu^2 + \frac{\kappa t}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

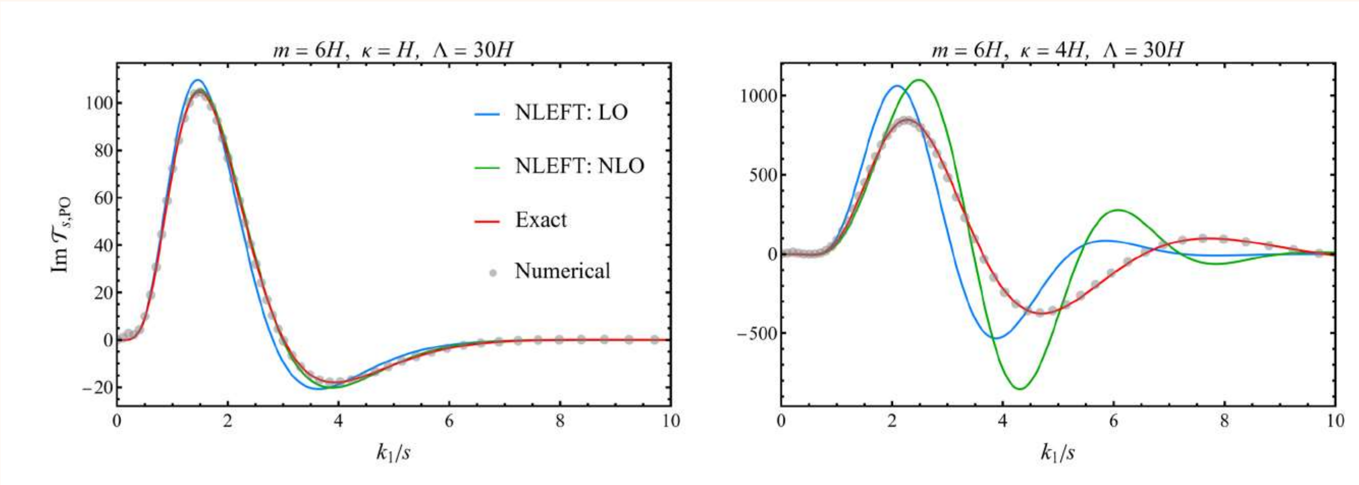
EoM: $A_{\pm1}'' + (k^2 \pm 2a\kappa k + a^2m^2)A_{\pm1} = 0$

Mode function: $A_h(\eta, k) = \frac{e^{-\pi\tilde{\kappa}/2}}{\sqrt{2k}}W_{i\tilde{\kappa},\nu}(2ik\eta)$

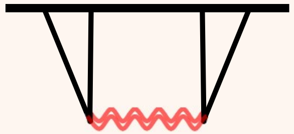
Exact solution:

with $\mathcal{I}_n^h(a,b,\nu) \sim {}_2\tilde{\mathrm{F}}_1\left[\begin{matrix}\frac{3}{2}+n-\nu, \frac{3}{2}+n+\nu \\ 2+n+i\tilde{\kappa}\end{matrix}\middle|\frac{1}{2}-\frac{b}{2a}\right]$

$$\begin{aligned}B_4^{\zeta,\mathrm{PO}} = &\, i\left(\frac{H}{\Lambda}\right)^6\frac{\pi^4\Delta_\zeta^4}{2c_s^2}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_3\cdot\mathbf{k}_4)}{k_1k_2k_3k_4}\frac{\mathbf{s}\cdot(\mathbf{k}_1\times\mathbf{k}_3)}{k_1^2k_3^2s^8}\left(1-k_1\frac{\partial}{\partial k_1}\right)\left(1-k_3\frac{\partial}{\partial k_3}\right) \\ &\times\left\{\frac{\pi i}{\Gamma\left(\frac{1}{2}-i\tilde{\kappa}+\nu\right)\Gamma\left(\frac{1}{2}-i\tilde{\kappa}-\nu\right)}\mathcal{I}_2^{(+1)}(s,c_sk_{12},\nu)\mathcal{I}_2^{(+1)}(s,c_sk_{34},\nu)\right. \\ &\left.+e^{\pi\tilde{\kappa}}\mathrm{Re}\left[\mathcal{I}_2^{(+1)}(s,c_sk_{12},\nu)\mathcal{I}_2^{(-1)}(s,c_sk_{34},\nu)\right]-(\tilde{\kappa}\to-\tilde{\kappa})\right\}+3\text{ perms} \\ &+(t\text{-channel})+(u\text{-channel}),\end{aligned}$$



Example 2: Exchange spin-2



$$S_{\text{int}} = \int d^3x d\eta \left(\frac{a}{\Lambda_1^2} \pi_c' \partial_i \partial_j \pi_c \sigma_{ij} + \frac{1}{\Lambda_2^3} \epsilon_{ijk} \partial_i \pi_c' \partial_j \partial_l \pi_c \sigma_{kl} \right)$$

Exact solution:

$$B^{\zeta,\text{PO}}_{4,h=\pm 1} = 2i\pi^4 \Delta_\zeta^4 \cos(\pi\nu) \left(\frac{H}{\Lambda_1}\right)^2 \left(\frac{H}{\Lambda_2}\right)^3 \frac{(\mathbf{k}_2 \cdot \mathbf{s})(\mathbf{k}_4 \cdot \mathbf{s})}{k_1 k_2 k_3 k_4} \frac{[\mathbf{s} \cdot (\mathbf{k}_1 \times \mathbf{k}_3)]}{c_{1,2} k_2^2 k_4^2 s^8} \times \left(1 - k_2 \frac{\partial}{\partial k_2}\right) \left(1 - k_4 \frac{\partial}{\partial k_4}\right) \mathcal{I}_1^0(c_{1,2}s, k_{12}, \nu) \mathcal{I}_2^0(c_{1,2}s, k_{34}, \nu) + 7 \text{ perms} + (t\text{-channel}) + (u\text{-channel}).$$

$$B^{\zeta,\text{PO}}_{4,h=\pm 2} = 2i\pi^4 \Delta_\zeta^4 \cos(\pi\nu) \left(\frac{H}{\Lambda_1}\right)^2 \left(\frac{H}{\Lambda_2}\right)^3 \frac{[s^2(\mathbf{k}_2 \cdot \mathbf{k}_4) - (\mathbf{k}_2 \cdot \mathbf{s})(\mathbf{k}_4 \cdot \mathbf{s})]}{k_1 k_2 k_3 k_4} \frac{[\mathbf{s} \cdot (\mathbf{k}_1 \times \mathbf{k}_3)]}{c_{2,2} k_2^2 k_4^2 s^8} \times \left(1 - k_2 \frac{\partial}{\partial k_2}\right) \left(1 - k_4 \frac{\partial}{\partial k_4}\right) \mathcal{I}_1^0(c_{2,2}s, k_{12}, \nu) \mathcal{I}_2^0(c_{2,2}s, k_{34}, \nu) + 7 \text{ perms} + (t\text{-channel}) + (u\text{-channel}).$$

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Conclusion & outlooks

Unitarity

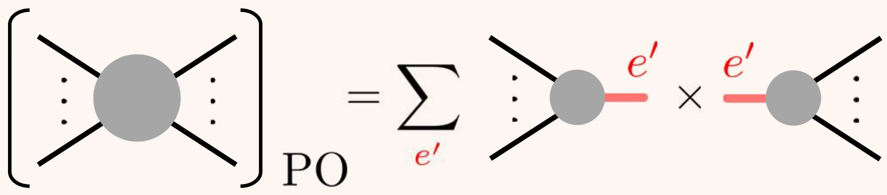
+

Scale invariance

+

Bunch-Davies

Factorization:



- Moving on to loops?
- Kramers-Kronig for correlators?
- EAdS perspective?
- Construct template for matching?

Thanks!