Gravitational wave background anisotropies as a probe of the early universe

Ema Dimastrogiovanni

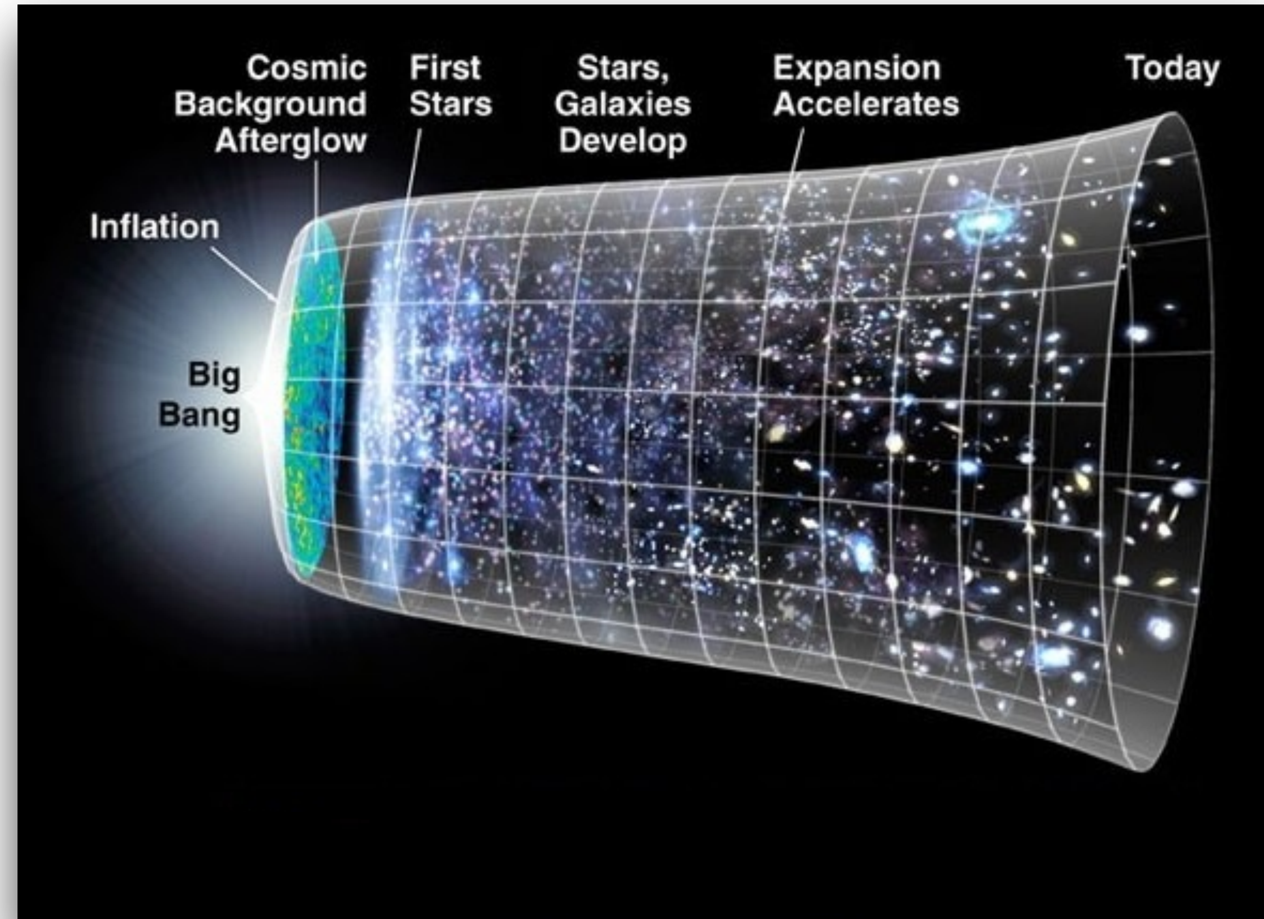
The University of Groningen

Gravity and Cosmology 2024 - YITP - February 2nd 2024

GW background from the early universe

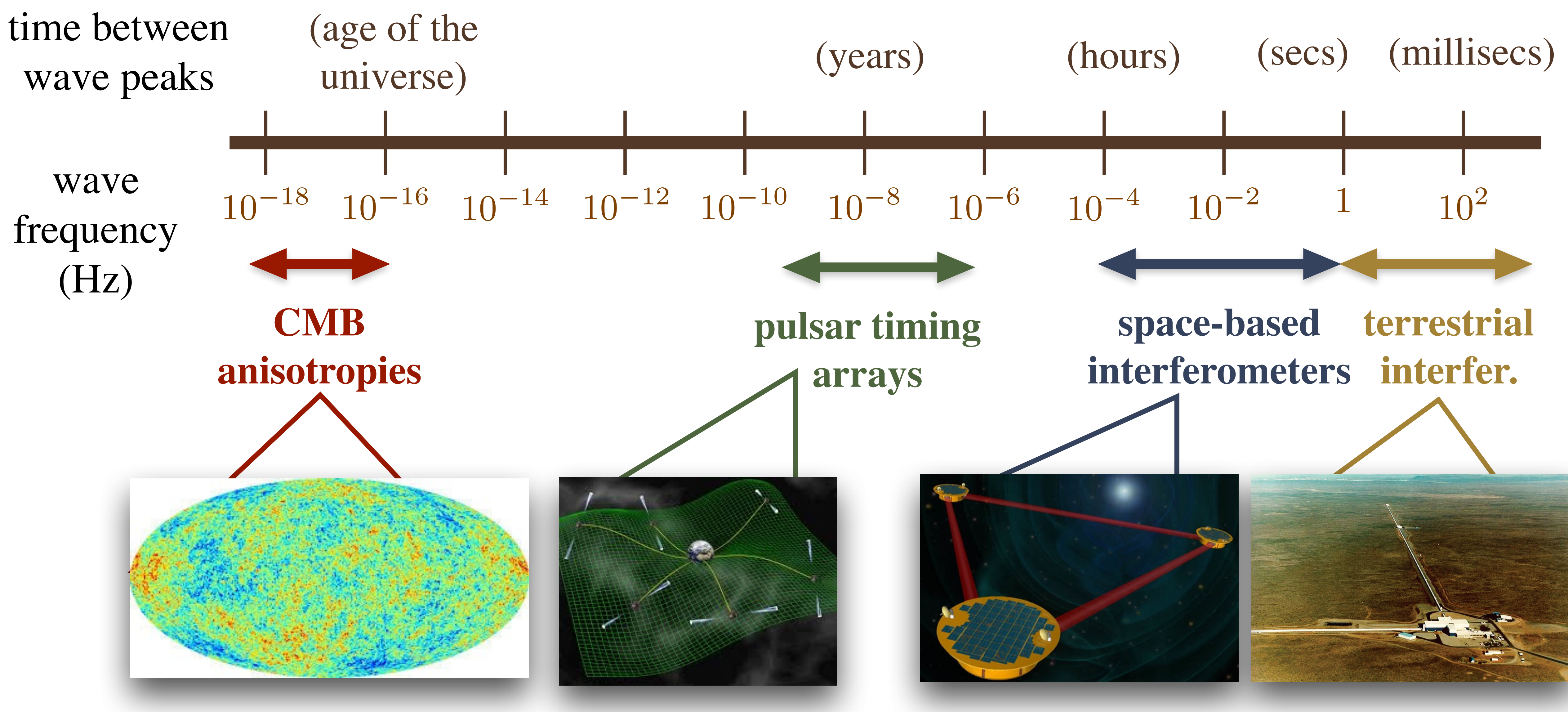
- Inflation
- Preheating
- Phase transitions
- Cosmic strings
- Alternatives to inflation…

Multiple potential GW production mechanisms within inflation alone:

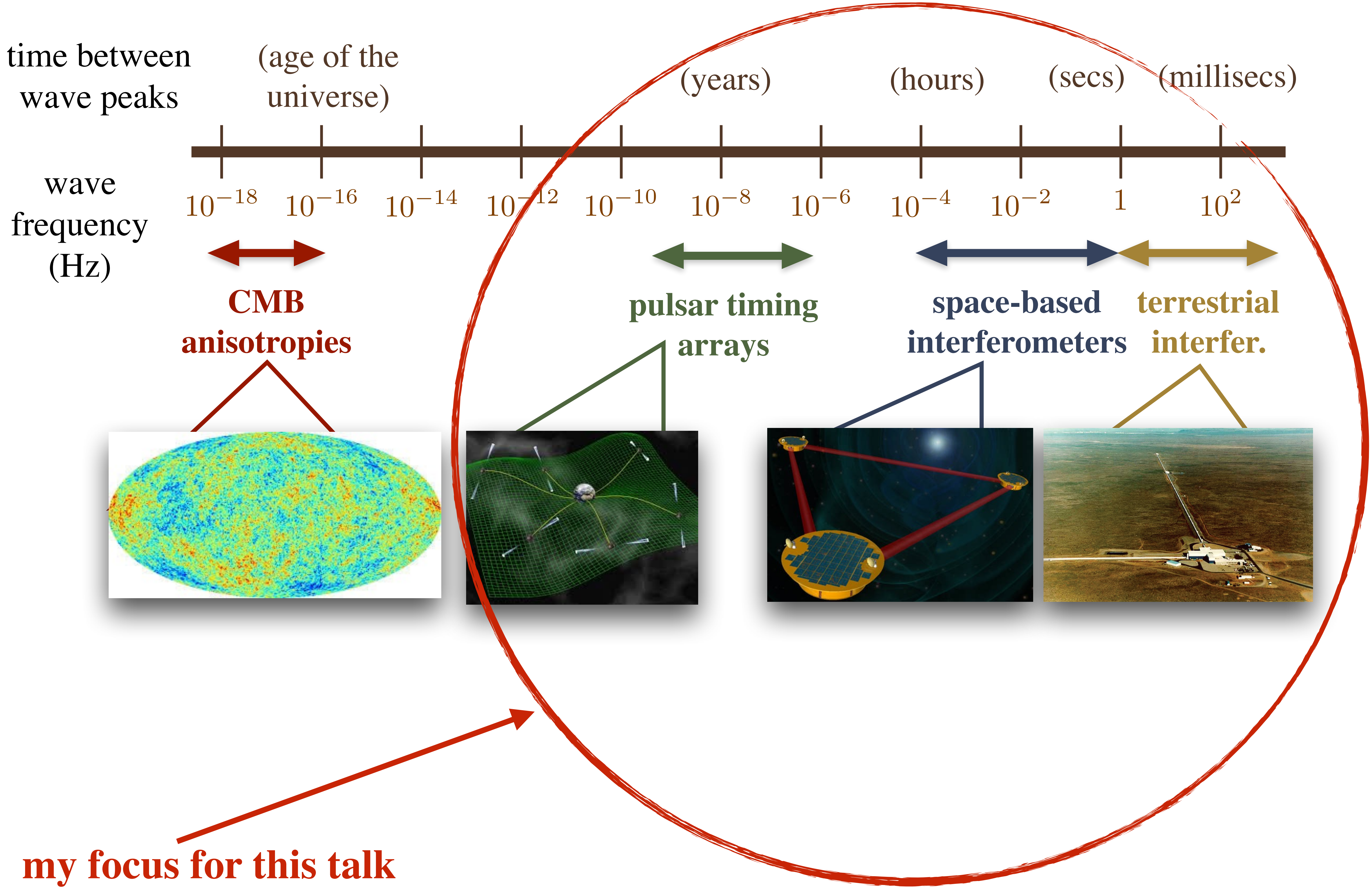


- GW from the amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation
- GW generated at second order from scalar fluctuations

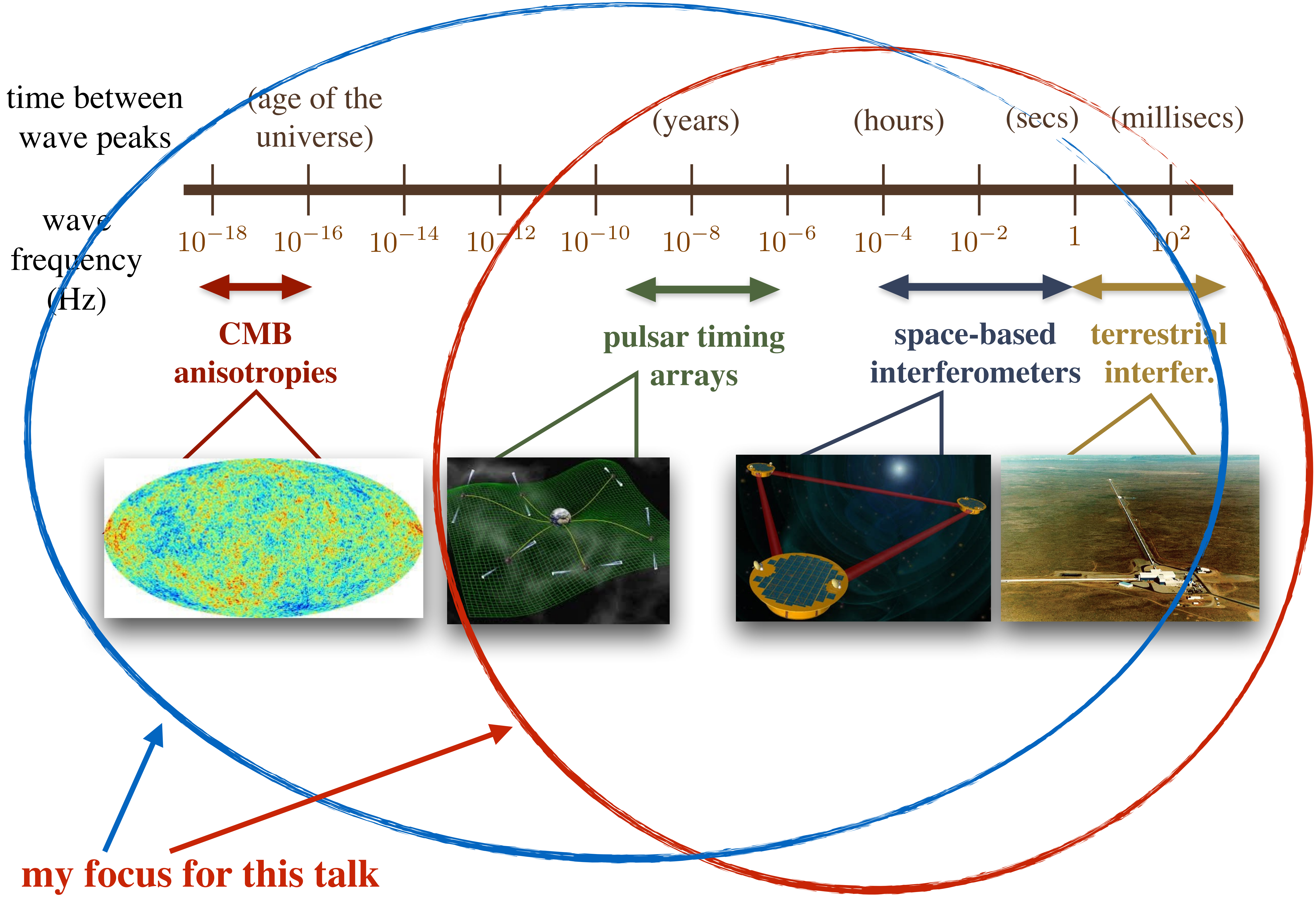
Scales — Experiments



Scales — Experiments

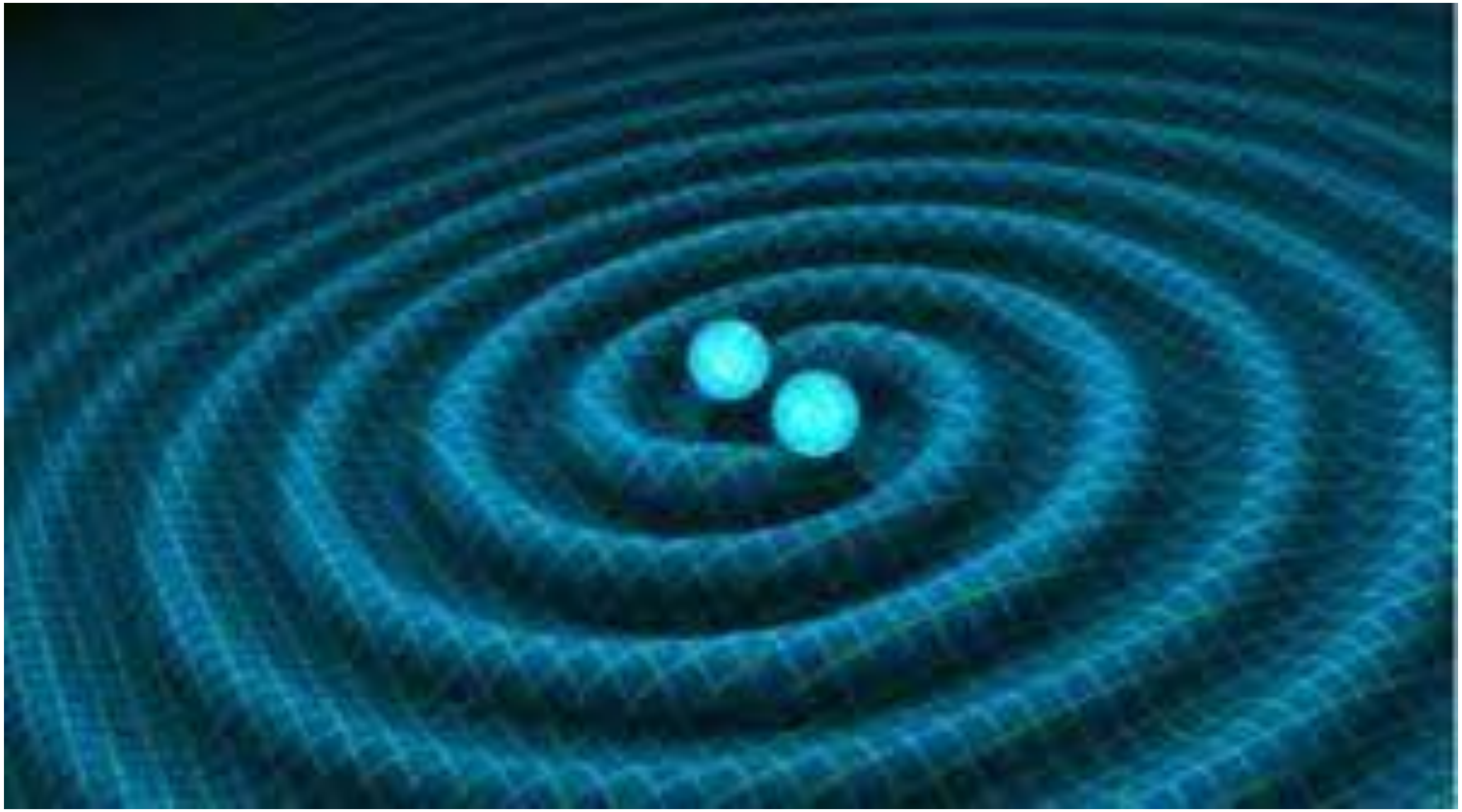


Scales — Experiments



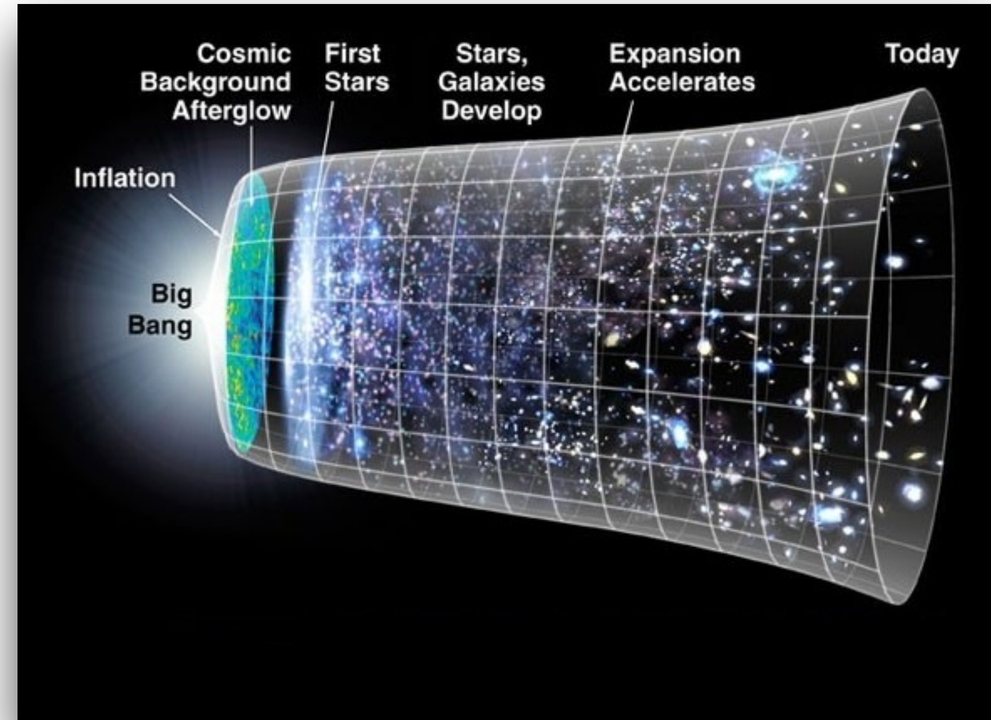
There is more: astrophysical background

expected from the superposition of signals from mergers
(black holes, neutron stars)



A puzzle to unravel …

Already the case for the observed gravitational wave background by PTA!

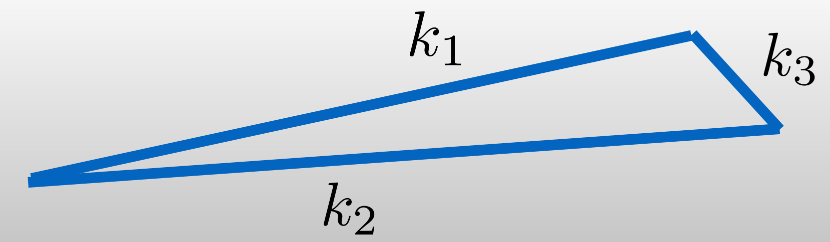


- How does it look like?
- What info does it provide on inflation?
- How do we characterise it (and distinguish it from other GWB)?

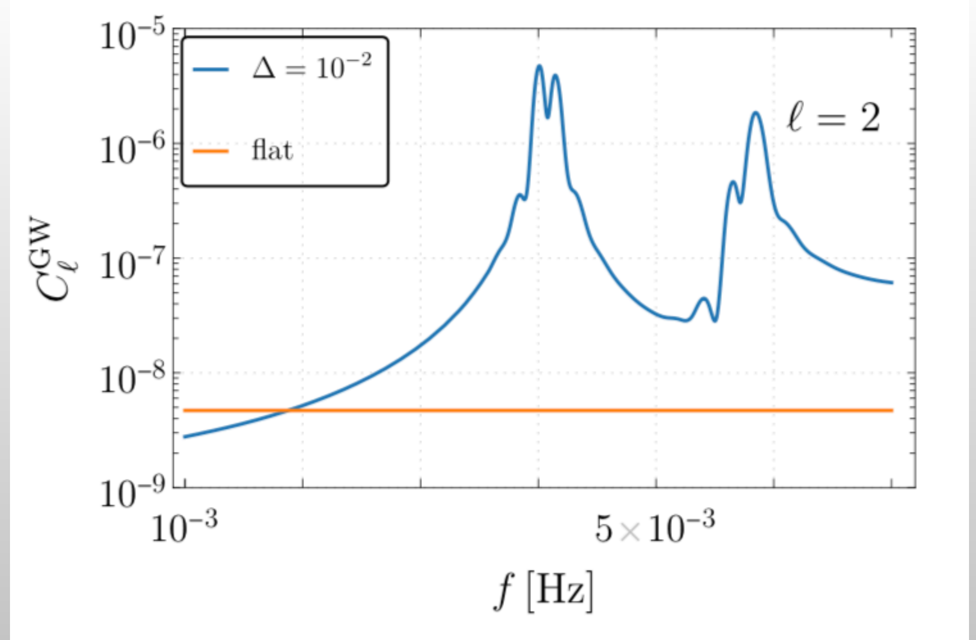
— Frequency profile
— Chirality
— Non-Gaussianity
— Anisotropies

Outline

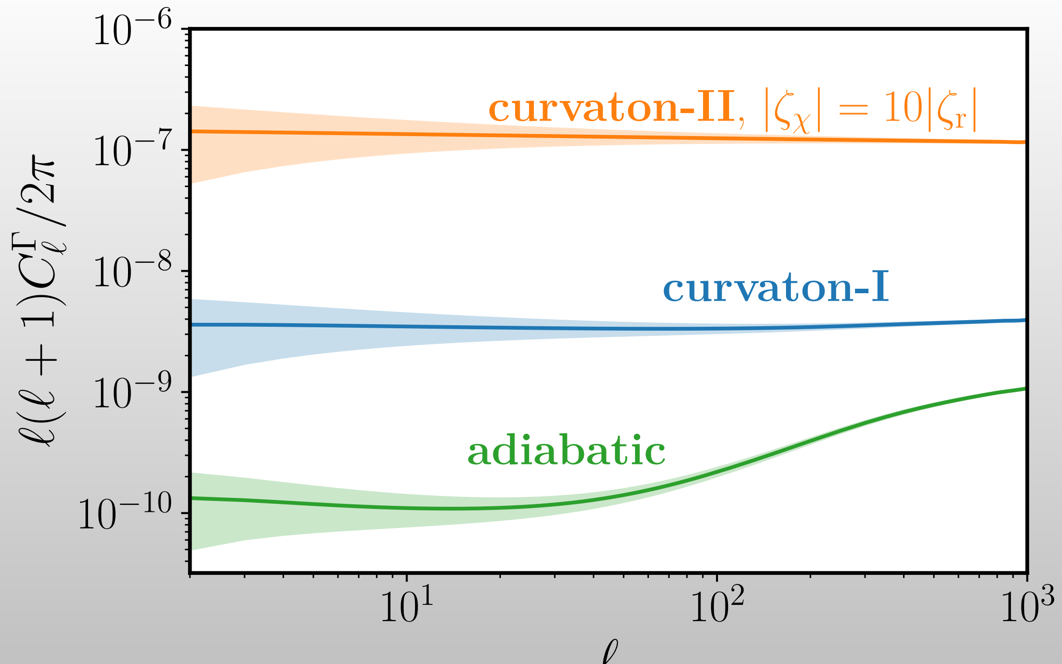
- Anisotropies as a probe of **squeezed non-Gaussianity** **(part I)**



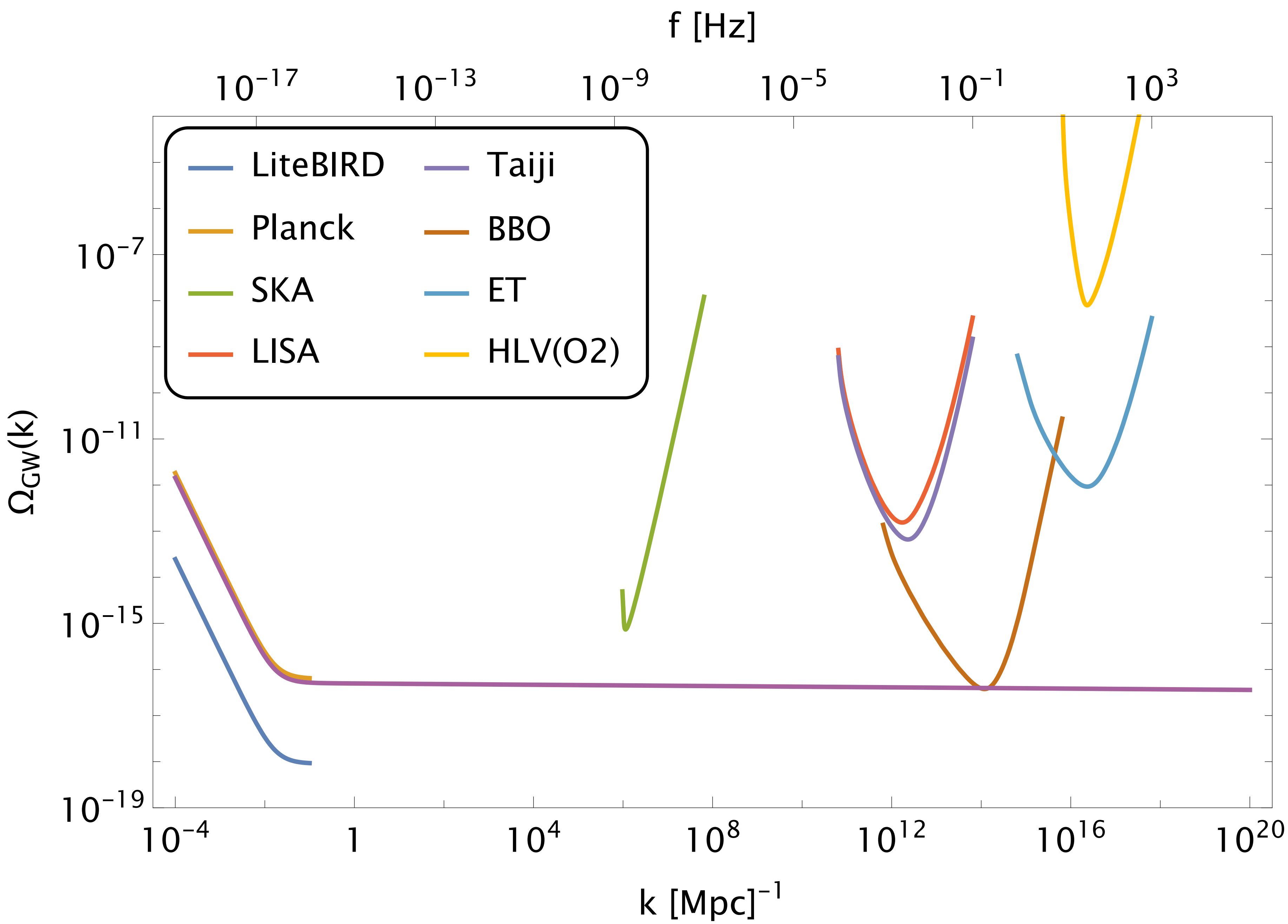
- Anisotropies as a probe of **primordial black holes** physics **(part II)**



- Anisotropies as a probe of **isocurvature perturbations** **(part III)**



Anisotropies in the stochastic GW background



Next step: <u>angular information</u>:

looking for spatial variations in the contributions to the energy density spectrum

Anisotropies in the stochastic GW background

$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k)\left[1 + \frac{1}{4\pi}\int d^2\hat{n}\, \delta_{\rm GW}(k,\hat{n})\right]$$

energy density spectrum for the GW background

isotropic component

directional intensity flux ("anisotropy")

analogous to $\frac{\Delta T}{T}$ for CMB anisotropies

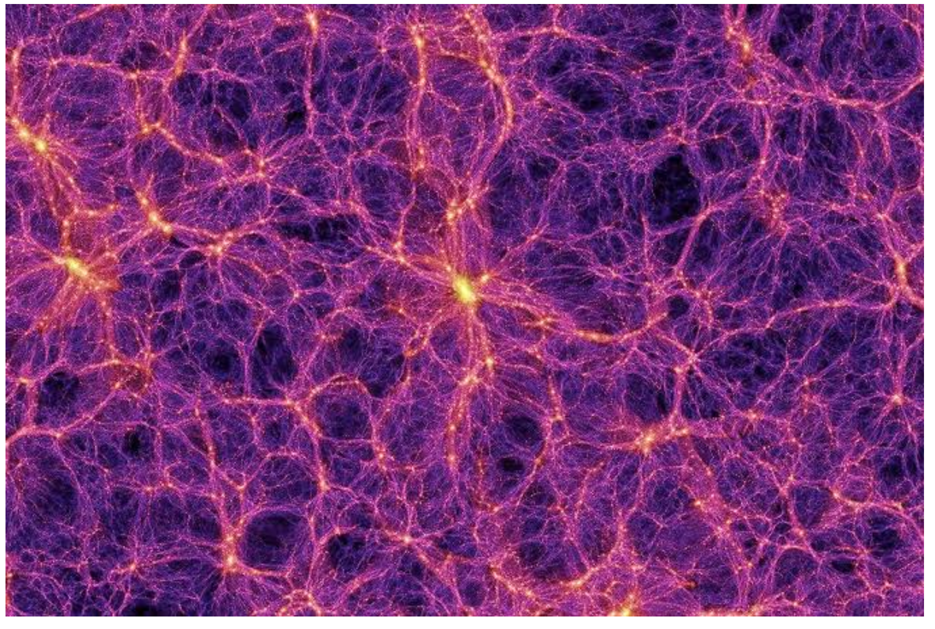
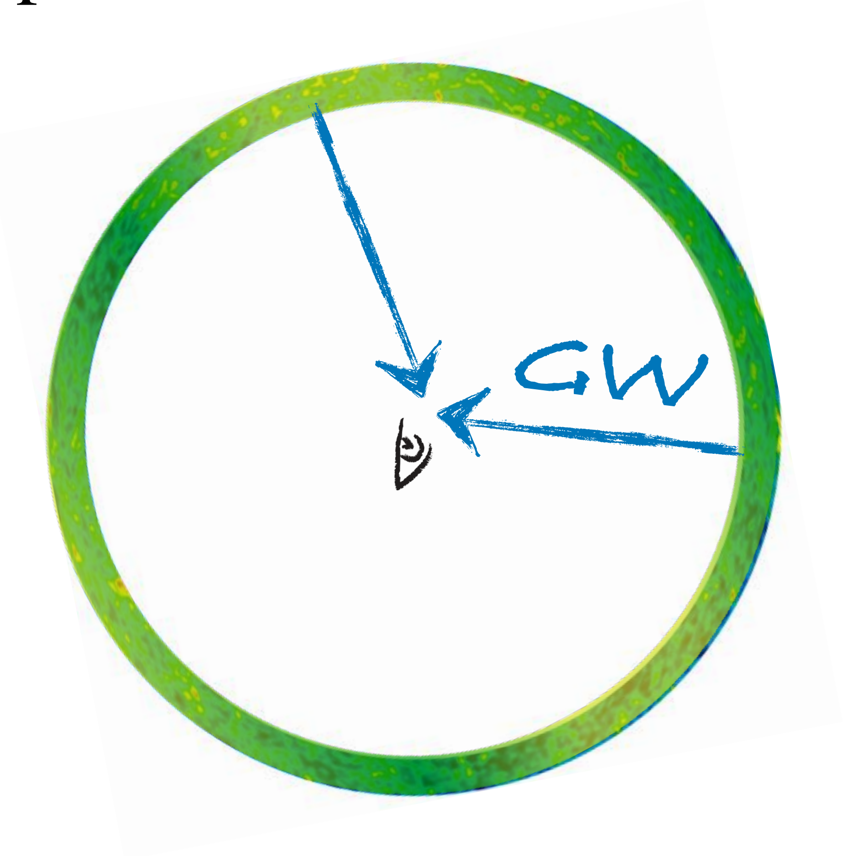
k = comoving wavenumber (~ observed frequency)

Angular power spectrum for the anisotropies:

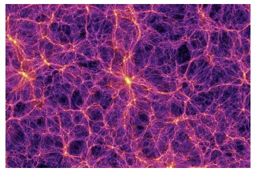
$$\delta_{\rm GW}(k,\hat{n}) = \sum_{\ell m} \delta^{\rm GW}_{\ell m} Y_{\ell m}(\hat{n})$$

$$\boxed{\langle \delta^{\rm GW}_{\ell m}\, \delta^{\rm GW*}_{\ell m}\rangle \equiv \delta_{\ell\ell'}\delta_{mm'}C^{\rm GW}_{\ell}}$$

Origin of the anisotropies

From propagation through the perturbed universe	Intrinsic to the production mechanism
	

Origin of the anisotropies: propagation through inhomogeneities

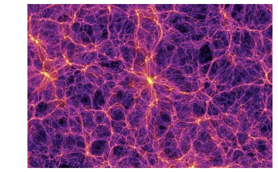


GW propagate through the perturbed universe

↓

subject to Sachs-Wolfe / integrated Sachs-Wolfe …,
similarly to CMB photons

Origin of the anisotropies: propagation through inhomogeneities



- Gravitational redshift:

Hierarchy of scales between GW frequency and scale of the perturbations
GW background = collection of massless particles emitted at early times

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (1+2\zeta)\delta_{ij}dx^i dx^j - \frac{4}{5aH}\partial_i\zeta d\eta dx^i\right]$$

(uniform density gauge in matter domination)

Geodesic equation for the graviton

graviton's 4-momentum: $P^\mu = \frac{dx^\mu}{d\lambda}$

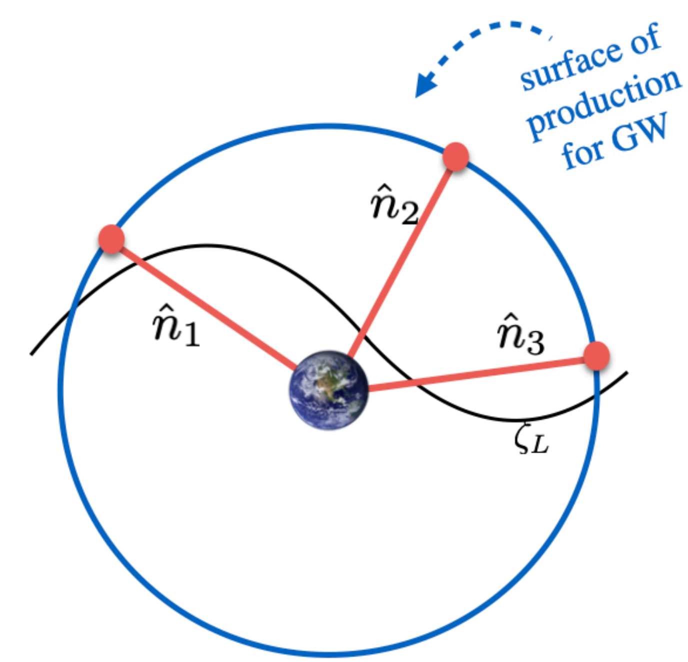
$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

affine parameter along the graviton's geodesic

$$P_0(\eta) = P_0(\eta_{\rm i})\left[1 + \frac{1}{5}\left(\underline{\zeta(\eta,\mathbf{x})} - \underline{\zeta(\eta_{\rm i},\mathbf{x}_i)}\right)\right]$$

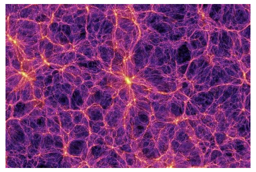
value at observer's location (common to emissions from all directions) -> it drops out

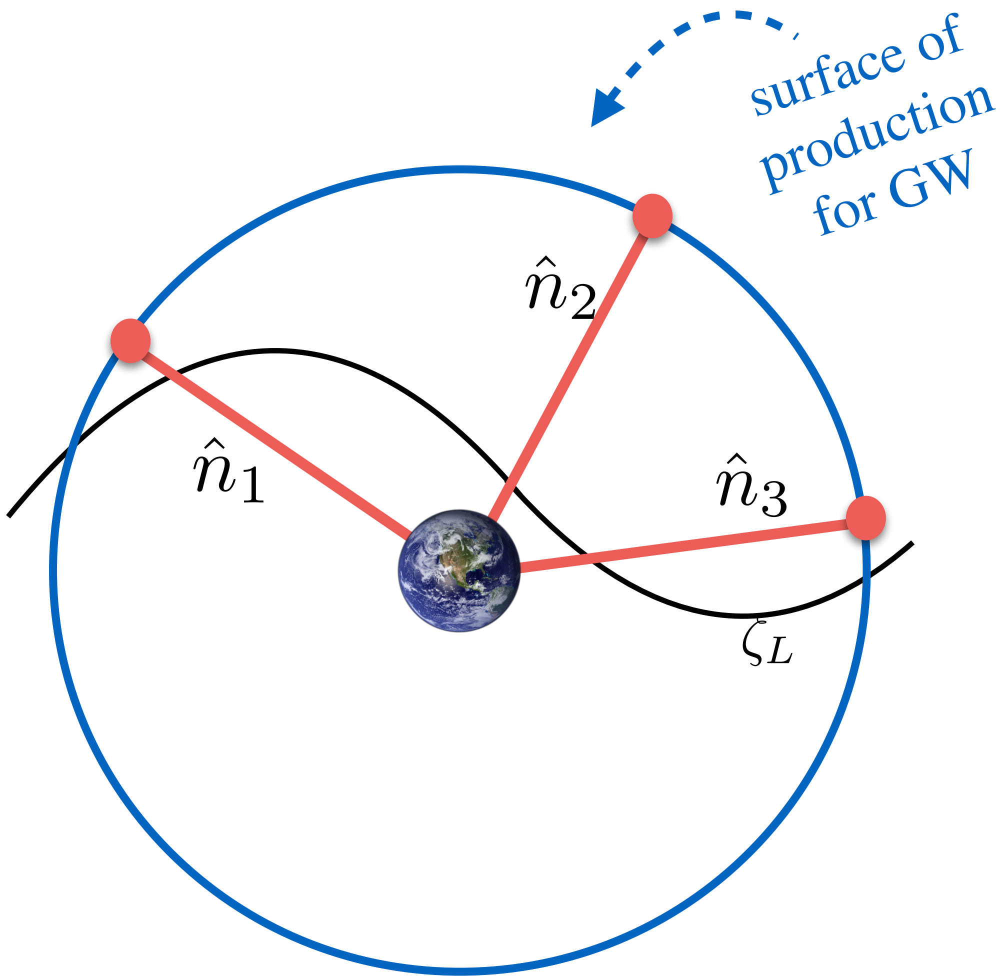
value at "emission" (direction dependent)



[Alba - Maldacena, 2015]

Origin of the anisotropies: propagation through inhomogeneities





Large-scales: SW dominates

$$\frac{\delta f(\hat{n})}{f} \propto \left[\zeta_{L(\text{today})} - \zeta_L(\hat{n}\cdot\eta_0)\right]$$

$$\zeta_L(\hat{n}_1\cdot\eta_0) \neq \zeta_L(\hat{n}_2\cdot\eta_0)$$

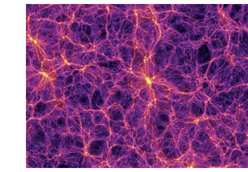
Direction-dependent frequency shift

Anisotropy in the GW energy density $\delta_{\rm GW} \sim \left(\frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k}\right) \zeta_{\rm L}$

$$\delta_{\rm GW} \simeq \mathcal{O}(1)\,\zeta_{\rm L} \simeq 10^{-5}$$ (for SFSR Inflation)

[Alba - Maldacena, 2015]

Origin of the anisotropies: original calculation (adiabatic perturbations)



Hierarchy of scales between GW frequency and scale of the perturbations

GW background = collection of massless particles emitted at early times

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (1+2\zeta)\delta_{ij}dx^i dx^j - \frac{4}{5aH}\partial_i\zeta d\eta dx^i\right]$$

(uniform density gauge in matter domination)

- Field theory derivation:

Massless scalar field h in a perturbed universe:

$$S = \frac{1}{2}\int d^3x\, d\eta\, a^2\,(1+3\zeta)\left[(-\partial_\eta h)^2 + (1-2\zeta)\left(\vec{\partial}h\right)^2 - \frac{4(\partial_\eta h)}{5aH}\vec{\partial}\zeta\cdot\vec{\partial}h\right]$$

$$\mathcal{L}_{\rm int} \propto a^2\zeta\left[3\,(\partial_\eta h)^2 - \left(\vec{\partial}h\right)^2\right] + \frac{4a(\partial_\eta h)}{5H}\vec{\partial}\zeta\cdot\vec{\partial}h$$

correction to the two point function of h proportional to ζ

[Alba - Maldacena, 2015]

Origin of the anisotropies: Boltzmann formalism

Hierarchy of scales between GW frequency and scale of the perturbations

GW background = collection of massless particles emitted at early times and described by a distribution function $f(x^\mu, p^\mu)$

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j\right]$$

$$\frac{df}{d\lambda} = \underbrace{C[f(\lambda)]}_{\text{collision term}} + \underbrace{I[f(\lambda)]}_{\text{injection term}} \simeq 0 \longrightarrow \frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\underbrace{\frac{dx^i}{d\eta}}_{n^i} + \frac{\partial f}{\partial q}\underbrace{\frac{dq}{d\eta}}_{q\left[\frac{\partial \Psi}{\partial \eta} - \frac{\partial \Phi}{\partial x^i}n^i\right]} = 0$$

collision term = 0 [gravitons basically decoupled below Mp]

injection term = 0 [accounted for as an initial condition for f]

(focusing on free-streaming, SW and iSW terms)

graviton's 4-momentum:

$$p^\mu = \frac{dx^\mu}{d\lambda} \qquad q \equiv |\vec{p}|a$$

[Contaldi, 2017- Bartolo et al 2019 see also: Pitrou et al, 2020]

Origin of the anisotropies: Boltzmann formalism

$$\boxed{\frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i} n^i + q \frac{\partial f}{\partial q}\left[\frac{\partial \Psi}{\partial \eta} - \frac{\partial \Phi}{\partial x^i} n^i\right] = 0}$$

$$f(\eta_0, \vec{x}_0, \hat{n}, q) = \bar{f}(q) - q\,\frac{\partial \bar{f}}{\partial q}\,\Gamma(\eta_0, \vec{x}_0, \hat{n}, q)$$

(linear expansion for the distribution function)

$$\rho_{\rm GW}(\underline{\eta_0, \vec{x}_0}) = \int d^3p\, p\, f(\eta_0, \vec{x}_0, q, \hat{n}) \;= \rho_{cr} \int d\ln q\, \Omega_{\rm GW}(\eta_0, \vec{x}_0, q)$$

observer's time/location

$$\Omega_{\rm GW}(\eta_0, \vec{x}_0, q) = \bar{\Omega}_{\rm GW}(\eta_0, \vec{x}_0, q)\left[1 + \frac{1}{4\pi}\int d^2n\, \delta_{\rm GW}(\hat{n}, q)\right]$$

(our previous definition for δ)

$$\boxed{\delta_{\rm GW}(\eta_0, \vec{x}_0, \hat{n}, q) \;= \left[4 - \frac{\partial \ln \bar{\Omega}_{\rm GW}(\eta_0, q)}{\partial \ln q}\right]\Gamma(\eta_0, \vec{x}_0, \hat{n}, q)}$$

$$\boxed{\Gamma(\eta_0, \vec{x}_0, \hat{n}, q) = \Gamma(\eta_{\rm i}, \vec{x}_i, q) + \Phi(\eta_{\rm i}, \vec{x}_i) + \int_{\eta_{\rm i}}^{\eta_0} d\eta(\Phi' + \Psi')}$$

initial SW iSW

[Contaldi, 2017- Bartolo et al 2019 see also: Pitrou et al, 2020]

Origin of the anisotropies: Boltzmann formalism

$$\Gamma(\eta_0, \vec{x}_0, \hat{n}, q) = \underbrace{\Gamma(\eta_{\rm i}, \vec{x}_i, q)}_{\Gamma_I} + \Phi(\eta_{\rm i}, \vec{x}_i) + \int_{\eta_{\rm i}}^{\eta_0} d\eta (\Phi' + \Psi')$$

For adiabatic primordial perturbations: $$\frac{\delta\rho_i}{(1+w_i)\bar{\rho}_i} = \frac{\delta\rho_j}{(1+w_j)\bar{\rho}_j}$$

(for any two species "i" and "j")

applying this to photons and gravitons fluids:

$$w_i = w_j = 1/3$$

$$\frac{\delta\rho_\gamma}{\bar{\rho}_\gamma} = -2\Phi$$ (from Einstein equ. in RD, superhorizon)

$$\Gamma_I = \frac{1}{4}\frac{\delta\rho_{\rm GW}}{\bar{\rho}_{\rm GW}}$$ (by definition)

$$\Gamma_I = -\frac{1}{2}\Phi$$

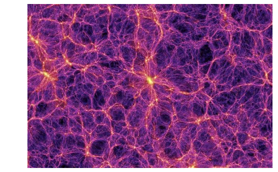
$$\Gamma = -\frac{1}{2}\Phi + \Phi + \int_{\eta_i}^{\eta_0} d\eta \left(\Phi^{'} + \Psi^{'}\right)$$

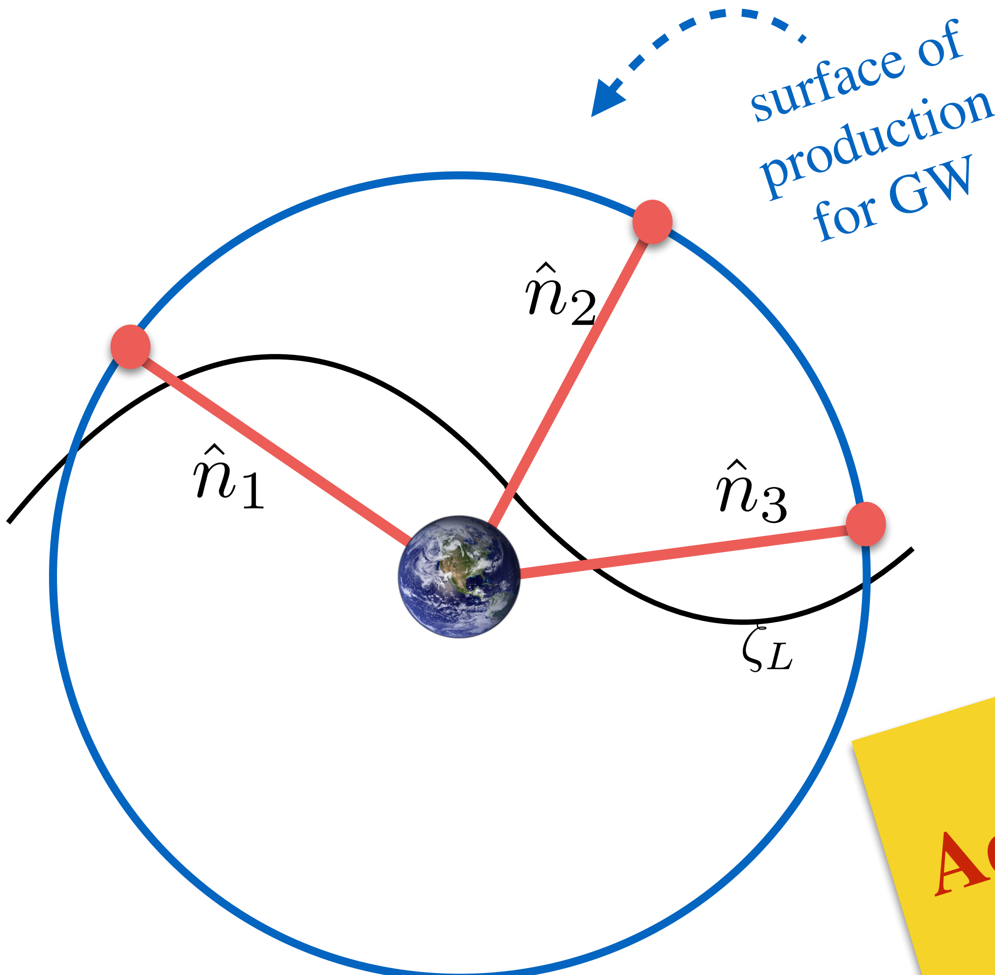
$$= \frac{1}{2}\Phi + \int_{\eta_i}^{\eta_0} d\eta \left(\Phi^{'} + \Psi^{'}\right)$$

$$= -\frac{1}{3}\zeta + \ldots$$

$$\Phi = -\frac{2}{3}\zeta$$

Origin of the anisotropies: propagation through inhomogeneities





Large-scales: SW dominates

$$\frac{\delta f(\hat{n})}{f} \propto [\zeta \ldots$$

Adiabatic primordial perturbations

Small spectral tilt for tensors

No intrinsic anisotropies

Anisotropy in the G... gy density

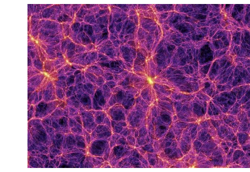
$$\delta_{\mathrm{GW}} \sim \left(\frac{\partial \ln \Omega_{\mathrm{GW}}}{\partial \ln k}\right) \zeta_{\mathrm{L}}$$

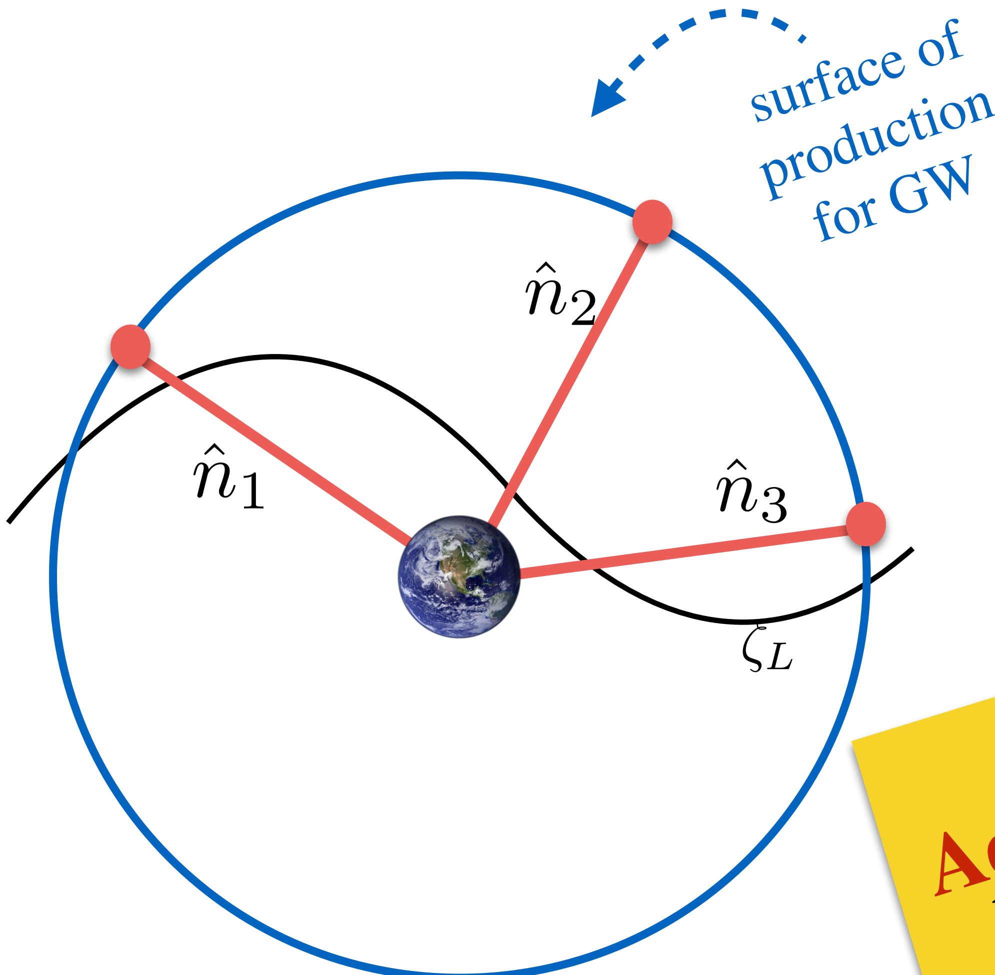
$$\delta_{\mathrm{GW}} \simeq \mathcal{O}(1)\, \zeta_{\mathrm{L}} \simeq 10^{-5}$$

(<u>for SFSR Inflation</u>)

[Alba - Maldacena, 2015]

Origin of the anisotropies: propagation through inhomogeneities





Large-scales: SW dominates

~~Adiabatic primordial perturbations~~ **(part III)**

~~Small spectral tilt for tensors~~ **(part II)**

~~No intrinsic anisotropies~~ **(part I)**

Anisotropy in the … density

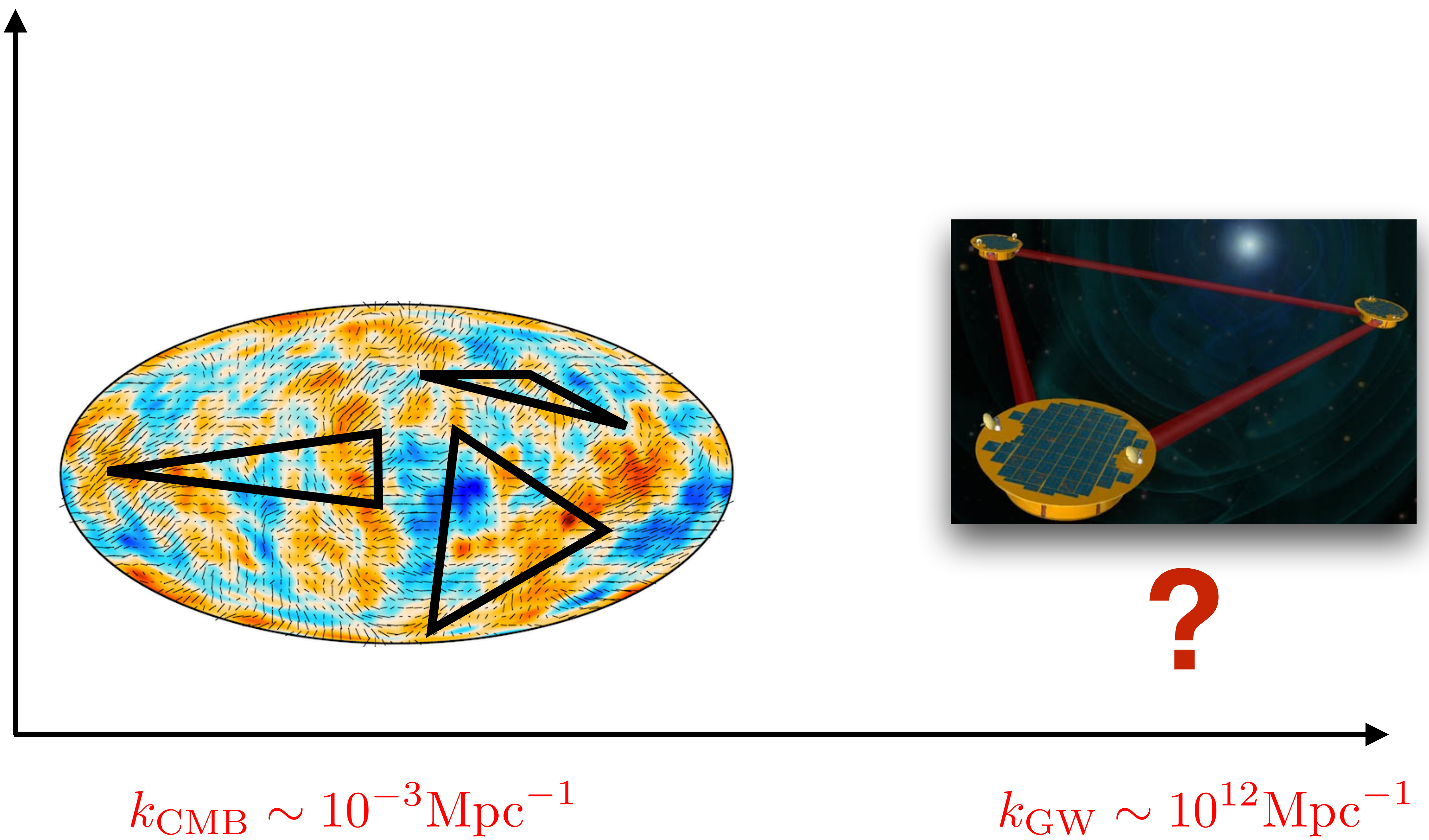
$$\delta_{\rm GW} \sim \left(\frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k} \right) \zeta_{\rm L}$$

$$\delta_{\rm GW} \simeq \,?$$

Anisotropies from **squeezed non-Gaussianity** (“intrinsic” type)

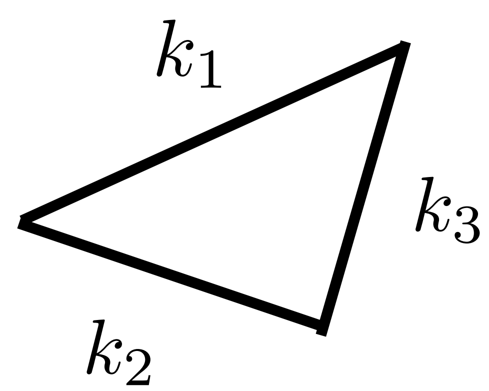
(part I)

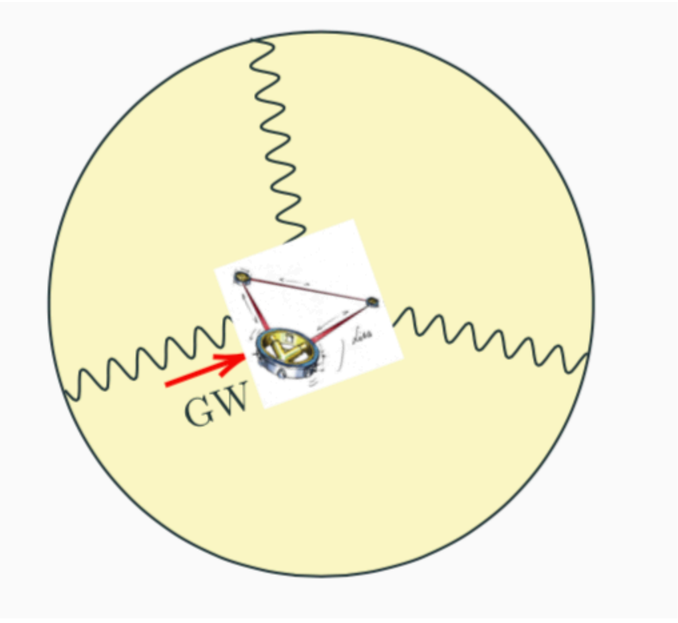
Primordial non-Gaussianity



Non-Gaussianity at interferometers

Measuring $\langle \gamma^3 \rangle$ directly is not possible: phase decorrelation from propagation in an inhomogeneous universe





[Bartolo et al. 2018]

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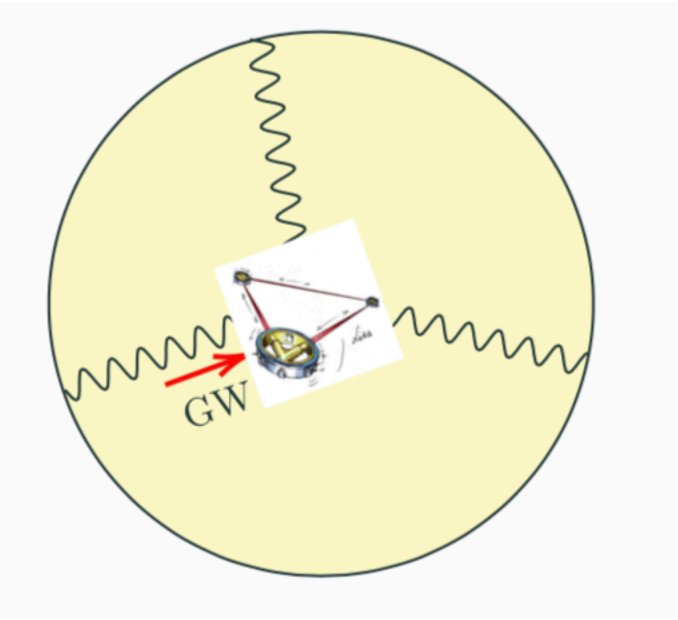


Shapiro time delay:

$$\gamma'' + 2\mathcal{H}\gamma' - [1 + (12/5)\zeta]\,\gamma_{,kk} = 0$$

GW propagating in FRW background
+ long-wavelength perturbations

$$\gamma_{ij} = A_{ij}\, e^{ik\tau + ik\cdot 2\int^{\tau} d\tau'\,\zeta[\tau',(\tau'-\tau_0)\hat{k}]}$$

GW from different directions
undergo different phase shifts
due to intervening structure

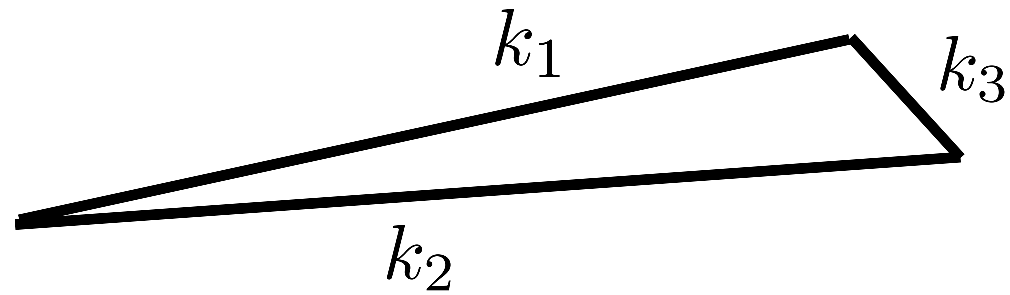
$\longrightarrow$ initial non-Gaussianity wiped out by propagation

[Bartolo et al. 2018]

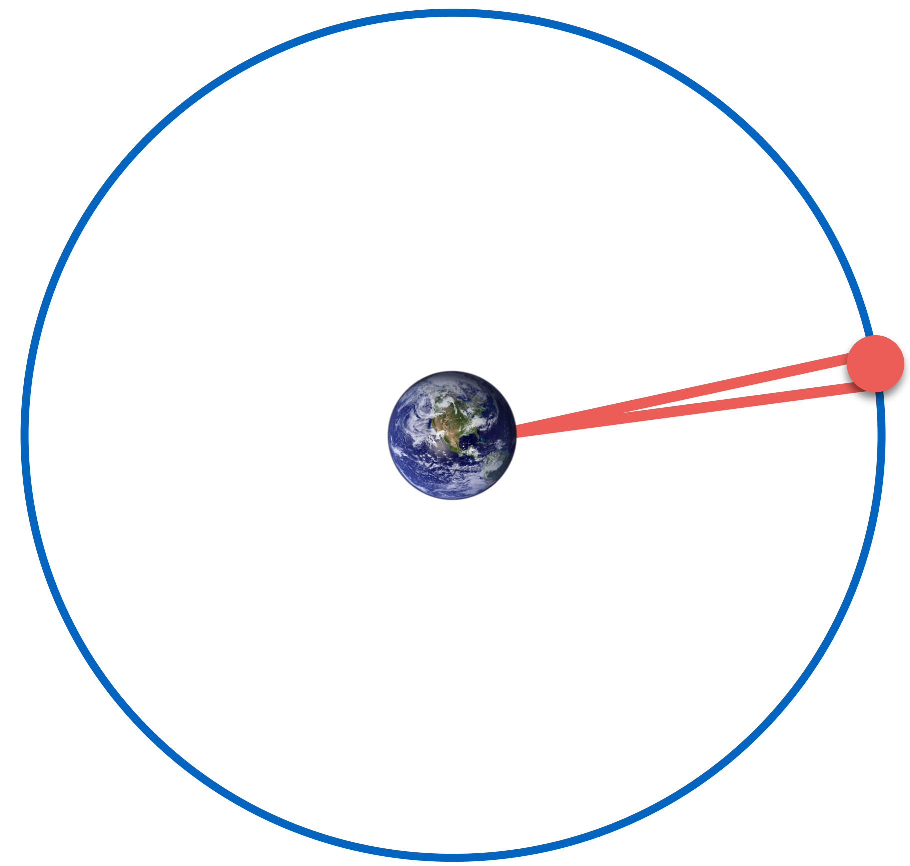
CLT: signal measured by an interferometer arises from the superposition of signals from a large number of Hubble patches at production

[Adshead, Lim 2009 — Caprini, Figueroa 2018 — Bartolo et al 2018]

Non-Gaussianity at interferometers



Correlation among two short-wavelength modes (e.g. interferometer scale) and 1 very long-wavelength mode: signals originate from the same patch!



[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

Based on:　　PRL 124(2020)6 061302

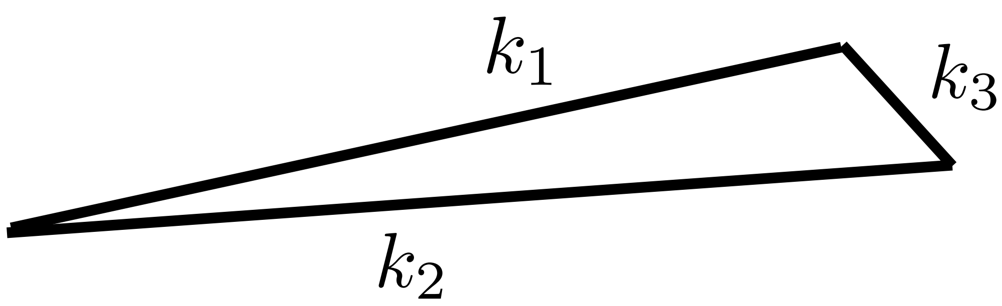
with

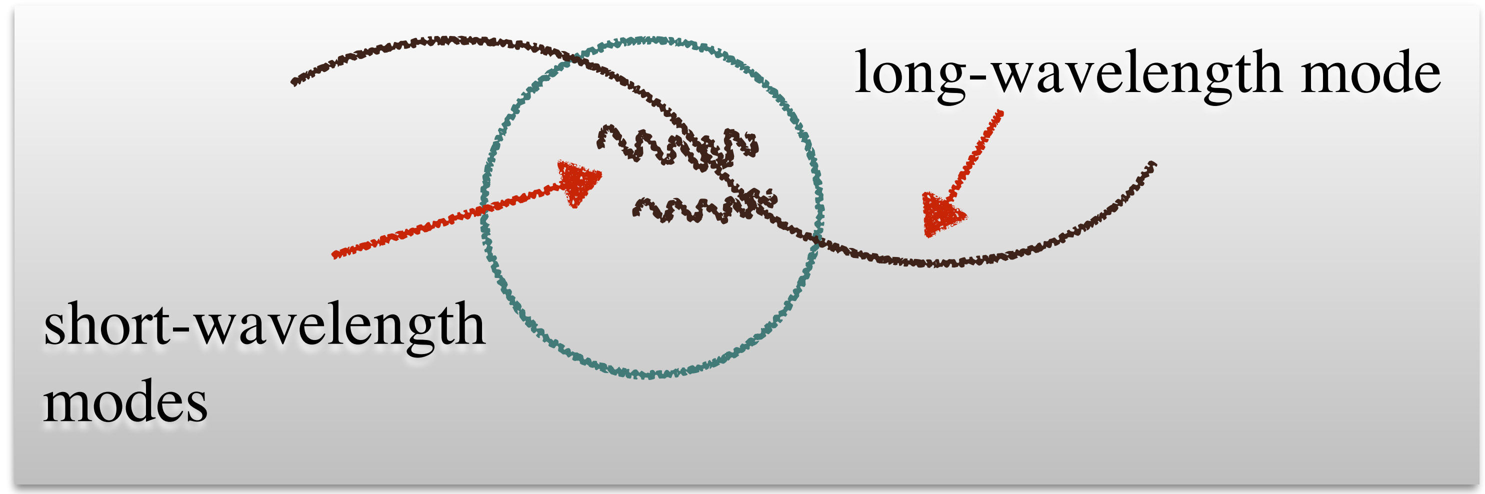
Matteo Fasiello
(IFT Madrid)

Gianmassimo Tasinato
(Swansea)

Soft limits and 'fossils'

$k_1 \simeq k_2 \gg k_3$





long wavelength modes introduces a modulation
in the primordial power spectrum of the short wavelength modes

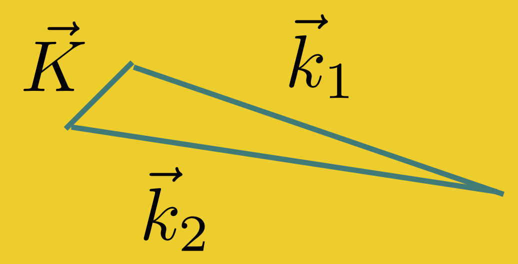
$$B^{F\gamma\gamma} \equiv \langle F_L \gamma_S \gamma_S \rangle' \sim F_L \cdot \langle \gamma_S \gamma_S \rangle'_{F_L}$$

$$\delta\langle \gamma_S \gamma_S \rangle \equiv \langle \gamma_S \gamma_S \rangle_{F_L} \sim \frac{B^{F\gamma\gamma}}{P_F(k_3)} \cdot F_L^* = P_\gamma(k_1) \cdot \underbrace{\frac{B^{F\gamma\gamma}}{P_F(k_3) P_\gamma(k_1)}}_{f_{\rm NL}^{F\gamma\gamma}} \cdot F_L^*$$

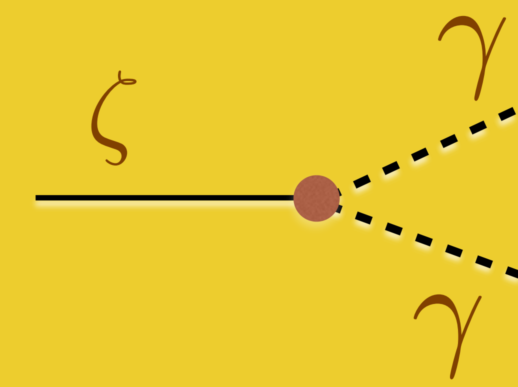
$$\boxed{\langle \gamma_S \gamma_S \rangle'_{\rm total} = P_\gamma(k_1) \left(1 + f_{\rm NL}^{F\gamma\gamma} \cdot F_L^*\right)}$$

[ED, Fasiello, Jeong, Kamionkowski - 2014, ED, Fasiello, Kamionkowski - 2015, …]

Soft limits and fossils



from



$$\delta_{\rm GW}(k,\hat{n}) = \int \frac{d^3q}{(2\pi)^3} e^{-id\hat{n}\cdot\mathbf{q}} \zeta(\mathbf{q}) F_{\rm NL}^{\rm stt}(\mathbf{k},\mathbf{q})$$

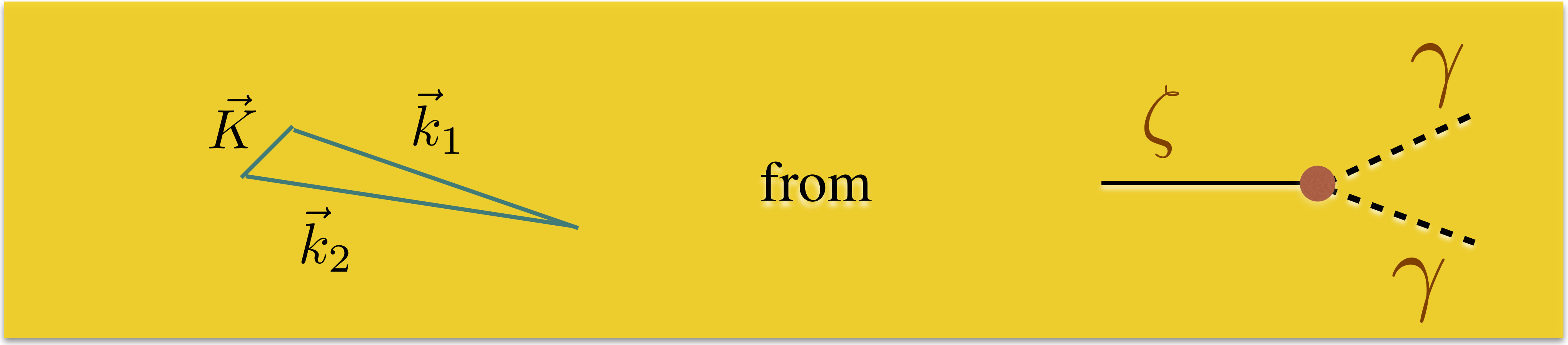
large scale variation in the energy density of GW

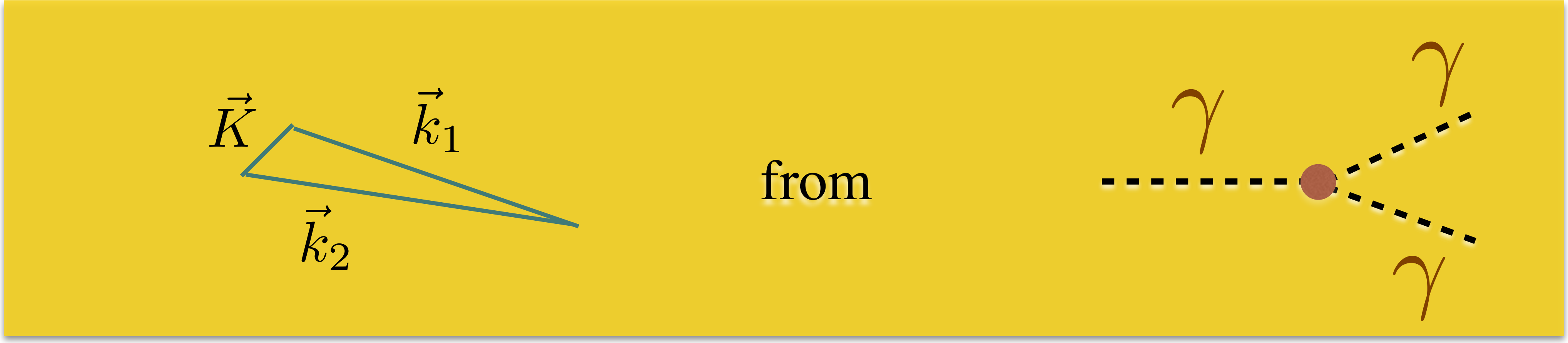
$$\mathbf{d} = -(\eta_0 - \eta_{\rm in})\hat{n}$$

$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k)\left[1 + \frac{1}{4\pi}\int d^2\hat{n}\,\delta_{\rm GW}(k,\hat{n})\right]$$

[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

Soft limits and fossils





[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

for derivation with in-in formalism and applications:

see: **[ED, Fasiello, Pinol, 2022]**

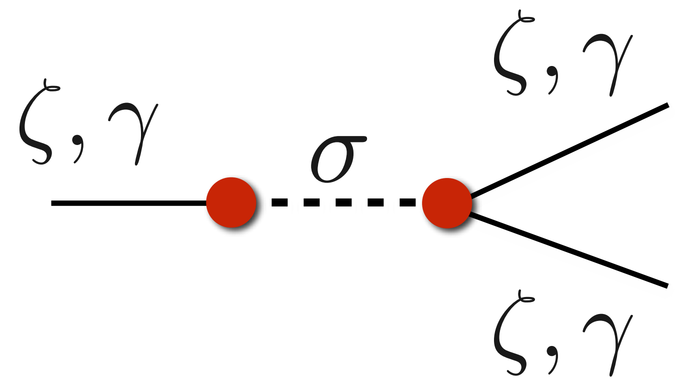
Soft limits in inflation

- ***Extra*** *fields* / superhorizon ***evolution***

  [Maldacena 2003, Creminelli - Zaldarriaga 2004, Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013, Pajer et al. 2013, ED - Fasiello - Kamionkowski 2015, …]

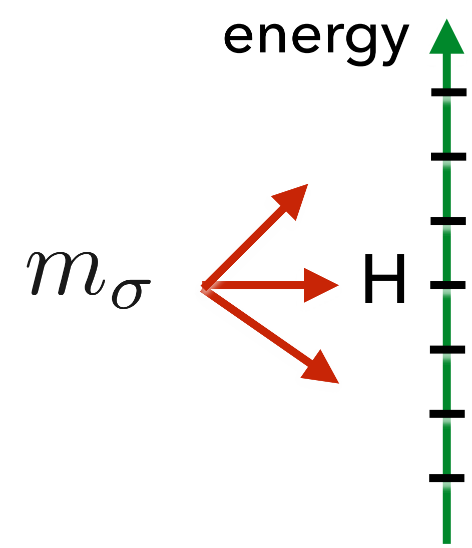
Soft limits in inflation

- ***Extra** fields*



Soft limits reveal (extra) fields mediating inflaton or graviton interactions

squeezed bispectrum delivers info on mass spectrum!!!



Soft limits in inflation

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- ***Non-Bunch Davies*** initial states

  [Holman - Tolley 2007, Ganc - Komatsu 2012, Brahma - Nelson - Shandera 2013, …]

- ***Broken space diffs***
  (e.g. space-dependent background)

  [Endlich et al. 2013, ED - Fasiello - Jeong - Kamionkowski 2014, Celoria - Comelli - Pilo - Rollo 2021…]

Ideal probe for (extra) fields, pre-inflationary dynamics, (non-standard) symmetry patterns

Based on work with:

Matteo Fasiello
(IFT Madrid)

Ameek Malhotra
(UNSW → Swansea)

JCAP 03 (2021) 088

Maresuke Shiraishi
(Suwa University)

JCAP 02 (2022) 02, 040

Daan Meerburg
(Groningen)

Giorgio Orlando
(Jagiellonian University)

GW anisotropies from squeezed non-Gaussianity

- Typical amplitude of these anisotropies:

$$\delta_{\rm GW}^{\rm tss} \sim F_{\rm NL}^{\rm tss}\sqrt{A_{\rm S}}$$

$$\delta_{\rm GW}^{\rm ttt} \sim F_{\rm NL}^{\rm ttt}\sqrt{r\, A_{\rm S}}$$

scalar power spectrum amplitude at CMB scales

tensor-to-scalar ratio

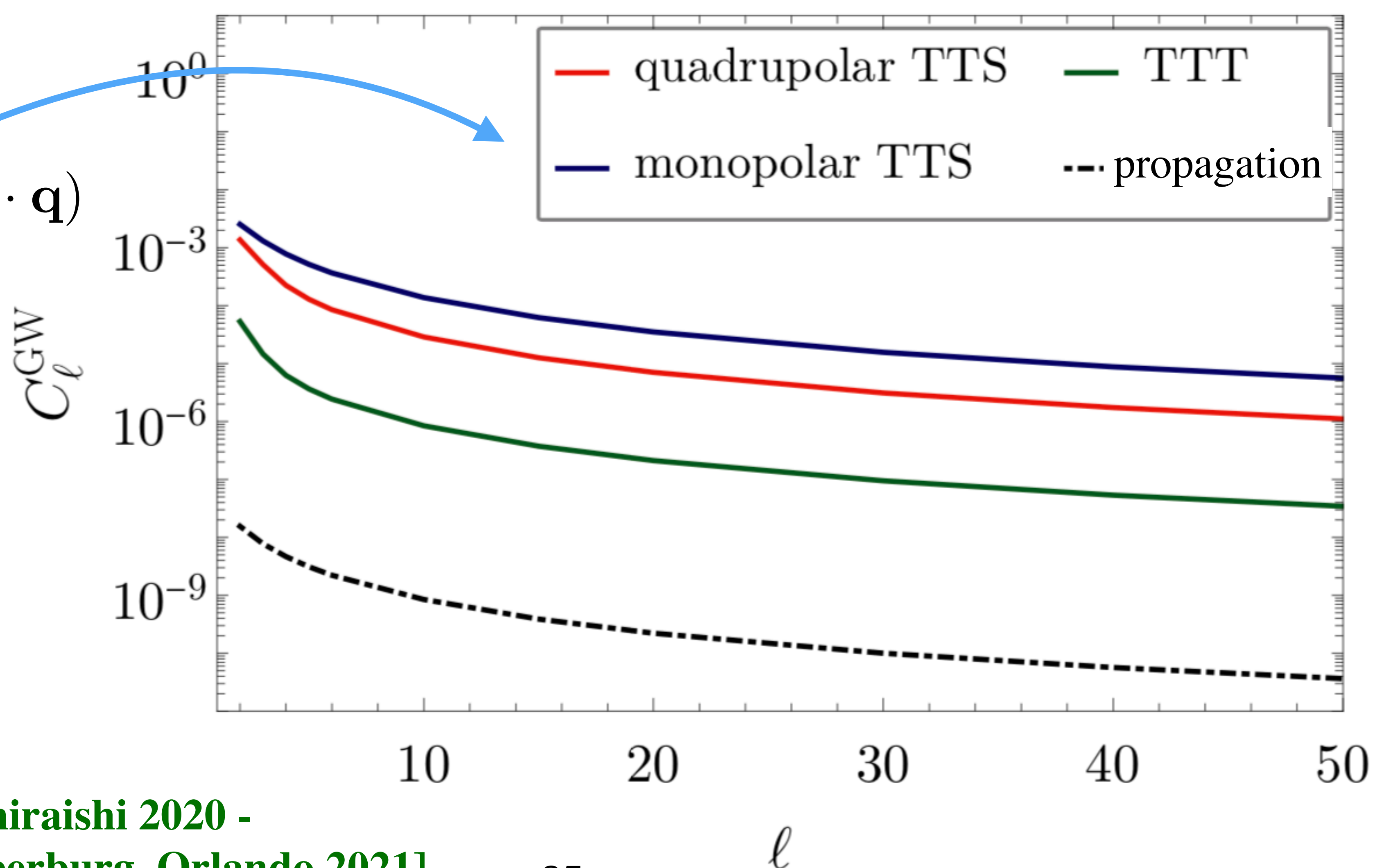
angular dependence:

$$F_{\rm NL}(\mathbf{k},\mathbf{q}) = \tilde{F}_{\rm NL}\mathcal{P}_\ell(\mathbf{k}\cdot\mathbf{q})$$

$$\begin{cases} \ell = 0 & \text{monopole} \\ \ell = 2 & \text{quadrupole} \end{cases}$$

(model dependent)

* scale-invariant case



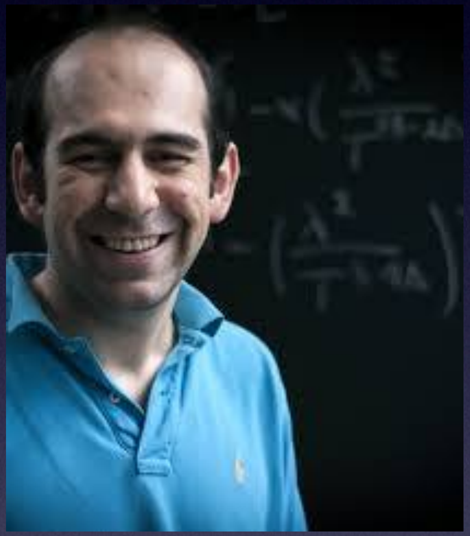
[Malhotra, ED, Fasiello, Shiraishi 2020 -
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Based on: Phys.Rev.D 103 (2021) 2, 023532

with

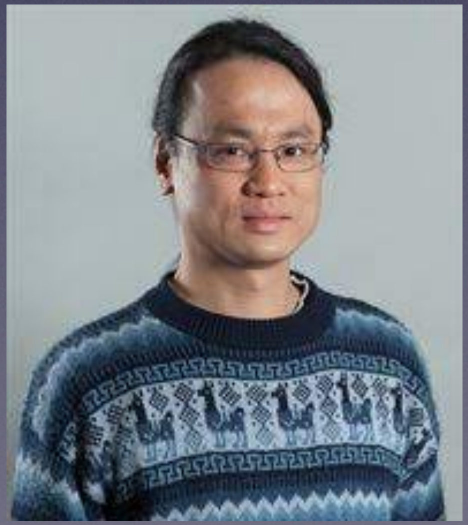
Peter Adshead
(UIUC)

Niayesh Afshordi
(Waterloo/PI)



Matteo Fasiello
(IFT Madrid)

Eugene Lim
(UCL)



Gianmassimo Tasinato
(Swansea)

Cross-correlations of GW and CMB anisotropies

$$\delta_{\rm GW}^{\rm stt} \sim F_{\rm NL}^{\rm stt} \cdot \zeta_L$$

$$\frac{\Delta T}{T} \sim \zeta_L$$

$$C_\ell^{\text{GW-T}} \sim F_{\rm NL}^{\rm stt} \cdot C_\ell^{TT}$$

[Adshead, Afshordi, ED, Fasiello, Lim, Tasinato 2020]

For forecasts (combining auto- and cross-correlations) and applications to specific models see:

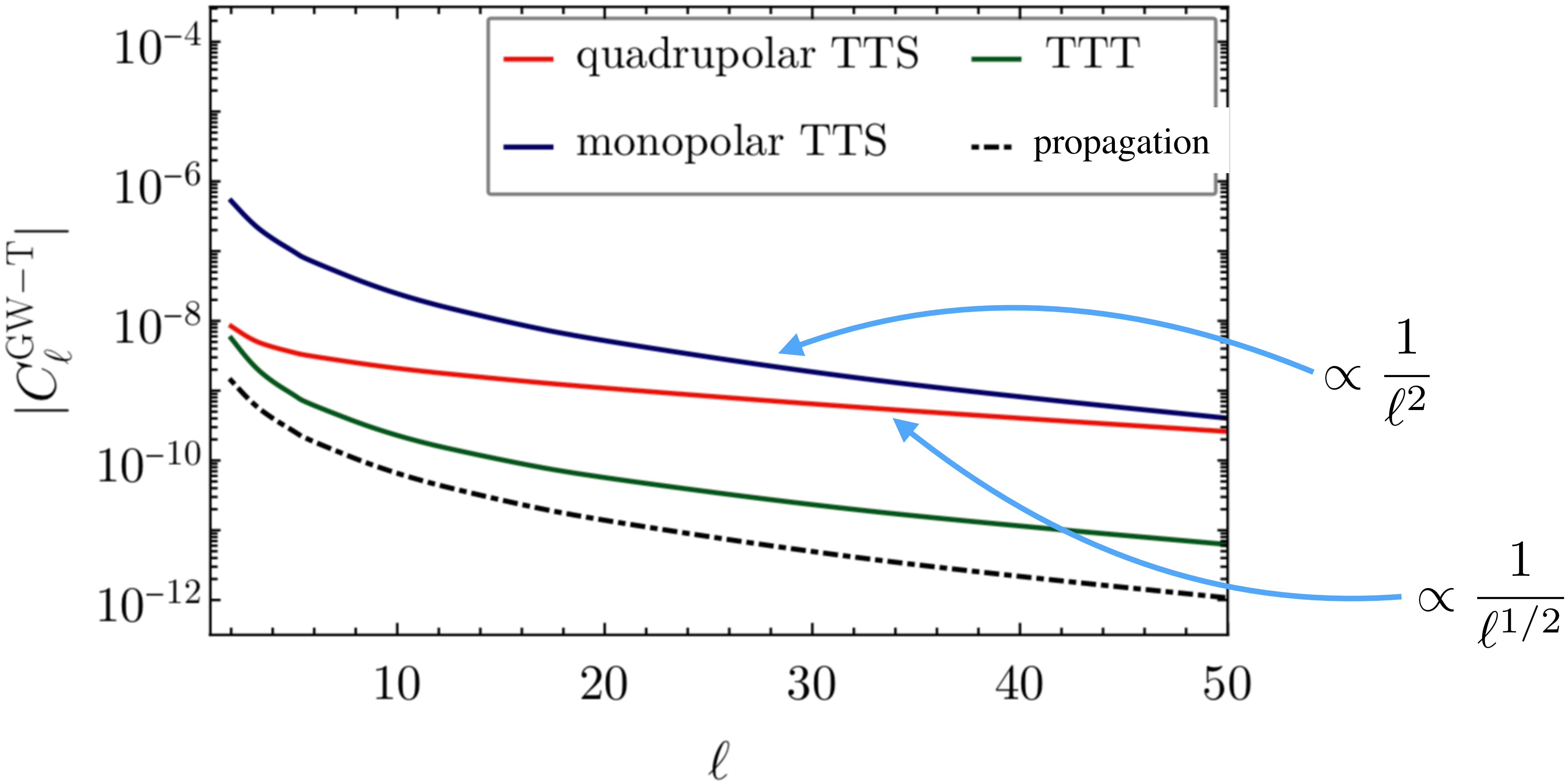
[Malhotra, ED, Fasiello, Shiraishi 2020]
[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

On large scales, anisotropies in the astrophysical GW background do not correlate strongly with CMB (cross-correlations with LSS observables much more effective) **[Ricciardone et al, 2021]**

↓

There is great potential in GW-CMB correlations as a probe of cosmological GW background!

Cross-correlations of GW and CMB anisotropies

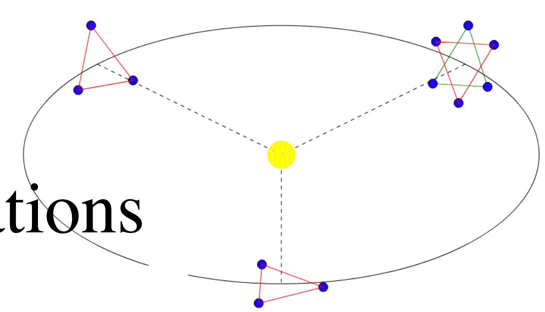


[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

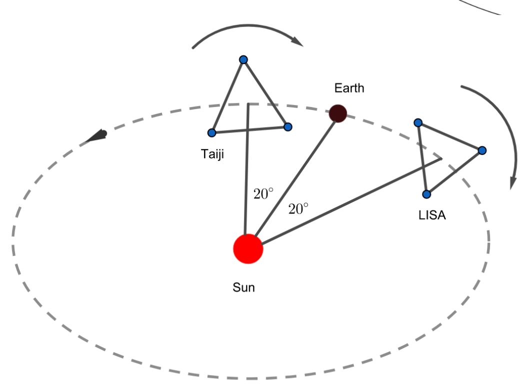
Projected constraints on $F_{\rm NL}^{\rm tss}$

$$F_{ij} = \sum_{XY} \sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}} \frac{\partial C_\ell^X}{\partial \theta_i} \left(\mathscr{C}_\ell^{XY}\right)^{-1} \frac{\partial C_\ell^Y}{\partial \theta_j} \qquad X, Y = \{\text{TT,GW,GW-T}\}$$

$$\mathscr{C}_\ell = \frac{2}{2\ell+1} \begin{bmatrix} (C_\ell^{\rm TT})^2 & (C_\ell^{\rm GW-T})^2 & C_\ell^{\rm TT} C_\ell^{\rm GW-T} \\ (C_\ell^{\rm GW-T})^2 & (C_\ell^{\rm GW})^2 & C_\ell^{\rm GW} C_\ell^{\rm GW-T} \\ C_\ell^{\rm TT} C_\ell^{\rm GW-T} & C_\ell^{\rm GW} C_\ell^{\rm GW-T} & \frac{1}{2}(C_\ell^{\rm GW-T})^2 + \frac{1}{2} C_\ell^{\rm TT} C_\ell^{\rm GW} \end{bmatrix}$$



- BBO: 4 LISA-like constellations



- LISA+Taiji
- ET + CE
- SKA (assumed 50 identical pulsars)

$$C_\ell^{\rm TT} \simeq \frac{2\pi A_S}{25\ell(\ell+1)},$$

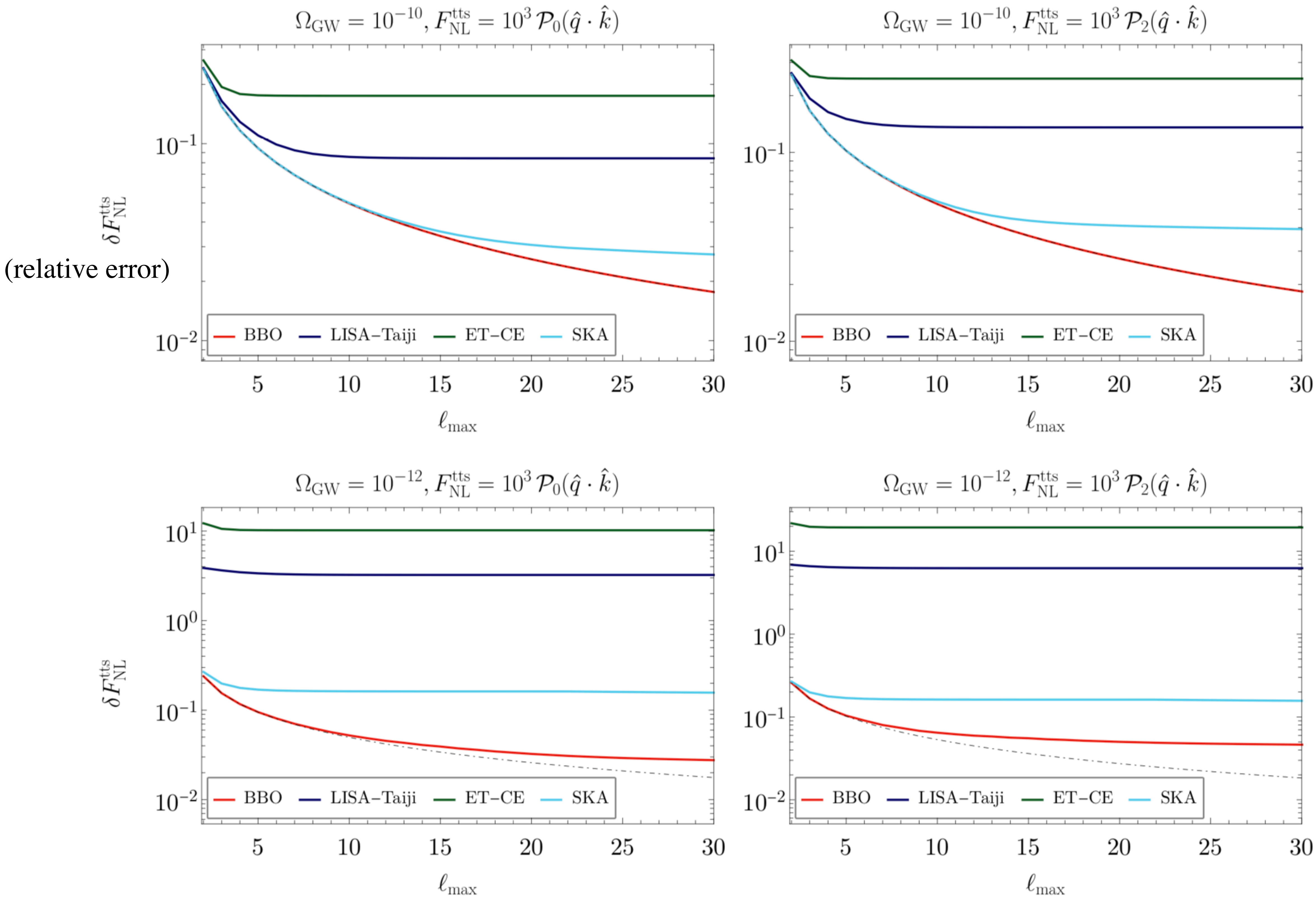
$$C_\ell^{\rm GW} = C_\ell^{\rm GW,tts} + C_\ell^{\rm GW,ind} + N_\ell^{\rm GW}$$

$$C_\ell^{\rm GW-T} = C_\ell^{\rm GW-T,tts} + C_\ell^{\rm GW-T,ind}$$

Noise angular power spectra computations based on **[Alonso et al. 2020]**

[Malhotra, ED, Fasiello, Shiraishi 2020]
[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Projected constraints on $F_{\mathrm{NL}}^{\mathrm{tss}}$



 [ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Conclusions (Part I)

- Tensor non-Gaussianity not directly observable with interferometers due to propagation of GW through the inhomogeneous universe

- (Tensor and mixed) primordial non-Gaussianity of the squeezed type induces anisotropies in the GW background

- Inflationary models with detectable GW background and significant squeezed non-Gaussianity can be tested at interferometer scales thanks to these anisotropies
  $\longrightarrow$ GW anisotropies as a probe of inflationary fields and interactions

Anisotropies from **peaked spectra**
(“propagation” type)

(part II)

Based on: JCAP 01 (2023) 018

with

Matteo Fasiello
(IFT Madrid)

Ameek Malhotra
(Swansea)

Gianmassimo Tasinato
(Swansea)

Anisotropies for peaked spectra

Anisotropies from propagation:
amplitude proportional to the slope of the spectrum

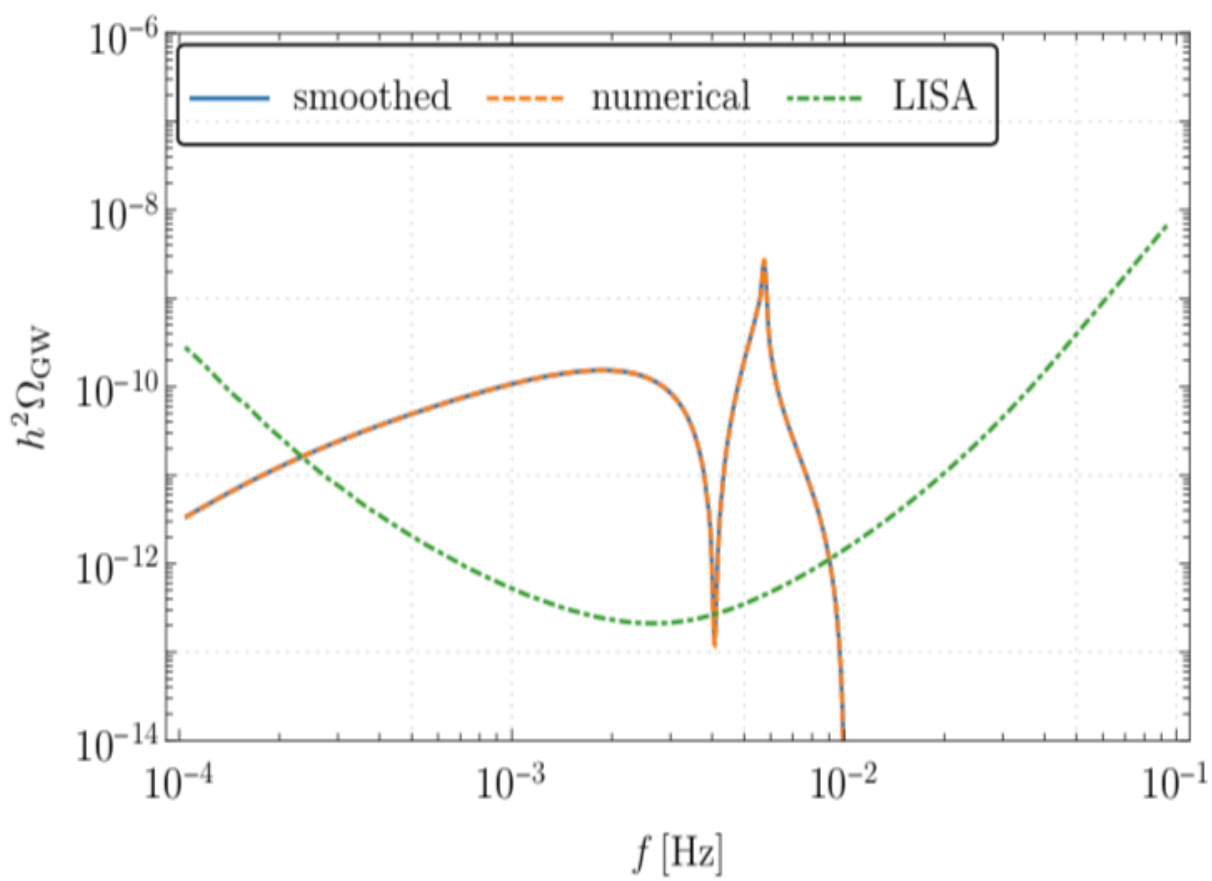
$$\delta_{\rm GW} \sim \left(\frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k} \right) \zeta_{\rm L}$$

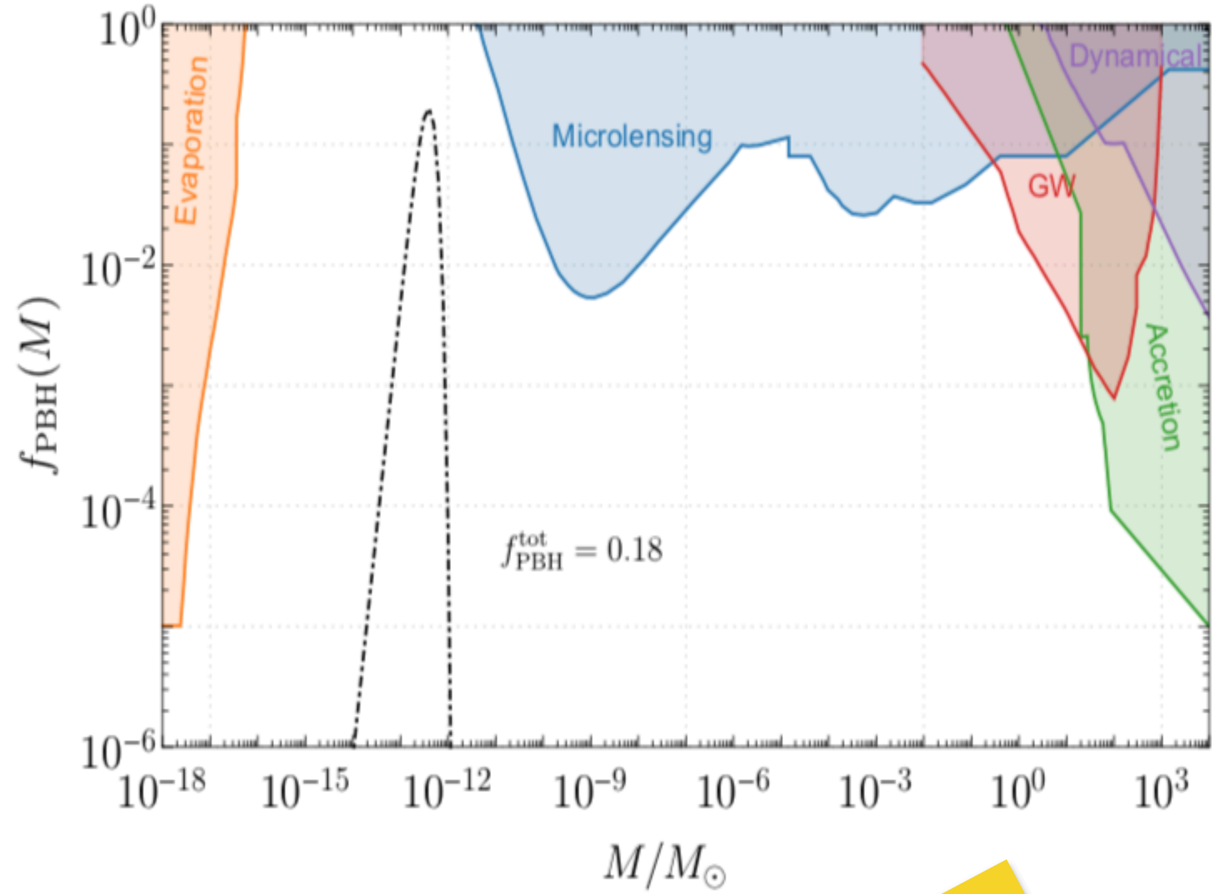
pronounced slope $\longrightarrow$ enhanced anisotropies

Anisotropies for peaked spectra

Models with sharp peaks in the scalar power spectrum (e.g. PBH production)

- a large GW background with sharp peaks induced at second order from scalar perturbations **[Ananda - Clarkson - Wands 2006, Baumann et al. 2007]**





$$\Omega_{\rm GW,r}(k) = 3\int_0^\infty dv \int_{|1-v|}^{1+v} du \frac{\mathcal{T}(u,v)}{u^2v^2}\mathcal{P}_\mathcal{R}(vk)\mathcal{P}_\mathcal{R}(uk)\,,$$

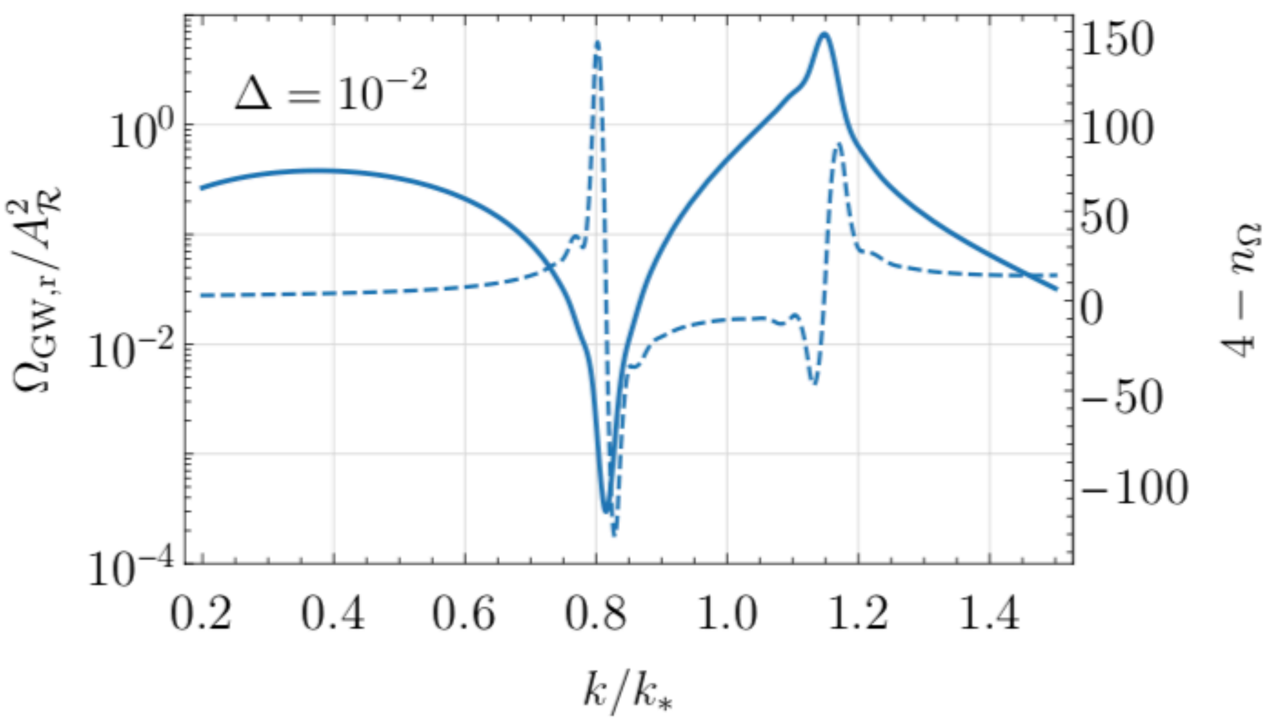
$$\mathcal{P}_\mathcal{R}(k)|_{k\gg k_{\rm CMB}} = \frac{A_\mathcal{R}}{\sqrt{2\pi}\Delta}\exp\left[-\frac{\ln^2(k/k_*)}{2\Delta^2}\right]$$

Δ	10^{-2}
f_*	5×10^{-3} Hz
k_*	3×10^{12} Mpc^{-1}
$A_\mathcal{R}$	7.5×10^{-3}

case study!

Anisotropies for peaked spectra

Models with sharp peaks in the scalar power spectrum (e.g. PBH production)



$$\Omega_{\rm GW,r}(k) = 3\int_0^\infty dv \int_{|1-v|}^{1+v} du \frac{\mathcal{T}(u,v)}{u^2v^2}\mathcal{P}_{\mathcal{R}}(vk)\mathcal{P}_{\mathcal{R}}(uk)\,,$$

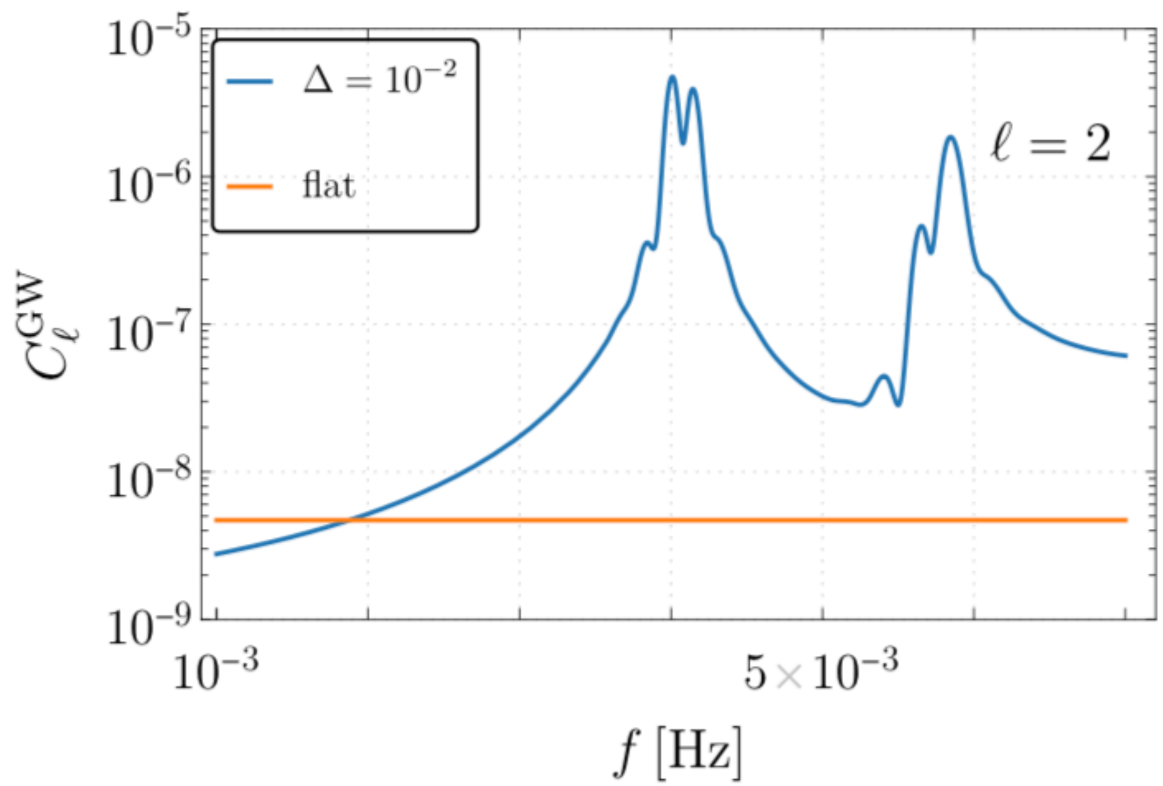
$$\mathcal{P}_{\mathcal{R}}(k)|_{k\gg k_{\rm CMB}} = \frac{A_{\mathcal{R}}}{\sqrt{2\pi}\Delta}\exp\left[-\frac{\ln^2(k/k_*)}{2\Delta^2}\right]$$

Δ	10^{-2}
f_*	5×10^{-3} Hz
k_*	$3\times10^{12}\,{\rm Mpc}^{-1}$
$A_{\mathcal{R}}$	7.5×10^{-3}

- the anisotropies can be typically enhanced by O(10-100)

$$\delta_{\rm GW} \sim \left(\frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k}\right)\zeta_{\rm L}$$

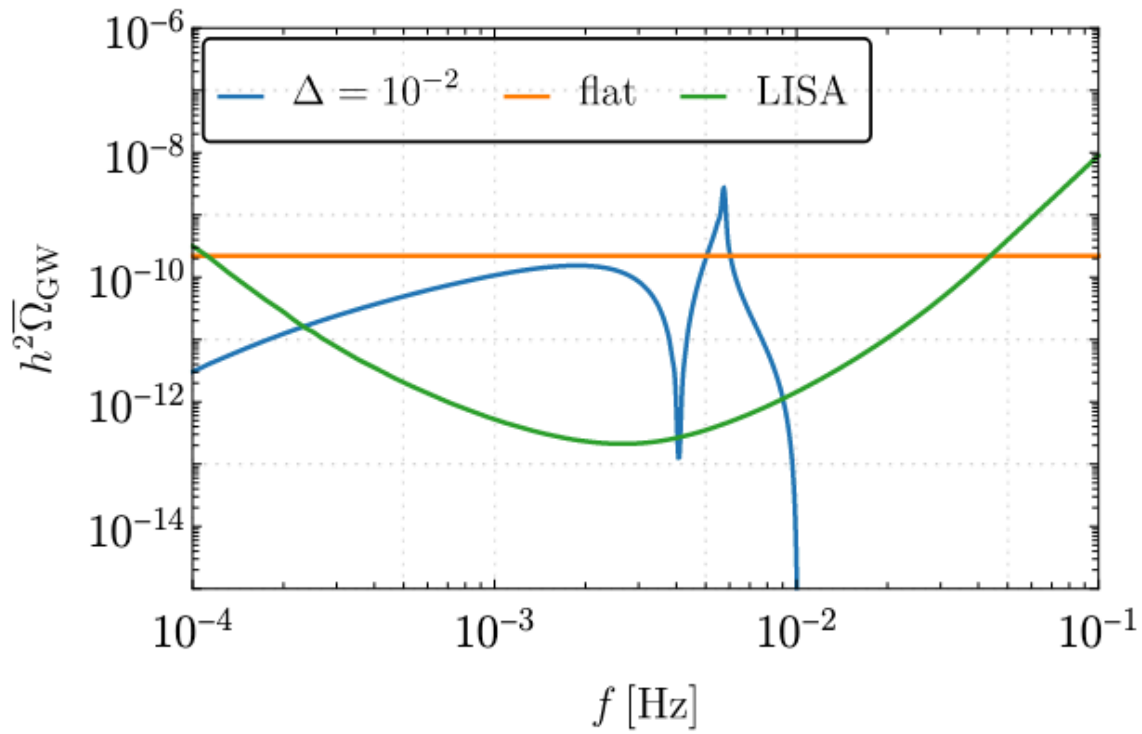
- the angular power spectrum of the GW anisotropies inherits the frequency dependence



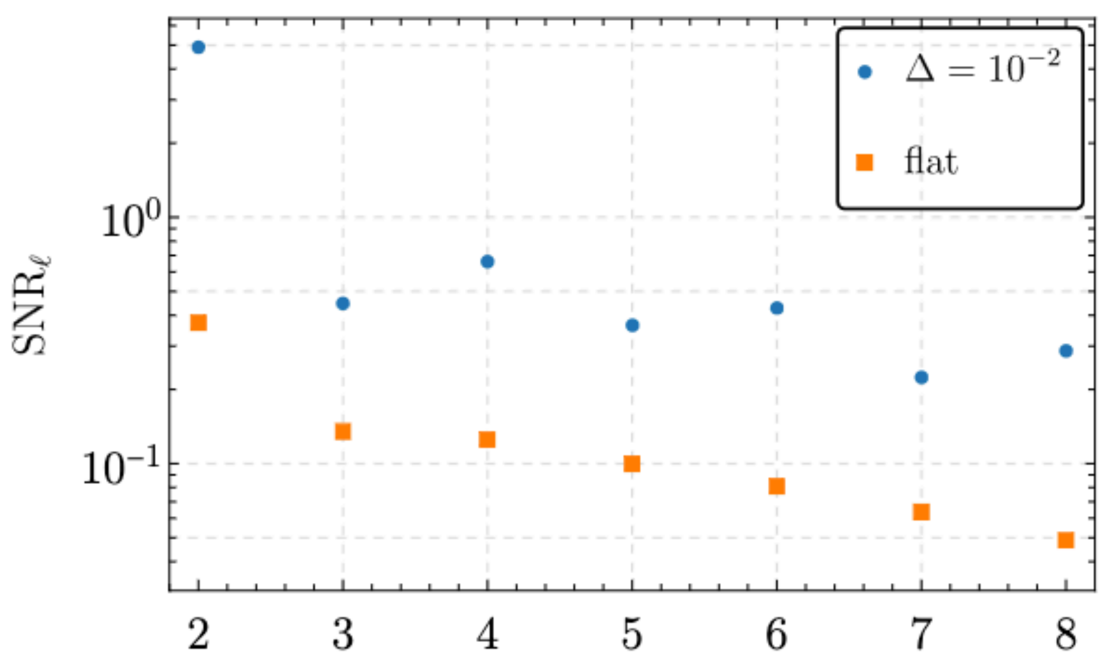
[ED, Fasiello, Malhotra, Tasinato 2022]

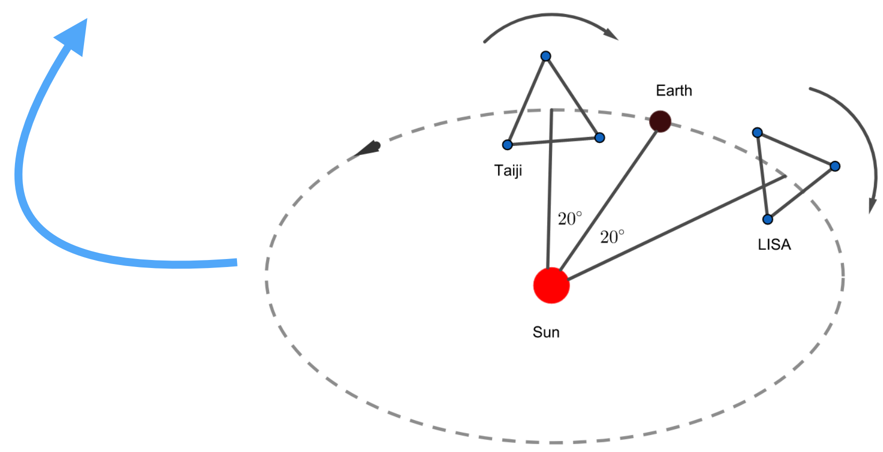
Anisotropies for peaked spectra

Considering a flat spectrum with the same SNR for monopole as peaked spectrum:



Signal to noise ratio for the individual multipoles:





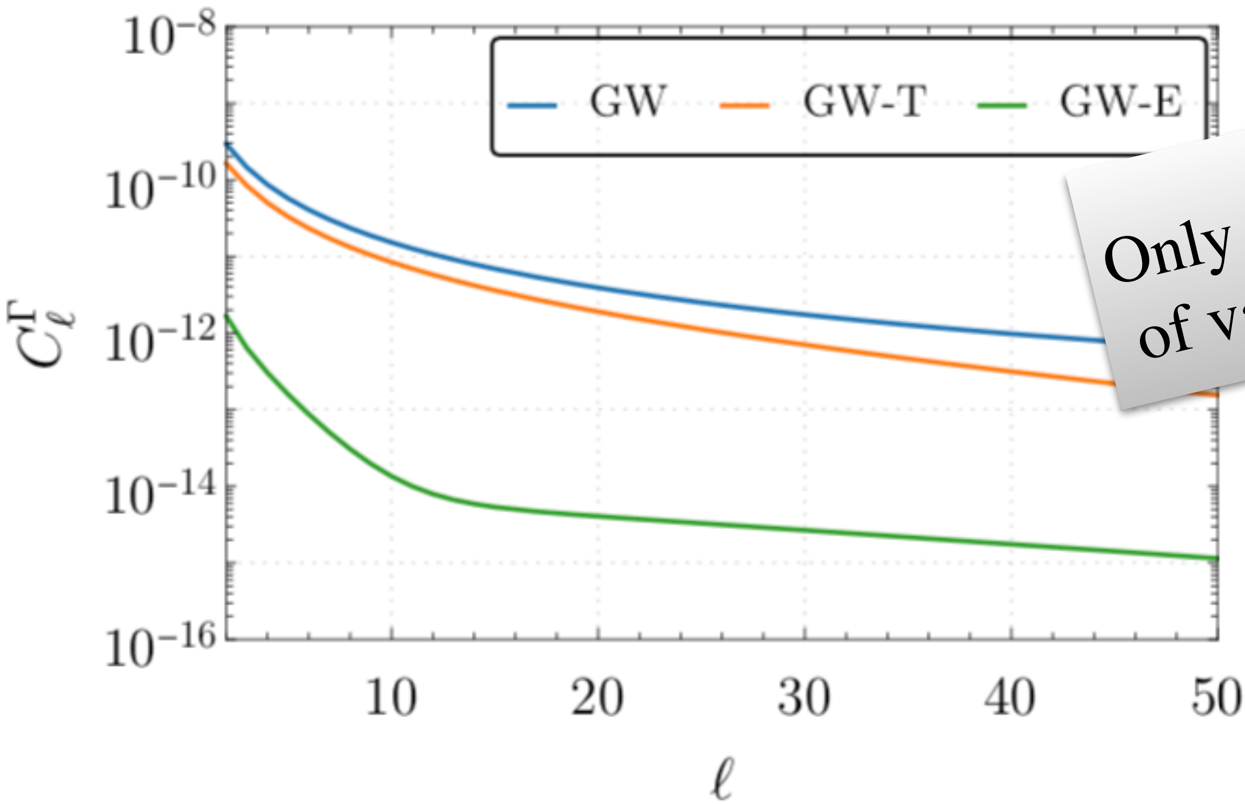
anisotropies are easier to detect compared to those of a flat spectrum!

[ED, Fasiello, Malhotra, Tasinato 2022]

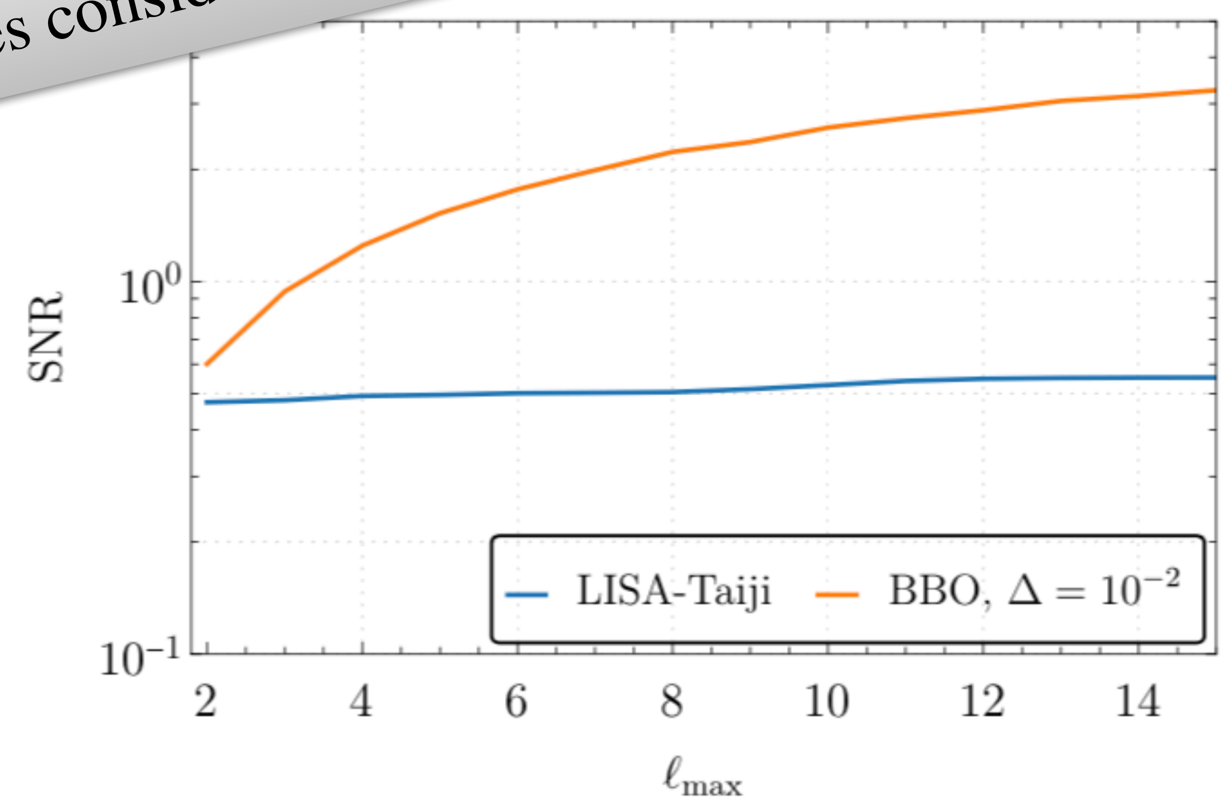
Cross-correlations

$$\mathrm{SNR}^2 = \sum_{\ell=2}^{\ell_{\max}} \sum_{\mathrm{X=T,E}} (2\ell+1) \frac{\left(C^{\mathrm{X}\Gamma}\right)^2}{\left(C^{\mathrm{X}\Gamma}\right)^2 + \left(C_\ell^\Gamma + N_\ell^\Gamma\right) C_\ell^{\mathrm{X}}}$$

(assuming full sky maps for both CMB and GW anisotropies)



Only valid for the specific set of values considered earlier!



[ED, Fasiello, Malhotra, Tasinato 2022]

Conclusions (Part II)

- For sharply peaked GW spectra (e.g. common to PBH formation scenarios), GW background anisotropies are enhanced w.r.t. power-law GW spectra

- In spite of enhancement being limited to a small range of scales, anisotropies easier to detect relative to those of power-law spectra.

- For a representative case, a LISA-Taiji network would be able to detect quadrupole, BBO would detect GW-CMB cross-correlations

- The distinct frequency dependence of these GW anisotropies potentially useful to separate these anisotropies from those associated with other (cosmological or astrophysical) GW backgrounds

Anisotropies from **isocurvature perturbations**

(part III)

Based on: Phys.Rev.D 107 (2023) 10, 103502

Guillem Domenech
(Hannover)

with

Ameek Malhotra
(Swansea)

Matteo Fasiello
(IFT Madrid)

Gianmassimo Tasinato
(Swansea)

Our initial question:

The observed GW background is in general determined both by:
(1) the specific source
(2) the evolution of the universe after production

In particular, an early non-standard cosmic history (e.g. early matter domination) could affect the frequency profile of Ω_{GW} in ways that are fully degenerate w.r.t. the imprints of the GW source.

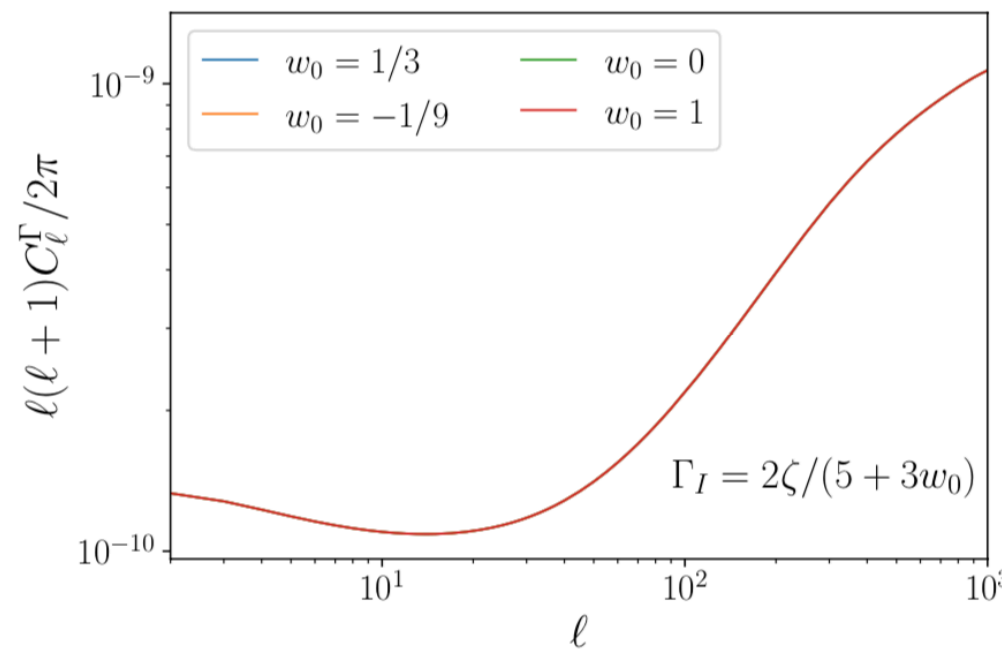
Can GW background anisotropies break this degeneracy?

<u>How do propagation anisotropies respond to a modified cosmic evolution?</u>

Our findings:

<u>universality of behaviour of anisotropies for adiabatic modes</u>

Anisotropies unaffected by an early non-standard phase of cosmic evolution so long as the primordial fluctuations are adiabatic



unsurprinsing: conservation of superhorizon curvature perturbation

↓ in turn:

A <u>departure from adiabaticity</u> would break universality condition

<u>Anisotropies as a probe of isocurvature modes</u>

Anisotropies in the presence of isocurvature perturbations

Discussed earlier: for adiabatic primordial perturbations: $\boxed{\Gamma_I = -\frac{1}{2}\Phi = \frac{\zeta}{3}}$

If there is an isocurvature perturbation of GW w.r.t. another component (e.g. radiation):

$$S_{\mathrm{GW},r} = 3\left(\zeta_{\mathrm{GW}} - \zeta_r\right) = \frac{3}{4}\left(\delta_{\mathrm{GW}} - \delta_{\mathrm{r}}\right)$$

$$\zeta = \sum_i f_i \zeta_i$$

$$f_i \equiv \frac{\rho_i}{\rho_{\mathrm{tot}}}$$ (fractional density for the "i" component)

$$\zeta_{\mathrm{GW}} = -\Psi - \mathcal{H}\frac{\delta\rho_{\mathrm{GW}}}{\rho'_{\mathrm{GW}}}$$
(curvature perturbation of GW)

$$\Gamma_I = \frac{1}{4}\frac{\delta\rho_{\mathrm{GW}}}{\bar{\rho}_{\mathrm{GW}}}$$ (Initial condition for GW anisotropies)

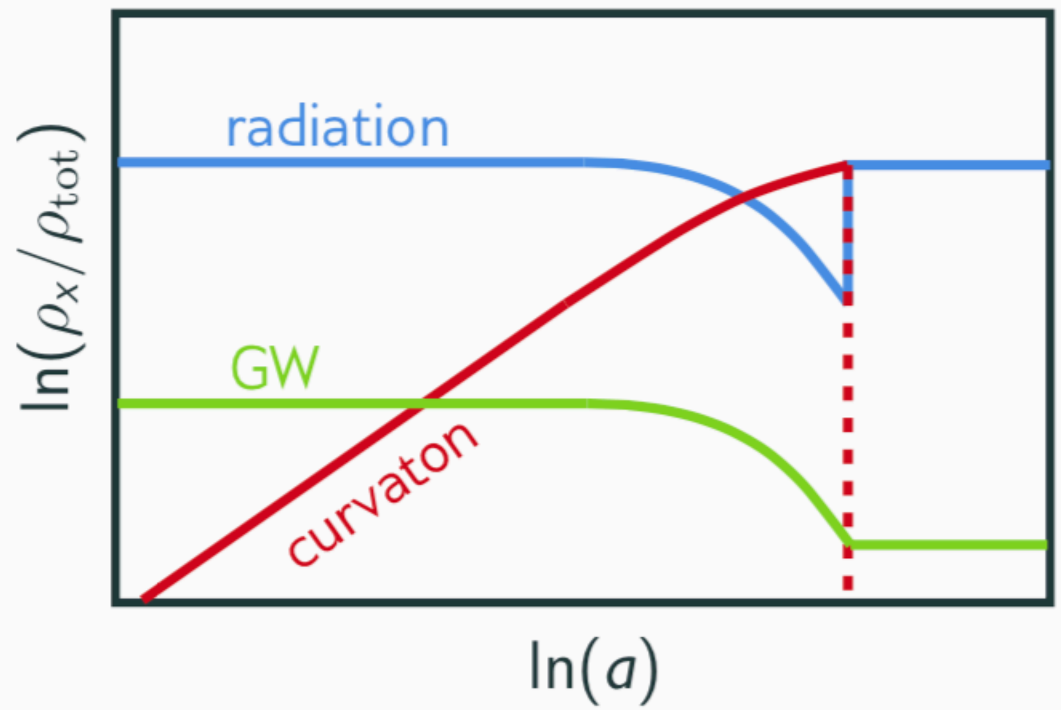
$$\longrightarrow \boxed{\Gamma_I = \frac{1}{3}\left(1 - f_{\mathrm{GW}}\right) S_{\mathrm{GW},r} + \frac{\zeta}{3}}$$

[Kumar, Sundrum, Tsai 2021]

Anisotropies in the presence of isocurvature perturbations

Case I:

- Curvaton dominates $\rho_{\rm tot}$ then decays *entirely* into radiation
- Fluctuation amplitude *fixed* by CMB normalisation



$$C_\ell \propto \Bigg[\underbrace{-\frac{4}{3}\zeta_r j_\ell(k\eta_0)}_{4\times\ \text{adiabatic term}}\Bigg]^2 + \text{standard ISW}$$

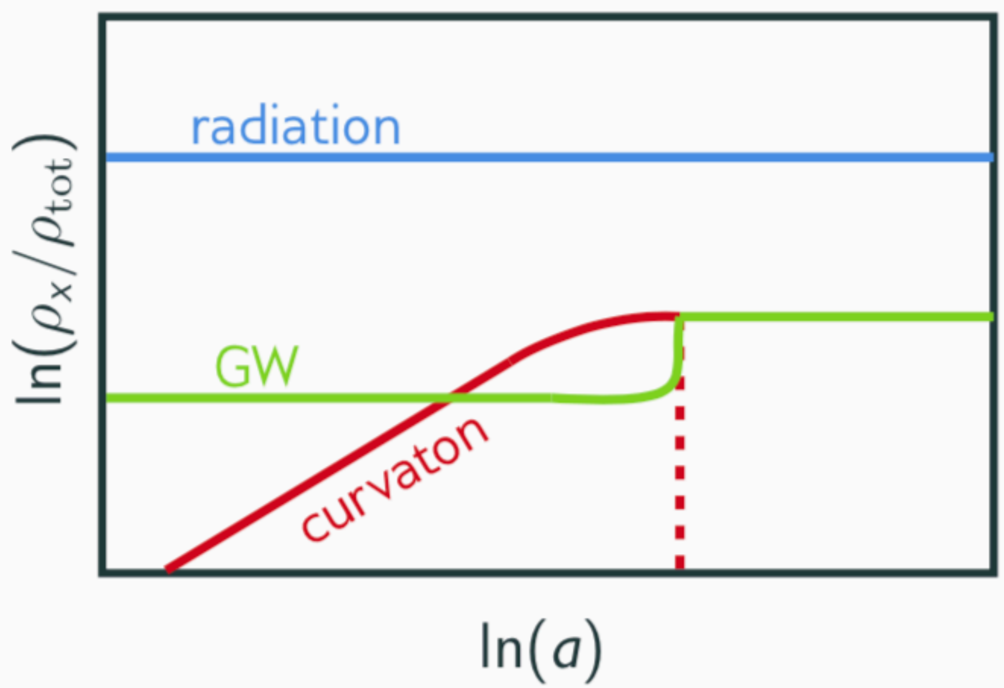
* Expect the GW background anisotropy map to be fully correlated with CMB

[Ameek Malhotra, ED, Guillem Domènech, Matteo Fasiello, Gianmassimo Tasinato 2023]

Anisotropies in the presence of isocurvature perturbations

Case II:

- Curvaton remains subdominant and decays *entirely* into GW
- Fluctuation amplitude *not fixed*



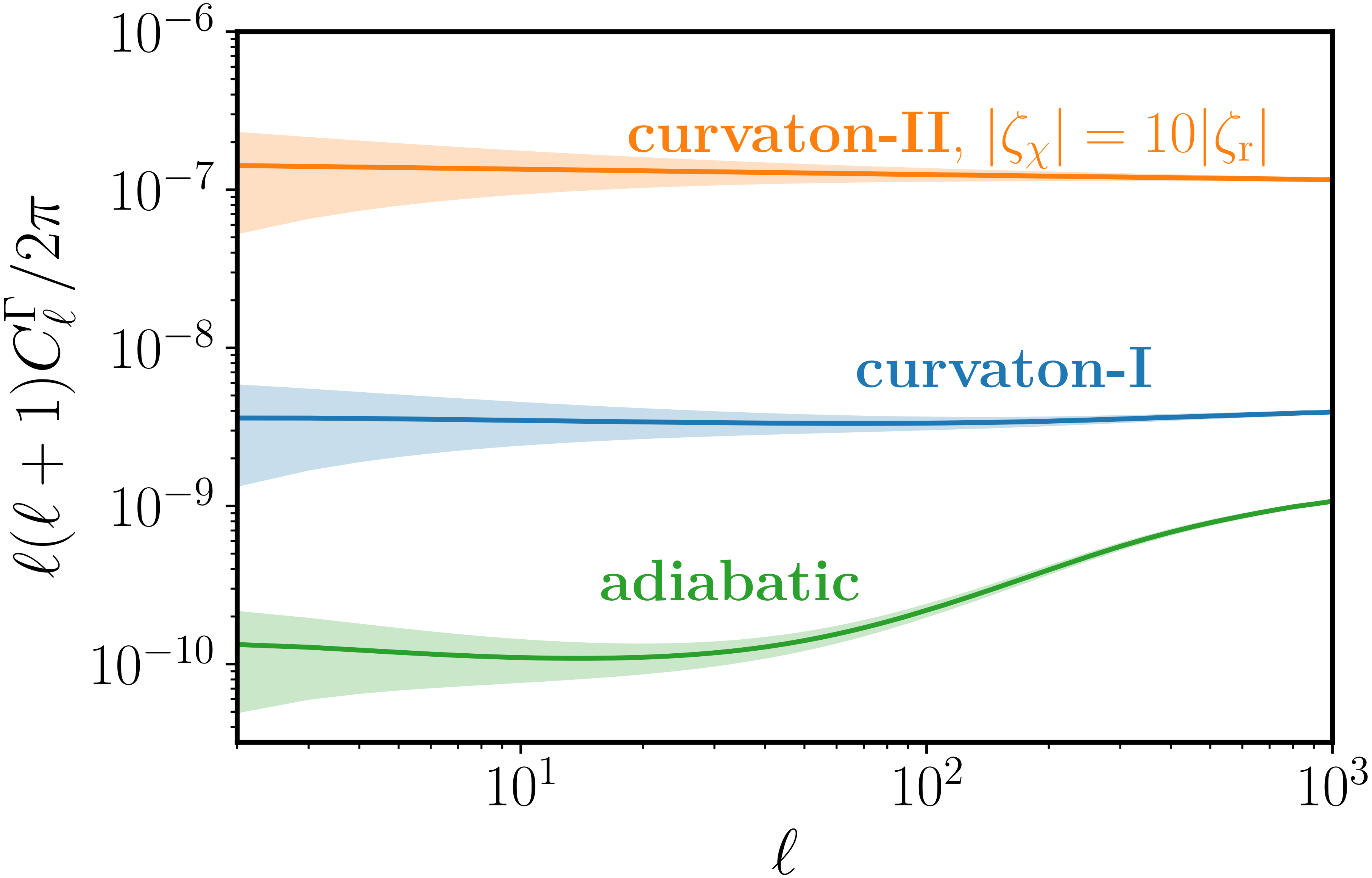
- $f^b_\chi \zeta_{\chi,\mathrm{ini}} \ll \zeta_r$, $\zeta_{\chi,\mathrm{ini}} \gg \zeta_r$

$$C_\ell \propto \left\{ \left[\frac{(1+w_\chi)}{(1+w_r)} \zeta_{\chi,\mathrm{ini}} - \frac{1}{3}\zeta_r \right] j_\ell(k\eta_0) \right\}^2 + \text{standard ISW}$$

* Expect little correlations between GW background and CMB anisotropies

[Ameek Malhotra, ED, Guillem Domènech, Matteo Fasiello, Gianmassimo Tasinato 2023]

Anisotropies in the presence of isocurvature perturbations



[Ameek Malhotra, ED, Guillem Domènech, Matteo Fasiello, Gianmassimo Tasinato 2023]

Conclusions (Part III)

- For adiabatic primordial fluctuations, GW anisotropies are insensitive to the EoS of the early universe $\longrightarrow$ universal behaviour

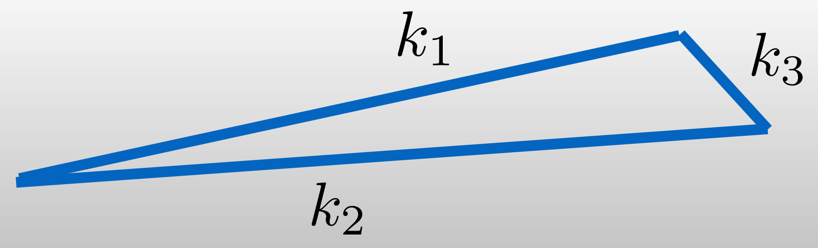
- Deviations from this universal behaviour is an indication of the presence of isocurvature fluctuations

- Isocurvature fluctuation can a sizeable enhancement of the GW anisotropies w.r.t. the adiabatic case

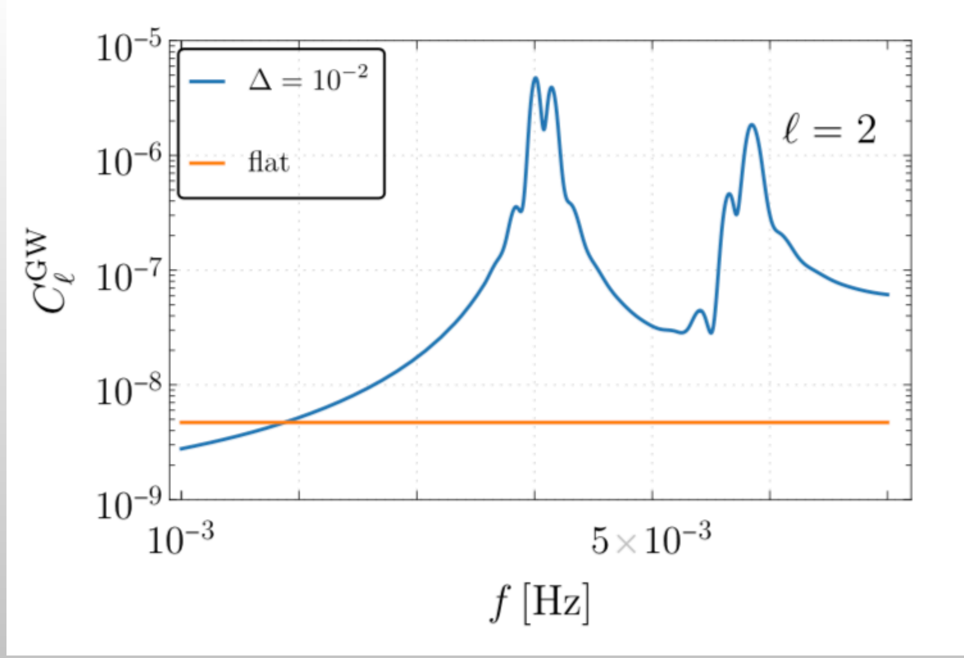
- GW-CMB cross-correlations can also be an effective probe for curvaton models

Outline

- Anisotropies as a probe of **squeezed non-Gaussianity**



- Anisotropies as a probe of **primordial black holes** physics



- Anisotropies as a probe of **isocurvature perturbations**