Primordial Black Holes and Induced Gravitational Waves from a Smooth Crossover beyond Standard Model

Albert Escrivà
postdoc at QG lab. Nagoya University (Japan)

YITP workshop on gravity and cosmology. 02/02/2024

Based on: A. Escrivà, Y. Tada and C.M Yoo. ArXiv:2311.17760

Motivation

A lot of literature on the topic

Currently, we have a very important connexion between

Gravitational waves <—> Primordial Black Holes

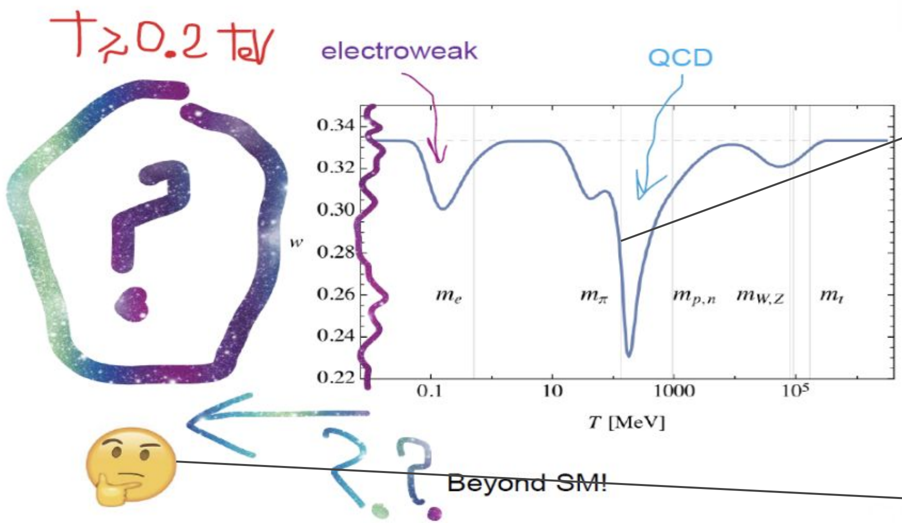
- Induced GWs-> can be a direct probe of the existence of primordial scalar curvature fluctuations, in particular at much smaller scales than the cosmic microwave background (CMB) scale.

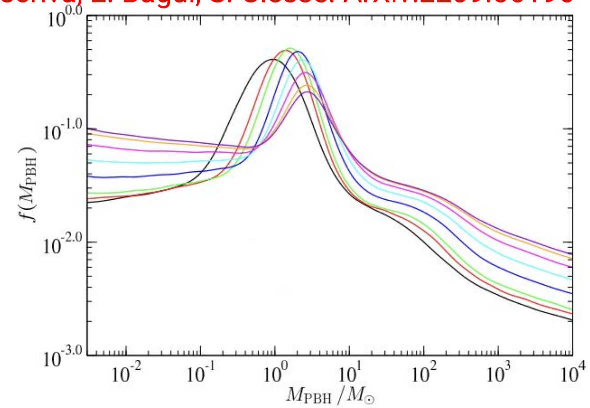
- But also, it can be an indirect probe of the existence of primordial black holes (scenario of PBH formation from the collapse of super-horizon curvature fluctuations).

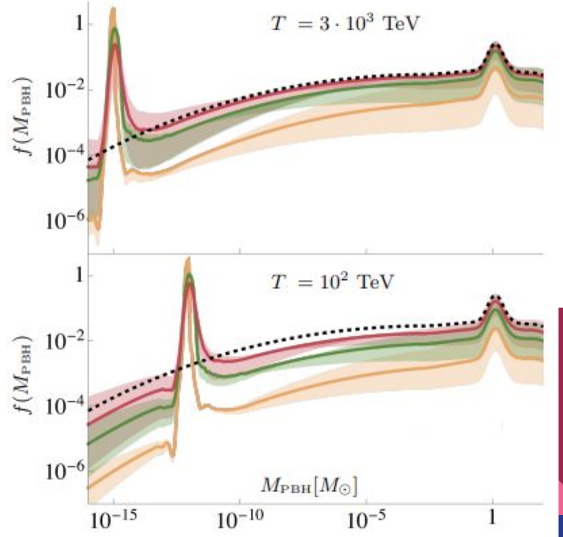
Motivation

A. Escrivà, E. Bagui, S. Clesse. ArXiv:2209.06196

In general, we focus on the radiation-dominated era, assuming a radiation-perfect fluid w=⅓







In the PBH scenario, actually can have important implications!

QCD

C. T. Byrnes, M. Hindmarsh, S. Young, M. R. S. Hawkins. Arxiv:1801.06138

B. Carr, S. Clesse, J. Garcia-Bellido, F. Kuhnel. ArXiv:1906.08217

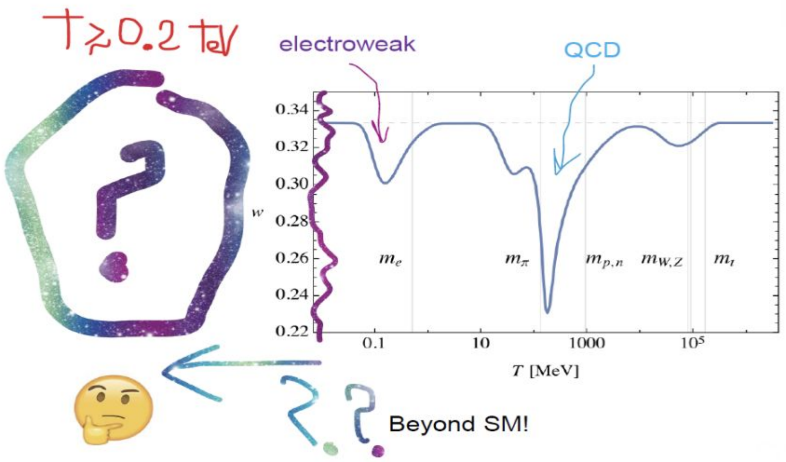
etc…

A. Escrivà, J. G. Subils. ArXiv:2211.15674

Motivation

Basically,

We want to obtain an observational signature (which could be tested with future space-based GW interferometers) from a hypothetical crossover beyond SM and make the corresponding connection with the PBH scenario.



Let's consider a flat power spectrum

The model

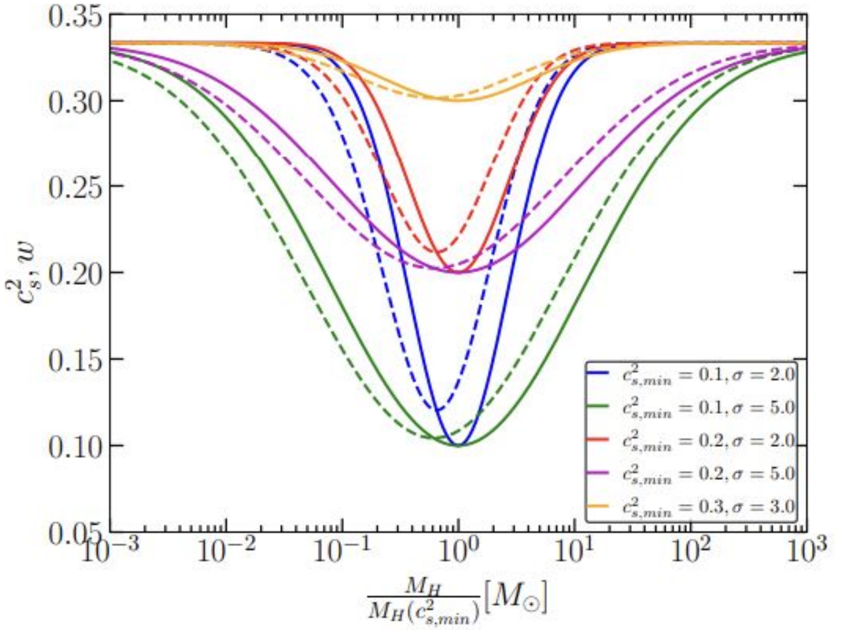
- Consider a hypothetical softening beyond SM with the following crossover template:

Model with two parameters

$$c_s^2(\rho) = w_0 - (w_0 - c^2_{\rm s,min}) \exp\left[\frac{-(\ln\tilde{\rho})^2}{2\sigma^2}\right]$$

$$w(\rho) = w_0 - \frac{\sigma}{\tilde{\rho}}\sqrt{\frac{\pi}{2}} e^{\sigma^2/2}(w_0 - c^2_{\rm s,min})\,{\rm erfc}\left[\frac{\sigma^2 - \ln(\tilde{\rho})}{\sqrt{2}\sigma}\right]$$

$$c_s^2(\rho) = \frac{\partial w(\rho)}{\partial\rho}\rho + w(\rho) \Leftrightarrow w(\rho) = \frac{1}{\rho}\int_{\bar{\rho}=0}^{\bar{\rho}=\rho} c_s^2(\bar{\rho})d\bar{\rho}.$$



BKG dynamics

$$\rho'(\eta) = -\sqrt{24\pi}(1 + w(\rho))a(\eta)\rho^{3/2}(\eta),$$

$$a'(\eta) = \sqrt{\frac{8\pi\rho(\eta)}{3}}a^2(\eta),$$

Gravitational collapse of super-horizon adiabatic curvature fluctuations

Sufficiently large fluctuations generated during inflation (very rare events) will collapse during the radiation epoch after they reenter the cosmological horizon.

(at super-horizon scales) $ds^2 = -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2\left(d\theta^2 + \sin^2(\theta)\, d\phi^2\right)\right]$

$(\nu = \mu/\sigma \gg 1)$

$\zeta(r) = \mu\, g(r) \quad \zeta \Rightarrow \frac{\delta\rho}{\rho}$

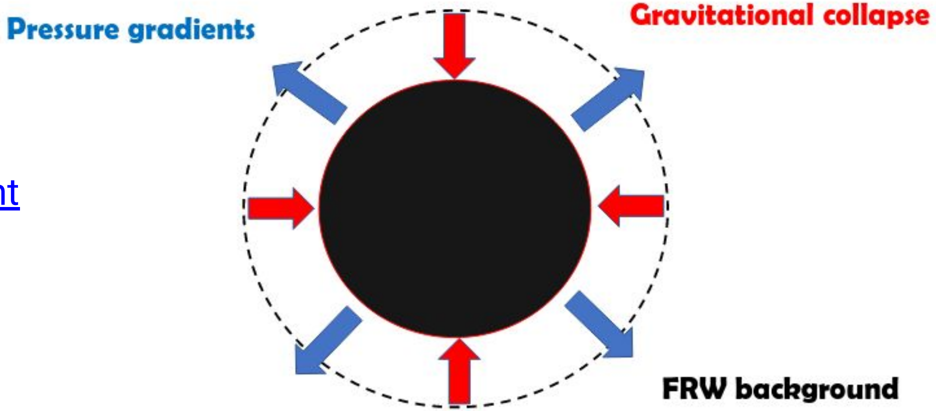
Large peaks ⟶ approximately spherically symmetric (BBKS paper. *Astrophys.J.* 304 (1986) 15-61)

$\mu > \mu_c$ Collapse of the fluctuations

$\mu < \mu_c$ Dispersion of the fluctuations

We need relativistic numerical simulations to determine the threshold value (most important quantity for estimating the PBH abundance)

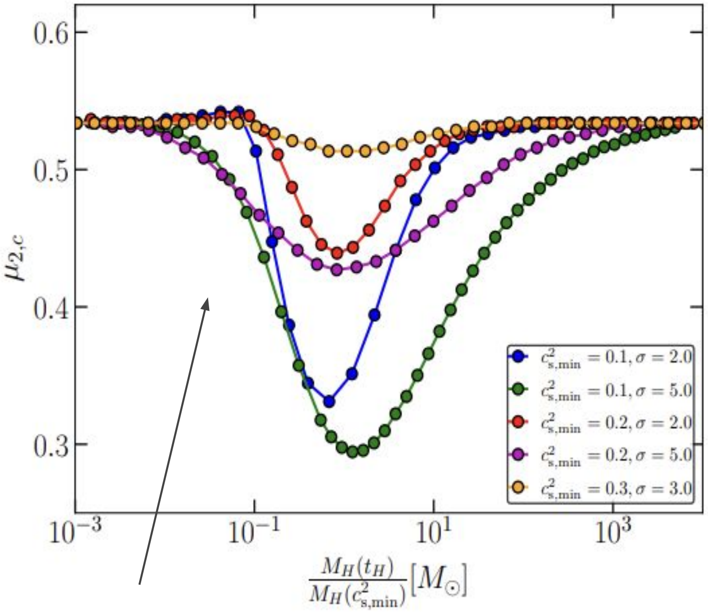
We use A. Escrivà. ArXiv:1907.13065



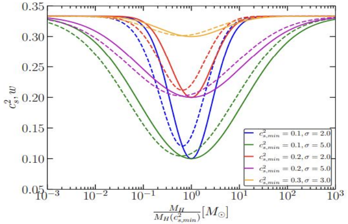
PBH threshold and abundance estimation

Let's consider peak theory to statistically estimate the PBH abundance

C.M Yoo, T. Harada, S. Hirano, K. Kohri. Arxiv:2008.02425



Numerical thresholds for PBH formation



"Typical profile" of the curvature fluctuation for flat spectrum

$$g(x;\kappa) = \frac{6}{x^4\lambda^4}\Big(-12\kappa^2 + x^2\lambda^2(3-4\kappa^2) + 4\left(3\kappa^2-2\right)x\lambda\sin\left(x\lambda\right) + 8 + \left[12\kappa^2 - 8 + x^2\lambda^2(1-2\kappa^2)\right]\cos\left(x\lambda\right)\Big)$$

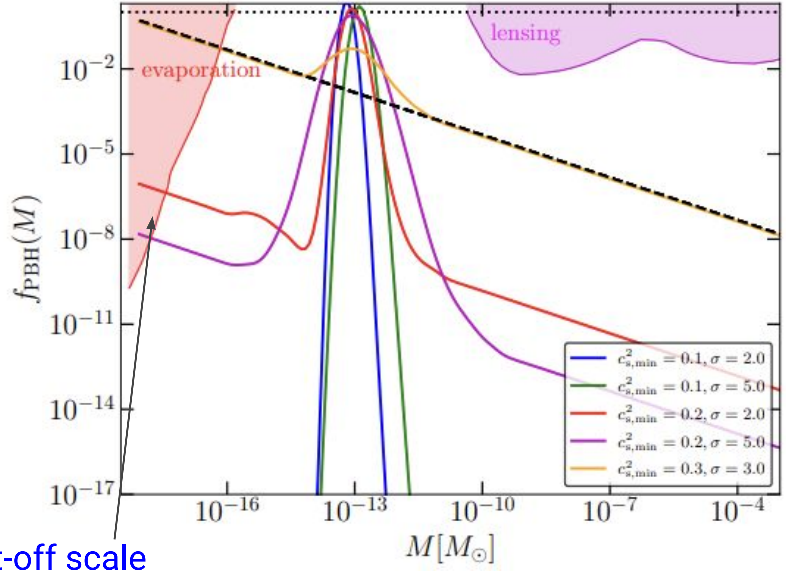
PBH abundance

$$\beta^{\rm approx}_{0,\max} := \left[\frac{3\kappa\lambda^3}{4\sqrt{\pi}(6\kappa^4-8\kappa^2+3)}e^{3\mu_2 g(1;\kappa)} \times f\left(\sqrt{\frac{2}{A}}\kappa^2\mu_2\right) P_1\left(\frac{\mu_2}{\sqrt{A}}, \sqrt{\frac{2}{A}}\kappa^2\mu_2\right) \times \left|\frac{\rm d}{{\rm d}\kappa}\ln\lambda + \mu_2\frac{\rm d}{{\rm d}\kappa}g_{\rm m}\right|^{-1}\right]_{\kappa=\kappa_{\rm t},\ \mu_2=\mu_{2,\rm c}(k_{\rm t})}$$

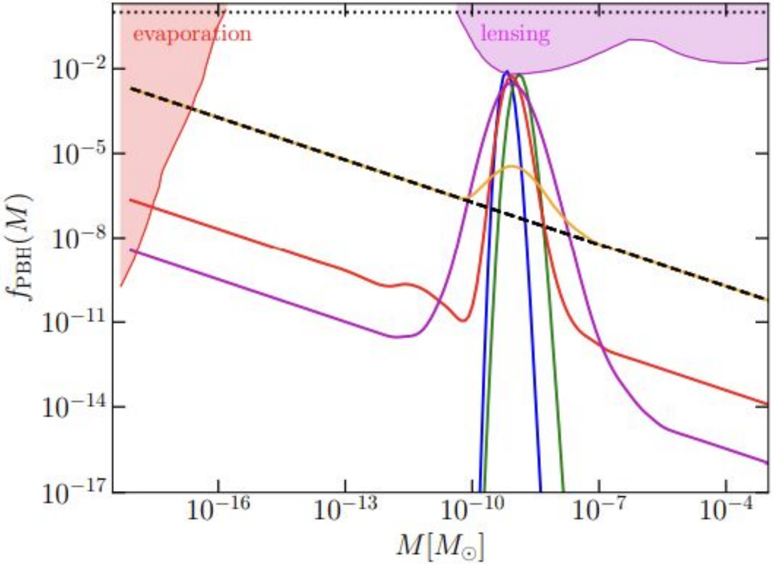
We make some simplifications

PBH mass function

Mass function highly peaked at the mass scale of the crossover



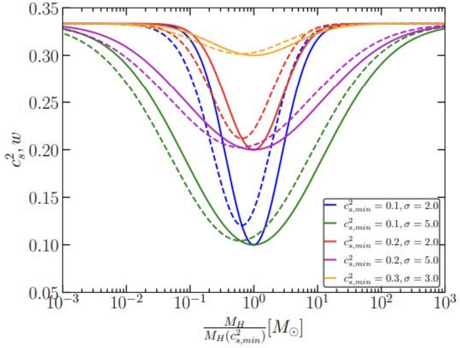
Cut-off scale



Model (A)-> $M_H \approx 10^{-13} M_\odot\,,\ \rho^{1/4} \approx 1.7 \cdot 10^6 \mathrm{GeV}$

Model (B)-> $M_H \approx 10^{-9} M_\odot\,,\ \rho^{1/4} \approx 17\,\mathrm{TeV}$

$c^2_{s,\rm min}$	σ	$\mathcal{A}^{\rm A}/10^{-3}$	$\mathcal{A}^{\rm B}/10^{-3}$
0.1	2.0	1.972	1.920
0.1	5.0	1.539	1.504
0.2	2.0	3.393	3.320
0.2	5.0	3.156	3.086
0.3	3.0	4.268	3.840



Computation of the induced GWs

New code using "Julia" language

QCD case-> K. T. Abe, Y. Tada, I. Ueda. ArXiv:2010.06193

The numerical computation is actually quite hard...

$$\Omega_{\rm GW}(k,\eta_0)h^2 = \Omega_{r,0}h^2 \left(\frac{a_{\rm sh}\mathcal{H}_{\rm sh}}{a_{\rm f}\mathcal{H}_{\rm f}}\right)^2 \frac{1}{24}\left(\frac{k}{\mathcal{H}_{\rm sh}}\right)^2 \overline{\mathcal{P}_h(k,\eta_{\rm sh})}.$$

$$\mathcal{P}_h(k,\eta) = \frac{64}{81a^2(\eta)}\int_{|k_1-k_2|\le k\le k_1+k_2} \mathrm{d}\ln k_1\,\mathrm{d}\ln k_2\, I^2(k,k_1,k_2,\eta) \times \frac{\left(k_1^2-(k^2-k_2^2+k_1^2)^2/(4k^2)\right)^2}{k_1k_2k^2}\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2), \quad (11)$$

$$I(k,k_1,k_2,\eta) = k^2\int_0^\eta \mathrm{d}\tilde\eta\; a(\tilde\eta)G_k(\eta,\tilde\eta)\left[2\Phi_{k_1}(\tilde\eta)\Phi_{k_2}(\tilde\eta) + \frac{4}{3(1+w(\tilde\eta))}\left(\Phi_{k_1}(\tilde\eta)+\frac{\Phi'_{k_1}(\tilde\eta)}{\mathcal{H}(\tilde\eta)}\right)\left(\Phi_{k_2}(\tilde\eta)+\frac{\Phi'_{k_2}(\tilde\eta)}{\mathcal{H}(\tilde\eta)}\right)\right]$$

Barden equation

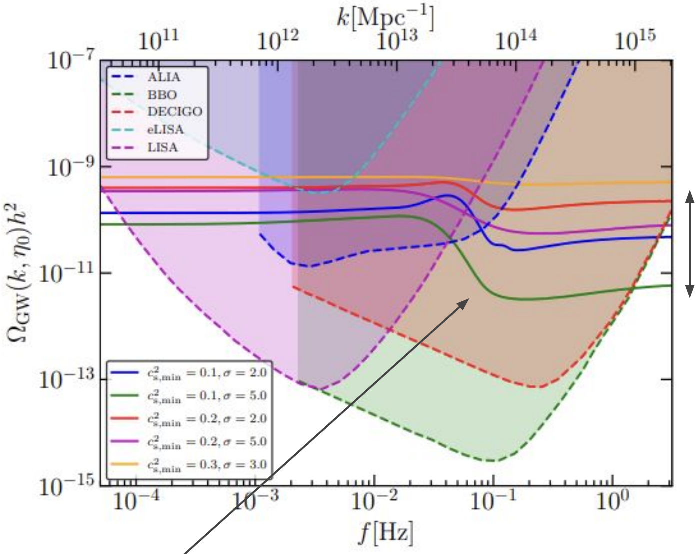
$$\Phi''_k(\eta) + 3\mathcal{H}(1+c_s^2)\Phi'_k(\eta) + \left[c_s^2k^2 + 3\mathcal{H}^2(c_s^2-w)\right]\Phi_k(\eta) = 0,$$

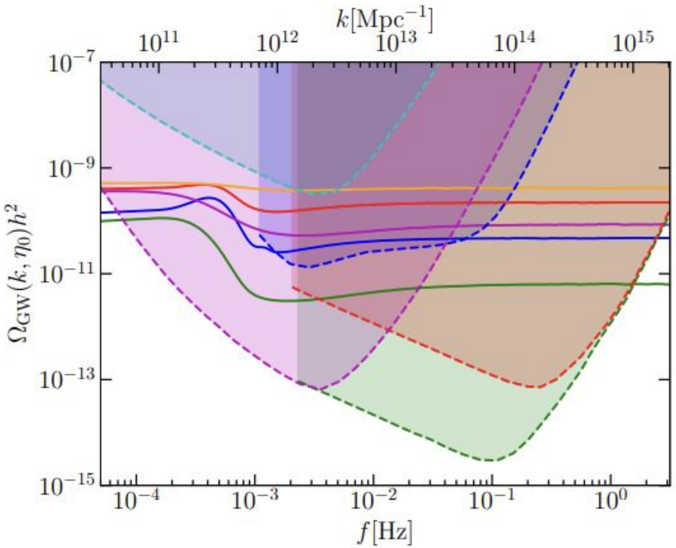
Green function

$$G_k(\eta,\tilde\eta) = \frac{1}{\mathcal{N}_k}\left[g_{1k}(\eta)g_{2k}(\tilde\eta) - g_{1k}(\tilde\eta)g_{2k}(\eta)\right]\Theta(\eta-\tilde\eta)$$

$$\left(\partial_\eta^2 + k^2 - \frac{1-3w(\eta)}{2}\mathcal{H}^2(\eta)\right)g_{jk}(\eta) = 0.$$

Gravitational Wave signature

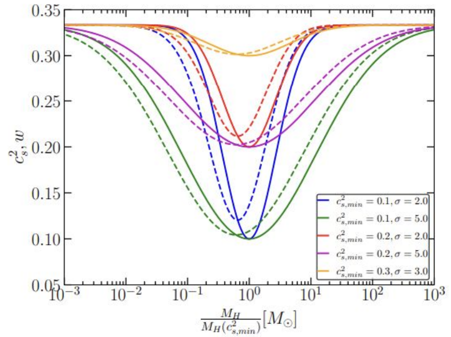




The "jump" between the two levels is caused by the crossover

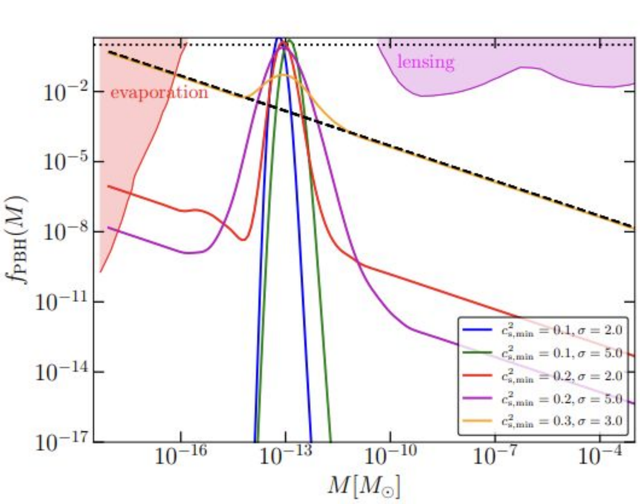
$a_{sh}\mathcal{H}_{sh}/a_f\mathcal{H}_f$

Difference between the two levels

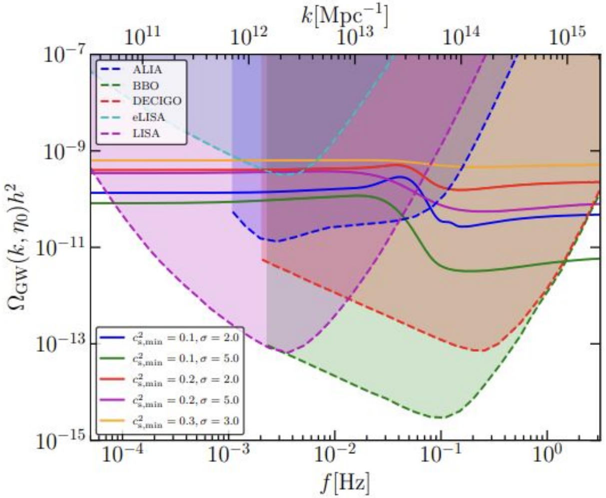


Solving the degeneracy

Our results show that the GW signal can be used to resolve the existing degeneracy of sharply peaked mass function caused by peaked power spectrums and broad ones in the presence of softening crossovers.



Theoretical prediction



Observational signature

Conclusions and messages to take home:

- Crossovers softenings beyond SM are usually not considered, but they can have important implications!

- We provide a potential observational signature for detecting a crossover beyond SM in future space based GW interferometers

- Such scenario can allow PBHs to be the dark matter, with a mass function specifically peaked at the mass scale of the minimum of the crossover.

- We solve the degeneracy of peaked mass functions caused by peaked power spectrums and broad ones in the presence of crossover softenings.

Extra slides 1

$$ds^2 = -A(r,t)^2\,dt^2 + B(r,t)^2\,dr^2 + R(r,t)^2\left(d\theta^2 + \sin^2(\theta)\,d\phi^2\right)$$

$$\dot{U} = -A\left[\frac{c_s^2(\rho)}{1+w(\rho)}\frac{\Gamma^2}{\rho}\frac{\rho'}{R'} + \frac{M}{R^2} + 4\pi R w(\rho)\rho\right],$$

$$\dot{R} = AU,$$

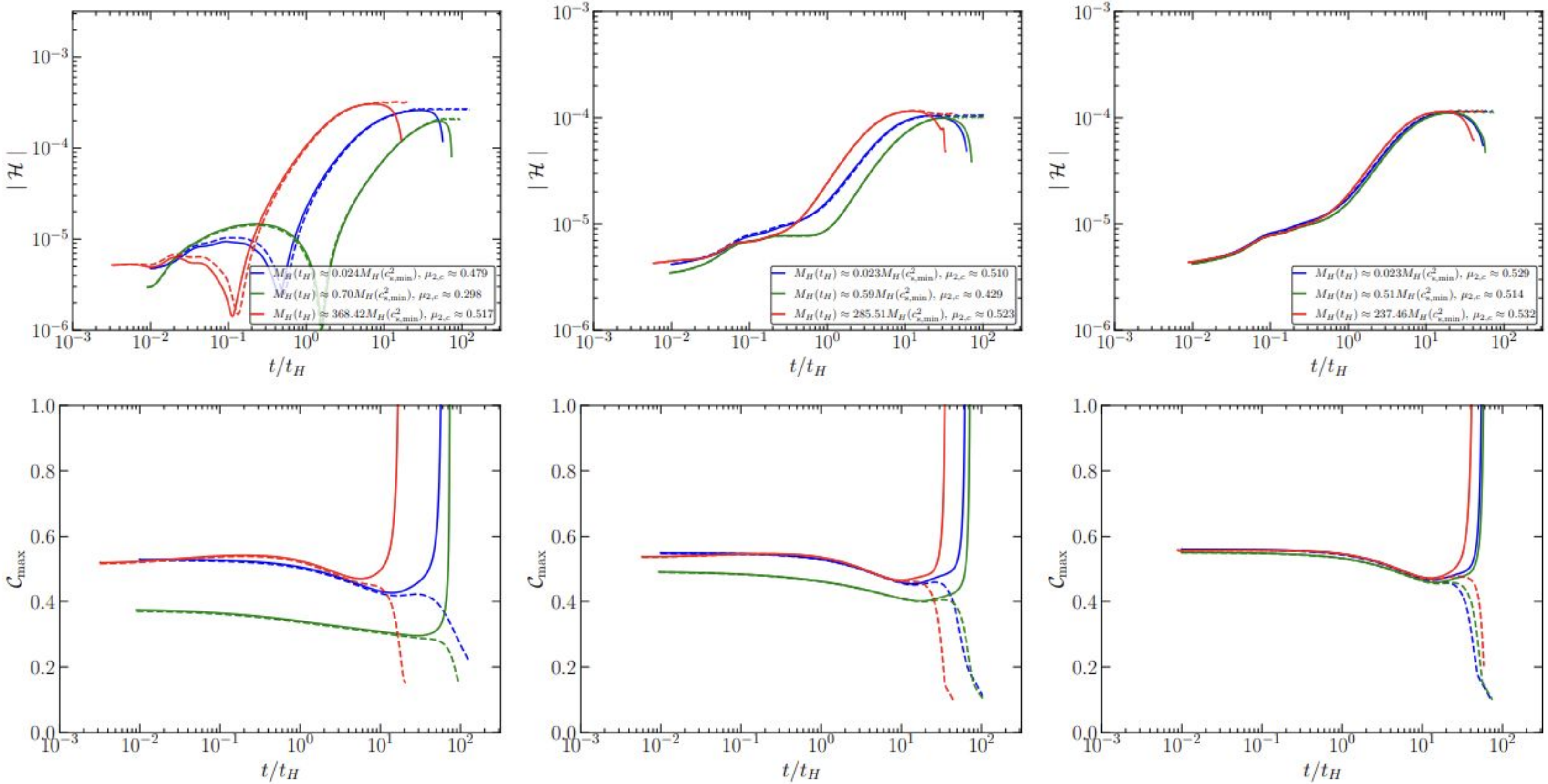
$$\dot{\rho} = -A\rho\left[1+w(\rho)\right]\left(2\frac{U}{R} + \frac{U'}{R'}\right),$$

$$\dot{M} = -4\pi A w(\rho)\rho U R^2,$$

$$A' = -A\frac{\rho'}{\rho}\frac{c_s^2(\rho)}{1+w(\rho)},$$

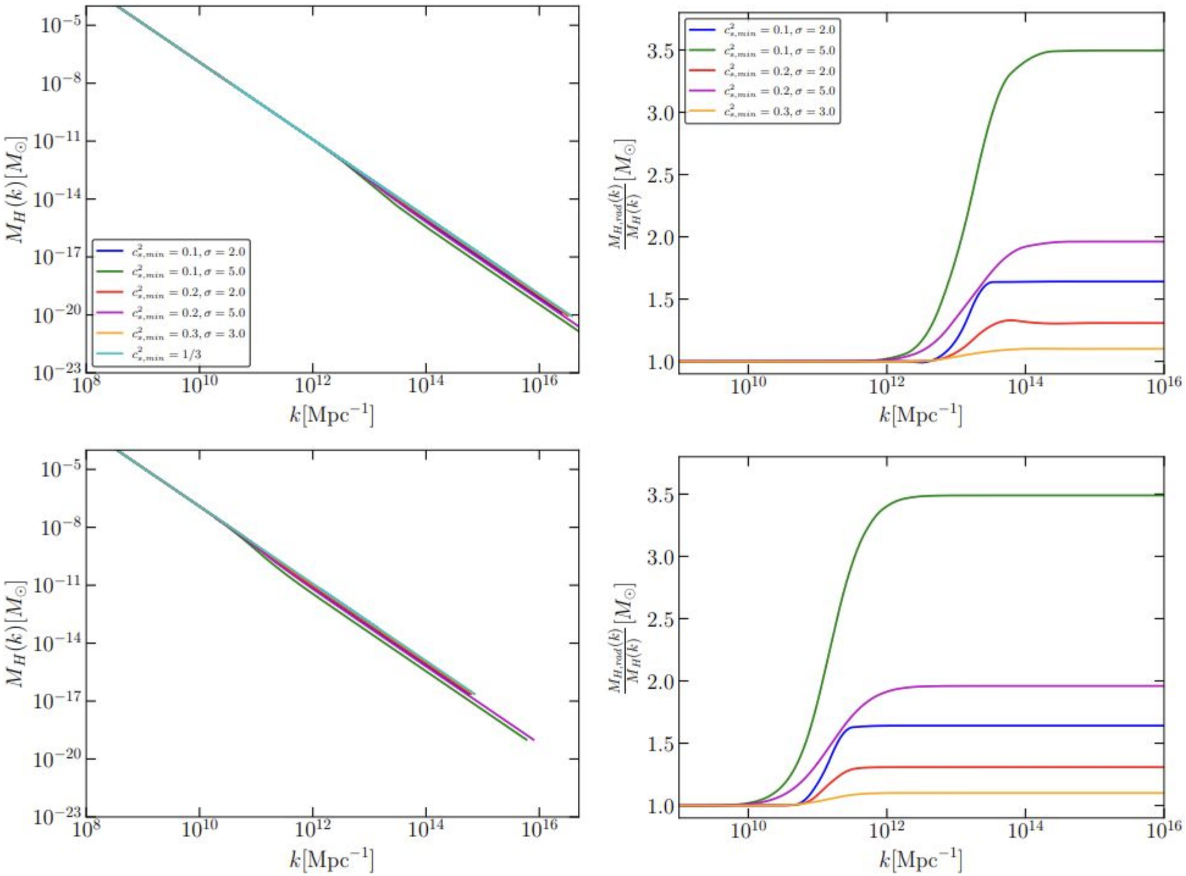
$$M' = 4\pi\rho R^2 R',$$

Extra slides 2



Extra slides 3

$$a^2(t_H)\rho(t_H)\tau^2 = \rho_{\rm b}(t_0),$$



Extra slides 4