Decoherence of cosmological perturbations: boundary terms and the WKB wave functional

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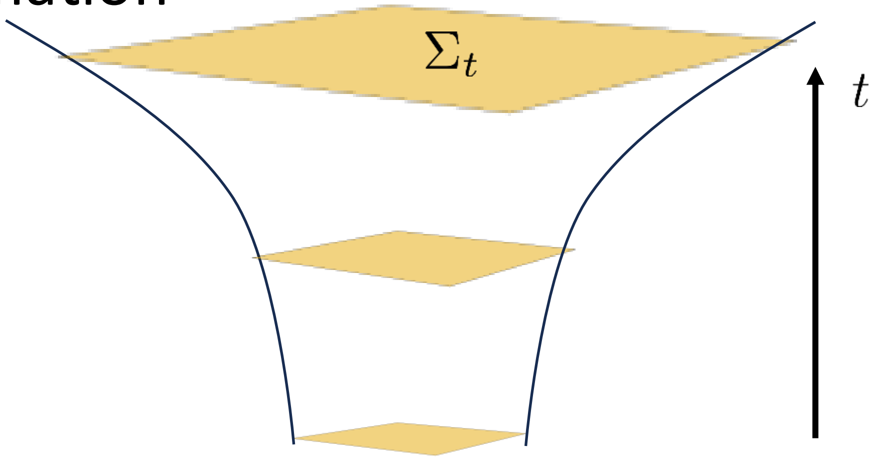
Based on: JHEP 06 (2023) 101 (arXiv:2305.08071) with **Sirui Ning** & **Yi Wang**

JHEP 04 (2023) 092 (arXiv:2207.04435) with **Duc Huy Tran** & **Yi Wang**

Cosmological perturbations from quantum fluctuation

Spatial metric of 3d hypersurface at time *t* during inflation



$$h_{ij}(\mathbf{x},t) = a(t)^2 e^{2\zeta(\mathbf{x},t)} \left(e^{\gamma(\mathbf{x},t)}\right)_{ij}$$

Maldacena's convention

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

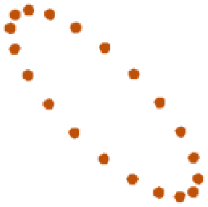
- **Scalar curvature perturbation** ζ is caused by inflaton's **quantum fluctuation**

$$\delta\phi = \phi(\mathbf{x},t) - \phi(t)$$

$^{(3)}R$

Perturb 3d Ricci scalar

- **Tensor perturbation** γ_{ij} is related to primordial gravitational wave
  - **Primordial gravitons** as the quantum fluctuation



- They are conserved outside the Hubble horizon

The two-mode squeezed state of cosmological perturbations

With canonical quantization of cosmological perturbations

$$\zeta_{\mathbf{k}} = u_k a_{\mathbf{k}} + u_k^* a^\dagger_{-\mathbf{k}} \ , \ \gamma_{ij}(\mathbf{k}) = \sum_{s=\pm} (v_k b^s_{\mathbf{k}} + v_k^* b^{s\dagger}_{-\mathbf{k}}) e^s_{ij}(\mathbf{k})$$

The vacuum becomes squeezed during inflation, e.g. for scalar

- (Grishchuk & Sidorov, PRD 42, 3413, 1990) (Albrecht, Ferreira, Joyce & Prokopec, astro-ph/9303001)

$$\mathcal{H}_{\mathbf{k}} = \underbrace{\frac{k}{2}(a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + a^\dagger_{-\mathbf{k}} a_{-\mathbf{k}} + 1)}_{\text{Harmonic oscillator}} - \frac{i}{2}\frac{(a\sqrt{\epsilon})'}{a\sqrt{\epsilon}} \underbrace{(a_{\mathbf{k}} a_{-\mathbf{k}} - \text{h.c.})}_{\text{squeezing}}$$

It evolves into a **two-mode squeezed state**, entangling two modes $(\mathbf{k}, -\mathbf{k})$

Squeezing angle

Particle number eigenstate

$$|\Psi_{\mathbf{k},-\mathbf{k}}\rangle = \frac{1}{\cosh r_k} \sum_{n=0}^{\infty} e^{-2in\varphi_k} \tanh^n r_k |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

Squeezing parameter

Some proposals in the literature to probe the quantum nature

The **squeezing parameter** grows as **e-folds after horizon rossing**

$$r_k \sim \log(aH/k) = N_{\text{cross}}$$

For inflation which can **last 60 e-folds**

$$r_k \sim 60$$

in decibel

$$-10\log_{10}\left(e^{-r_k}\right)\,\text{dB} \approx 260\ \text{dB}$$

squeezed states produced in lab (Hosten et al, Nature 529, 7587, 2016):

$$\sim 20\ \text{dB}$$

Cosmological Bell test

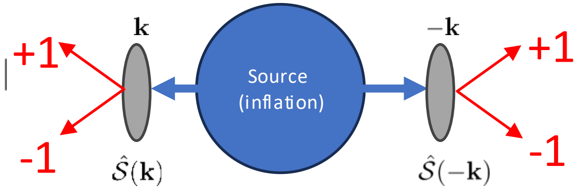
(Martin & Vennin, 1706.05001)

- Utilize the **entanglement** between $\mathbf{k}, -\mathbf{k}$ modes
- Construct **pseudo spins** with the eigenstates of $\hat{q}_{\mathbf{k}} \propto a_{\mathbf{k}} + a^{\dagger}_{\mathbf{k}}$ and **the Bell inequality**

$$\hat{S}_1(\mathbf{k}) = \int_0^{+\infty} dq_{\mathbf{k}}\left(|q_{\mathbf{k}}\rangle\langle q_{\mathbf{k}}| - |-q_{\mathbf{k}}\rangle\langle -q_{\mathbf{k}}|\right)$$

$$\hat{S}_2(\mathbf{k}) = -\int_0^{+\infty} dq_{\mathbf{k}}\left(|q_{\mathbf{k}}\rangle\langle -q_{\mathbf{k}}| - |-q_{\mathbf{k}}\rangle\langle q_{\mathbf{k}}|\right)$$

$$\hat{S}_3(\mathbf{k}) = -\int_{-\infty}^{+\infty} dq_{\mathbf{k}}|q_{\mathbf{k}}\rangle\langle -q_{\mathbf{k}}|$$



Quantum noise by gravitons

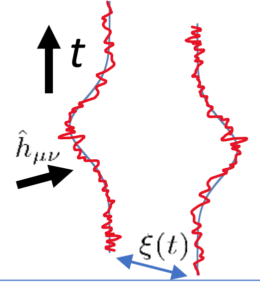
(Parikh, Wilczek & Zahariade, 2010.08208)

- gravitons can produce a **noise term** to the detector's EOM

$$\ddot{\xi}(t) - \frac{1}{2}\left[\underbrace{\ddot{h}(t) - \frac{m_0 G}{c^5}\frac{d^5\xi^2(t)}{dt^5}}_{\text{Classical GW effect}} + \underset{\substack{\uparrow \\ \text{Quantum noise}}}{\ddot{N}(t)}\right]\xi(t) = 0$$

- Power spectrum of noise **enhanced exponentially with squeezing**

$$S_{\text{squeezed}} = 4G\omega\sqrt{\cosh(2r_k)}$$



Motivation of studying cosmic decoherence during inflation

Not only **explaining** the quantum-to-classical transition, but also **constraining** the probe of quantum origin

Quantumness vs Decoherence:
Quantitative results are important

Outline

Boundary term

Non-Gaussian phase

Decoherence for ζ and γ_{ij}

Boundary terms (total time derivative, independent to $\dot{\zeta}$,$\dot{\gamma}_{ij}$), usually neglected in the non-Gaussianity literature, e.g.

$$\mathcal{L}_{\mathrm{bd},\zeta} = M_p^2 \frac{d}{dt}(-2Ha^3 e^{3\zeta}) \qquad \mathcal{L}_{\mathrm{bd},\zeta-\gamma} = M_p^2 \frac{d}{dt}\left[-\frac{a\partial_i\zeta\partial_j\zeta\gamma_{ij}}{H} - \frac{a\zeta\left(\partial_l\gamma_{ij}\right)^2}{8H}\right]$$

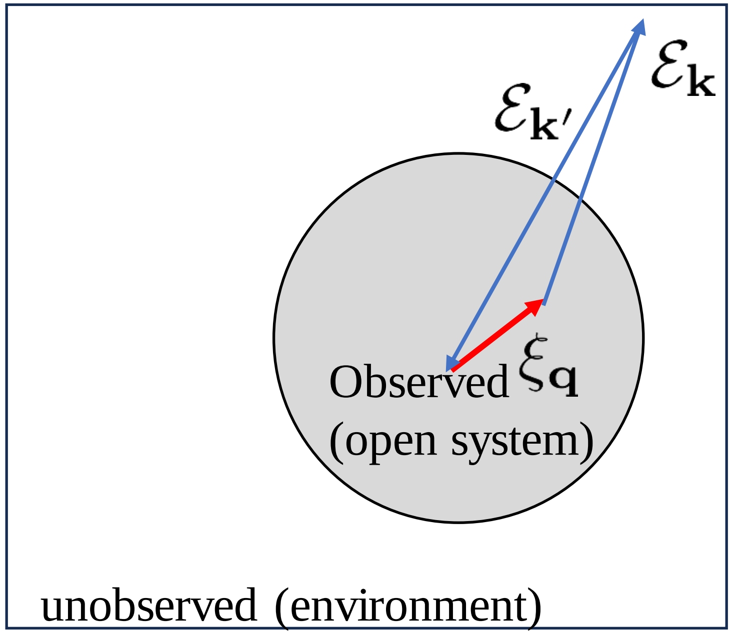
- From the standard integration by parts (IBP), the boundary terms cause **slow-roll unsuppressed** NG phase to the wave functional $\Psi(\zeta, \gamma_{ij})$
- **Independent to the IBP**, seen from the WKB approximation of the Wheeler-DeWitt equation

Lead to **great improvement** of estimating the cosmic decoherence

Subtleties of expanding the single-field action perturbatively

Decoherence starts from cubic

Interaction makes **observable modes** coupling with **unobserved modes**



e.g. the observable range for Cosmic Microwave Background (Planck, 2018)

$$10^{-4} < q < 10^{-1}\mathrm{Mpc}^{-1}$$

System-environment coupling **starts from cubic interaction**

$$\int d^3x \zeta(\mathbf{x}, t)^2 = \int \frac{d^3p}{(2\pi)^3} \zeta_{\mathbf{p}} \zeta_{-\mathbf{p}}$$

quadratic term only couples modes with opposite directions

$$\int d^3x \zeta(\mathbf{x}, t)^3 = \int_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3} \zeta_{\mathbf{p}_1} \zeta_{\mathbf{p}_2} \zeta_{\mathbf{p}_3}$$

$$\int \frac{d^3p_1}{(2\pi)^3} \cdots \frac{d^3p_n}{(2\pi)^3} (2\pi)^3 \delta^3 \left(\sum \mathbf{p}_i \right)$$

Deriving the cubic interactions from the gravitational action is **complicated**

Follow the famous paper of non-Gaussianity (Maldacena, astro-ph/0210603)

- Consider the simplest single-field inflation

$$S = \int d^4x \mathcal{L} = \int d^4x \sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_{\mathrm{GHY}} \qquad \textbf{(1)}$$

Gibbons-Hawking-York boundary term
Adding it = Ignoring covariance derivative (as usual) in
$R = {}^{(3)}R - K^2 + K^\mu_\nu K^\nu_\mu - 2\nabla_\mu\left(-Kn^\mu + n^\nu\nabla_\nu n^\mu\right)$

- The correct non-Gaussian correlators like $\langle\zeta\zeta\zeta\rangle$ **is NOT simply from expanding (1)**
- Need **integration by parts** and **rearrange with EOM** to select bulk interactions with the **correct slow-roll order**:

$$\mathcal{L}^{(3)} = \underbrace{\mathcal{L}_{\zeta\zeta\zeta} + \mathcal{L}_{\zeta\zeta\gamma} + \mathcal{L}_{\zeta\gamma\gamma} + \mathcal{L}_{\gamma\gamma\gamma}}_{\text{Bulk interaction}} + \underbrace{f(\zeta,\gamma)\frac{\delta L_2}{\delta\zeta} + f_{ij}(\zeta,\gamma)\frac{\delta L_2}{\delta\gamma_{ij}}}_{\text{Equation of motion (EOM)}} + \underbrace{\mathcal{L}_{\mathrm{bd},\zeta\zeta\zeta} + \mathcal{L}_{\mathrm{bd},\zeta\zeta\gamma} + \mathcal{L}_{\mathrm{bd},\zeta\gamma\gamma}}_{\text{Boundary interaction}}$$

Deriving the cubic interactions from the gravitational action is <u>**complicated**</u>

Done with the Mathematica package *MathGR* (Ning, **Sou** & Wang, 2305.08071)

- Bulk terms

$$\mathcal{L}_{\zeta\zeta\zeta} = M_p^2\left[a^3\epsilon(\epsilon-\eta)\zeta\dot{\zeta}^2 + a\epsilon(\epsilon+\eta)\zeta(\partial_i\zeta)^2 + \left(\frac{\epsilon}{2}-2\right)\frac{\partial^2\chi}{a}\partial_i\chi\partial_i\zeta + \frac{\epsilon}{4a}\partial^2\zeta\left(\partial_i\chi\right)^2\right]$$

$$\mathcal{L}_{\zeta\zeta\gamma} = M_p^2\left[-\frac{1}{2}a\epsilon\chi\partial_i\partial_j\zeta\dot{\gamma}_{ij} + \frac{\partial_i\chi\partial_j\chi\partial^2\gamma_{ij}}{4a} + a\epsilon\partial_i\zeta\partial_j\zeta\gamma_{ij}\right]$$

slow-roll suppressed

$$\mathcal{L}_{\zeta\gamma\gamma} = M_p^2\left[\frac{1}{8}a^3\epsilon\zeta{\dot{\gamma}_{ij}}^2 - \frac{1}{4}a\partial_l\chi\dot{\gamma}_{ij}\partial_l\gamma_{ij} + \frac{1}{8}a\epsilon\zeta\left(\partial_l\gamma_{ij}\right)^2\right]$$

$$\mathcal{L}_{\gamma\gamma\gamma} = M_p^2\left[\frac{1}{4}a\partial_m\gamma_{il}\partial_l\gamma_{jm}\gamma_{ij} + \frac{1}{8}a\partial_i\gamma_{lm}\partial_j\gamma_{lm}\gamma_{ij}\right],$$

slow-roll unsuppressed

- EOM terms

$$f(\zeta,\gamma) = -\frac{\dot{\zeta}\zeta}{H} + \frac{1}{4a^2H^2}\left[(\partial_i\zeta)^2 - \partial^{-2}\partial_i\partial_j\left(\partial_i\zeta\partial_j\zeta\right)\right] - \frac{1}{2a^2H}\left[\partial_i\zeta\partial_i\chi - \partial^{-2}\partial_i\partial_j\left(\partial_i\zeta\partial_j\chi\right)\right]$$
$$+\frac{\partial_i\partial_j\zeta\dot{\gamma}_{ij}}{4H}\partial^{-2}$$

$$f_{ij}(\zeta,\gamma) = -\frac{\zeta\dot{\gamma}_{ij}}{H} + \frac{\partial_i\zeta\partial_j\zeta}{a^2H^2} + \frac{2\chi\partial_i\partial_j\zeta}{a^2H}$$

$$\frac{\delta L_2}{\delta\zeta} = 2M_p^2\left[-\frac{d}{dt}\left(\epsilon a^3\dot{\zeta}\right) + \epsilon a\partial^2\zeta\right]$$

$$\frac{\delta L_2}{\delta\gamma_{ij}} = \frac{M_p^2}{4}\left[-\frac{d}{dt}\left(a^3\dot{\gamma}_{ij}\right) + a\partial^2\gamma_{ij}\right],$$

EOM terms are zero at the leading order

- <u>**Boundary terms**</u>

We focus on these

$$\mathcal{L}_{\mathrm{bd},\zeta\zeta\zeta} = M_p^2\frac{d}{dt}\left\{\boxed{-9a^3H\zeta^3 + \frac{a}{H}\left(1-\epsilon\right)\zeta\left(\partial_i\zeta\right)^2 - \frac{1}{4aH^3}\left(\partial_i\zeta\right)^2\partial^2\zeta}\right.$$
$$\left.\boxed{-\frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 - \frac{\zeta}{2aH}\left[(\partial_i\partial_j\chi)^2 - \left(\partial^2\chi\right)^2\right] + \frac{\zeta}{2aH^2}\left(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi\right)}\right\}$$

$$\mathcal{L}_{\mathrm{bd},\zeta\zeta\gamma} = M_p^2\frac{d}{dt}\left(\boxed{-\frac{a\partial_i\zeta\partial_j\zeta\gamma_{ij}}{H}} + \boxed{\frac{a\partial_i\zeta\partial_j\zeta\dot{\gamma}_{ij}}{4H^2} + \frac{a\chi\partial_i\partial_j\zeta\dot{\gamma}_{ij}}{2H}}\right)$$

$$\mathcal{L}_{\mathrm{bd},\zeta\gamma\gamma} = M_p^2\frac{d}{dt}\left[\boxed{-\frac{a\zeta\left(\partial_l\gamma_{ij}\right)^2}{8H}} - \boxed{\frac{a^3\zeta{\dot{\gamma}_{ij}}^2}{8H}}\right].$$

$$\chi = a^2\epsilon\partial^{-2}\dot{\zeta}$$

Depends on $\dot{\zeta}$ or $\dot{\gamma}_{ij}$

<u>**red boxes**</u> equivalent to non-linear field redefinitions (Burrage, Ribeiro & Serry, 1103.4126) (Arroja & Tanaka, 1103.1102)

Slow-roll order estimation of cubic interaction terms

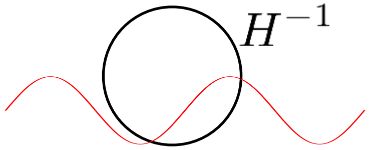
Bulk/Boundary	Type	Leading interaction of each type	Slow-roll order
Bulk	$\zeta\zeta\zeta$	$\epsilon(\epsilon+\eta)a(\partial_i\zeta)^2\zeta$	$\epsilon(\epsilon+\eta)\zeta^3$
Bulk	$\zeta\zeta\gamma$	$\epsilon a\partial_i\zeta\partial_j\zeta\gamma_{ij}$	$\epsilon^{\frac{3}{2}}\zeta^3$
Bulk	$\zeta\gamma\gamma$	$\epsilon a\zeta\partial_l\gamma_{ij}\partial_l\gamma_{ij}$	$\epsilon^2\zeta^3$
Bulk	$\gamma\gamma\gamma$	$a\partial_i\gamma_{lm}\partial_j\gamma_{lm}\gamma_{ij}$	$\epsilon^{\frac{3}{2}}\zeta^3$
Boundary	$\zeta\zeta\zeta$	$\partial_t\left(a^3\zeta^3\right)$	ζ^3
Boundary	$\zeta\zeta\gamma$	$\partial_t\left(a\partial_i\zeta\partial_j\zeta\gamma_{ij}\right)$	$\epsilon^{\frac{1}{2}}\zeta^3$
Boundary	$\zeta\gamma\gamma$	$\partial_t\left(a\zeta\partial_l\gamma_{ij}\partial_l\gamma_{ij}\right)$	$\epsilon\zeta^3$

- The **slow-roll order** is estimated with $\Delta_\gamma^2 \sim \mathcal{O}(\epsilon)\Delta_\zeta^2 \implies \gamma \sim \mathcal{O}\left(\sqrt{\epsilon}\right)\zeta$
- Boundary terms are **less slow-roll suppressed**

Revisit the boundary term of ζ: contribute a phase

The dynamics **is dominated by a boundary term** in long wavelength limit (Maldacena, astro-ph/0210603)

$$\mathcal{L} \to M_p^2 \frac{d}{dt}\left(-2Ha^3 e^{3\zeta}\right)$$



Check with **the in-in formalism**, it **does not** contribute to correlators

$$\langle \zeta^n \rangle = \int D\zeta |\Psi(\zeta)|^2 \zeta^n$$

It **contributes** conjugate momentum $\Pi_\zeta = -i\frac{\delta}{\delta\zeta}$

$$\langle \Pi_\zeta^n \rangle = \int D\zeta \Psi^*(\zeta)\left(-i\frac{\delta}{\delta\zeta}\right)^n \Psi(\zeta)$$

$$\Psi(\zeta) = \sqrt{\mathcal{P}(\zeta)}e^{iS(\zeta)}$$

Pure phase

Probability distribution

the boundary term contributes a pure phase

The non-Gaussian phase of wave functional

Wave functional with the boundary term

Several ways to see that the boundary terms contribute a non-Gaussian phase to the wave functional (**Sou**, Tran & Wang, 2207.04435) (Ning, **Sou** & Wang, 2305.08071)

$$\mathcal{L} = \mathcal{L}_2 - \partial_t \mathcal{K} = f_{aa}(t)\dot{\alpha}^a\dot{\alpha}^a + j_{aa}(t)\alpha^a\alpha^a - \partial_t\left(F_{abc}(t)\alpha^a\alpha^b\alpha^c\right)$$

Include spatial-derivative terms

1. Calculate evolution operator at the cubic order $|\Psi(t)\rangle = U(t,t_i)|\Psi(t_i)\rangle$

$$H_{\rm bd}(\zeta,\gamma,t) = \int \partial_t \mathcal{K}(\zeta,\gamma,t)$$

Spatial integral

$$U(t,t_i) = \exp\left(-i\int \mathcal{K}\right) U_{\rm free}(t,t_i)$$

$$\langle \zeta,\gamma|\Psi(t)\rangle = \exp\left(-i\int \mathcal{K}(\zeta,\gamma,t)\right)\Psi_G(\zeta,\gamma,t)$$

2. Canonical quantization in the Schrödinger picture

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{\alpha}^a} = 2f_{bb}\delta^b_a\dot{\alpha}^b - \left(F_{dbc}+F_{bdc}+F_{bcd}\right)\delta^d_a\alpha^b\alpha^c$$

$$\Psi(\vec{\alpha}) = e^{-i\int F_{abc}\alpha^a\alpha^b\alpha^c}\,\Psi_{\rm free}(\vec{\alpha})$$

3. The WKB limit of the Wheeler-DeWitt equation

Systematic way to find out the slow-roll unsuppressed phase?

So far our analysis is based on integration by parts (IBP) and rearrangement with EOM terms (Maldacena, astro-ph/0210603)

- Question: there are **(infinitely) many ways to do IBP to the action**, how to **ensure the correct phase factor** in the wave functional?

Goal: finding a method independent to integration by parts

The form of wave functional with long wavelength

At the long wavelength limit $a(t) \to +\infty$, the wave functional looks like (Pimentel, 1309.1793):

$$\Psi(h_{ij},\phi) = e^{iW(h_{ij},\phi)} Z(h_{ij},\phi)$$

Real, local, grows as $\mathcal{O}(a^n)$

Non-local, converges at large $a(t)$

- Only $Z(h_{ij},\phi)$ contributes to usual cosmological correlators

$$\langle O(h_{ij})\rangle = \int Dh_{ij}\, |\Psi(h_{ij},\phi)|^2\, O(h_{ij}) = \int Dh_{ij}\, |Z(h_{ij},\phi)|^2\, O(h_{ij})$$

- e.g. the free wave functional of scalar curvature perturbation

$$\Psi(\zeta) \propto \exp\left[-\epsilon\frac{M_p^2}{H^2}\int_{\mathbf{k}} \left(k^3 + ik^2 Ha\right)\zeta_{\mathbf{k}}\zeta_{-\mathbf{k}}\right] \Longrightarrow \langle\zeta_{\mathbf{k}}\zeta_{-\mathbf{k}}\rangle \propto \frac{H^2}{4\epsilon M_p^2 k^3}$$

$\subset Z(h_{ij},\phi)$ $\quad \propto {}^{(3)}R \subset W(h_{ij},\phi)$

The WKB approximation of Wheeler-DeWitt equation

To obtain the phase dominated at long wavelength, apply the WKB approximation to the Wheeler-DeWitt equation

$$\mathcal{H}\left(\phi, h_{ab}, \frac{\delta}{\delta\phi}, \frac{\delta}{\delta h_{ab}}\right)\Psi(h_{ij},\phi) = 0 \qquad \Psi(h_{ij},\phi) \sim e^{i\frac{W(h_{ij},\phi)}{\hbar}}$$

Hamiltonian constraint

- the leading order $\mathcal{O}(\hbar^0)$ is **the solution of the Hamilton-Jacobi equation** (Salopek & Stewart, Class. Quantum Grav., 9 1943, 1992)

$$W(h_{ij},\phi) \approx M_p^2 \int_\Sigma d^3x\sqrt{h}\left(U(\phi) + M(\phi)h^{ij}\partial_i\phi\partial_j\phi + \Phi(\phi){}^{(3)}R\right) + \mathcal{O}\left(a^0\right)$$

Only include ζ

Include terms with γ_{ij}

$$\approx M_p^2 \int_\Sigma d^3x a^3 e^{3\zeta}\left(-2H + \frac{1}{2H}{}^{(3)}R\right) + \mathcal{O}(\epsilon,\eta)$$

$$\supset M_p^2 \int_\Sigma d^3x\left[\underbrace{-9a^3H\zeta^3 + \frac{a\zeta\left(\partial_i\zeta\right)^2}{H} - \frac{a\zeta\left(\partial_l\gamma_{ij}\right)^2}{8H} - \frac{a\partial_i\zeta\partial_j\zeta\gamma_{ij}}{H}}_{\text{Slow-roll unsuppressed boundary terms}} + \underbrace{\frac{a\partial_m\gamma_{il}\partial_l\gamma_{jm}\gamma_{ij}}{4H} + \frac{a\partial_i\gamma_{lm}\partial_j\gamma_{lm}\gamma_{ij}}{8H}}_{\text{Phase from the slow-roll unsuppressed bulk interaction } \mathcal{L}_{\gamma\gamma\gamma}}\right]$$

Independent to integration by parts (not needed)! (Ning, **Sou** & Wang, 2305.08071)

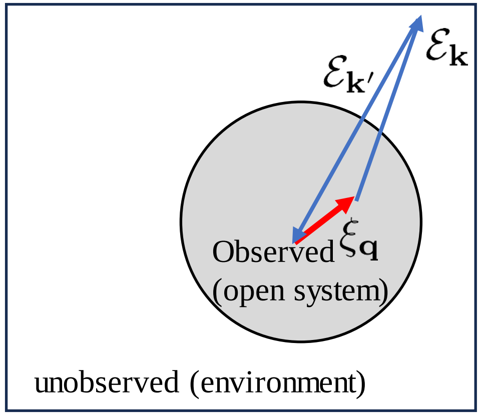
Improve the estimation of cosmic decoherence

Decoherence by tracing out unobserved modes

Through the cubic interaction, wave functional has cubic term (Nelson, 1601.03734)

$$\Psi(\xi,\mathcal{E}) \propto \exp\left(\int_{\mathbf{k},\mathbf{k}',\mathbf{q}} \mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}}\mathcal{E}_{\mathbf{k}}\mathcal{E}_{\mathbf{k}'}\xi_{\mathbf{q}}\right)\Psi_G(\mathcal{E},\xi)$$

← Gaussian part



Property which **turns out to be general**

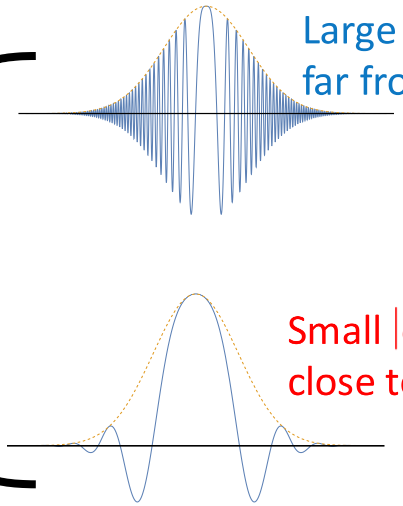
$$\mathrm{Re}\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \to \mathcal{O}(a^0) \quad \mathrm{Im}\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \to \mathcal{O}(a^n)$$

Loss of interference when environment are traced out (taking average)

Reduced density matrix:

$$\rho_R(\xi_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}}) = \langle\xi_{\mathbf{q}}|\mathrm{Tr}_{\mathcal{E}}\left(|\Psi\rangle\langle\Psi|\right)|\tilde{\xi}_{\mathbf{q}}\rangle$$

$$= \langle\Psi(\xi_{\mathbf{q}},\mathcal{E})\Psi^*(\tilde{\xi}_{\mathbf{q}},\mathcal{E})\rangle_{\mathcal{E}} = \int D\mathcal{E}\ \Bigg\{$$



Large $|\xi_{\mathbf{q}} - \tilde{\xi}_{\mathbf{q}}|$
far from diagonal

Small $|\xi_{\mathbf{q}} - \tilde{\xi}_{\mathbf{q}}|$
close to diagonal

$$\propto e^{\underbrace{-(\mathrm{Im}\mathcal{F})^2|\xi_{\mathbf{q}}-\tilde{\xi}_{\mathbf{q}}|^2\times\ldots}}$$

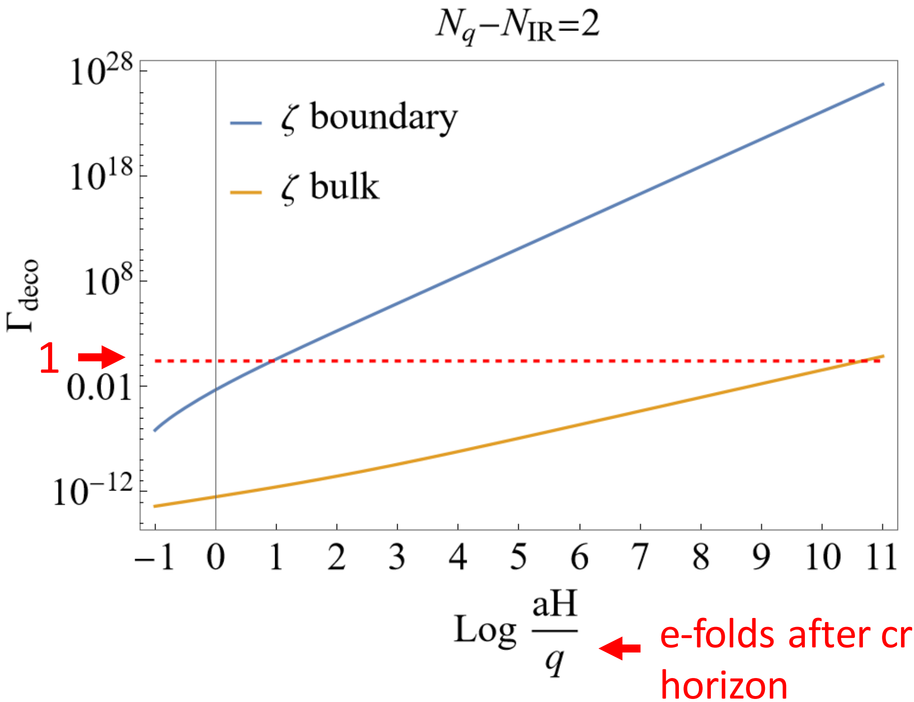
Decoherence factor

$$D(\xi_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}}) \sim e^{-\Gamma_{\mathrm{deco}}}$$

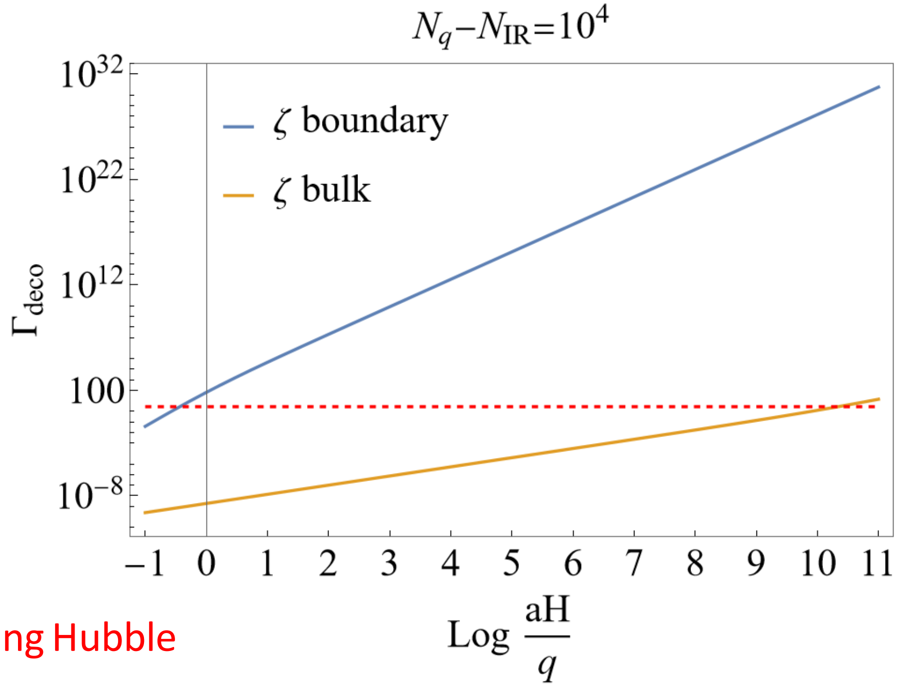
↑
Decoherence exponent

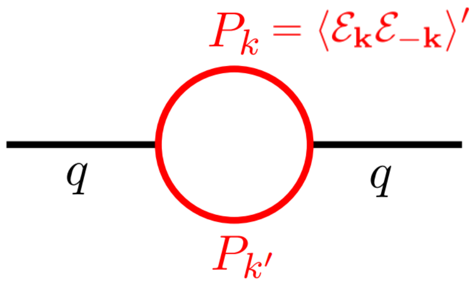
Non-Gaussian phase is important!

Compare the decoherence exponent for scalar curvature perturbation ζ



e-folds after crossing Hubble horizon





Our result with the dominated three-scalar boundary term (**Sou**, Tran & Wang, 2207.04435):

$$\Gamma_{\mathrm{bd},\zeta} \approx \frac{729}{4\epsilon^2} \Delta_\zeta^2 (N_q - N_{\rm IR} - 1) \left(\frac{aH}{q}\right)^6$$

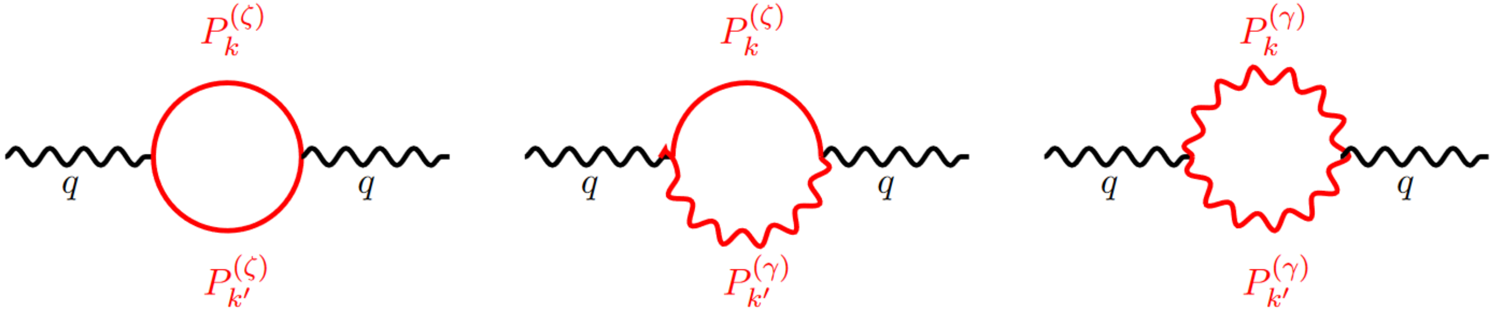
The previous result with bulk term (Nelson, 1601.03734):

$$\Gamma_{\mathrm{bulk},\zeta} \approx \boxed{\left(\frac{\epsilon+\eta}{12}\right)^2} \Delta_\zeta^2 \left(\frac{aH}{q}\right)^3 + \frac{(\epsilon+\eta)^2 \Delta_\zeta^2}{9\pi} \left(\frac{aH}{q}\right)^2 \left[N_q - N_{\rm IR} - \frac{19}{48}\right]$$

Slow-roll suppressed

Decoherence of gravitons γ_{ij}

Solid: scalar curvature perturbation ζ, wavy: primordial graviton γ_{ij}



3 decoherence exponents

(Ning, **Sou** & Wang, 2305.08071)

$$\Gamma^{\rm bd}_{\zeta\zeta\gamma} \approx \frac{\pi\Delta_\zeta^2}{15\epsilon}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bd}_{\zeta\gamma\gamma} \approx \frac{\Delta_\zeta^2}{120}\left[60(N_q - N_{\rm IR}) - 79\right]\left(\frac{aH}{q}\right)^2$$

$$\Gamma^{\rm bulk}_{\gamma\gamma\gamma} \approx \boxed{\epsilon\Delta_\zeta^2}\left[\frac{\pi}{4}\left(\frac{aH}{q}\right)^3 + 2(N_q - N_{\rm IR})\left(\frac{aH}{q}\right)^2\right]$$

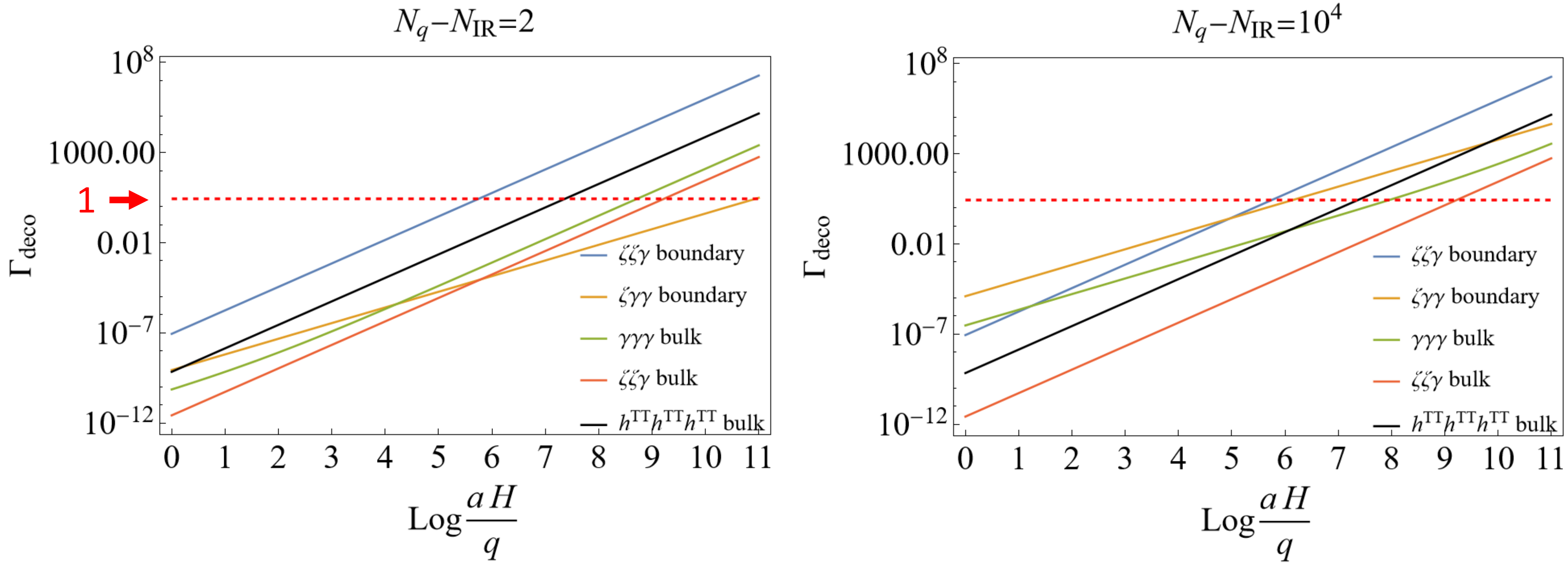
Previous results with bulk interactions (Gong & Seo, 1903.12295) (Burgess et al., 2211.11046)

$$\Gamma^{\rm bulk}_{hhh} \approx \boxed{\frac{2048\epsilon\Delta_\zeta^2}{45}}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bulk}_{\zeta\zeta\gamma} \approx \boxed{\frac{\pi\epsilon\Delta_\zeta^2}{18}}\left(\frac{aH}{q}\right)^3$$

Bulk terms give slow-roll suppressed

Decoherence of primordial gravitons γ_{ij} by different interactions



$$\Gamma^{\rm bd}_{\zeta\zeta\gamma} \approx \frac{\pi\Delta_\zeta^2}{15\epsilon}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bd}_{\zeta\gamma\gamma} \approx \frac{\Delta_\zeta^2}{120}\left[60(N_q - N_{\rm IR}) - 79\right]\left(\frac{aH}{q}\right)^2$$

$$\Gamma^{\rm bulk}_{\gamma\gamma\gamma} \approx \boxed{\epsilon\Delta_\zeta^2}\left[\frac{\pi}{4}\left(\frac{aH}{q}\right)^3 + 2(N_q - N_{\rm IR})\left(\frac{aH}{q}\right)^2\right]$$

$$\Gamma^{\rm bulk}_{hhh} \approx \boxed{\frac{2048\epsilon\Delta_\zeta^2}{45}}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bulk}_{\zeta\zeta\gamma} \approx \boxed{\frac{\pi\epsilon\Delta_\zeta^2}{18}}\left(\frac{aH}{q}\right)^3$$

Bulk terms give slow-roll suppressed

Summary

- **Quantifying** cosmic decoherence is **essential** for probing the quantum nature of cosmological perturbations

- The **boundary terms**, naturally exist in the action of cosmological perturbations, can **contribute faster decoherence effect** by trancing out unobserved modes
  - ➢Improve the decoherence calculations for both scalar curvature perturbation and primordial gravitons

- The non-Gaussian phase can be analyzed systematically with **the WKB approximation of the Wheeler-DeWitt equation, a way independent to IBP**