Relativistic evolution of the self-interacting axion condensate

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Work in progress with
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Introduction

- Axion is attracting many interests!
  - Solution to strong CP problem
  - "String Axiverse." String theory predicts the plentitude of axions. (Arvanitaki+,2009)
  - Candidate of the dark matter.
  - Can be observed by the astrophysical/cosmological phenomena.

Axion mass (eV)

$\sim 10^{-33}$ | $\sim 10^{-26}$ | $\sim 10^{-20}$ | $\sim 10^{-10}$

Can be probed by cosmological phenomena

Size of astrophysical black hole

→**Black hole as axion detector**

$\sim H_0^{-1}$ | $\sim 1\mathrm{kpc}$ | $\sim 10^{10} M_\odot$ | $\sim M_\odot$

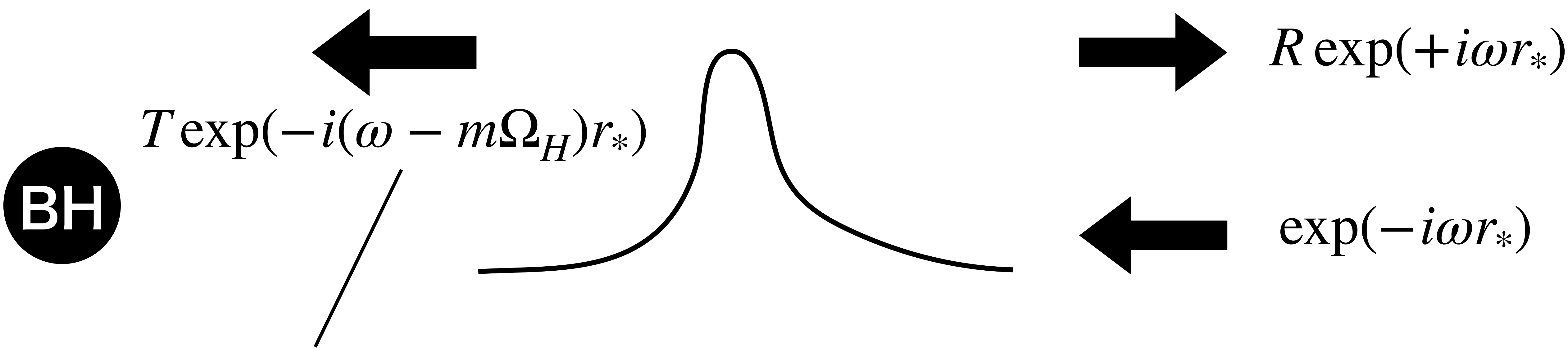
Compton wavelength

Searching axion with black hole

Superradiant instability turns black holes into the axion detector. (Arvanitaki+,2010)

Superradiance

Energy and angular momentum extraction from black hole via wave (Press&Teukolsky,1972,……)



Wave number changes due to black hole spin

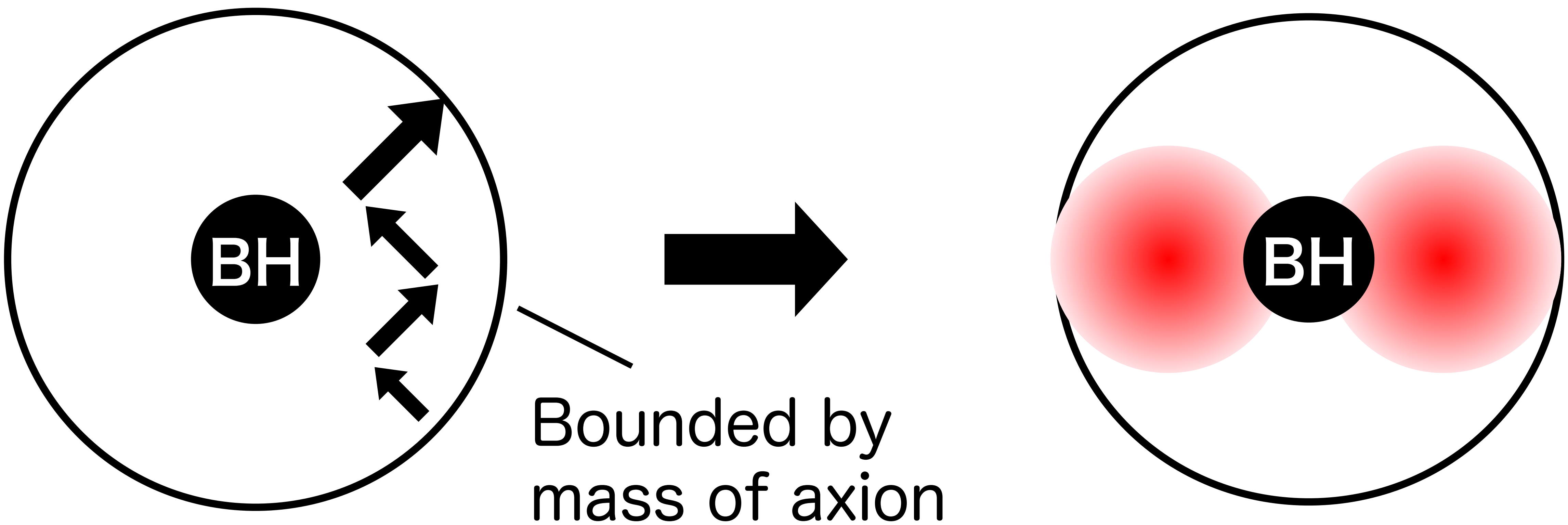
Particle number conservation,

$$|R|^2 = 1 - \frac{\omega - m\Omega_H}{\omega}|T|^2 > 1$$

Superradiant instability

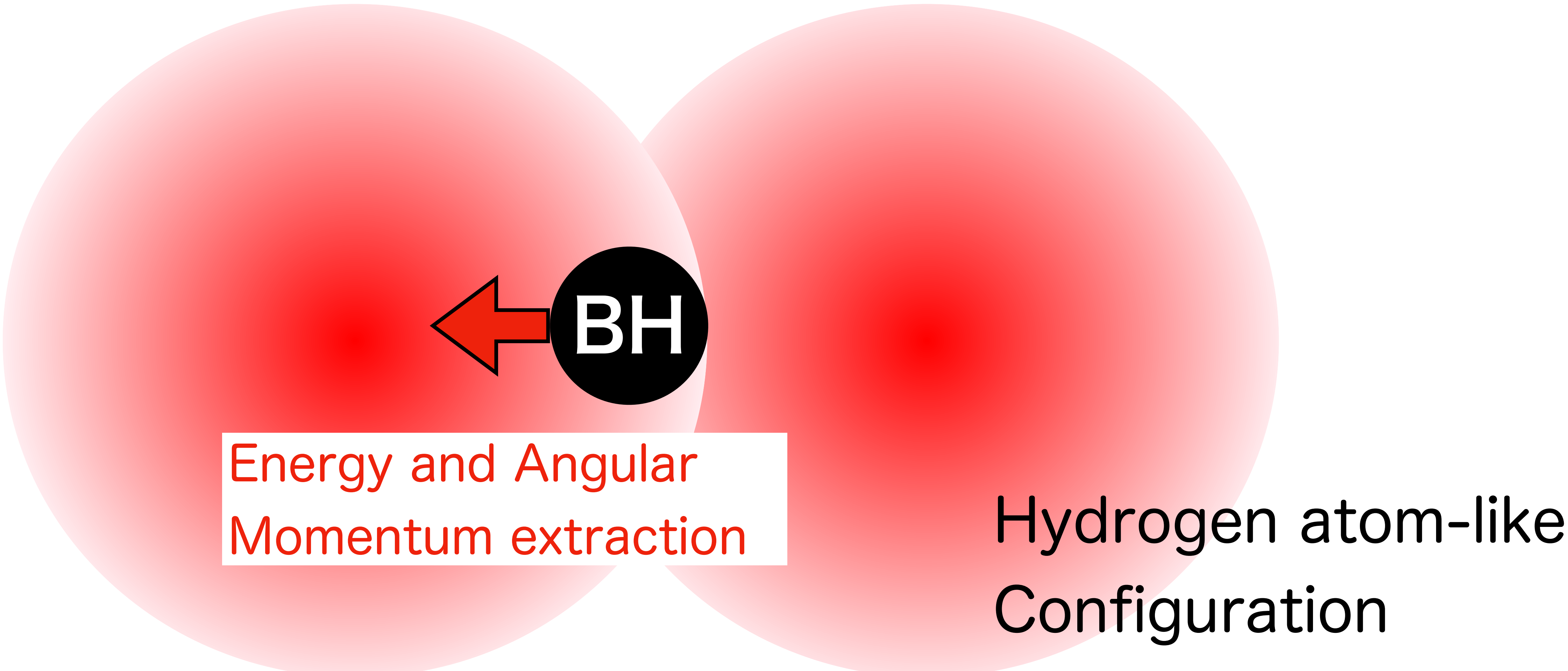
(Press&Teukolsky,1972,……)

Instability due to multiple occurence of superradiance



Spontaneous formation of the condensate

Axion cloud



$$\phi = R_{lm\omega}(r)S_{lm\omega}(\theta)e^{-i(\omega t - m\varphi)} + \mathrm{c.c.}$$

$$\omega = \omega_R + i\omega_I \qquad \tau = \omega_I^{-1} \sim 1\mathrm{min} \ (\mu M \sim 0.42, M \sim M_\odot)$$

$$\underline{\omega_I > 0}, \ \underline{\omega_R \sim \mu \gg \omega_I}, \ \underline{\omega_R - m\Omega_H < 0}$$

Instability — Adiabatic growth — Supperradiance condition

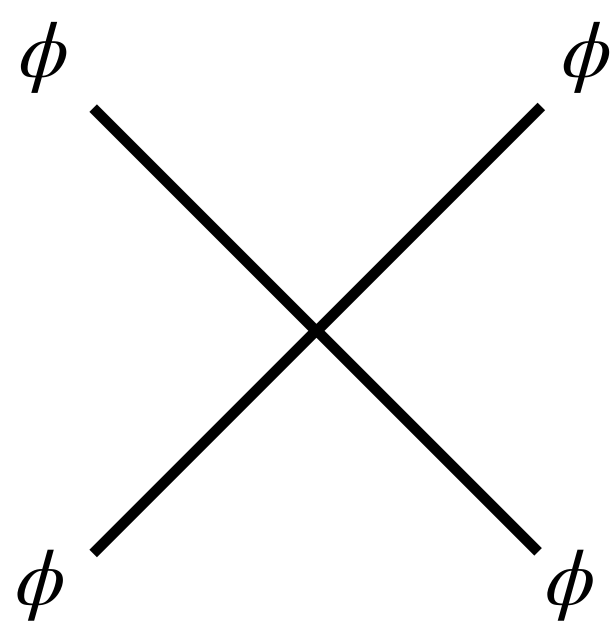
Self-interaction

$$S = F_a^2 \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial_\mu \phi)^2 - \mu^2 \left(1 - \cos\phi\right) \right]$$

g:Kerr metric ϕ:Axion μ:mass F_a:decay constant

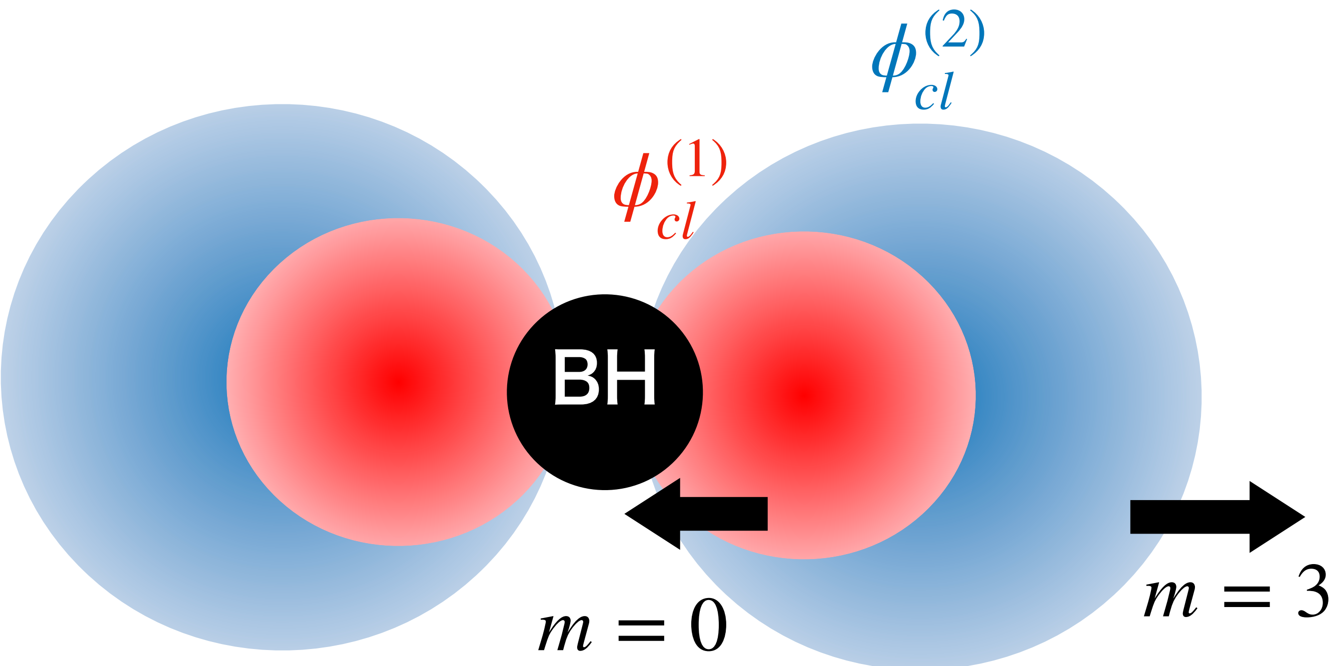
When condensate becomes dense,
self-interaction affects evolution.

The main effect is the dissipation of the condensate.



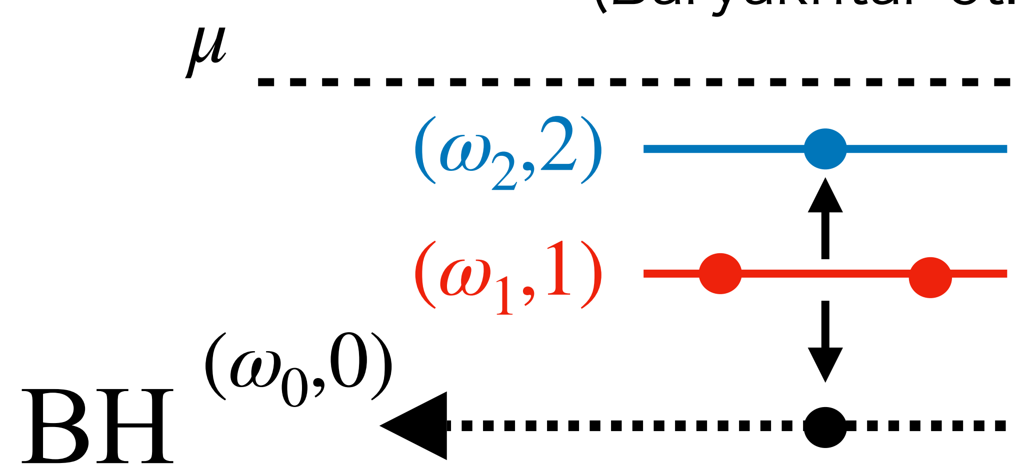
Dissipation by mode coupling

(Baryakhtar et. al. 2020)



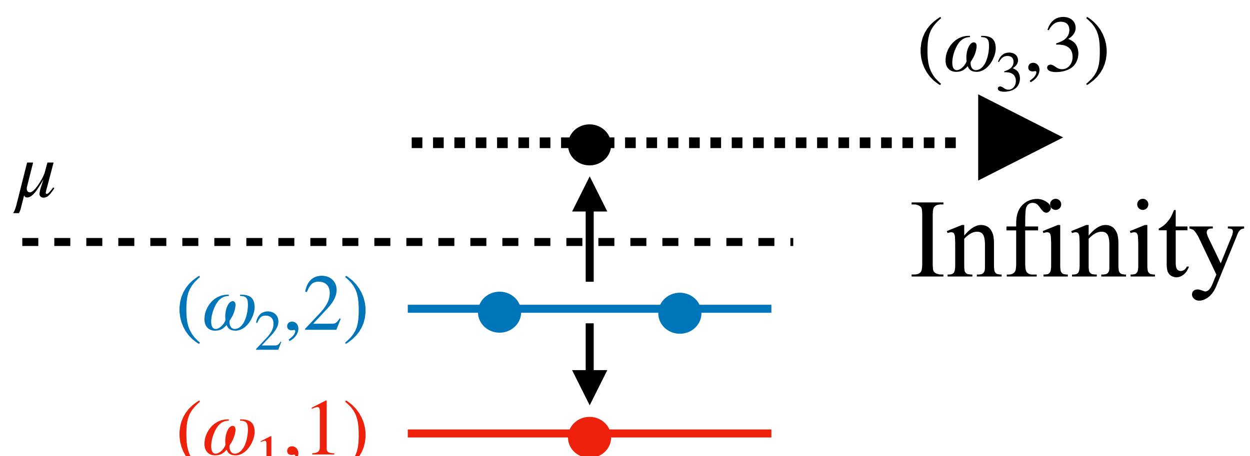
$\phi_{cl}^{(1)} : l = m = 1, \omega_R^{(1)} < \mu$

$\phi_{cl}^{(2)} : l = m = 2, \omega_R^{(1)} < \omega_R^{(2)} < \mu$



$l = m = 1$ transit to $l = m = 2$.

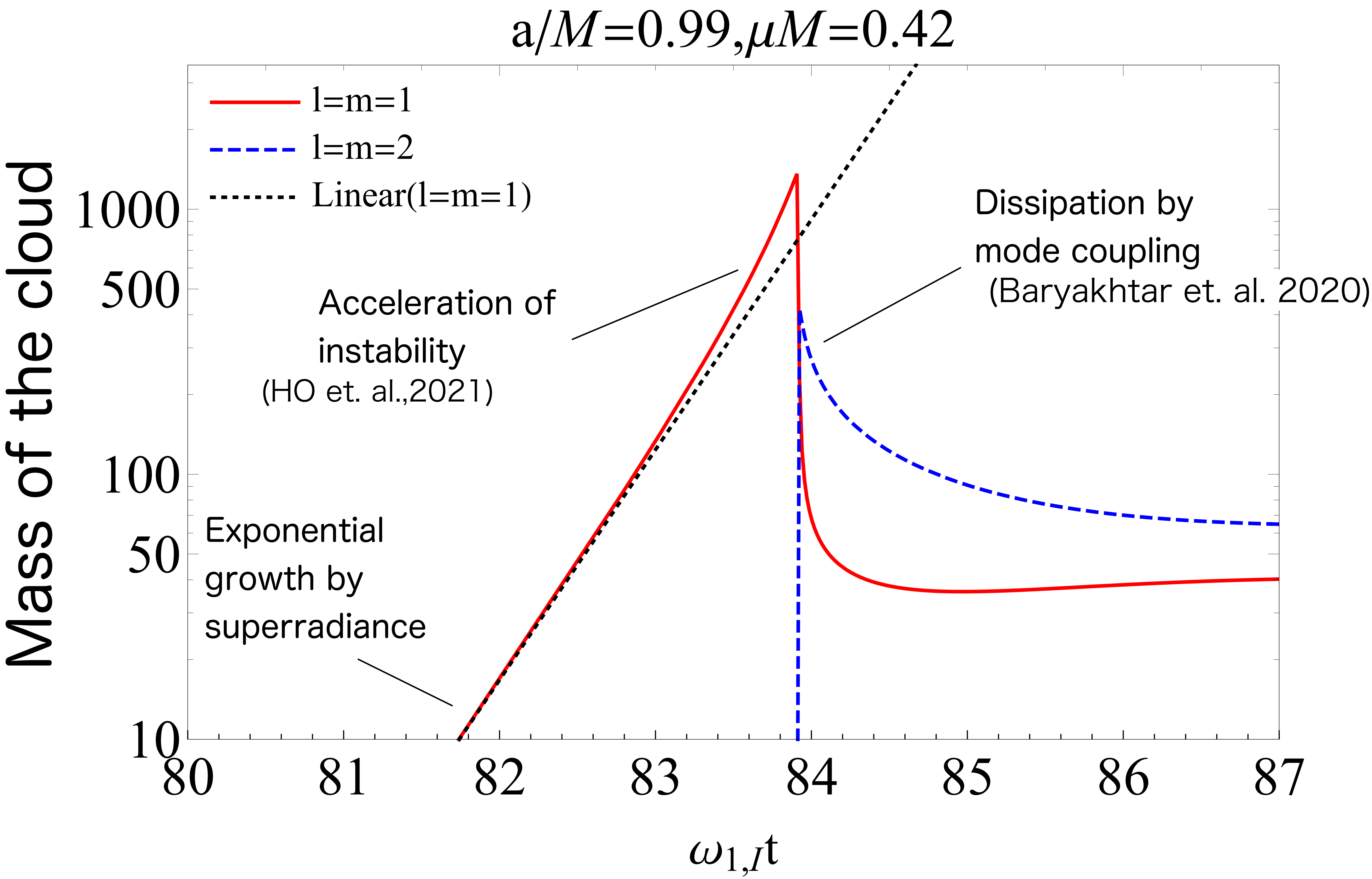
$m = 0$ mode dissipates energy to black hole.



$l = m = 2$ transit to $l = m = 1$.

$m = 3$ mode dissipates energy to infinity.

Evolution of the cloud



Situation so far

- Non-relativistic calculation with two mode.

(Baryakhtar et. al. 2020)

Non-relativistic means leading in $G\mu M_{\rm BH}$.

※Velocity of a particle with mass μ in Newton potential: $v \sim G\mu M_{\rm BH}$

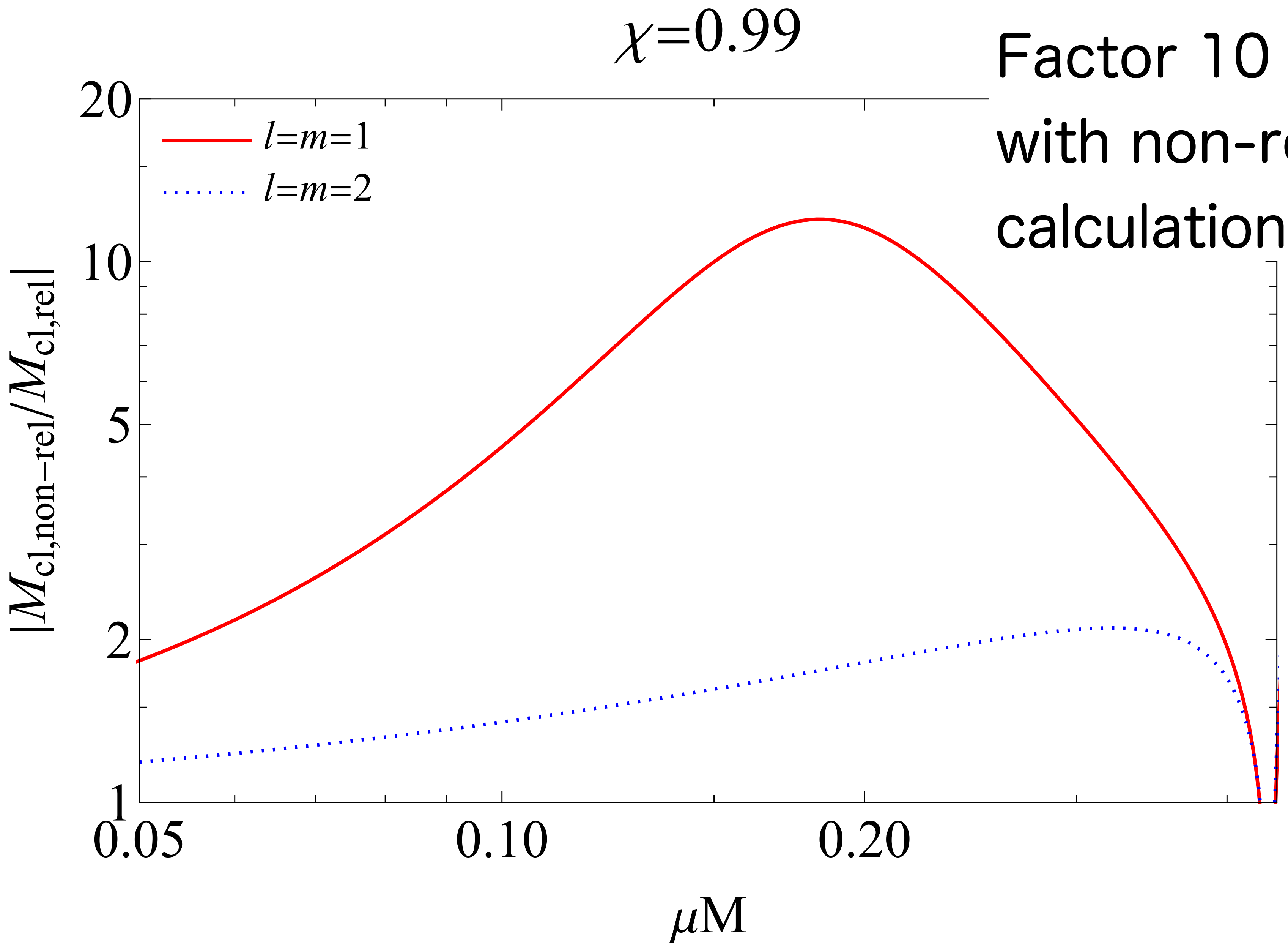
- Relativistic calculation with three modes without backreaction to black hole spin. (HO et. al. 2022)

Suggest multiple modes excite at the same time.
But, the cloud never decays. No spin down.

We perform calculation including effects, not included in the previous calculation.

Importance of the relativistic effect

For example, mass of the cloud at saturation.



Factor 10 overestimation with non-relativistic calculation!

- The relativistic collection also affects the estimation of gravitational waves. (Yoshino&Kodama, 2013)

Four mode analysis

(Work in progress)

$$\frac{dM_1}{dt} = 2\omega_{1,I}M_1 - 2F_0M_1^2M_2 + F_3M_1M_2^2 + \cdots \ ,$$

$$\frac{dM_2}{dt} = 2\omega_{2,I}M_2 + F_0M_1M_2^2 - 2F_3M_1M_2^2 + \cdots \ ,$$

$$\frac{dM_3}{dt} = 2\omega_{3,I}M_3 + \cdots \ , \quad \frac{dM_4}{dt} = 2\omega_{4,I}M_4 + \cdots \ ,$$

Evolution equation of condensate

$$\frac{dM}{dt} = -F_a^2\left(2\omega_{1,I}M_1 + \cdots\right) \ ,$$

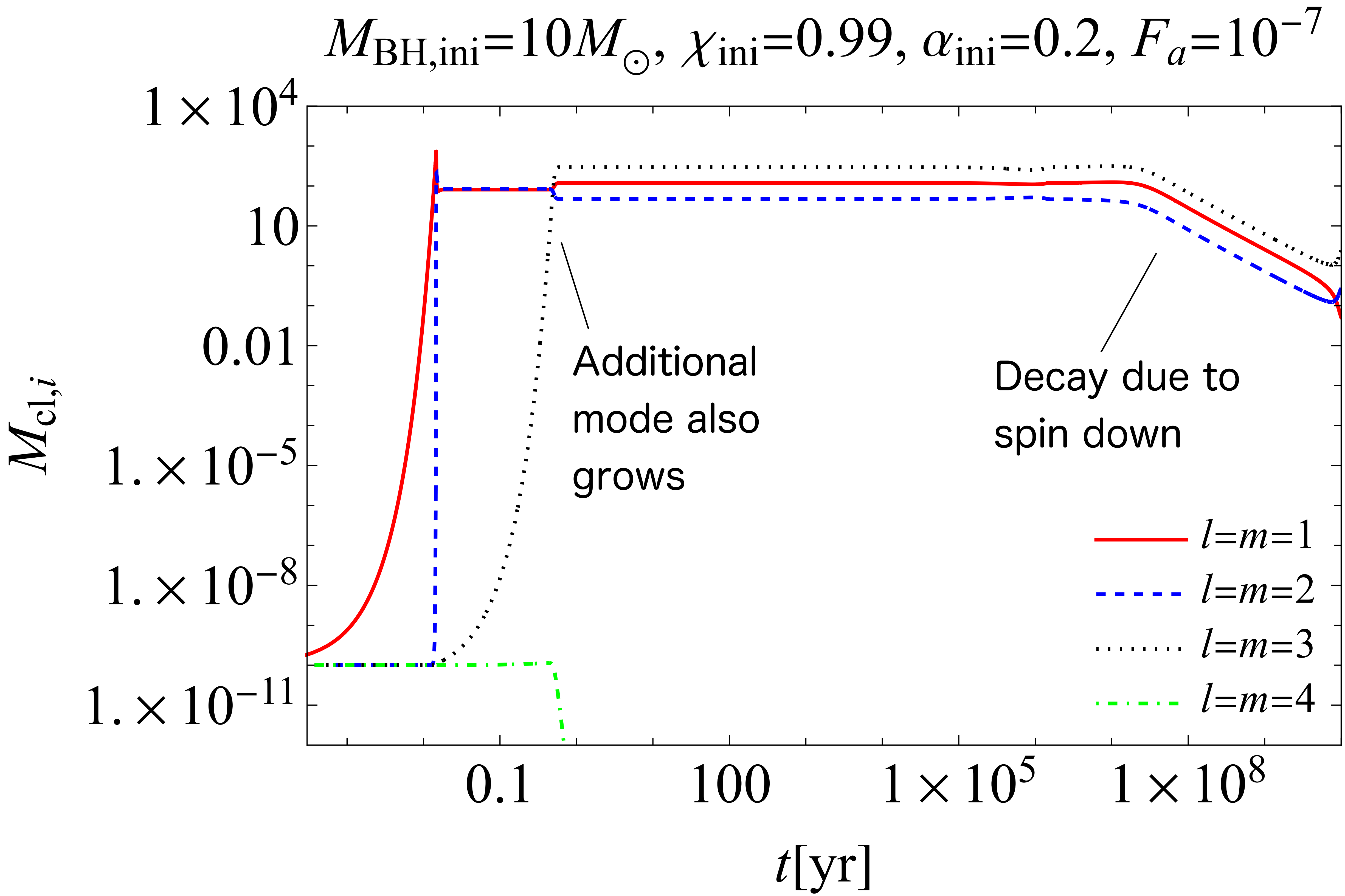
$$\frac{dJ}{dt} = -F_a^2\left(2\omega_{1,I}\frac{m_1M_1}{\omega_{1,R}} + \cdots\right) \ ,$$

Evolution equation of black hole

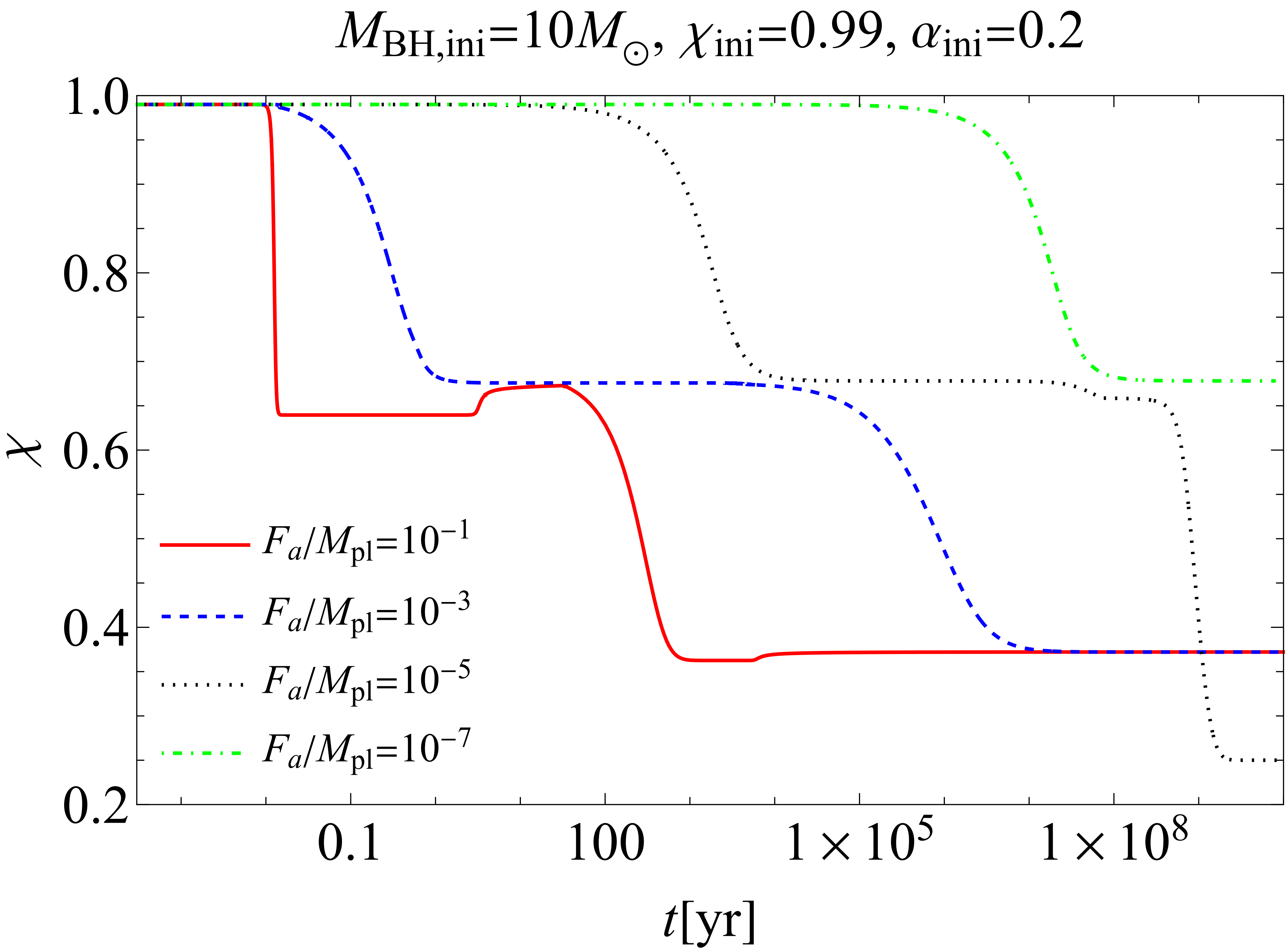
$M_{1,2,3,4}$: Mass of $l = m = 1,2,3,4$ cloud

M, J: Mass and Angular momentum of the black hole

Example of time evolution

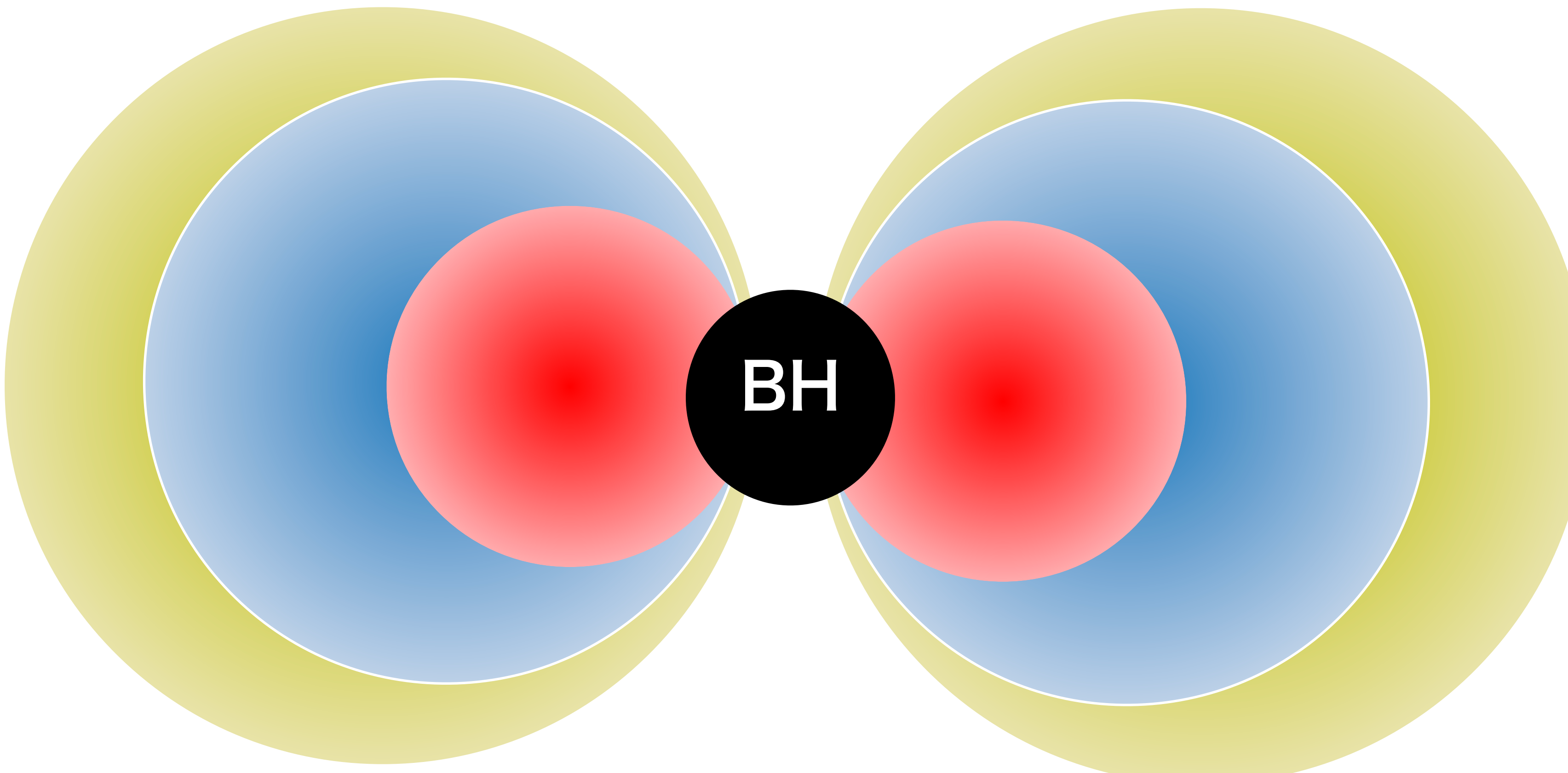


Example of time evolution



Observational Signals

Cloud spends some time with multiple modes simultaneously excited.



Expected Signals

- Gravitational wave
- Axion wave

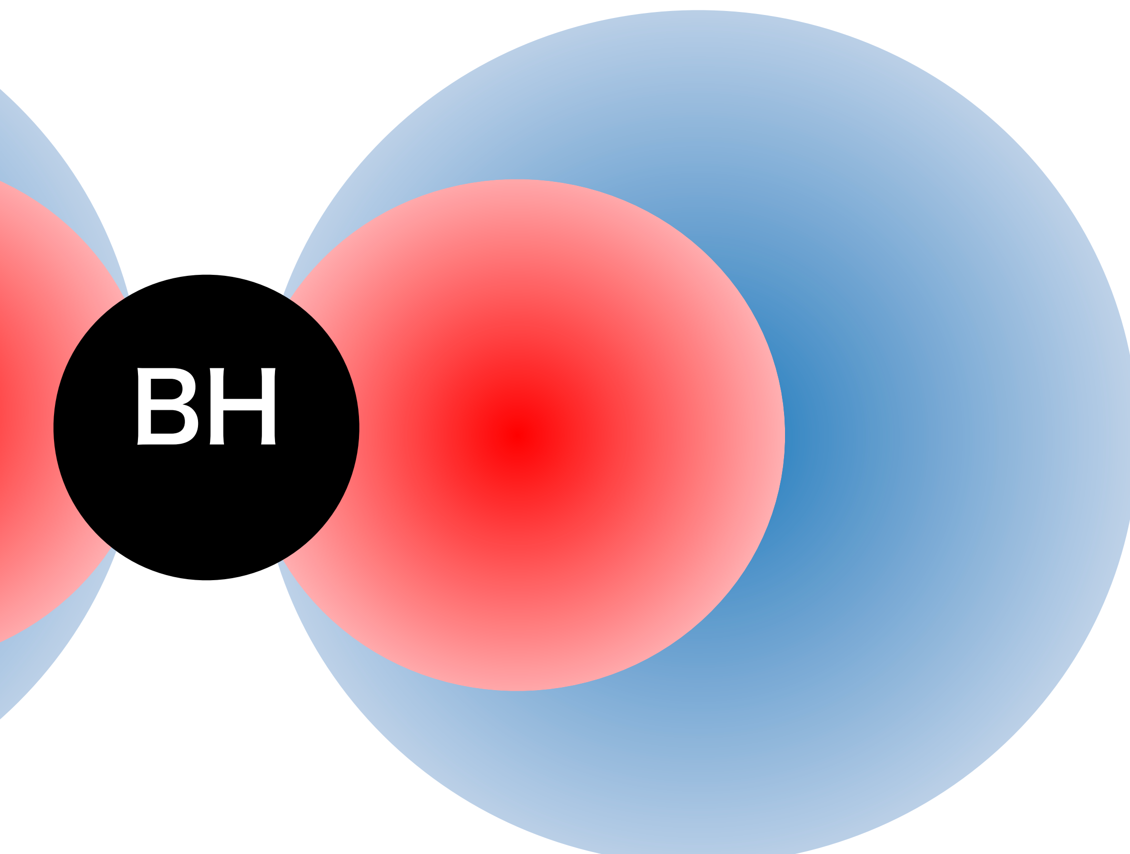
Gravitational waves

Two possible signal

$$\phi \sim e^{-i(\omega_1 t - m_1 \varphi)} \psi_1 + \mathrm{c.c.}$$

$$+ e^{-i(\omega_2 t - m_2 \varphi)} \psi_2 + \mathrm{c.c.}$$

$$T_{\mu\nu} \sim \partial_\mu \phi \partial_\nu \phi$$



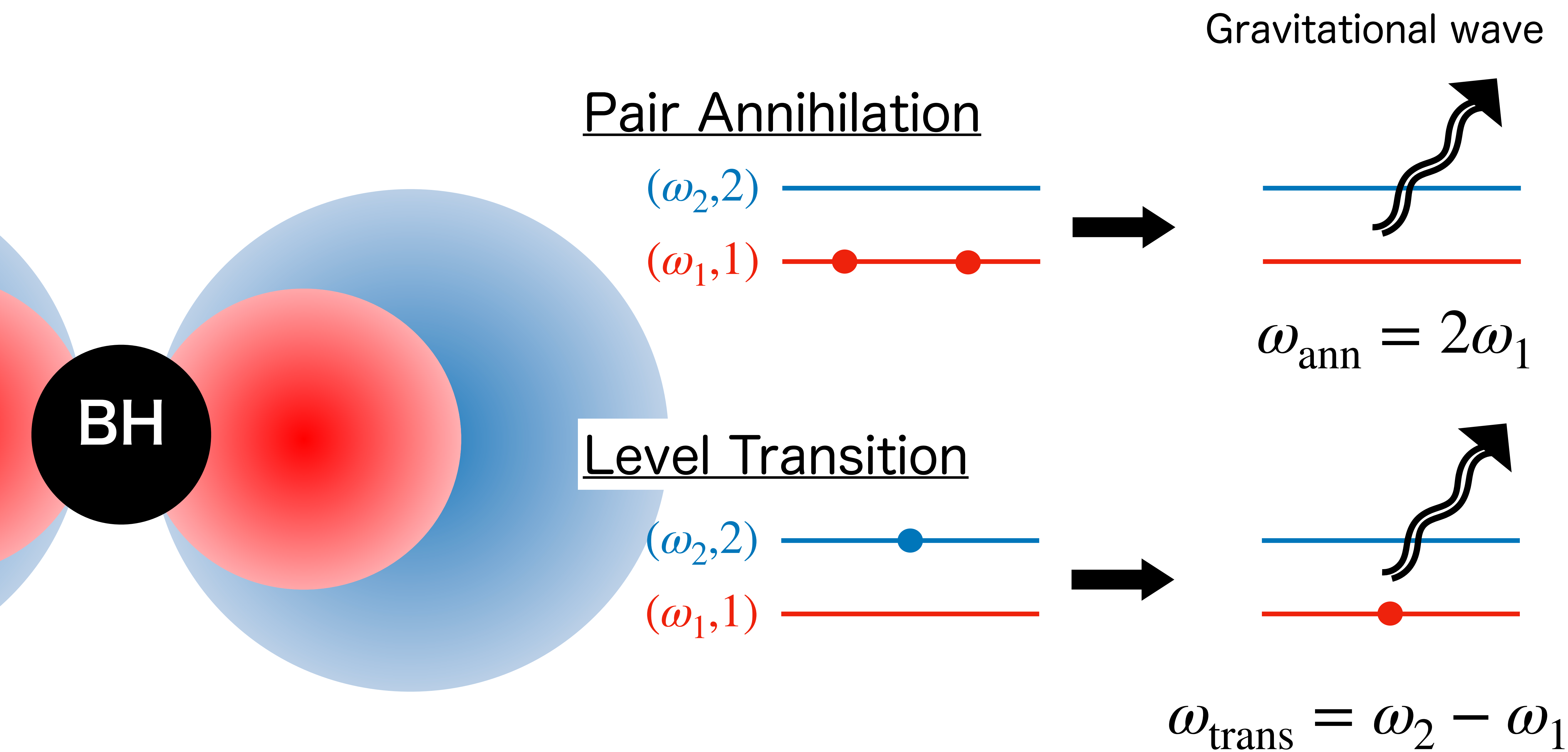
Energy momentum tensor contains

$$e^{-i(2\omega_i t - 2m_i \varphi)} \,,$$

$$e^{-i((\omega_2 - \omega_1)t - (m_2 - m_1)\varphi)} \,,$$

Radiation with several frequencies

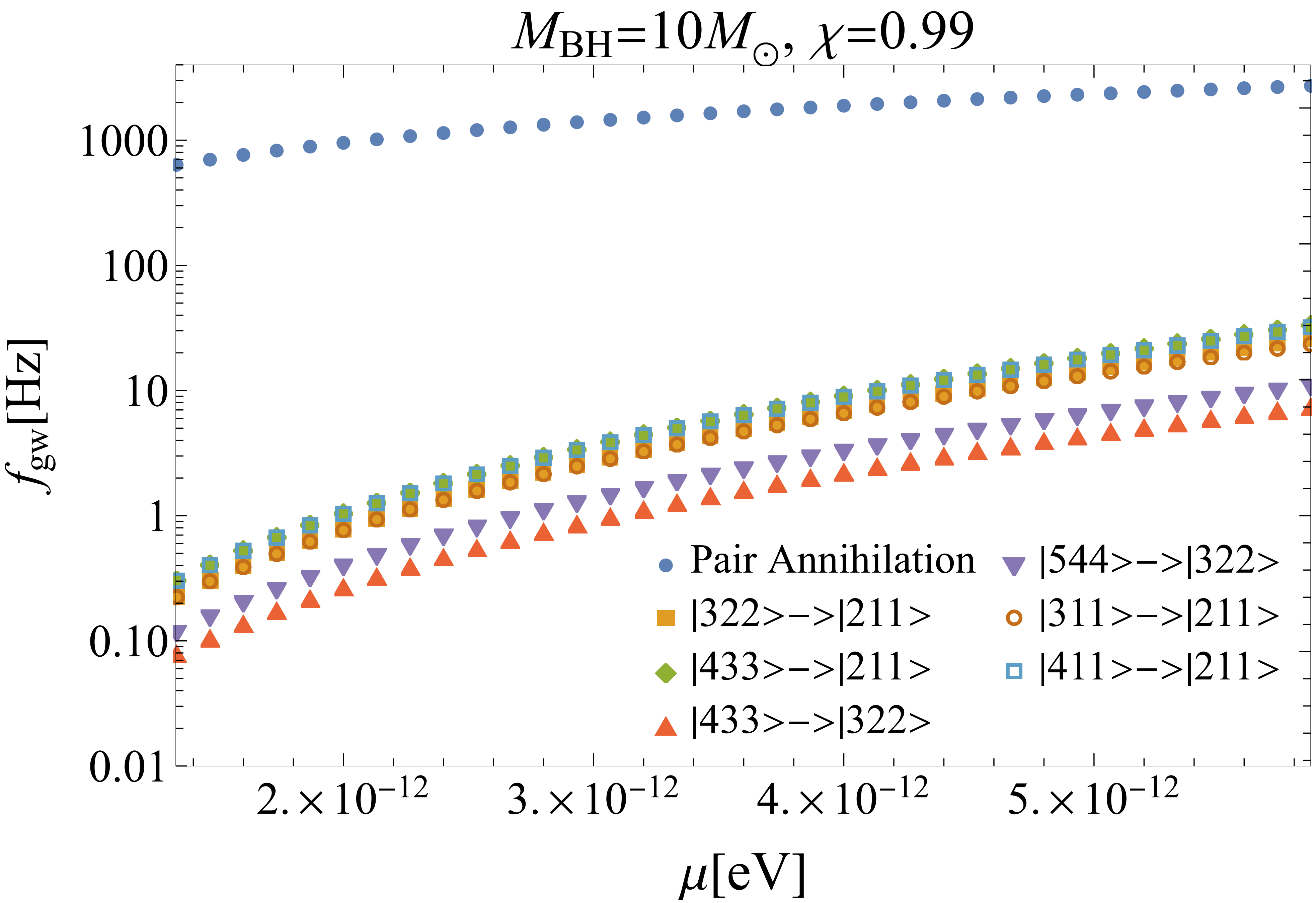
Gravitational waves



Multiple mode excitation

→Rich level transition signal

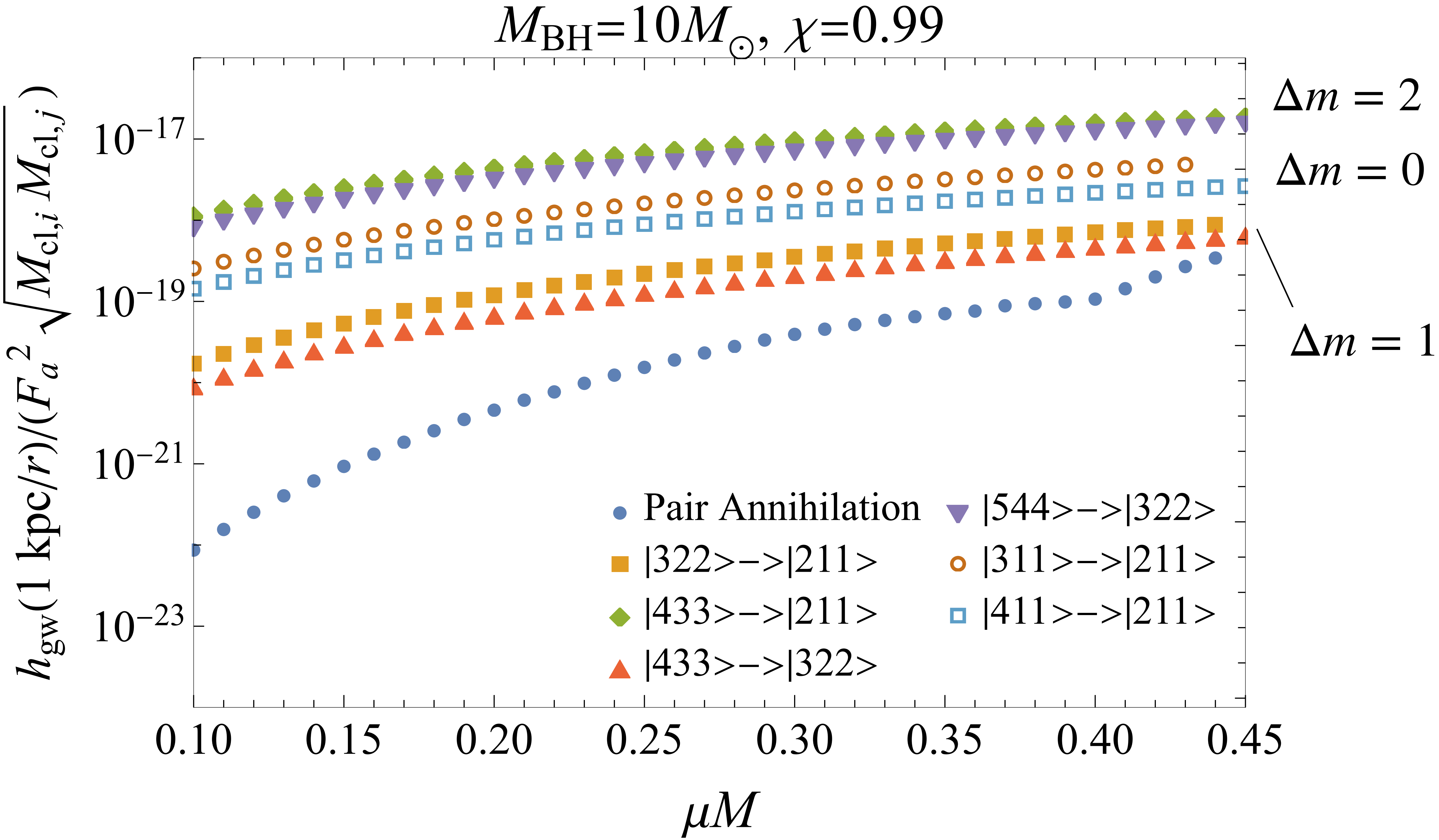
Gravitational wave frequency



Pair Annihilation:kHz (LIGO,⋯⋯)

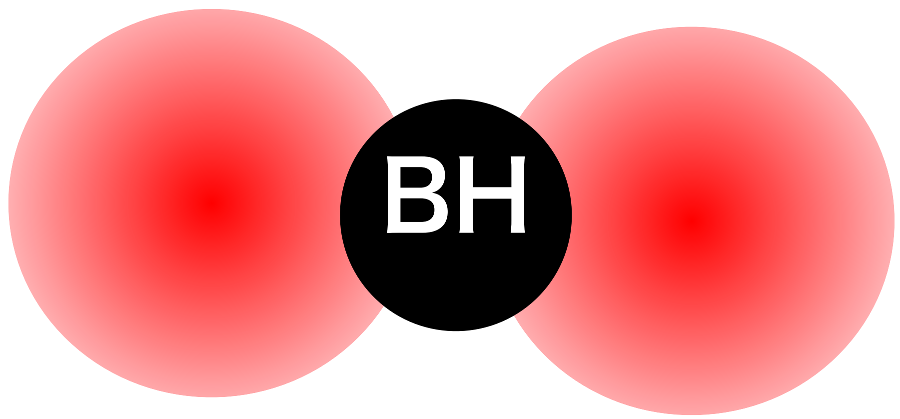
Level Transition:Hz (DECIGO, Atom Interferometer, ⋯⋯)

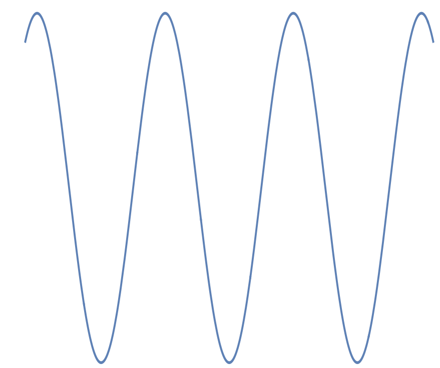
Gravitational wave amplitude



Gravitational wave amplitude

Suppression of pair annihilation signal:





$$\lambda_{\rm cloud} \sim 1/\mu/(G\mu M_{\rm BH}) \gg \lambda_{\rm gw} \sim 1/\mu$$

Hierarchy between level transition signal:

Quadrupole transition or not

$$< 211 | Q_{ij} | 433 > \neq 0$$

$$< 211 | Q_{ij} | 322 > = 0$$

Q_{ij}:mass quadrupole

Order Estimate

Amplitude depends on the mass of the cloud.

Maximum amplitude for GUT scale decay constant,

$\underline{l = m = 3 \to l = m = 1}$

$$h_{\rm gw} \sim \begin{cases} 1.0 \times 10^{-21} \left(\frac{1\rm kpc}{r}\right) \left(\frac{M_{\rm BH}}{10 M_\odot}\right) \left(\frac{F_a}{10^{-3}}\right)^2 \left(\frac{\sqrt{M_{\rm cl,1} M_{\rm cl,3}}}{380}\right) . & (\chi = 0.99, \mu M_{\rm BH} \sim 0.15) \\ 7.9 \times 10^{-22} \left(\frac{1\rm kpc}{r}\right) \left(\frac{M_{\rm BH}}{10 M_\odot}\right) \left(\frac{F_a}{10^{-3}}\right)^2 \left(\frac{\sqrt{M_{\rm cl,1} M_{\rm cl,2}}}{290}\right) . & (\chi = 0.7, \mu M_{\rm BH} \sim 0.15) \end{cases}$$

Duration of the signal: $\sim 2\rm yr$

$\underline{l = m = 2 \to l = m = 1}$

$$h_{\rm gw} \sim \begin{cases} 4.0 \times 10^{-23} \left(\frac{1\rm kpc}{r}\right) \left(\frac{M_{\rm BH}}{10 M_\odot}\right) \left(\frac{F_a}{10^{-3}}\right)^2 \left(\frac{\sqrt{M_{\rm cl,1} M_{\rm cl,2}}}{54}\right) , & (\chi = 0.99, \mu M_{\rm BH} \sim 0.42) \\ 4.8 \times 10^{-24} \left(\frac{1\rm kpc}{r}\right) \left(\frac{M_{\rm BH}}{10 M_\odot}\right) \left(\frac{F_a}{10^{-3}}\right)^2 \left(\frac{\sqrt{M_{\rm cl,1} M_{\rm cl,2}}}{64}\right) . & (\chi = 0.7, \mu M_{\rm BH} \sim 0.2) \end{cases}$$

Duration of the signal: $\sim 0.1\rm yr$

※Duration becomes longer if F_a is reduced.

Summary

- Investigated the evolution of the axion around the black hole, taking into account the relativistic correction, higher multipoles, and spin down.

- Self-interaction excites higher multipole moments, leading to a rich gravitational wave signal.
- If the black hole with $M_{\rm BH} \sim 10 M_\odot$, $\chi \gtrsim 0.7$ exists near us ($\lesssim 10 \rm Mpc$), we may observe gravitational waves from the axion in the Hz range.

- Ignored tidal effect from the environment, coupling to the photon,……. Maybe spoiled by these effects.

Back up

Time evolution