Testing scalar decoherence by entangled initial state of cosmic inflation

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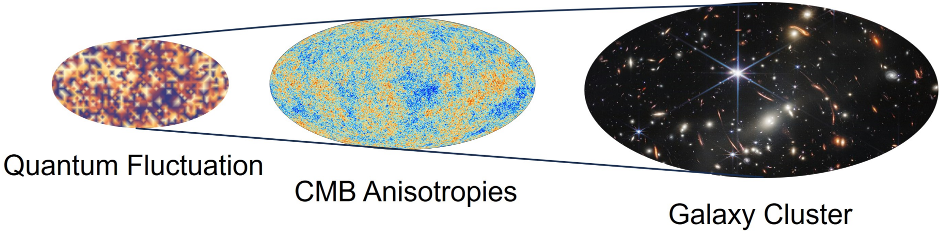
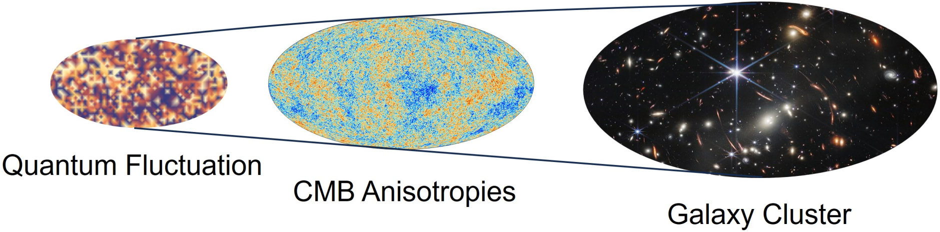
with Chon Man Sou, Yi Wang

Large scale structure from quantum fluctuation



Large scale structures have a Quantum origin

Large scale structure from quantum fluctuation



Large scale structures have a Quantum origin

Quantum State

Quantum Criterion

[CMB figure from Planck, Galaxy figure from JWST]

Two Mode Squeeze State

$$S = \frac{1}{2}\int d^4x 2\epsilon M_{pl}^2 a^3 \left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right]$$

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In Fock space, at initial time, $|\Omega\rangle = |0\rangle$. Due to the expansion of the Universe

$$|\Omega\rangle = \Pi_{\boldsymbol{k}\in\mathbb{R}^{3+}} \frac{1}{\cosh r_k} \sum_{n=0}^{\infty} e^{-2in\varphi_k} \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$$

[L. P. Grishchuk and Y. V. Sidorov (1990)]

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In Schrödinger picture

The vacuum will be evolved by Hamiltonian, then particles are generated.

In Heisenberg picture

The creation operator and annihilation operator are evolved and described by Bogoliubov transformation.

Bell Inequality in spin-1/2 system

The Bell operator in spin-1/2 system

$$\hat{\mathcal{B}} = \hat{A} \otimes \hat{B} + \hat{A}' \otimes \hat{B} + \hat{A} \otimes \hat{B}' - \hat{A}' \otimes \hat{B}' \,.$$

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For local hidden variable theory

$$\hat{\mathcal{B}} = \hat{A} \otimes \underbrace{\left(\hat{B} + \hat{B}'\right)}_{a} + \hat{A}' \otimes \underbrace{\left(\hat{B} - \hat{B}'\right)}_{b}$$

Either $a = 2, b = 0$ or $a = 0, b = 2$, so

$$|\langle \hat{\mathcal{B}} \rangle| \leq 2$$

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For quantum theory

$$\hat{\mathcal{B}}^2 = 4I - \left[\hat{A}, \hat{A}'\right]\left[\hat{B}, \hat{B}'\right]$$

since $\left|\left[\hat{A}, \hat{A}'\right]\right| \leq 2$ and $\left|\left[\hat{B}, \hat{B}'\right]\right| \leq 2$,

$$|\langle \hat{\mathcal{B}} \rangle| \leq 2\sqrt{2}$$

non-commuting observables are important!

Construction of Bell Operators for Cosmological Bell test

We introduce GKMR pseudo-spin operator[G. Gour, F.C. Khanna, A. Mann,M. Revzen (2004)] Since we want to measure the entanglement between $\boldsymbol{k}$ and $-\boldsymbol{k}$. It is nature to define the following auxiliary operator.

$$\hat{x}_{\boldsymbol{k}} = (\hat{a}_{\boldsymbol{k}} + \hat{a}^{\dagger}_{\boldsymbol{k}})/\sqrt{2k}$$

$$|\mathcal{E}_{\boldsymbol{k}}\rangle = \frac{1}{\sqrt{2}}\left(|x_{\boldsymbol{k}}\rangle + |-x_{\boldsymbol{k}}\rangle\right)$$

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GKMR pseudo-spin Operators

$$\hat{S}_x(\boldsymbol{k}) = \int_0^{+\infty} dx_{\boldsymbol{k}} \left(|\mathcal{E}_{\boldsymbol{k}}\rangle\langle\mathcal{O}_{\boldsymbol{k}}| + |\mathcal{O}_{\boldsymbol{k}}\rangle\langle\mathcal{E}_{\boldsymbol{k}}|\right)$$

$$\hat{S}_z(\boldsymbol{k}) = -\int_0^{+\infty} dx_{\boldsymbol{k}} \left(|\mathcal{E}_{\boldsymbol{k}}\rangle\langle\mathcal{E}_{\boldsymbol{k}}| - |\mathcal{O}_{\boldsymbol{k}}\rangle\langle\mathcal{O}_{\boldsymbol{k}}|\right)$$

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The observables are defined as

$$\hat{A} = \boldsymbol{n}\cdot\hat{\boldsymbol{S}}$$

$$\boldsymbol{n} = (\sin\theta_n, 0, \cos\theta_n)$$

For an optimal configuration

$$\boxed{\left\langle\hat{\mathcal{B}}\right\rangle = 2\sqrt{\left\langle\hat{S}_x(\boldsymbol{k})\hat{S}_x(-\boldsymbol{k})\right\rangle^2 + \left\langle\hat{S}_z(\boldsymbol{k})\hat{S}_z(-\boldsymbol{k})\right\rangle^2}}$$

Obstruction of Cosmological Bell test[J. Martin and V. Vennin (2017)]

Detectability

Can we access the non-commuting pair?

- What is it? We know ζ can relates to temperature perturbation. But how about its conjugate(in the meaning of non-commuting pair)?
- Can we observe? As we now, $\dot{\zeta}$ is decaying and exponentially suppressed. (Even for theorists are impossible[J. Maldacena (2016)])

Rubustness

Is there still violation of Bell inequality considering decoherence? Martin and Vennin discussed constant decoherence rate case.

- the form of density matrix is not $\rho = \rho_G \exp[-\Gamma(x_{\boldsymbol{k}} - \tilde{x}_{\boldsymbol{k}})^2 - \Gamma(x_{-\boldsymbol{k}} - \tilde{x}_{-\boldsymbol{k}})^2]$, but $\rho = \rho_G \exp[-\Gamma|(\zeta_{\boldsymbol{k}} - \tilde{\zeta}_{\boldsymbol{k}})|^2]$ which leads to difficulties in calculation.
- decoherence rate has a complicated form with time dependence

Decoherence

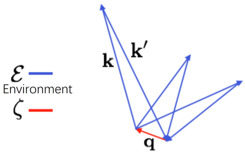
Quantum decoherence is the loss of quantum coherence and can be described by the loss of quantum information.

The information in the off-diagonal is lost.

The simplest way to describe it

$$\rho = \rho_G \exp[-\Gamma(\zeta - \tilde{\zeta})^2]$$

The reason for decoherence is interaction with other fields (Environment). We need to integrated out to get the reduced density matrix.



[figure from E.Nelson(2016)]

$$\boxed{\rho_R[\zeta_{\boldsymbol{q}}, \tilde{\zeta}_{\boldsymbol{q}}] = \Psi_G[\zeta_{\boldsymbol{q}}]\Psi^*_G[\tilde{\zeta}_{\boldsymbol{q}}] \exp\left(-\tilde{\Gamma}|\Delta\bar{\zeta}_{\boldsymbol{q}}|^2\right)}$$

Decoherence from the gravitational nonlinearity

For general interaction

$$\mathcal{L}_{\rm int} \supset \zeta f(\mathcal{E})$$

The contribution to the density matrix can be write as

$$\left\langle e^{f(\mathcal{E})\Delta\zeta} \right\rangle \approx \exp\left[\frac{1}{2}\left(\langle f(\mathcal{E})^2\rangle - \langle f(\mathcal{E})\rangle^2\right)\Delta\zeta^2\right]$$

Expanding the action of a scalar field with FRWL background up to 3rd order. The leading terms for decoherence are

Bulk term $$\mathcal{L}_{\rm int}^{\rm bulk} = \epsilon(\epsilon+\eta)a(t)\zeta\left(\partial_i\zeta\right)^2$$

Boundary term $$\mathcal{L}_{\rm int}^{\rm bd} = \partial_t\left(-2a^3HM_{pl}^2e^{3\zeta}\right)$$

Decoherence from the gravitational nonlinearity

The $\Gamma \equiv \frac{4\pi^2\Delta_{\mathcal{E}}^2}{q^3}\tilde{\Gamma}$

Bulk term induced decoherence rate

$$\Gamma_{\rm bulk} = \frac{4\pi^2\Delta_\zeta^2}{8\pi}\left\{\left(\frac{\epsilon+\eta}{12}\right)^2\left(\frac{aH}{q}\right)^3 + \frac{(\epsilon+\eta)^2}{9\pi}\left(\frac{aH}{q}\right)^2\left[\Delta N - \frac{19}{48}\right]\right\}$$

[E.Nelson(2016)]

Boundary term induced decoherence rate

$$\Gamma_{\rm bd} \approx \frac{729\Delta_\zeta^2}{16\epsilon^2}\left[4(\Delta N - 1)\left(\frac{aH}{q}\right)^6 - 4\Delta N\left(\frac{aH}{q}\right)^4 + \frac{5\pi}{2}\left(\frac{aH}{q}\right)^3 + (4\Delta N - 7)\left(\frac{aH}{q}\right)^2\right]$$

[C.M.Sou, D.H.Tran and Y. Wang(2023)]

GKMR operators in field eigenstates basis

$$\left\langle \hat{S}_z(\boldsymbol{k})\hat{S}_z(-\boldsymbol{k})\right\rangle = \frac{\mathrm{Re}\, A_\zeta(k,\tau)}{\mathrm{Re}\, A_\zeta(k,\tau) + 2\tilde{\Gamma}}$$

$$\left\langle \hat{S}_x(\boldsymbol{k})\hat{S}_x(-\boldsymbol{k})\right\rangle = \frac{2}{\pi}\arctan\left(\frac{|A_\zeta(k,\tau)|^2 + 2\,\mathrm{Re}\, A_\zeta(k,\tau)\tilde{\Gamma} - k^2 z^4}{2kz^2\sqrt{|A_\zeta(k,\tau)|^2 + 2\,\mathrm{Re}\, A_\zeta(k,\tau)\tilde{\Gamma}}}\right)$$

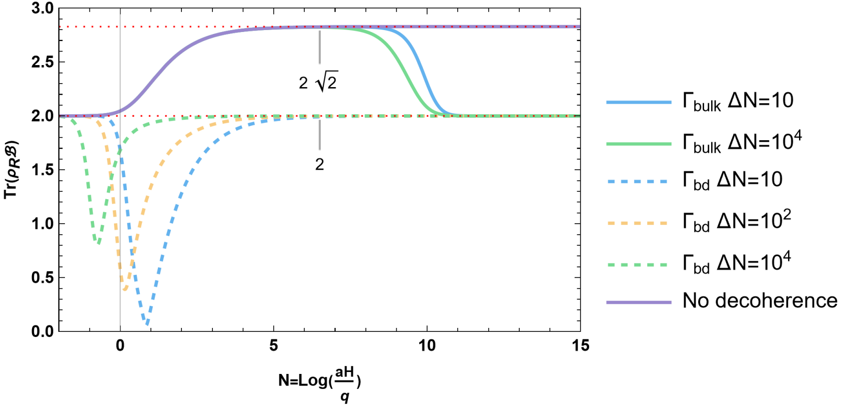
When $\tilde{\Gamma} = 0$ the result is consistent with [J. Martin and V. Vennin (2017)]

Take $A_\zeta(k,\tau) = 2k^3\frac{\epsilon M_{pl}^2}{H^2}\frac{1-\frac{i}{k\tau}}{1+k^2\tau^2}$ the quadratic coefficient of Gaussian wave functional into the expression of $\left\langle \hat{\mathcal{B}}\right\rangle$

$$\left\langle \hat{\mathcal{B}}\right\rangle = 2\sqrt{\left(\frac{1}{1+2\Gamma(\exp(-2N)+1)}\right)^2 + \frac{4}{\pi^2}\arctan^2\left(\frac{1-\frac{2\Gamma\exp(-2N)+1}{\exp(2N)+1}}{2\sqrt{\frac{2\Gamma\exp(-2N)+1}{\exp(2N)+1}}}\right)}$$

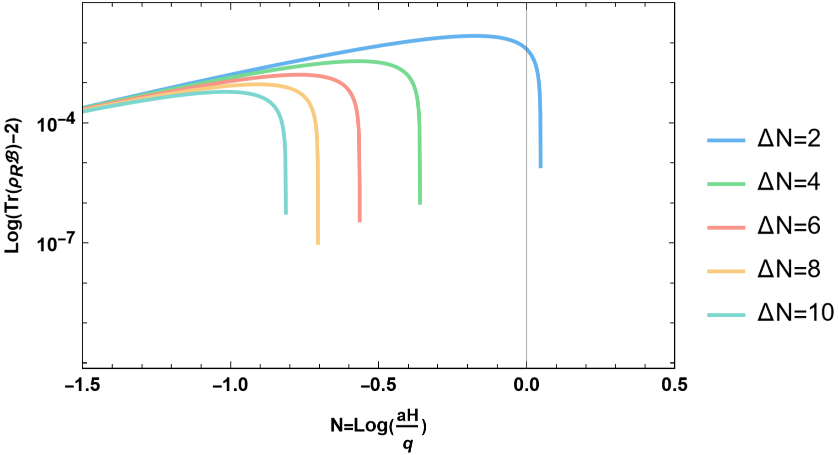
The expectation value of Bell Operator

The expectation value of Bell Operator vs e-folds after crossing horizon.



Violation of Bell-Inequality

Boundary induced decoherence case: the violation of Bell inequality



Quadratic Boundary Term Contribution to Bell Inequality

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$$\left\langle\hat{\mathcal{B}}\right\rangle = 2\sqrt{\left(\frac{1}{1+2\Gamma(e^{-2N}+1)}\right)^2 + \frac{4}{\pi^2}\arctan^2\left(\frac{1-\frac{(2\Gamma e^{-2N}+1)\epsilon^2+9e^{2N}(2\epsilon+9)+81e^{4N}}{(e^{2N}+1)\epsilon^2}}{2\sqrt{\frac{(2\Gamma e^{-2N}+1)\epsilon^2+9e^{2N}(2\epsilon+9)+81e^{4N}}{(e^{2N}+1)\epsilon^2}}}\right)}$$

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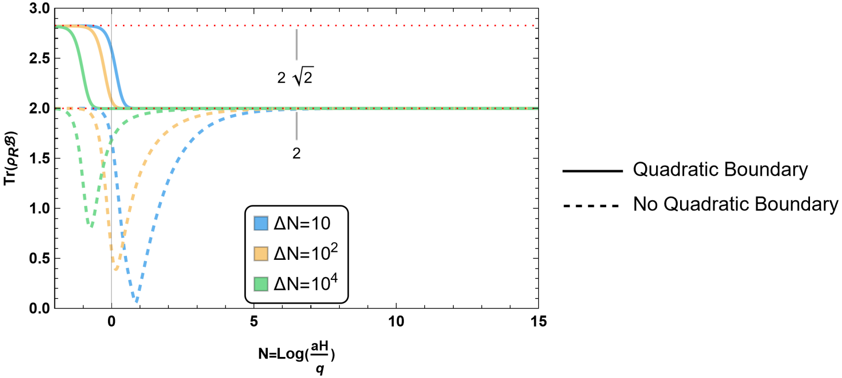

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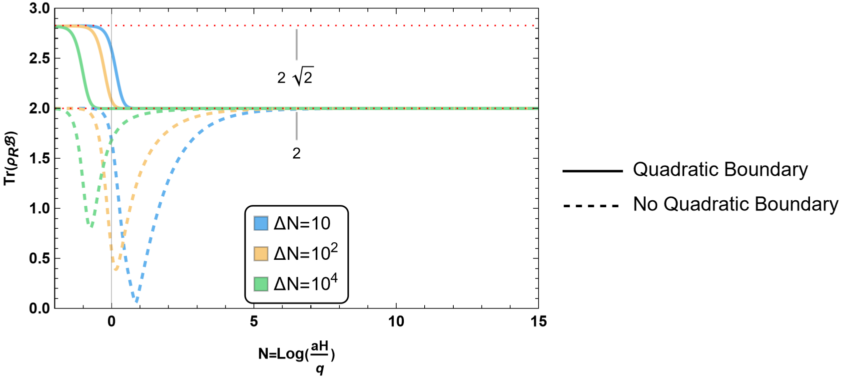

We can have maximal violation of Bell Inequality before decoherence.

What does that mean?

Does Quadratic Boundary Term Accelerate the Squeezing?

Squeezing Parameter

$$\sinh^2 r_k = \frac{1}{(2k\tau)^2} \to \frac{(k\tau)^2(\epsilon+18)^2+324}{4(k\tau)^4\epsilon^2}$$

Which is a **linear canonical transformation**

The definition of canonical momentum changed

$$p = y' - \frac{z'}{z}y + \frac{18}{\epsilon\tau}y$$

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Which is a **linear canonical transformation** $\Leftrightarrow$ **quadratic boundary term**

Ambiguity of the definition of particle

We need good a choice of canonical momentum, but from theoretical aspect we don't know what it is.
Simpler case to notice this problem is from $(a\zeta)'$ and $a\dot{\zeta}$. The corresponding squeezing parameter is $\sinh^2 r_k = \frac{1}{(2k\tau)^2}$ and $\sinh^2 r_k = \frac{1}{(2k^2\tau^2)^2}$.
[R.Laflamme, A. Matacz(1993)], [J.Grain,V.Vennin(2019)], [I.Agullo, et al(2022)]

Conclusion

- We can distinguish different decoherence sources by Bell test.
- In case of gravitational nonlinearity induced decoherence, we can still have violation of Bell Inequality. (Boundary term case, very slight violation)
- In Bell test, the ambiguity of the definition of particle still exists and has big effect.

Thank You