(P)reheating, Nonlinear Gravity and Primordial Black Holes

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Gravity and Cosmology 2024
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1907.10601

work done with Avery Tishue, Peter Adshead, Ryn Grutkoski, Mary Gerhardinger

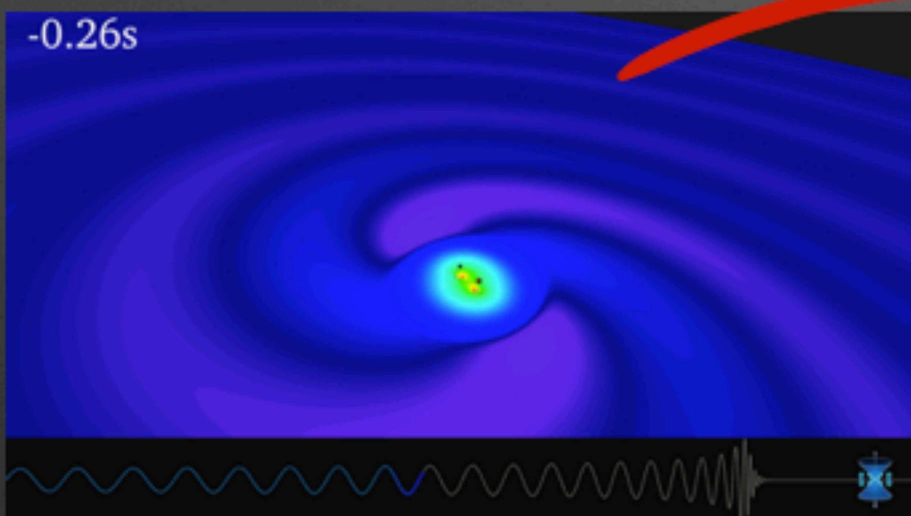
My group at Kenyon

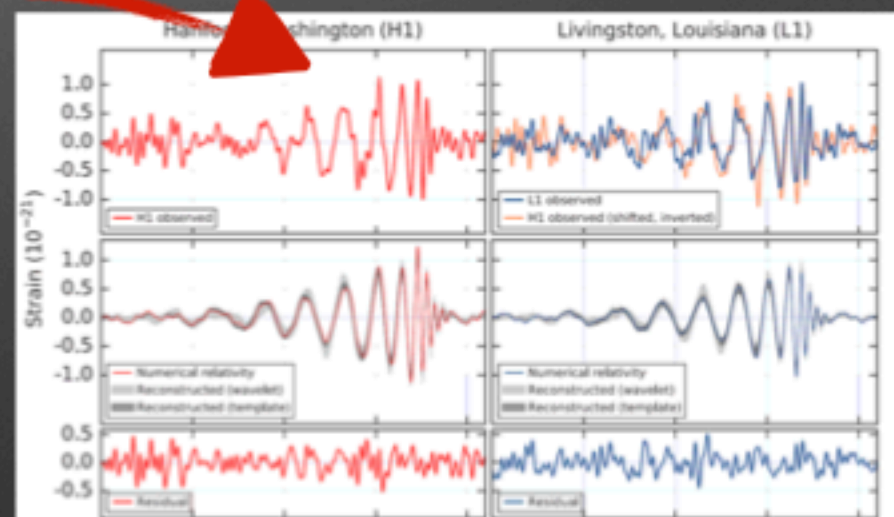
Gravity

- General Relativity appears to be one heck of a theory

Gravity

- An example:
  - Two black holes collide*
  - General Relativity *predicts*** a signal
  - We measure the signal***





*where'd they come from?
**Many contributors, this analysis from Simulating Extreme Space-time (not me)
**LIGO: Phys. Rev. Lett. 116, 061102 (absolutely not me)

Unfortunately

- No one seemed to tell the Universe

According to Concordance Cosmology here's what happened (mathematically speaking)

Dark Energy Dominated Universe (expansion of the universes seems to be accelerating)

The Universe today is a combination of Matter and Radiation (mostly matter)

PLUS Dark matter

The Universe cools enough to be transparent

Because matter dilutes slower than radiation, the earlier Universe was more radiation than matter

In the distant past, the Universe was very very dense

Inflation? Ekpyrosis? Bubbles? Gnomes?

The Main Point: Gravity is Non-Linear

- Being “non-linear” is *more* that just “not being small”
- We like to *separate scales* when doing physics problems (e.g. what happens here, stays here)
  - Non-linear physics can mix up scales - power transferred between scales through *cascades* or *inverse-cascades*

The Main Question: For the Universe, *does it matter?*

Averaging

- Generally a Hubble Volume is taken to be the region over which we do averaging — we all agree that different Hubble patches could have different expansion rates (causality, right?)

$$H^{-3} \approx (4000\,\mathrm{Mpc})^3$$

- Yet there is structure at (just) smaller scales
  - Galaxy Clusters $\sim 1 - 10\,\mathrm{Mpc}$
  - Inter-Cluster Distances $\sim 50\,\mathrm{Mpc}$

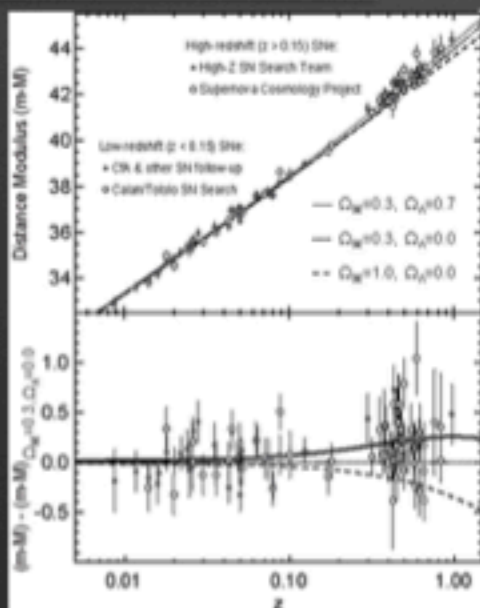
Work With

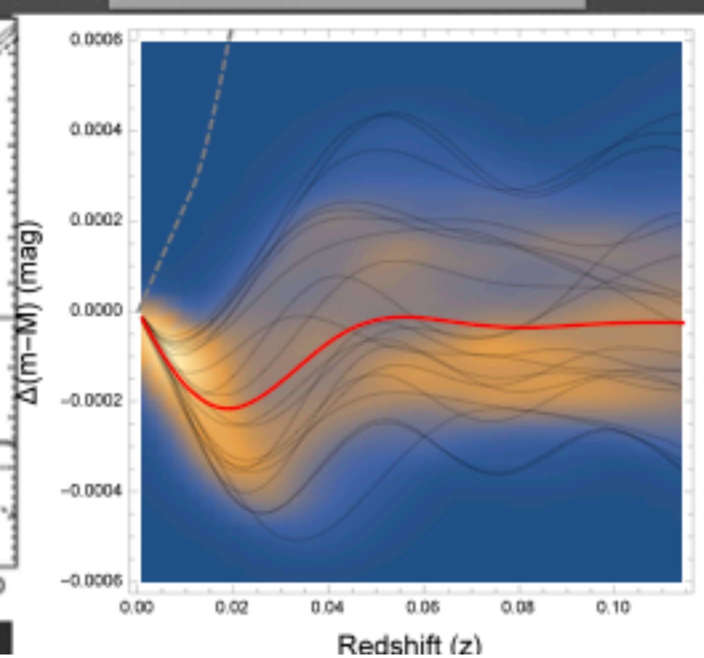
- For the Late Universe

Jim Mertens
Wash U/CWRU

Glenn Starkman
Case Western

Chi Tian
Anhui University





Scales at Reheating

- Generally a Hubble Volume is taken to be the region over which we do averaging — we all agree that different Hubble patches could have different expansion rates (causality, right?)

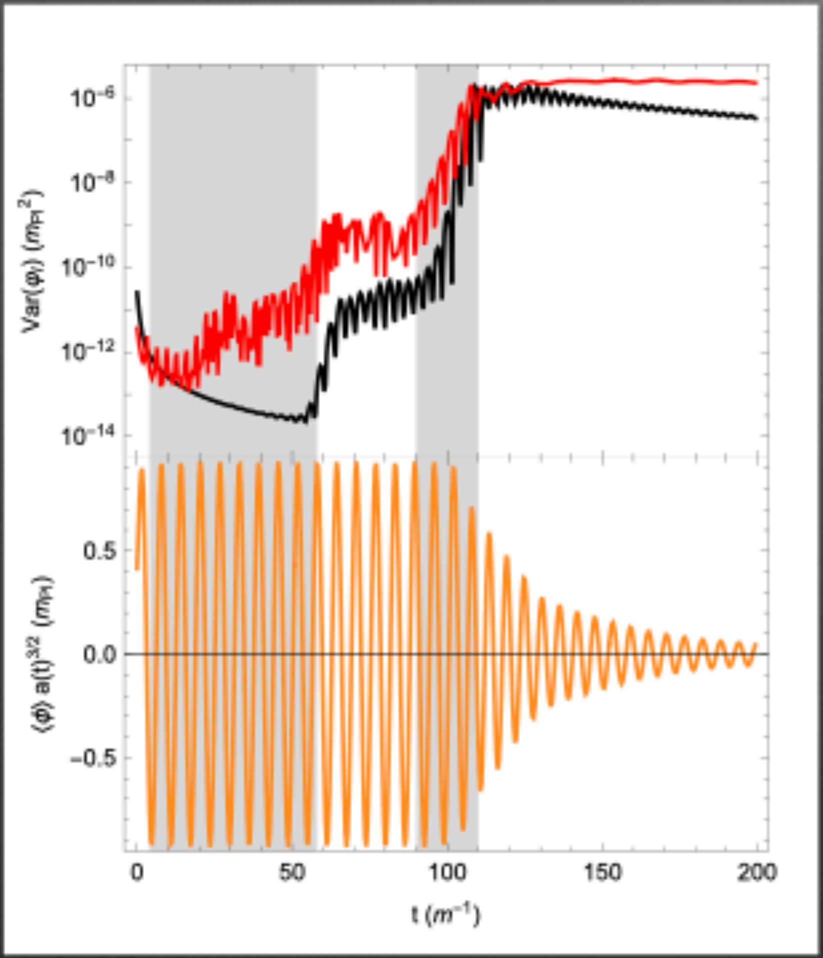
$$H^{-1} \propto \mathcal{O}(1) \times \frac{m_{\rm pl}^2}{m^2 \phi_0^2} = \mathcal{O}(1) \times m^{-1}$$

- YET: we talk about things at scales around this
  - Oscillons
  - Tachyonic/Parametric Resonance

$$\left.\begin{matrix}\text{Oscillons} \\ \text{Tachyonic/Parametric Resonance}\end{matrix}\right\} k \propto \mathcal{O}(1) \times m^{-1}$$

Can non-linear physics help explain the great mysteries of the Universe?

Preheating



The Inflationary field is coupled to a second “matter” field

Non-linear interactions case the energy to be transferred quickly and violently.

The End of Inflation

- most inflationary models are driven by a classical degree of freedom

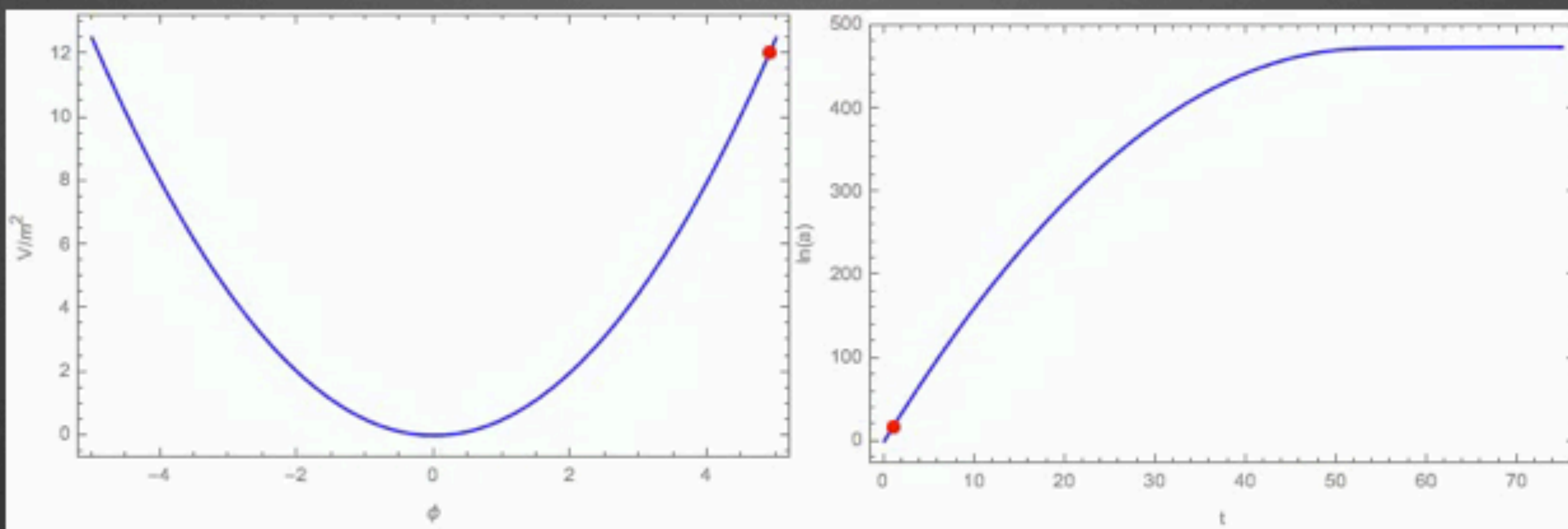
$$\phi$$

- and have an associated potential (we'd also like to know where this comes from)

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

The End of Inflation

- There is a phase transition at the end of inflation which might tell us *something* about the properties of the inflaton



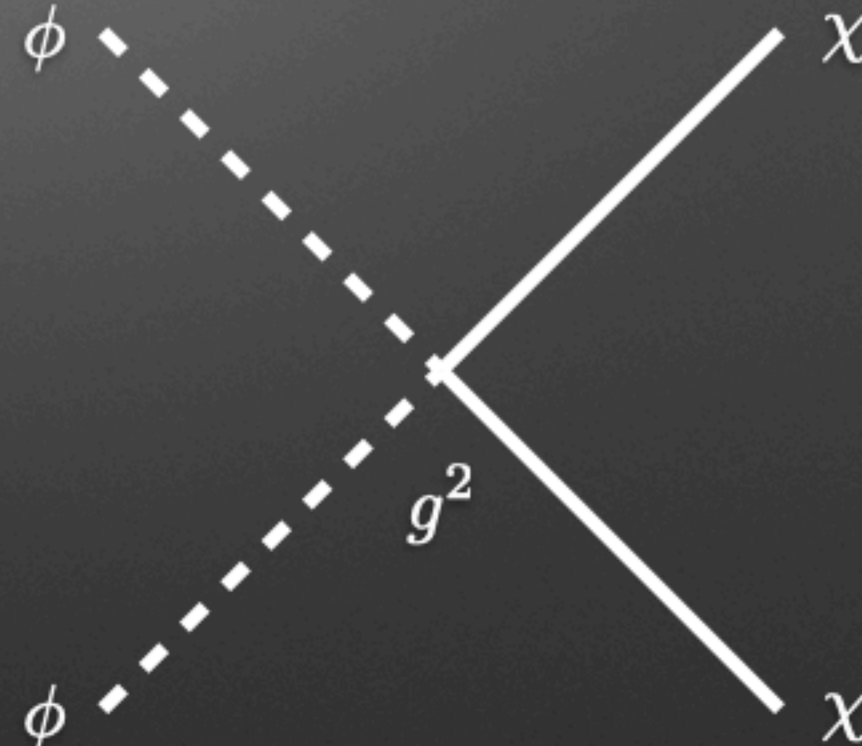
The End of Inflation

- What I normally worry about is whether inflationary models can create a radiation dominated final state
- In order to protect the inflation potential, *direct couplings* to other degrees of freedom have to be small
- Perturbative decay (the ‘old theory of reheating’) can sometimes take too long
- This is/was a problem for fans of extremely low-scale inflation

A simple coupling

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}m^2\phi^2 - \frac{g^2}{2}\phi^2\chi^2$$

- if g is too big - it messes up inflation
- if go s too small - the inflation never decays



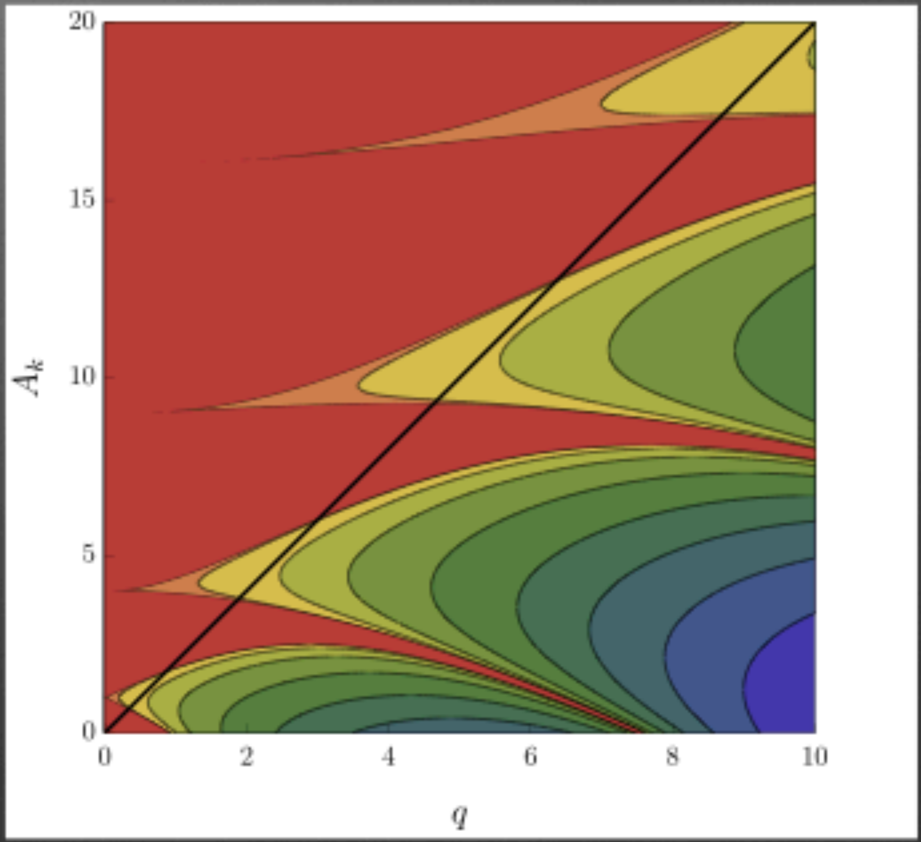
What can we see from this?

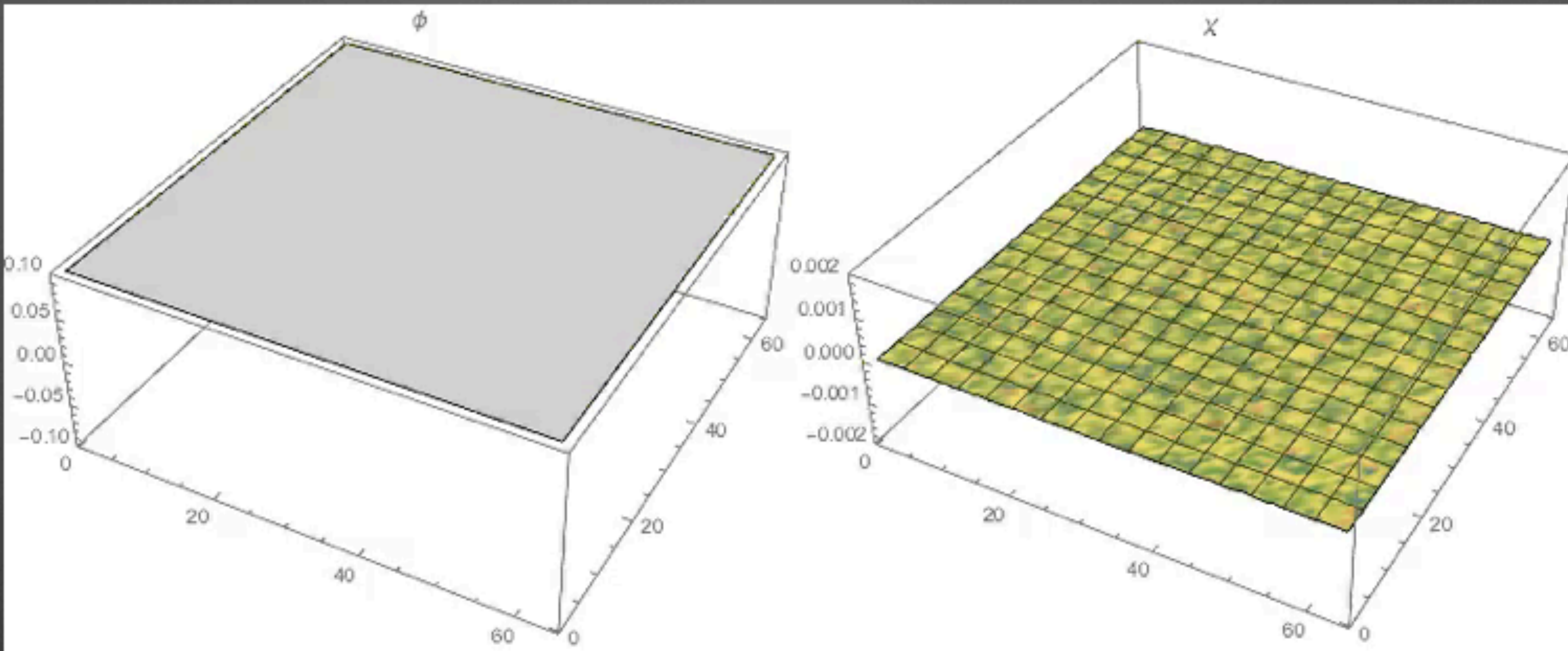
$$\ddot{\phi}_k + 3H\dot{\phi}_k + \frac{k^2}{a^2}\phi_k = -\frac{\partial V}{\partial \phi}$$

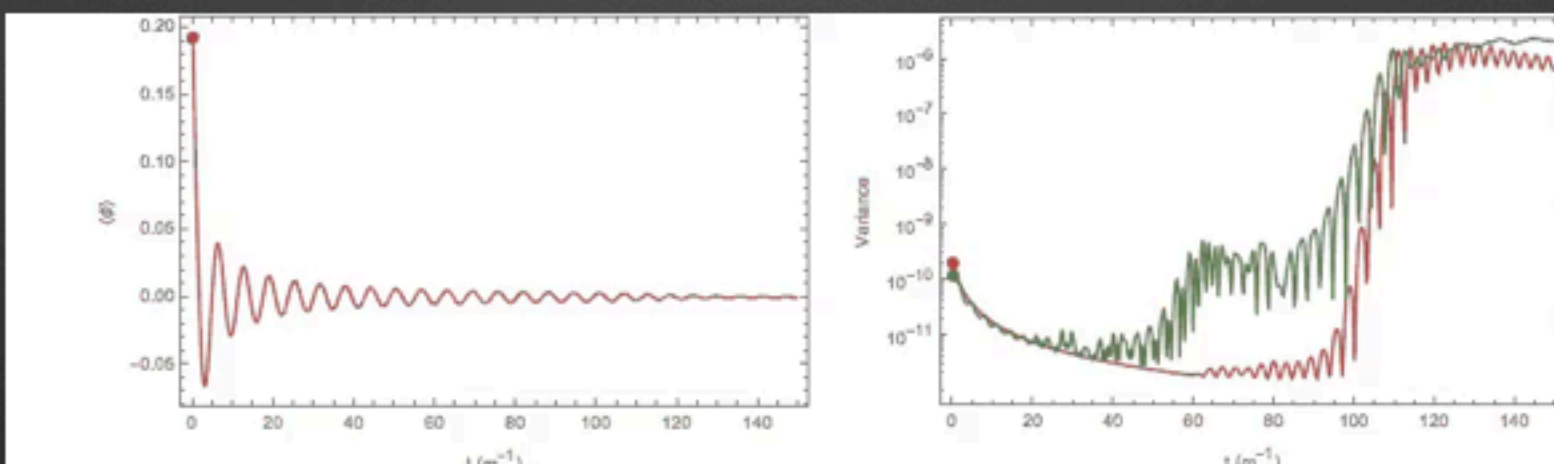
$$g^2 \left\langle \phi^2 \right\rangle \chi$$

time dependent mass

parametric resonance




ϕ
χ
0.10
0.05
0.00
-0.05
-0.10
0.002
0.001
0.000
-0.001
-0.002
0
20
40
60


⟨ϕ⟩
Variance
t (m⁻¹)
0.20
0.15
0.10
0.05
0.00
-0.05
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
0
20
40
60
80
100
120
140

What about gravity?

What you would *like* to do

- Write down the most general form of the metric,

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix}$$

- Plug it into Einstein's Equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- Solve the system of second order differential equations (subject to your gauge-constraints)

```
In[9]:= SetDirectory[NotebookDirectory[]];

In[10]:= << GREAT.m

GREAT functions are: IMetric, Christoffel,
  Riemann, Ricci, SCurvature, EinsteinTensor, SqRicci, SqRiemann.

Enter 'helpGREAT' for this list of functions

In[11]:= (metric = {{g00[x0, x1, x2, x3], g01[x0, x1, x2, x3], g02[x0, x1, x2, x3],
        g03[x0, x1, x2, x3]}, {g01[x0, x1, x2, x3], g11[x0, x1, x2, x3],
        g12[x0, x1, x2, x3], g03[x0, x1, x2, x3]},
       {g02[x0, x1, x2, x3], g12[x0, x1, x2, x3], g22[x0, x1, x2, x3],
        g23[x0, x1, x2, x3]}, {g03[x0, x1, x2, x3], g13[x0, x1, x2, x3],
        g23[x0, x1, x2, x3], g33[x0, x1, x2, x3]}}) // MatrixForm

Out[11]//MatrixForm=
 ( g00[x0, x1, x2, x3]  g01[x0, x1, x2, x3]  g02[x0, x1, x2, x3]  g03[x0, x1, x2, x3] )
 ( g01[x0, x1, x2, x3]  g11[x0, x1, x2, x3]  g12[x0, x1, x2, x3]  g03[x0, x1, x2, x3] )
 ( g02[x0, x1, x2, x3]  g12[x0, x1, x2, x3]  g22[x0, x1, x2, x3]  g23[x0, x1, x2, x3] )
 ( g03[x0, x1, x2, x3]  g13[x0, x1, x2, x3]  g23[x0, x1, x2, x3]  g33[x0, x1, x2, x3] )

In[12]:= coords = {x0, x1, x2, x3}

Out[12]= {x0, x1, x2, x3}

In[13]:= EinsteinTensor[metric, coords]
```

What can we do?

- You can do a little better by making gauge choices that reduce the number of parameters or (re)parameterize so that you have nice equations for.. some.. of them…

- Even then they are extremely difficult to numerically stabilize

arXiv:gr-qc/0211028v1 7 Nov 2002

Numerical Relativity and Compact Binaries

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Abstract

Numerical relativity is the most promising tool for theoretically modeling the inspiral and coalescence of neutron star and black hole binaries, which, in turn, are among the most promising sources of gravitational radiation for future detection by gravitational wave observatories. In this article we review numerical relativity approaches to modeling compact binaries. Starting with a brief introduction to the 3+1 decomposition of Einstein's equations, we discuss important components of numerical relativity, including the initial data problem, reformulations of Einstein's equations, coordinate conditions, and strategies for locating and handling black holes on numerical grids. We focus on those approaches which currently seem most relevant for the compact binary problem. We then outline how these methods are used to model binary neutron stars and black holes, and review the current status of inspiral and coalescence simulations.

Key words:

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1 Introduction 3

2 Decomposing Einstein's Equations 6

2.1 Foliations of Spacetime 6

What we have to do…

- Luckily there are a set of new approaches. We use the most common of these: the BSSN formalism.

- It is based on the ADM metric decomposition

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \gamma_{lk}\beta^l\beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$

- We we introduce more parameters than (minimally) necessary so that the equations are easier to solve

In Cosmology

- We can fix the gauge (we will give up being able to create black holes, as well as some other concessions) to focus on spatial slices

- We can then track the spatial 3-metric

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$

- as well as the extrinsic curvature

$$K_{ij} = e^{4\phi}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

In Cosmology

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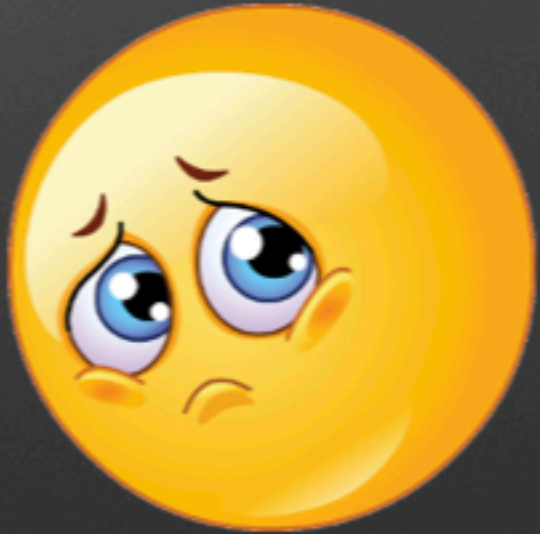


- as well as the extrinsic curvature

$$K_{ij} = e^{4\phi}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

In Cosmology

- We can fix the gauge (we will give up being able to create black holes, as well as some other concessions) to focus on spatial slices

- We can then track the spatial 3-metric

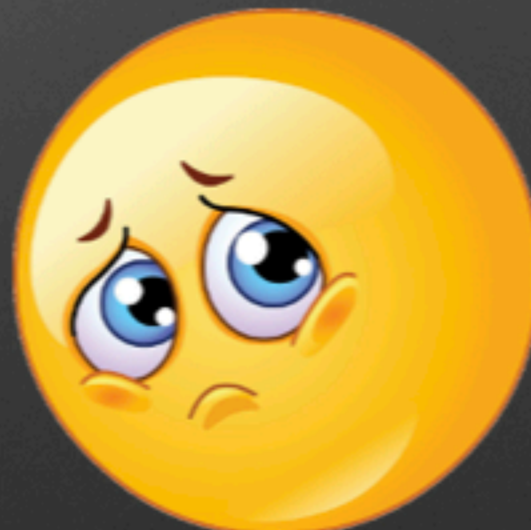
Think of this as keeping track of the size of local volumes

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$

- as well as the extrinsic curvature

Think of this as measuring the local expansion rate

$$K_{ij} = e^{4\phi}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$



Importantly

These variables have well-behaved differential equations and *are a complete description of GR without dimensional reductions or simplifications*

$$\partial_t \phi = -\frac{1}{6}\alpha K + \beta^i \partial_i \phi + \frac{1}{6}\partial_i \beta^i$$

$$\partial_t \bar{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \bar{\gamma}_{ij} + \bar{\gamma}_{ij}\partial_j \beta^k + \bar{\gamma}_{ij}\partial_i \beta^k - \frac{2}{3}\bar{\gamma}_{ij}\partial_k \beta^k$$

$$\partial_t K = \gamma^{ij} D_j D_i \alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i \partial_i K$$

$$\partial_t \tilde{A}_{ij} = e^{-4\phi}\left(-(D_i D_j \alpha)^{TF} + \alpha(R_{ij}^{TF} - 8\pi S_{ij}^{TF})\right) + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}$$

$$+ \beta^k \partial_k \tilde{A}_{ij} + \tilde{A}_{ik}\partial_j \beta^k + \tilde{A}_{kj}\partial_i \beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k \beta^k$$

$$\partial_t \bar{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha\left(\bar{\Gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K - 8\pi\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi\right)$$

$$+ \beta^j \partial_j \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{il}\partial_l \partial_j \beta^j + \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

Importantly

These variables have well-behaved differential equations and *are a complete description of GR without dimensional reductions or simplifications*

$$\partial_t \phi = -\frac{1}{6}\alpha K + \beta^i \partial_i \phi + \frac{1}{6}\partial_i \beta^i$$

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$$\partial_t K = -\gamma^{ij} D_j D_i \alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i \partial_i K$$

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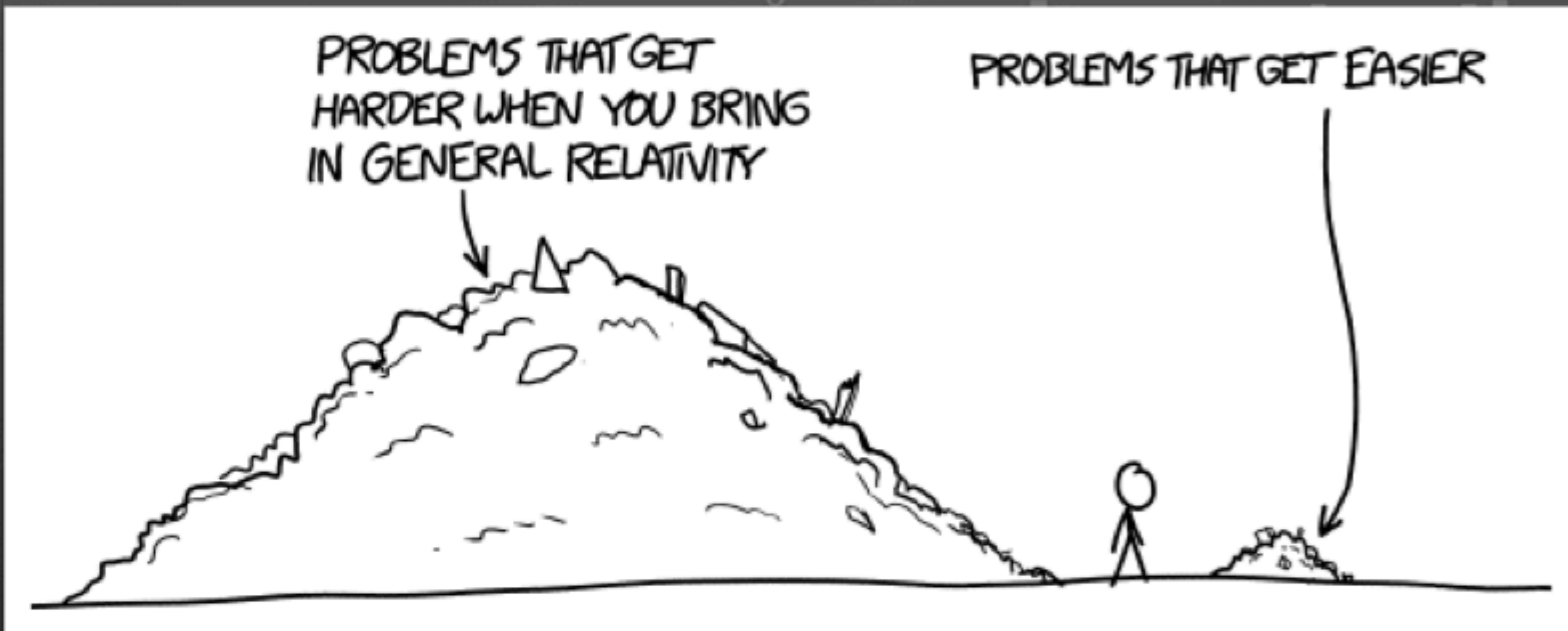
$$+ \beta^k \partial_k \tilde{A}_{ij} + \tilde{A}_{ik}\partial_j \beta^k + \tilde{A}_{kj}\partial_i \beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k \beta^k$$

$$\partial_t \bar{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha\left(\bar{\Gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K - 8\pi\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi\right)$$
$$+ \beta^j \partial_j \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{il}\partial_l \partial_j \beta^j + \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

The POWER is in the redundancy of the equations of motion

$$\partial_t \bar{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha(\bar{\Gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K$$
$$- 8\pi\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi) + \beta^j \partial_j \bar{\Gamma}^i$$
$$- \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{li}\partial_l \partial_j \beta^j$$
$$+ \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

$$\partial_t \phi = -\,\bar{\gamma}_{ij} rac16 \alpha K + \beta^i \partial_i \phi + \frac{1}{6} \partial_i \beta^i$$



$$+ A_{ik} \partial_j \beta^k + A_{kj} \partial_i \beta^k - \frac{}{3} A_{ij} \partial_k \beta^k$$

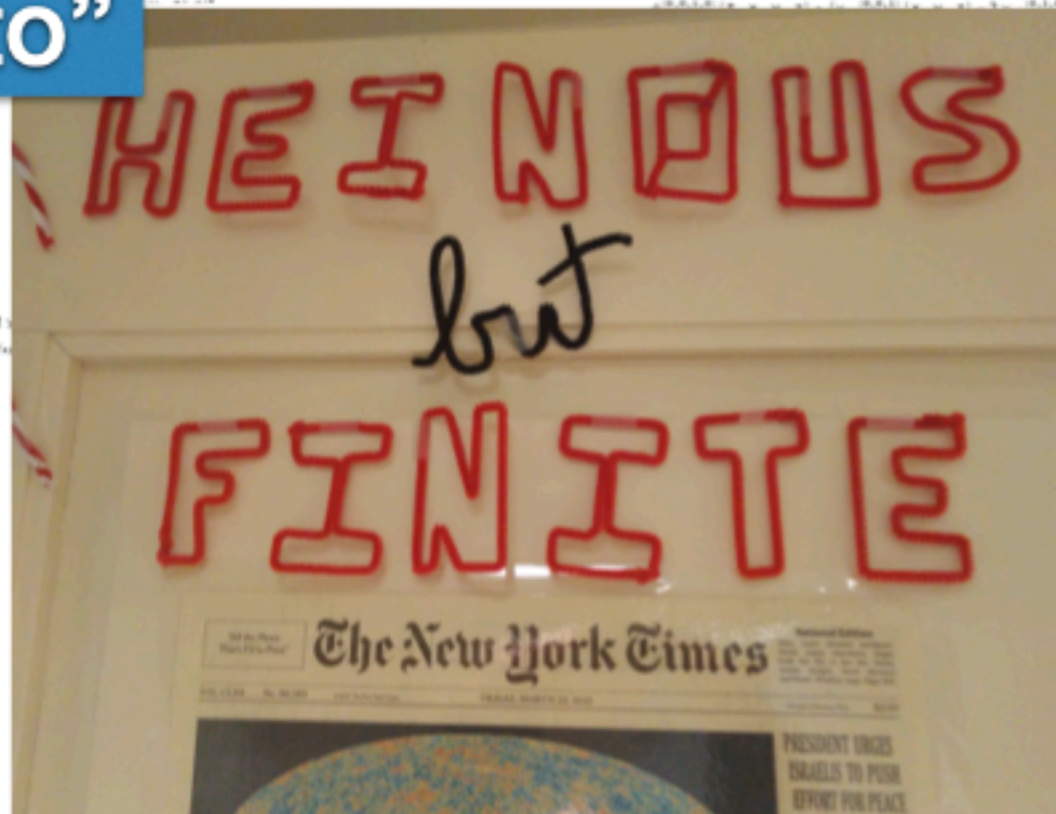
```
In[ ]:= eom = 1/sqrtg0 * (D[sqrtg0 *
      (gupper0[[1, 1]] * D[ϕ[t, x, y, z]
        gupper0[[1, 3]] * D[ϕ[t, x, y, z
      t] + D[sqrtg0 * (gupper0[[2, 1]] * D
        gupper0[[2, 2]] * D[ϕ[t, x, y, z
        gupper0[[2, 4]] * D[ϕ[t, x, y, z
      (gupper0[[3, 1]] * D[ϕ[t, x, y, z]
        gupper0[[3, 3]] * D[ϕ[t, x, y, z
      y] + D[sqrtg0 * (gupper0[[4, 1]] * D
        gupper0[[4, 2]] * D[ϕ[t, x, y, z
        gupper0[[4, 4]] * D[ϕ[t, x, y, z
```

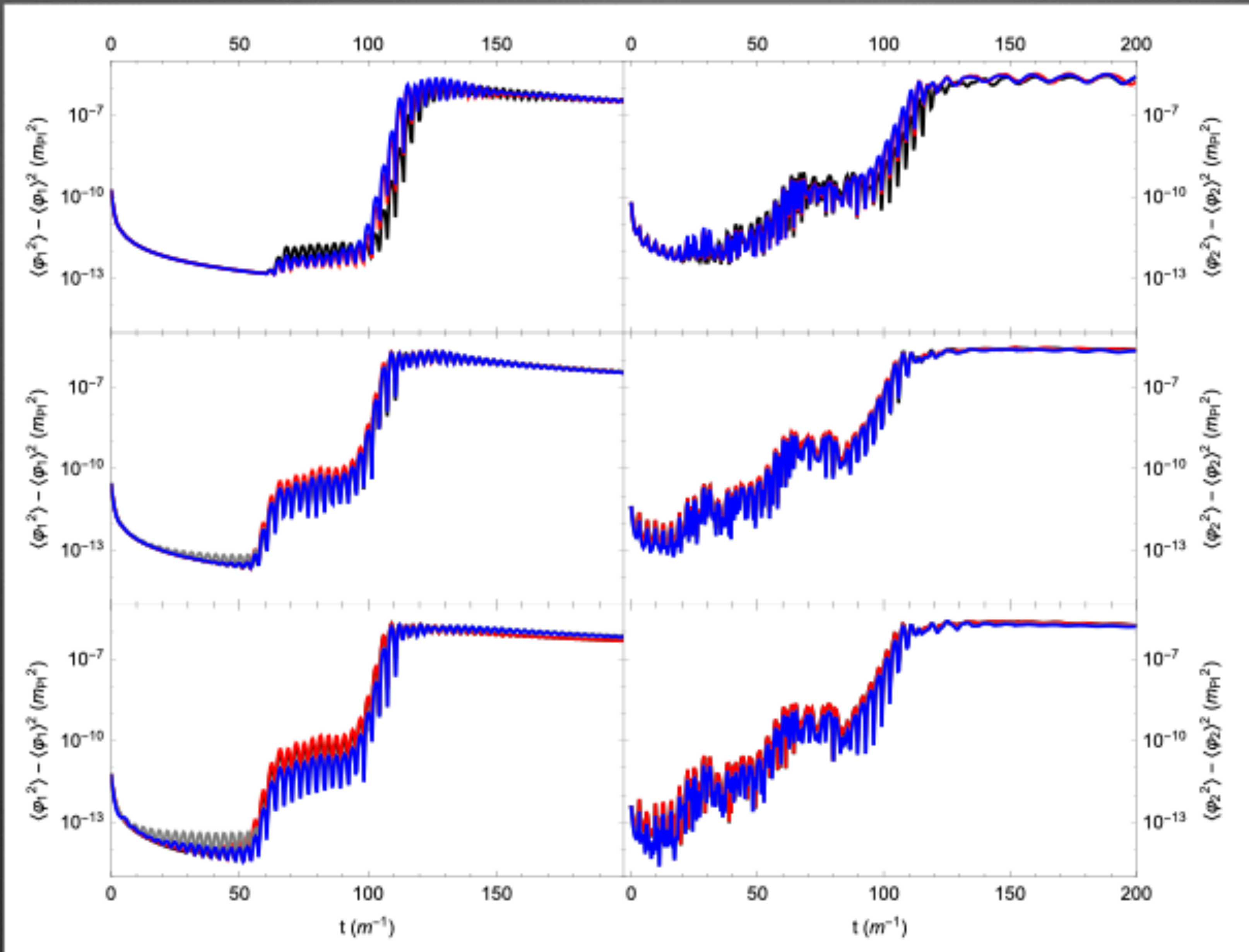
```
In[ ]:= FullSimplify
```



```
In[ ]:= teom = 1/sqrtg0 * (D[sqrtg0 * (gupper0[[1, 1]] * D[ϕ[t, x, y, z], t] +
        gupper0[[1, 2]] * D[ϕ[t, x, y, z], x] + gupper0[[1, 3]] * D[ϕ[t, x, y, z], y] +
        gupper0[[1, 4]] * D[ϕ[t, x, y, z], z]), t]) == 0;
```

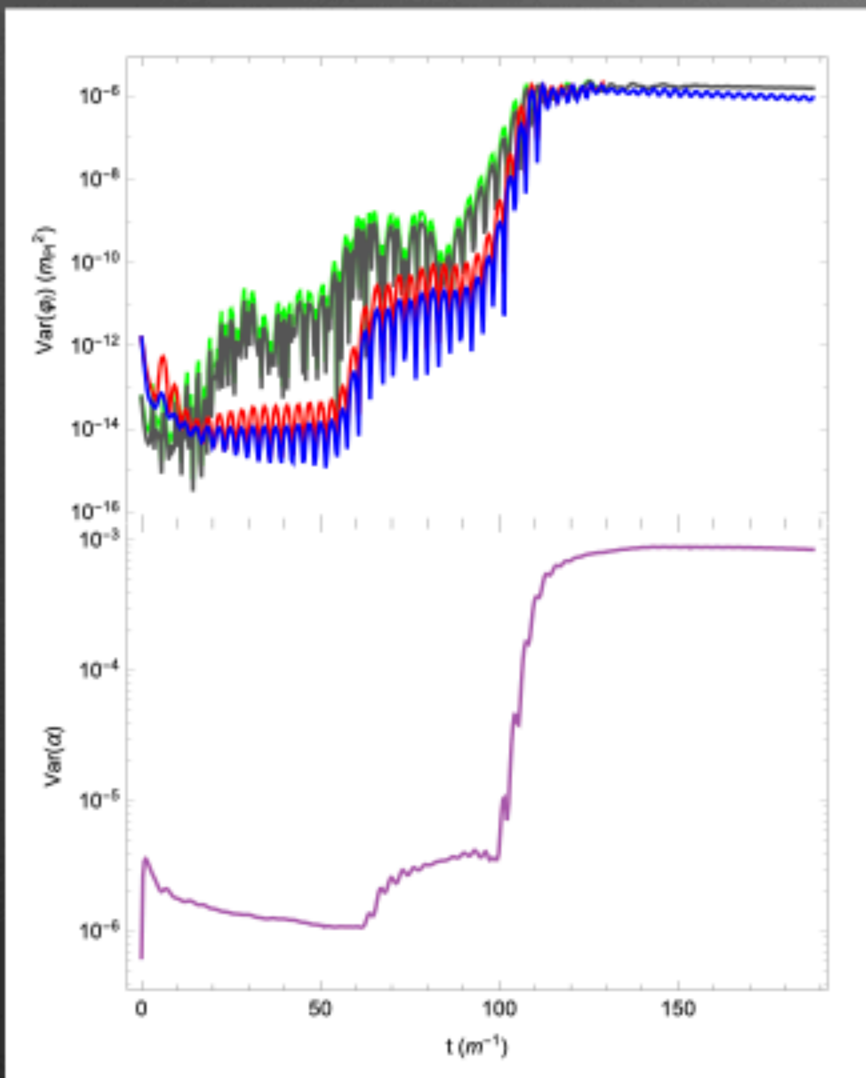
Sort of our "Group Motto"





Black = FLRW, Grey = Perturbative, Blue = BSSN

For the big box

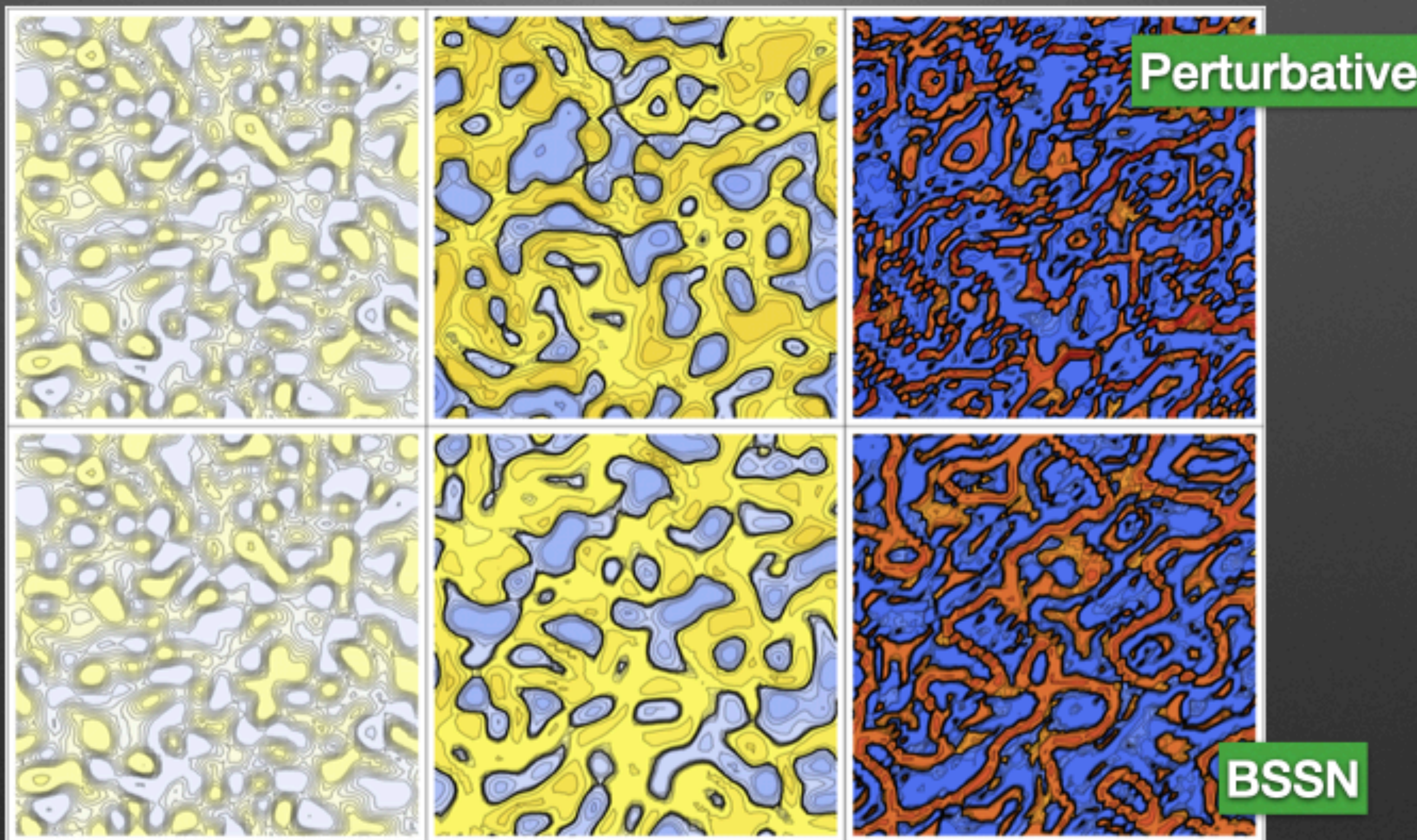


Red: inflaton Perturbative
Blue: inflaton BSSN

Green: decay field Perturbative
Black: decay field BSSN

The *variance* of the lapse does not show departures from homogeneity that indicate back hole formation

How do they look



So there’s no need to panic

- In these cases: non-linear physics seems to be a friend, not a foe
- But there’s still *much* left in parameter space: e.g. we know that collapse will happen

Gauge-Preheating

- There is a history of incorporating couplings of the inflation to gauge fields, generally with *charged* inflation fields (often in the context of Higgs inflation)
    - Coupling to U(1) fields by A. Rajantie , E. J. Copeland, and S. Pascal
    - Coupling to SU(2) fields by J. Garcia-Bellido et. al., Saffin et al.
- However using uncharged *scalar (or pseudo-scalar)* degrees of freedom were technically a bit more challenging

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- The “normal” Maxwell Stress-Tensor
- (but not for “normal” E/M)

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$W(\phi) = e^{-\phi/M}$$

$$X(\phi) = 0$$

- W is a *dilatonic* coupling that vanishes as the inflation decays to zero
- Possible generation of long-wavelength magnetic fields during inflation, e.g. Caldwell, Motta, Kamionkowski Phys. Rev. D 84, 123525 (2011).

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$W(\phi) = 1$$

$$X = \frac{\alpha_g}{f}\phi$$

- X is a *Chern Simons* coupling that couples the inflation to the curl of the vector field
- A coupling consistent with a shift-symmetric inflaton
- Also possible generation of polarized magnetic fields during inflation, e.g. Garretson, Field and Carroll, Phys. Rev. D 46 5346 (1992)

A Chern-Simons Coupling

- The form of the interaction begs for a decomposition of the gauge field onto its two helicity states,

$$\mathcal{L}_{\rm int} = -\frac{X(\phi)}{4} F_{\mu}\tilde{F}^{\mu\nu}$$

$$\vec{A}_{\mathbf{k}} = \sum_{\lambda=\pm} A^{\lambda}_{\mathbf{k}} \vec{\varepsilon}^{\lambda}(\mathbf{k})$$

which

$$k_i \varepsilon_i^{\pm}(\mathbf{k}) = 0, \quad i\epsilon^{ijk} k_j \varepsilon_k^{\pm}(\mathbf{k}) = \pm k\, \varepsilon_i^{\pm}(\mathbf{k}),$$

$$\varepsilon_i^{\pm}(\mathbf{k})^* = \varepsilon_i^{\pm}(-\mathbf{k}), \quad \varepsilon_i^{\lambda}(\mathbf{k})\varepsilon_i^{\lambda'}(-\mathbf{k}) = \delta_{\lambda\lambda'}.$$

so we see the two polarizations couple (with opposite sign) to the velocity of the inflation),

$$\left(\partial_\tau^2 + k^2 \pm \frac{\alpha}{f}\frac{\dot{\phi}}{H}\frac{k}{\tau}\right) A_k^{\pm} = 0$$

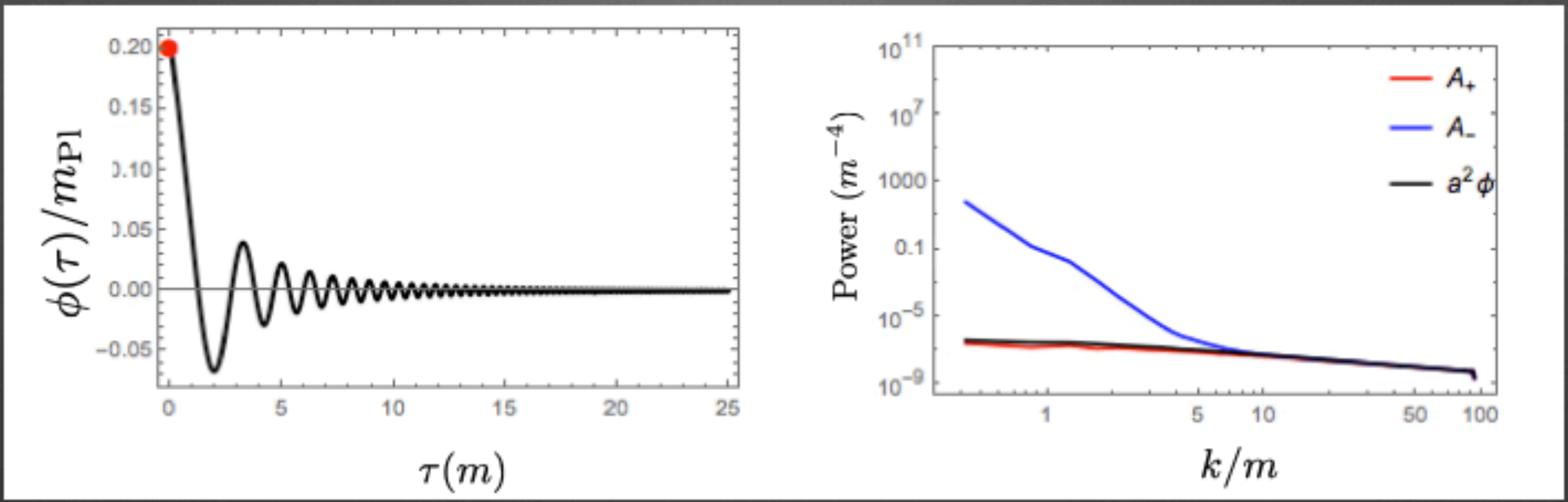
A Chern-Simons Coupling

- For large values of the velocity of the inflation (relative to small momenta), there is a clear tachyonic instability

- This instability should be polarized and present during each oscillation of the field :

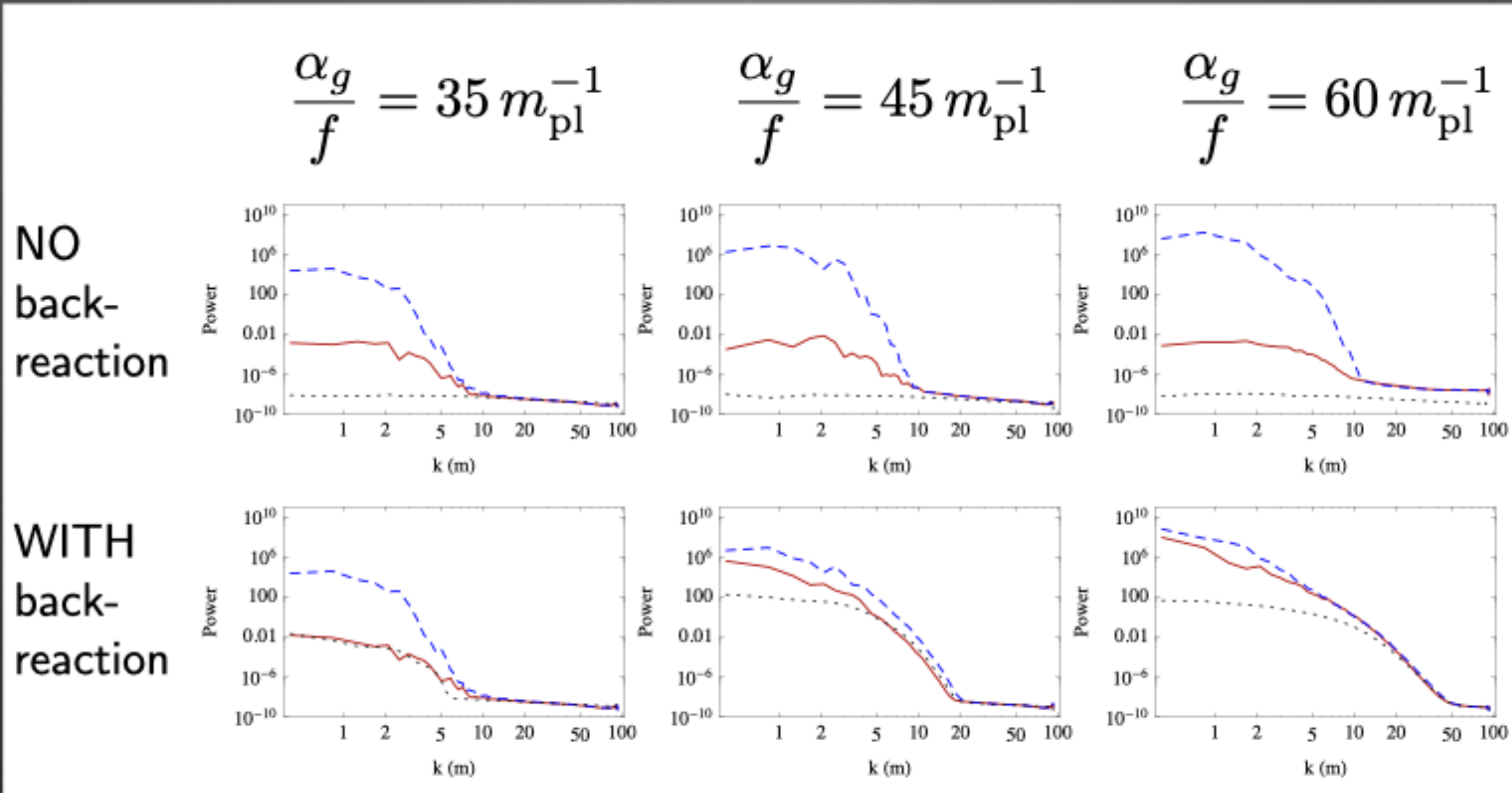
$$\left(\partial_\tau^2 + k^2 \pm \frac{\alpha}{f}\frac{\dot{\phi}}{H}\frac{k}{\tau}\right) A_k^\pm = 0$$

A Small Chern-Simons Coupling

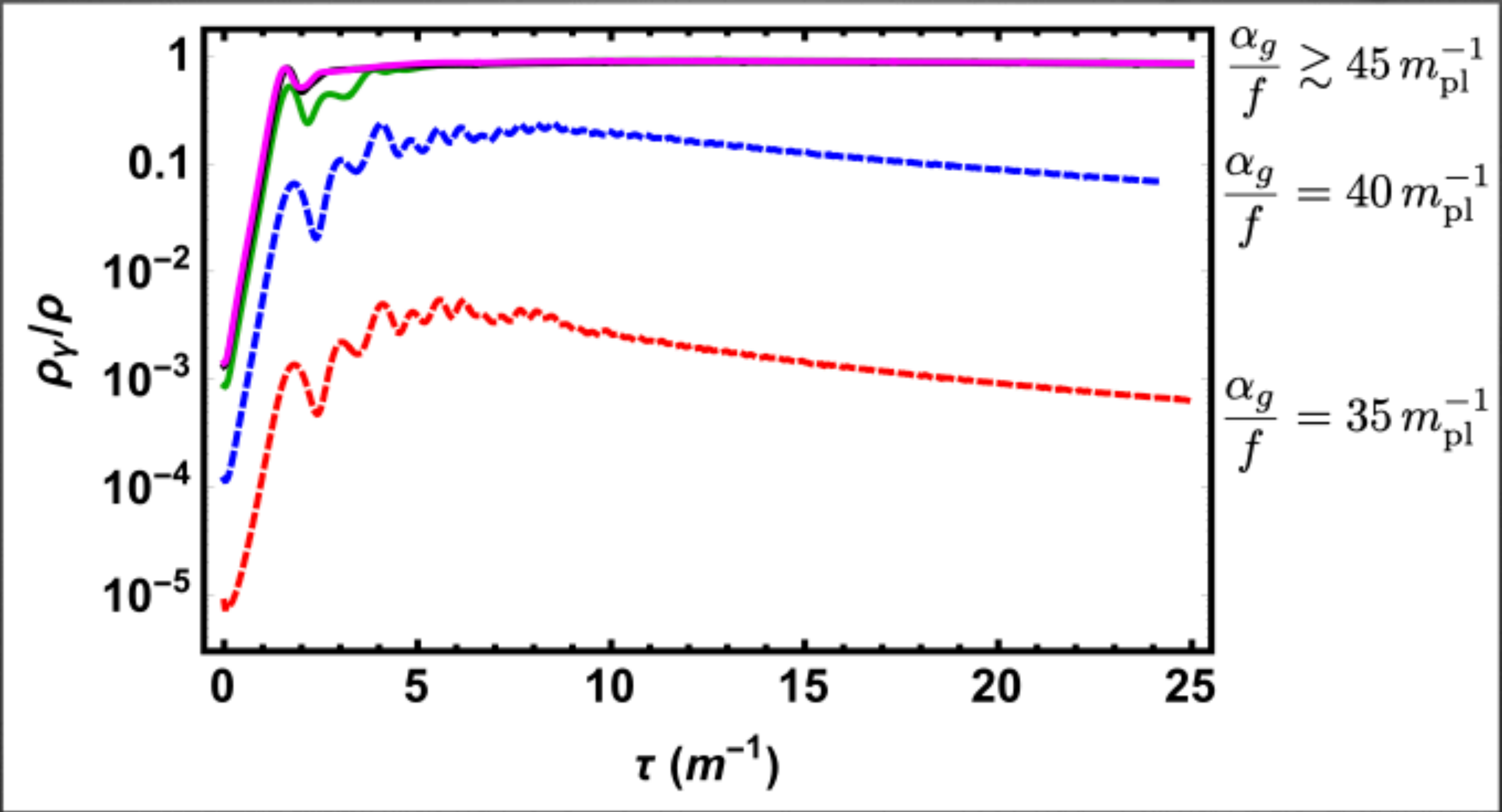


$$\frac{\alpha_g}{f} = 35 m_{\mathrm{pl}}^{-1}$$

The Final State

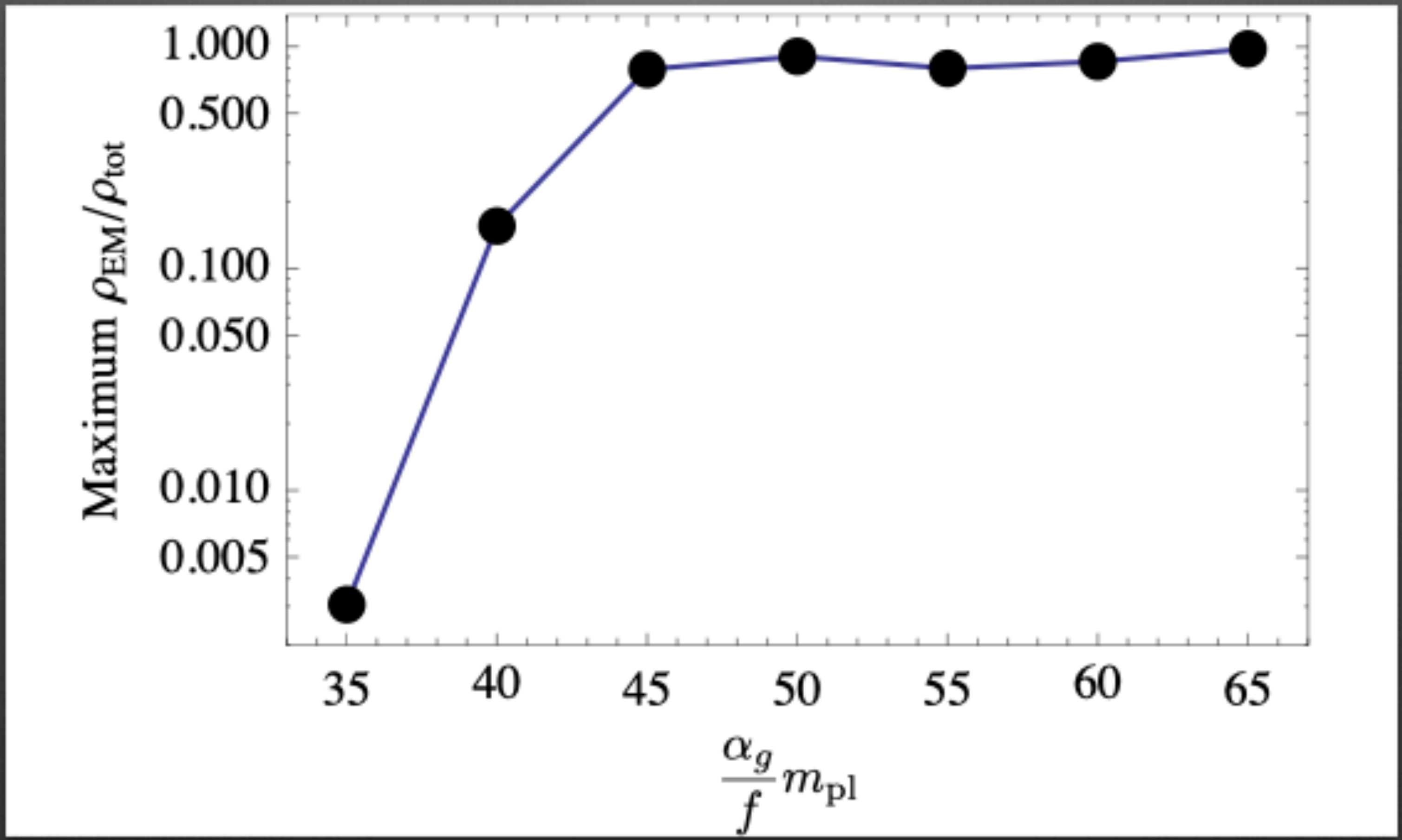


A Chern-Simons Coupling



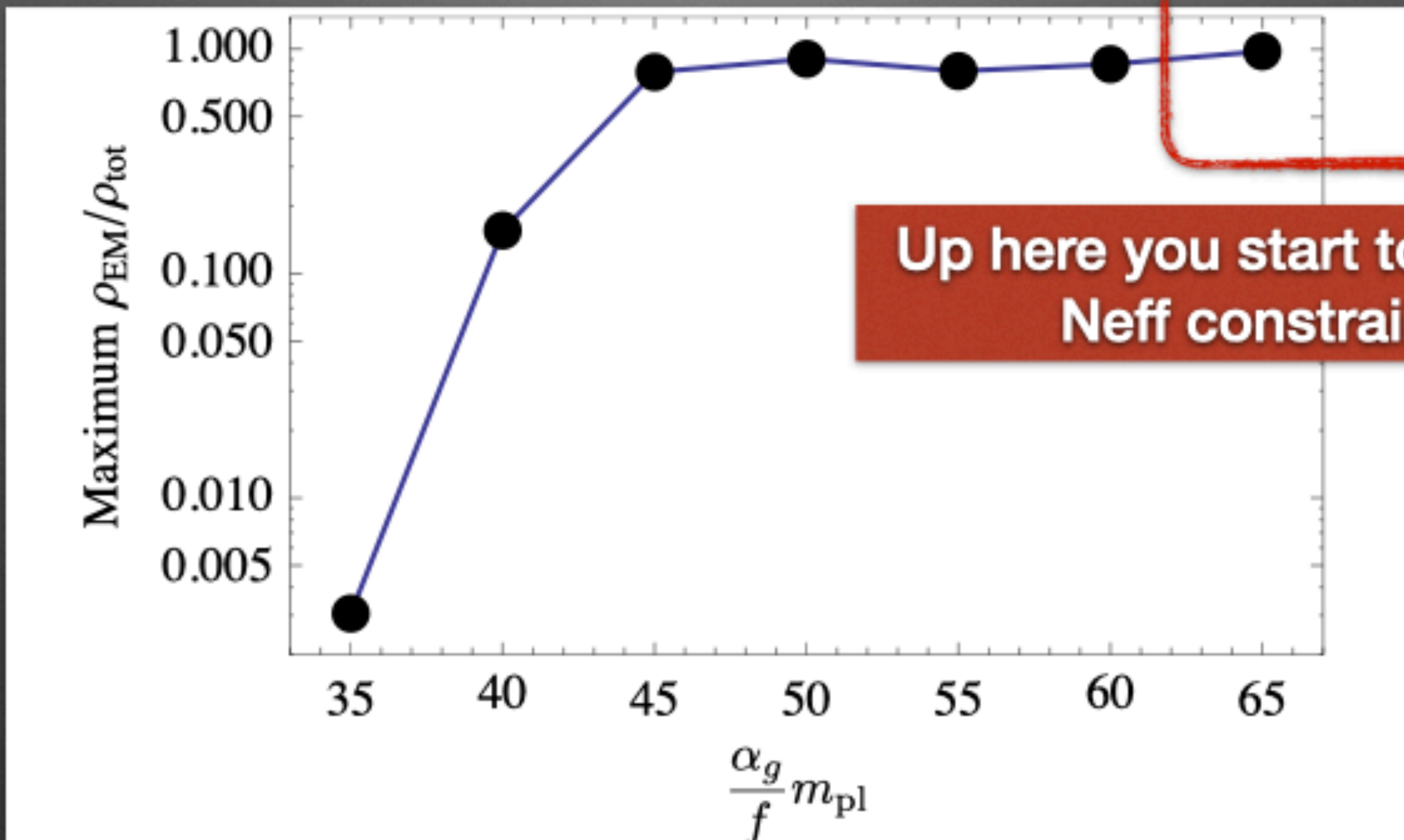
$$X = \frac{\alpha_g}{f}\phi$$

A Chern-Simons Coupling



$$X = \frac{\alpha_g}{f}\phi$$

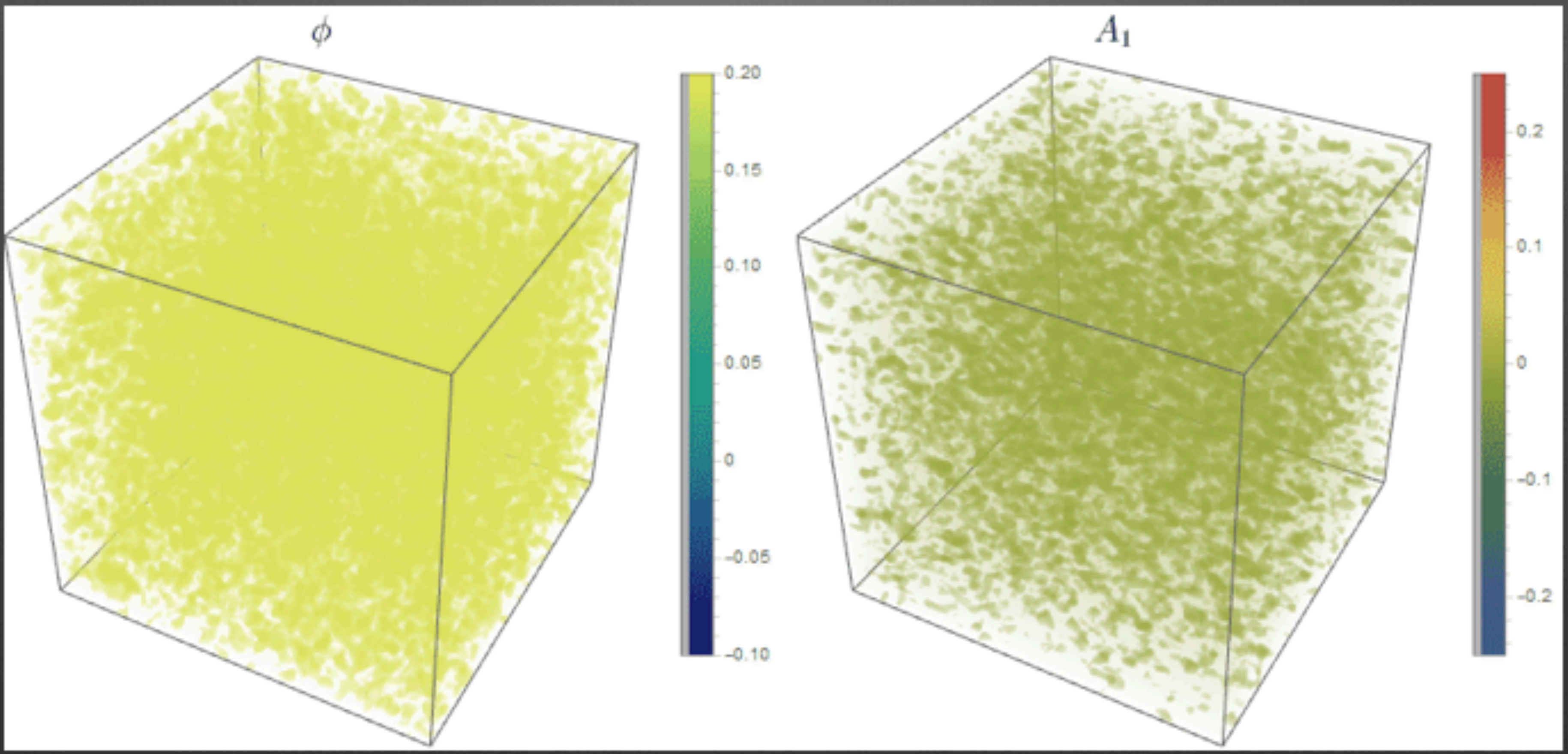
A Chern-Simons Coupling



Up here you start to run into Neff constraints

$$X = \frac{\alpha_g}{f}\phi$$

But… we get structure



The BSSN version

We can write down a set of evolution equations,

$$\partial_t E^m = \beta^o \partial_o E^m - E^o \partial_o \beta^m + \alpha \left(K E^m + \epsilon^{mno} D_n B_o - \mathcal{J}^m \right) + \epsilon^{mno} D_n \alpha B_o$$

$$\partial_t \mathcal{A}_m = \beta^o \partial_o \mathcal{A}_m + \mathcal{A}_o \partial_m \beta^o - \alpha \left(E_m + D_m \mathcal{A} \right) - \mathcal{A} D_m \alpha$$

$$\partial_t \mathcal{A} = \beta^o D_o \mathcal{A} + \alpha \left(K \mathcal{A} - D^m \mathcal{A}_m \right) - \mathcal{A}^m D_m \alpha$$

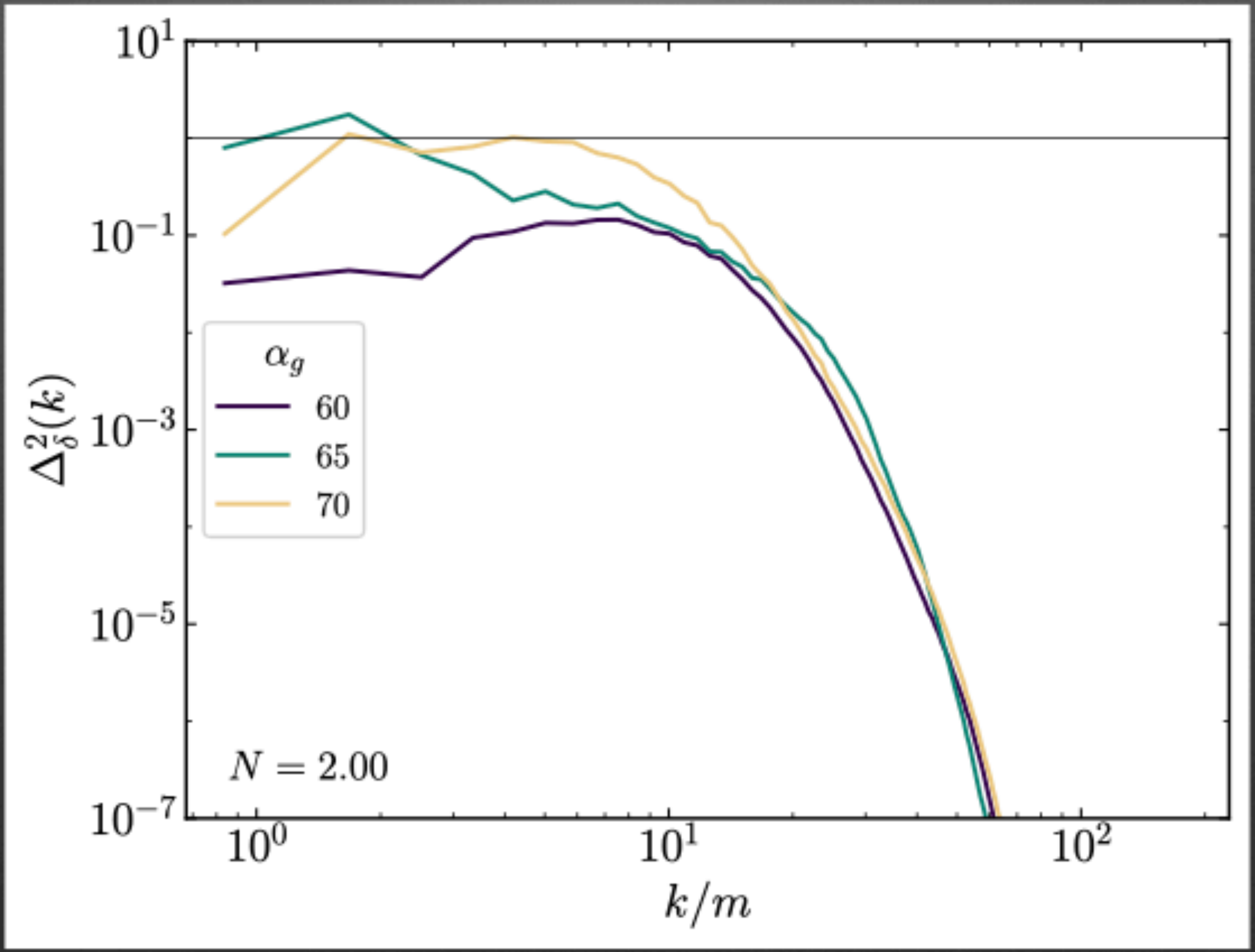
with…

$$\mathcal{J} = -\frac{1}{W(\varphi)} \left(W'(\varphi) E^m D_m \varphi - X'(\varphi) B^m D_m \varphi \right)$$

$$\mathcal{J}^m = \frac{1}{W(\varphi)} \left(W'(\varphi) \left[\Pi E^m - \epsilon^{mno} D_n \varphi B_o \right] - X'(\varphi) \left[\Pi B^m + \epsilon^{mno} D_n \varphi E_o \right] \right.$$

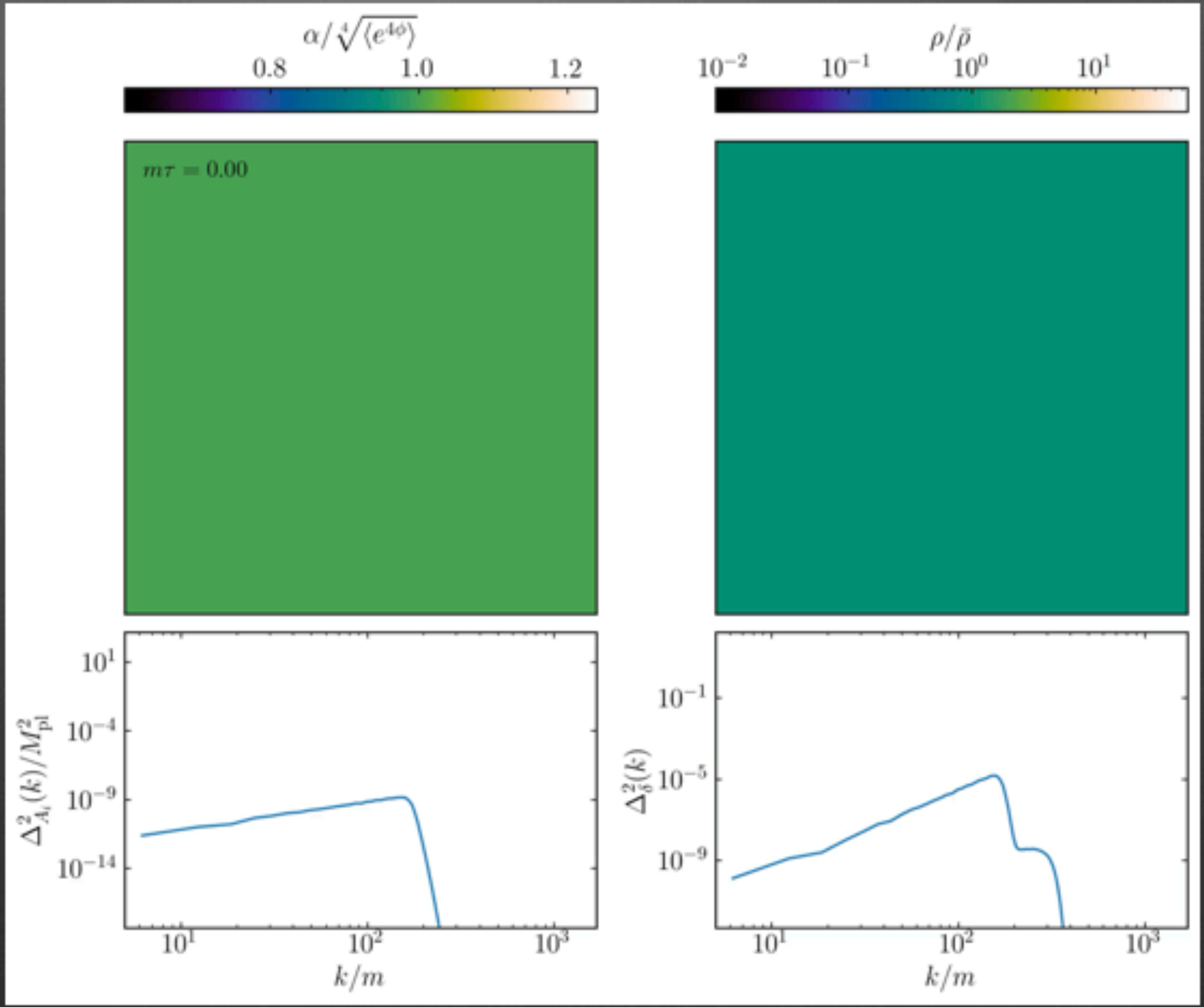
$$B^m = \epsilon^{mno} D_n \mathcal{A}_o = \epsilon^{mno} \partial_n \mathcal{A}_o, = e^{-6\phi} \varepsilon^{mno} \partial_n \mathcal{A}_o$$

We see very BIG density contrasts!



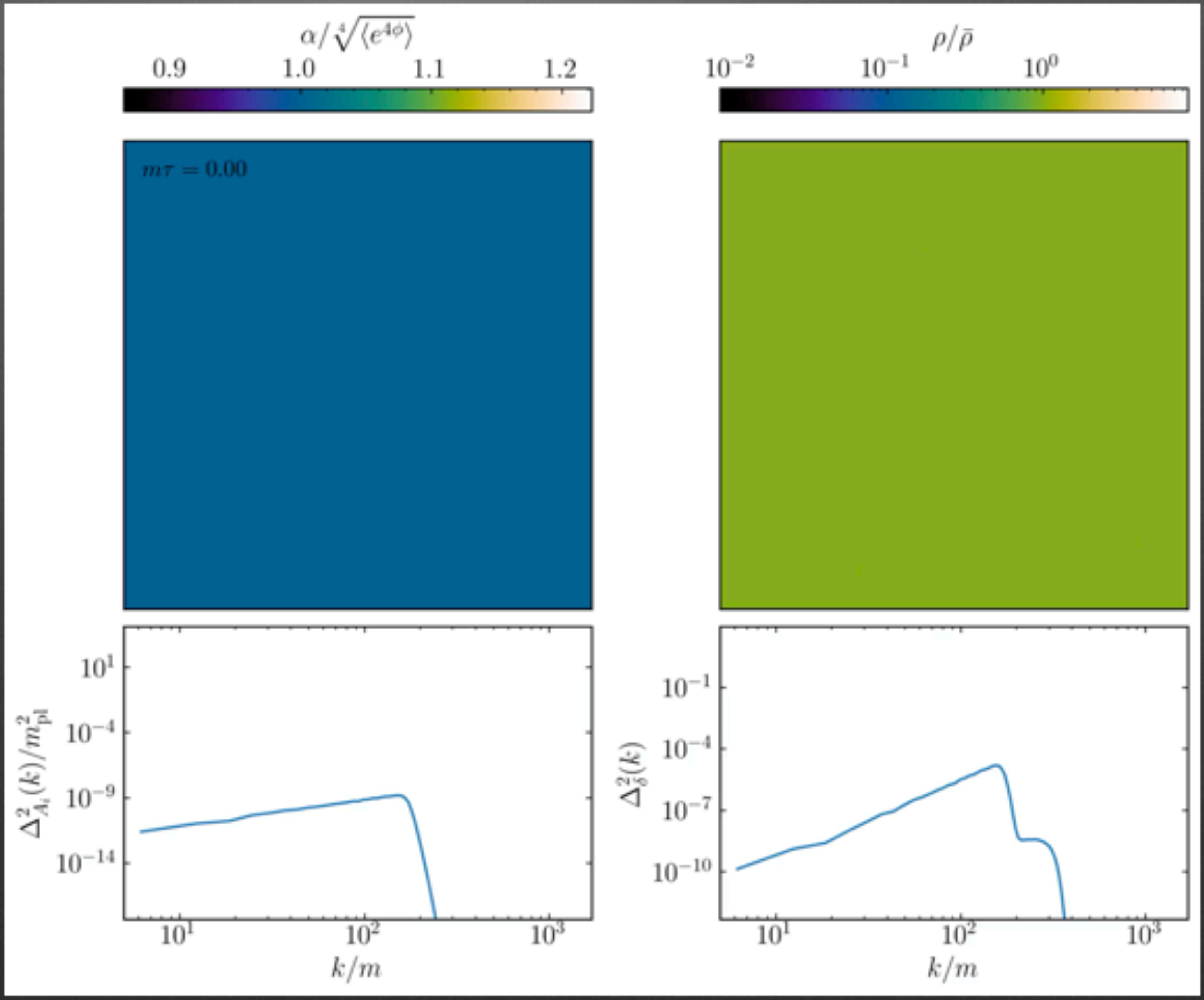
For *exciting* couplings

$$\frac{\alpha_g}{f} = 65\, m_{\rm pl}^{-1}$$

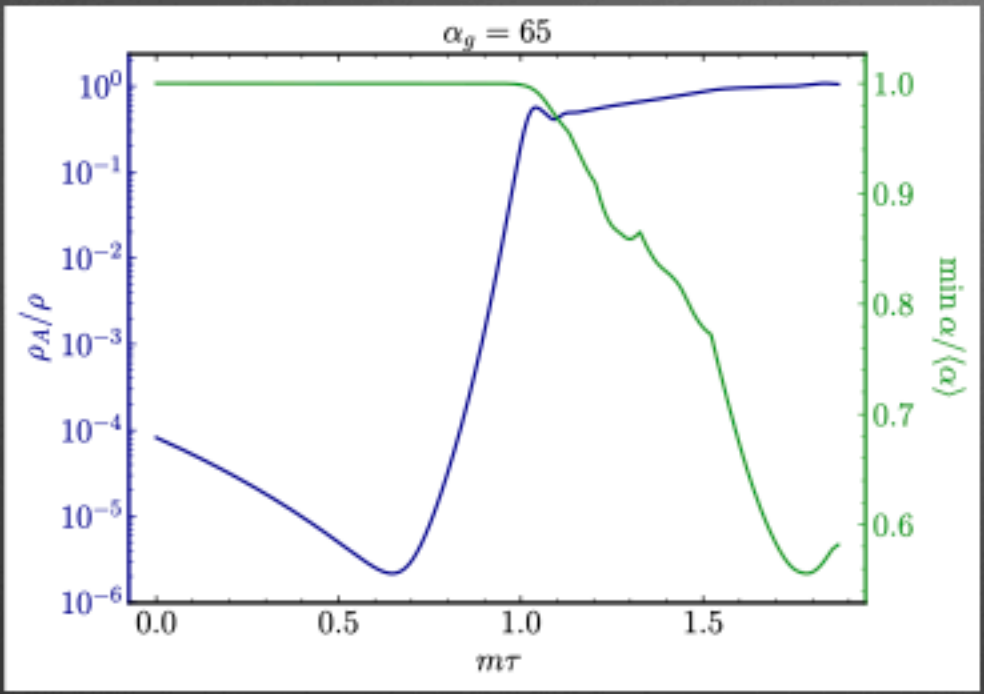


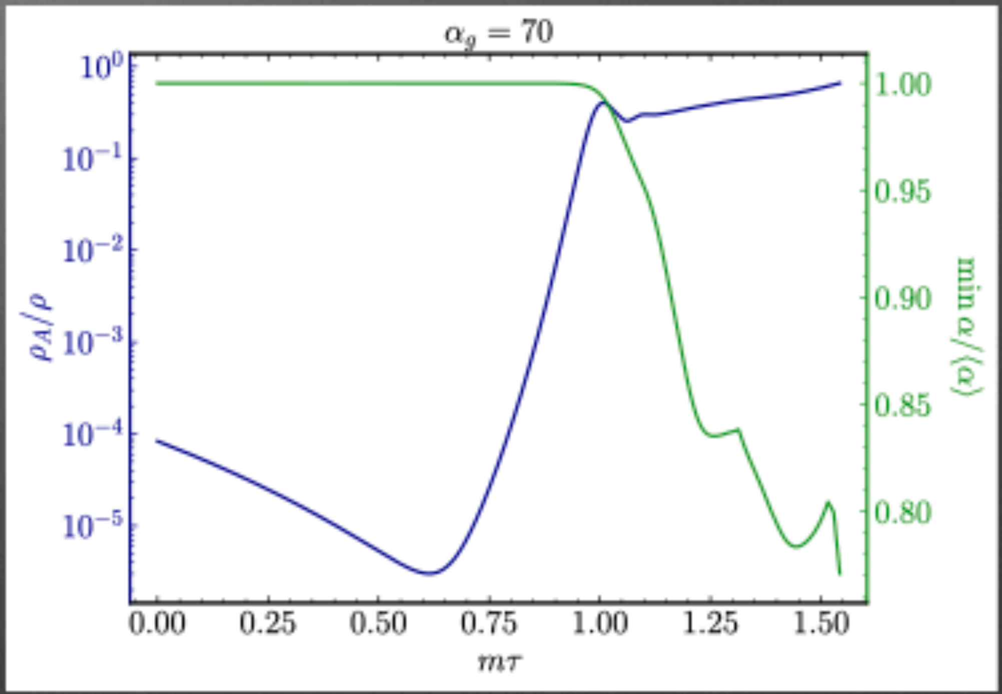
For *exciting* couplings

$$\frac{\alpha_g}{f} = 70\, m_{\rm pl}^{-1}$$



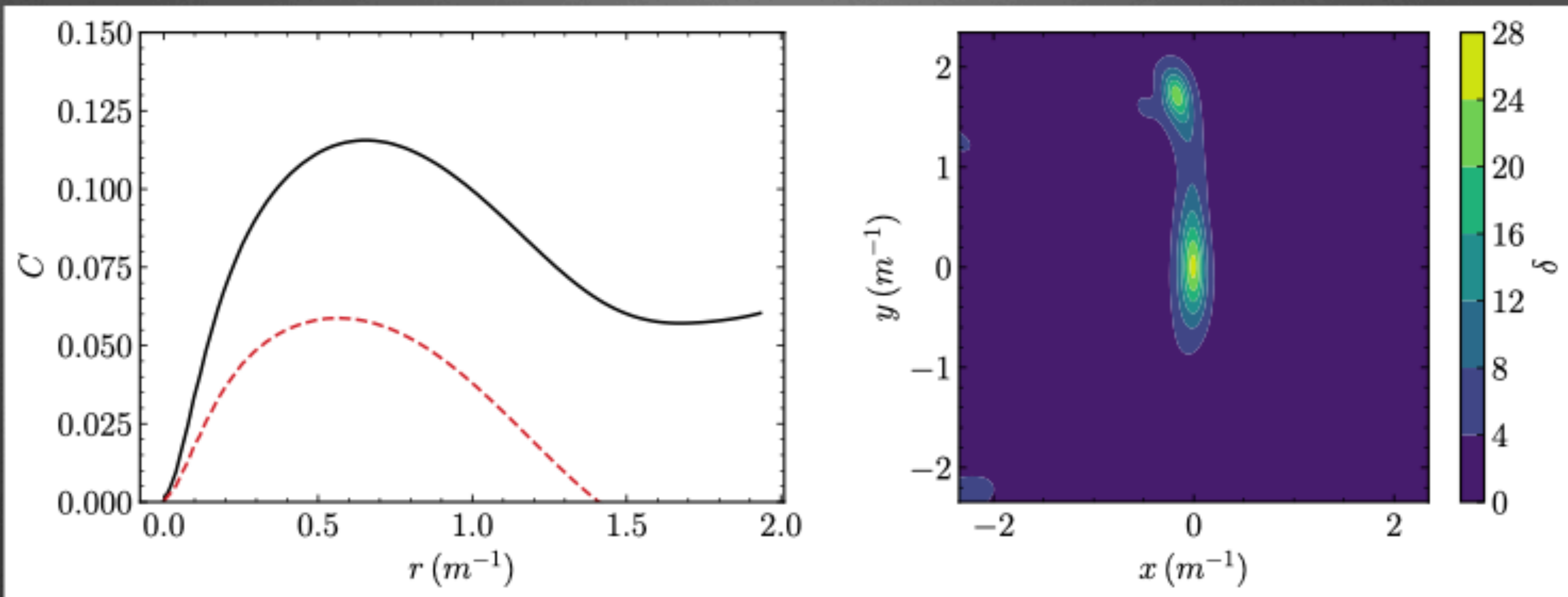
But … there are no PBH





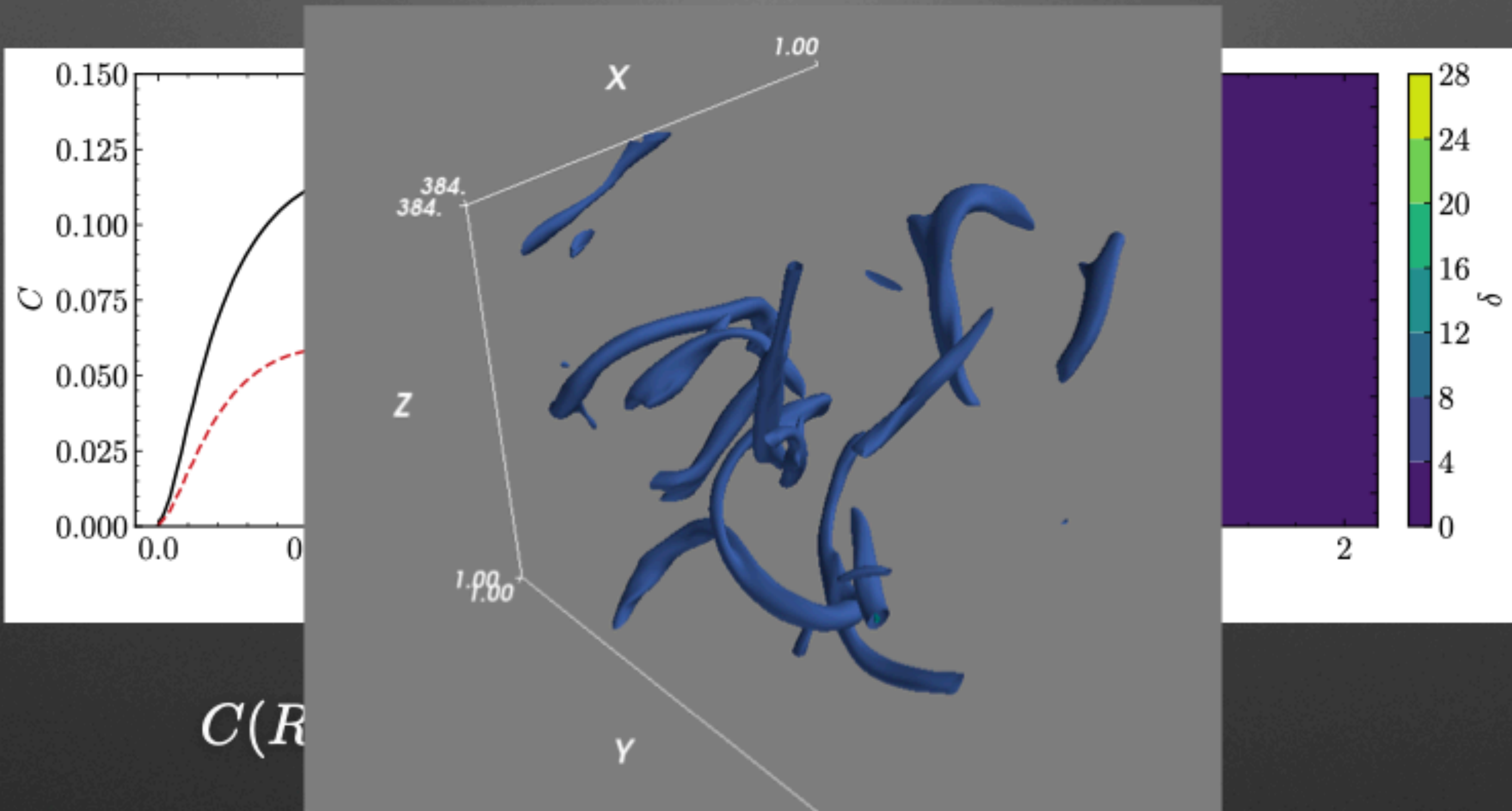
The minimum value of the lapse throughout the grid doesn't approach zero

What does it look like?



$$C(R) = \frac{G\delta M}{R}$$

What does it look like?

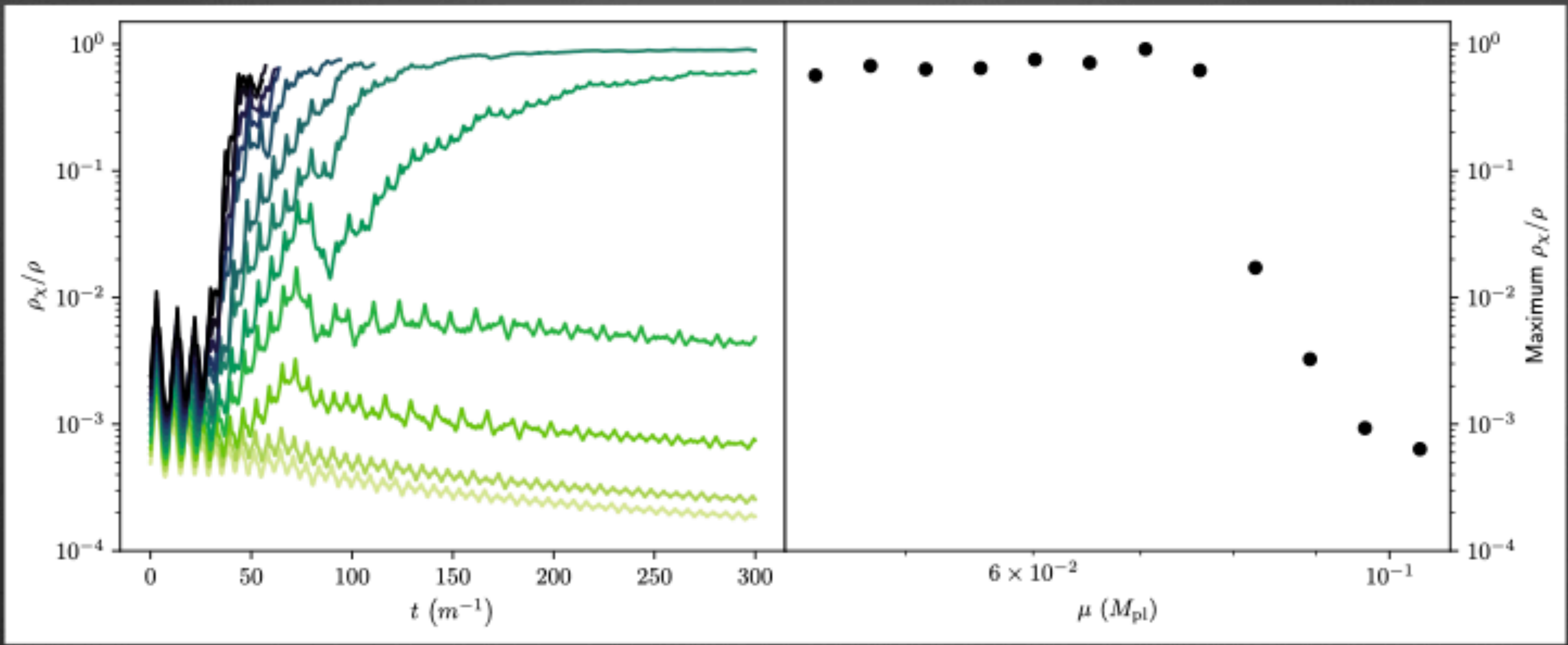


$C(R$

Next Steps

Alpha-attractors are a possibility (a la 2311.17237):

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{e^{2\phi/\mu}}{2}(\partial\chi)^2 - \frac{m^2\mu^2}{2}\left(1 - e^{-\phi/\mu}\right)^2$$



Next Steps

- What are *they* saying?

Primordial black hole formation with full numerical relativity

JCAP03(2022)

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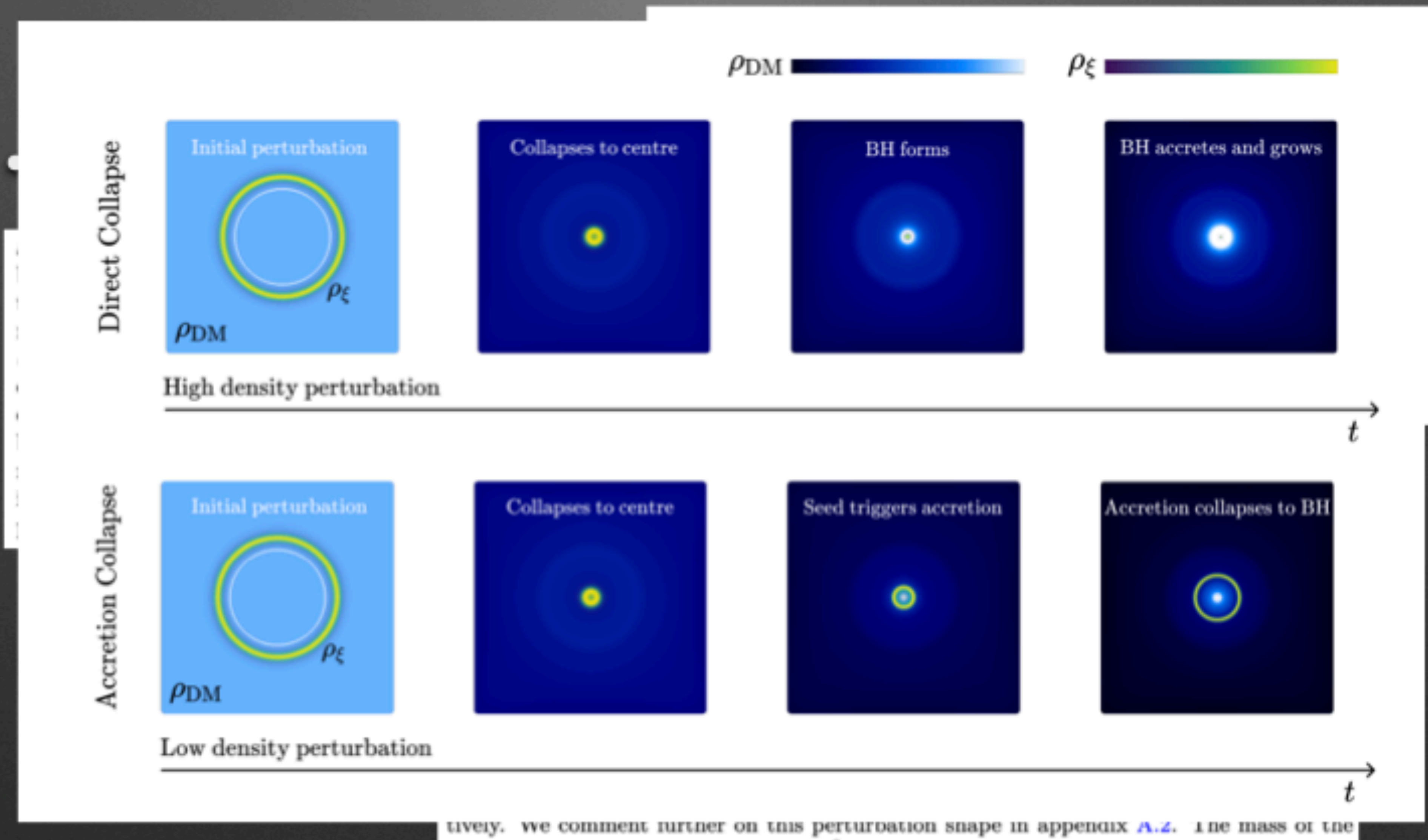
Abstract. We study the formation of black holes from subhorizon and superhorizon perturbations in a matter dominated universe with 3+1D numerical relativity simulations. We find that there are two primary mechanisms of formation depending on the initial perturbation's mass and geometry — via *direct collapse* of the initial overdensity and via *post-collapse accretion* of the ambient dark matter. In particular, for the latter case, the initial perturbation does not have to satisfy the hoop conjecture for a black hole to form. In both cases, the duration of the formation the process is around a Hubble time, and the initial mass of the black hole is $M_{\rm BH} \sim 10^{-2} H^{-1} M_{\rm Pl}^2$. Post formation, we find that the PBH undergoes rapid mass growth beyond the self-similar limit $M_{\rm BH} \propto H^{-1}$, at least initially. We argue that this implies that most of the final mass of the PBH is accreted from its ambient surroundings post formation.

Meanwhile, the massless scalar field ξ provides the energy density perturbation that will trigger BH formation. In this paper, we exclusively consider initially static spherically symmetric perturbations and we leave the generalisation to fewer degrees of symmetry for future work. We choose the initial configuration of ξ to be space dependent as

$$\xi(t=0,r) = \Delta\xi \tanh\left[\frac{r-R_0}{\sigma}\right], \tag{1.9}$$

where $\Delta\xi$, σ and R_0 are the amplitude, width and the initial size of the perturbation respectively. We comment further on this perturbation shape in appendix A.2. The mass of the

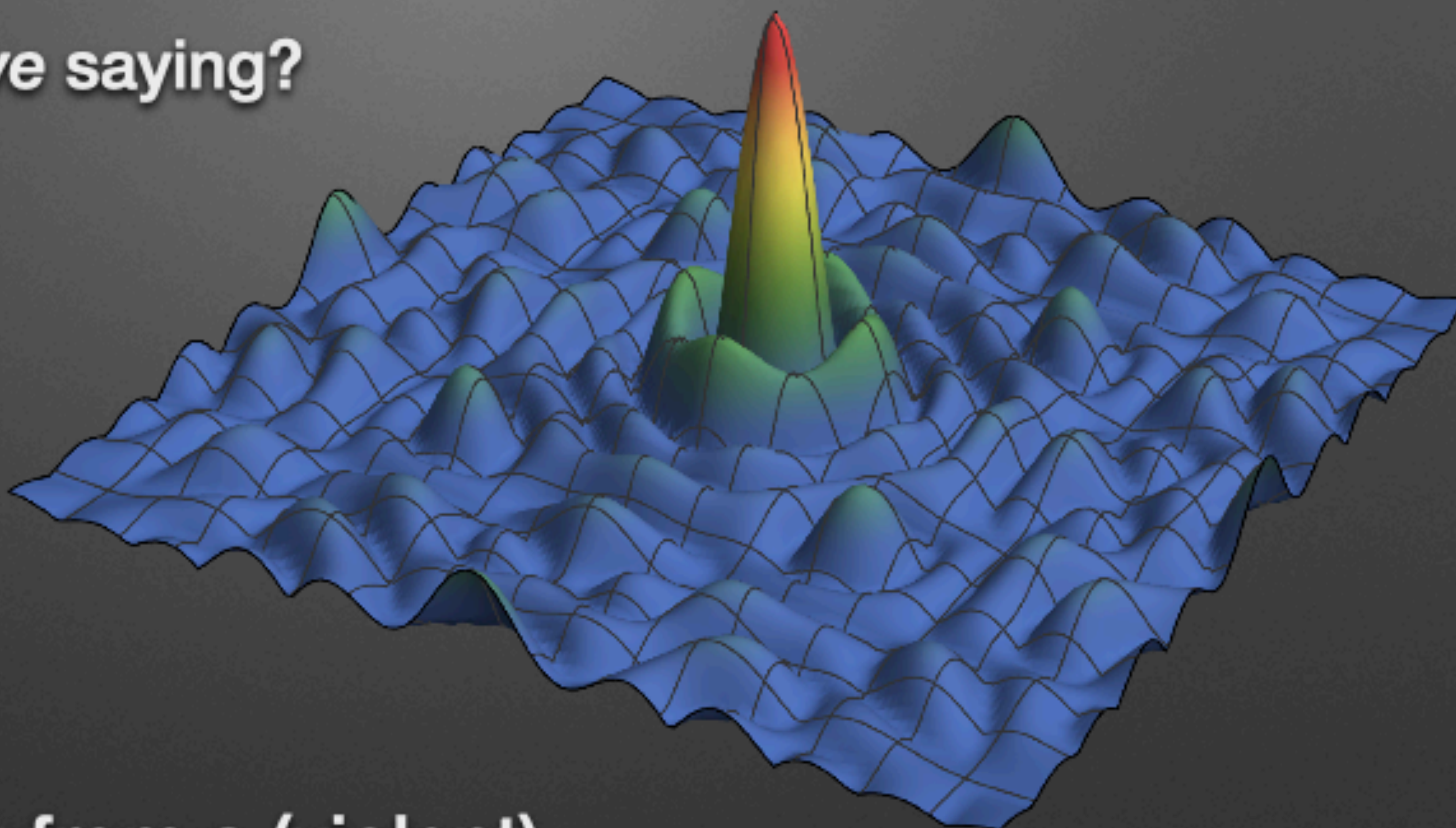
Next Steps



ρ_{DM}
ρ_ξ
Direct Collapse
Initial perturbation
Collapses to centre
BH forms
BH accretes and grows
ρ_ξ
ρ_{DM}
High density perturbation
t
Accretion Collapse
Initial perturbation
Collapses to centre
Seed triggers accretion
Accretion collapses to BH
ρ_ξ
ρ_{DM}
Low density perturbation
t
tively. We comment further on this perturbation shape in appendix A.2. The mass of the

Next Steps

- What are *we* saying?



Starting from a (violent) cosmological process does not (yet) seem to produce black holes

Next Steps

- Let's forget about *physical mechanisms* and focus on when *cosmological messiness* ruins everything!

$$T^{\mu\nu} = \left(\rho_0 + \frac{p}{c^2}\right) U^\mu U^\nu + p g^{\mu\nu}$$

$$E_* \equiv \sqrt{\gamma}\frac{\rho_0}{\alpha}\left[\frac{4}{3}W^2 - \frac{1}{3}\right] \qquad P_*^j \equiv \sqrt{\gamma}\frac{4}{3}W^2\rho_0 v^j$$

My Thoughts

- Gravity is important when studying black holes
  - Gravity is nonlinear
- We need to *do* the problem before we write down the answer
- Black hole formation does not occur when perturbation theory breaks down