YITP long-term workshop

Gravity and Cosmology 2024

# Termination of Superradiance from a Binary Companion

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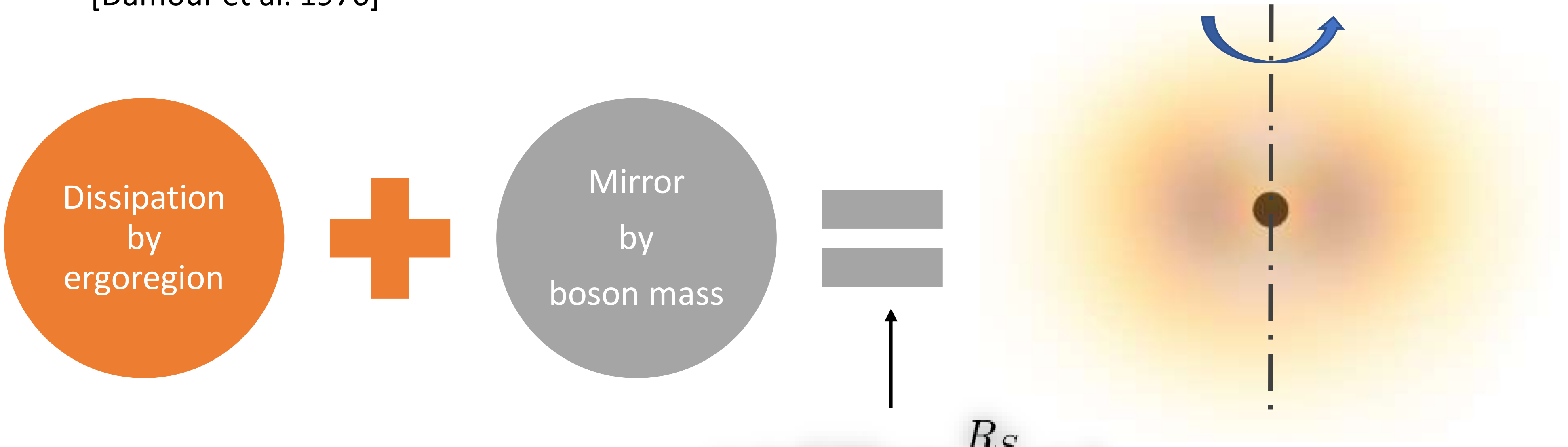
2311.17013

# Superradiance and the G-atom

- Superradiant instability ⇨ Kerr BH grows a ultralight boson cloud

[Press & Teukolsky, 1972]
[Damour et al. 1976]




$$\alpha \equiv GM\mu \sim \frac{R_S}{\lambda_C} < 1$$

$$(g^{ab}\nabla_a\nabla_b - \mu^2)\Phi = 0$$

→ Schrodinger Equation Form at leading order.

# Superradiance and the G-atom

[Press & Teukolsky, 1972]
[Damour et al., 1976]
[Detweiler, 1980]
[Baumann et al, 2019, 2020]

Solutions:

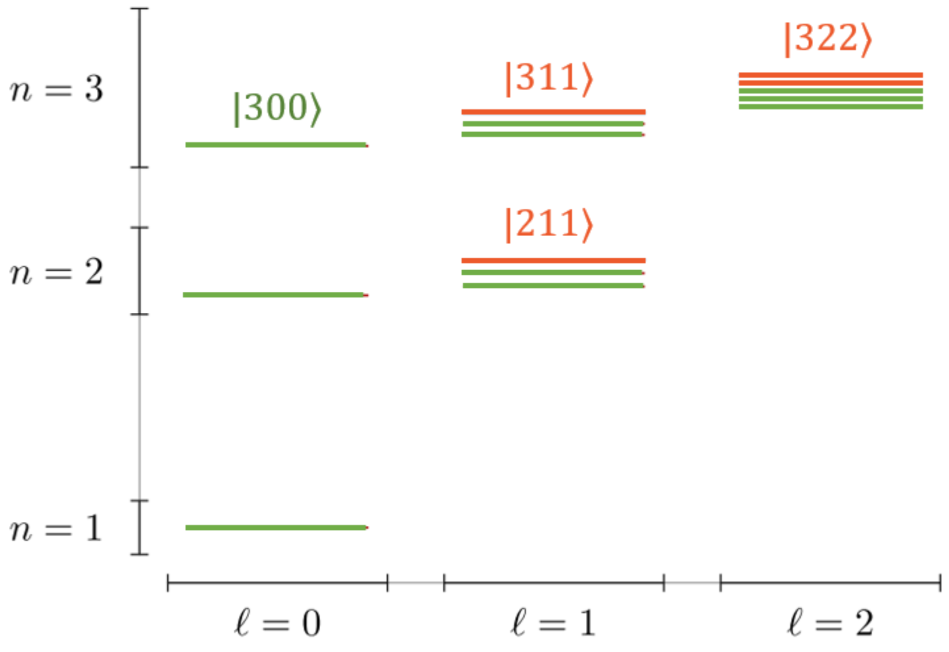
$|\psi_{nlm}\rangle$ **with** $\omega_{nlm} = E_{nlm} + i\Gamma_{nlm}\ , \quad \alpha \ll 1$

$$E_{nlm} = \mu\left[1 - \frac{\alpha^2}{2n^2} + \alpha^4 A(n,l) + \alpha^5 \tilde{a} m B(n,l) + \cdots\right]$$

Rest mass — Bohr — Fine — Hyperfine

$$\Gamma_{nlm} \propto (m\Omega_H - \mu)\alpha^{4l+5} \begin{cases} > 0 & \text{Superradiance} \\ < 0 & \text{Absorption} \end{cases}$$

$$\psi_{nlm} \sim e^{-i\omega_{nlm}t} \sim e^{\Gamma_{nlm}t}$$



Q: What phenomena does Gravitational Atom have?

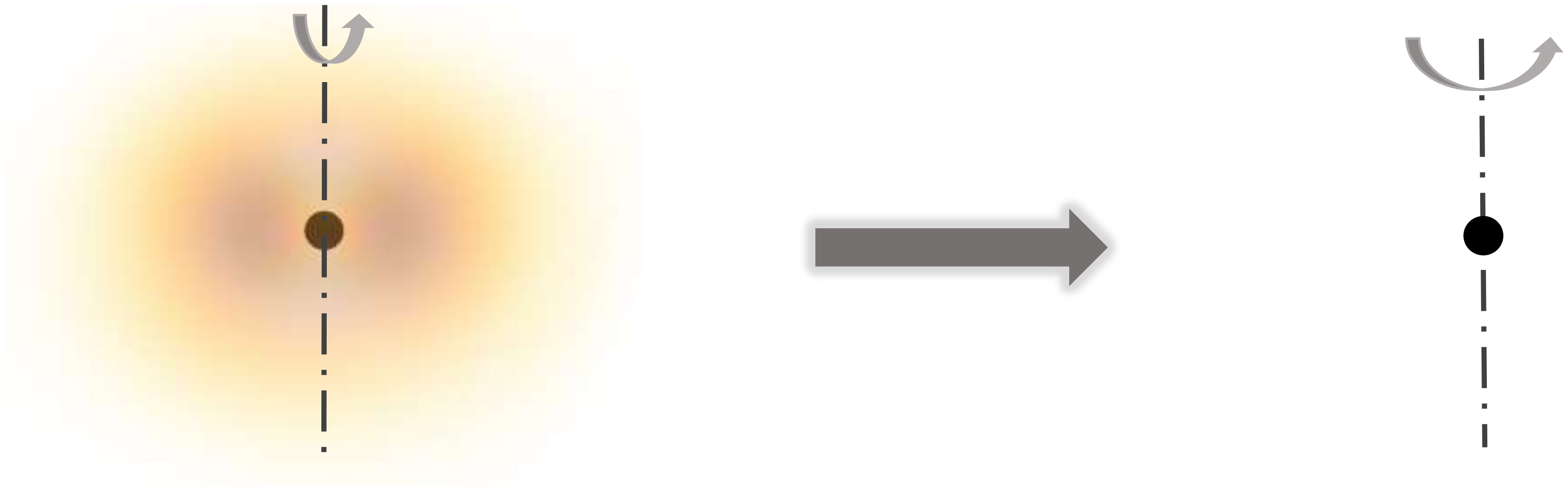
[Arvanitaki et al, 2011, 2017]
[Yoshino and Kodama,2014]
[Baumann et al, 2019, 2020]
[Tong et al, 2021]

For an isolated gravitational atom:

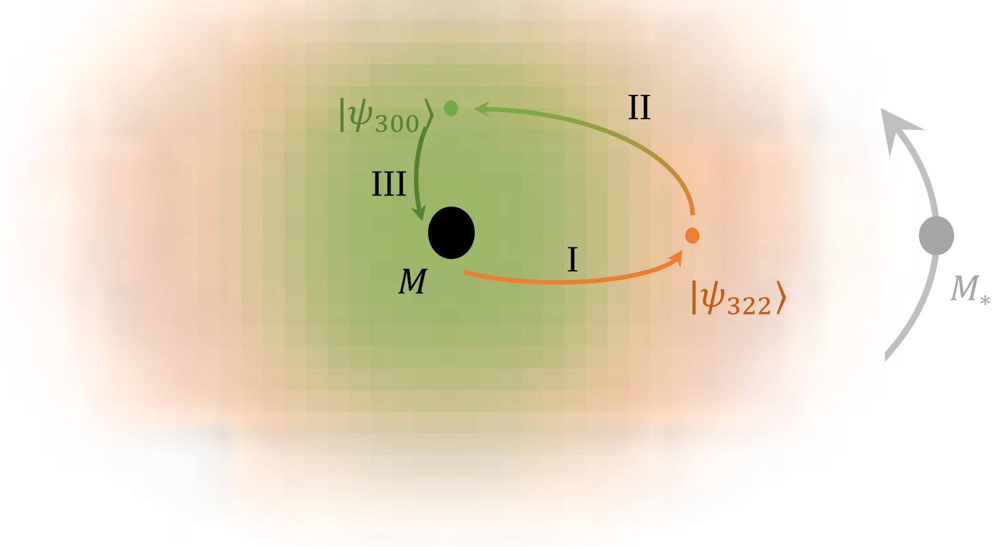
- Bosonic cloud emits monochromatic GW.
- Cloud extracts the BH spin.

For binary systems:

- Resonant transition triggered by orbital motion (GCP resonance transition), which can be detected by GW and Pulsar Timing.

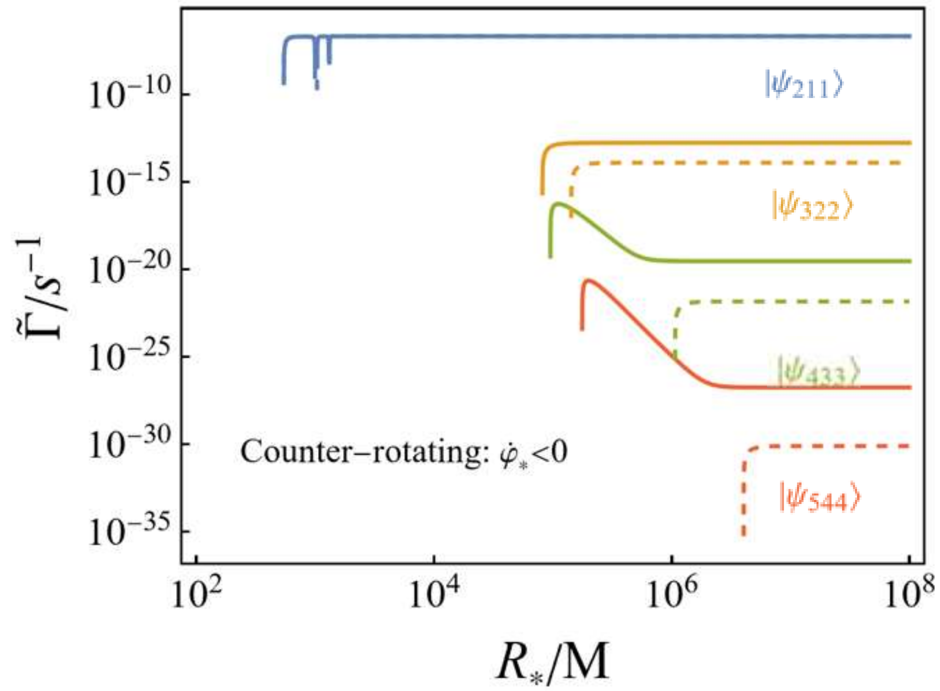


# G-atom in a binary




$$H = \begin{pmatrix} E_1 + i\Gamma_1 & 0 \\ 0 & E_2 + i\Gamma_2 \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$
$$= \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix}$$

$$\tilde{\Gamma}_1 = \Gamma_1 + \Delta\Gamma_1$$




$$\Delta\Gamma_1 \simeq \sum_{i=n'l'm'} \frac{\Gamma_1 - \Gamma_i}{[\bar{E}_1 - \bar{E}_i - (m_1 - m_i)\dot{\phi}_*(R_*)]^2} |\eta_{1i}(R_*)|^2$$

**With** $\eta_{ij} \equiv V_{ij} = \langle i|V_*|j\rangle$

[Tong, Wang & **Zhu**, 2022]

# Critical Distance

**Mass ratio:** $q = M_*/M$

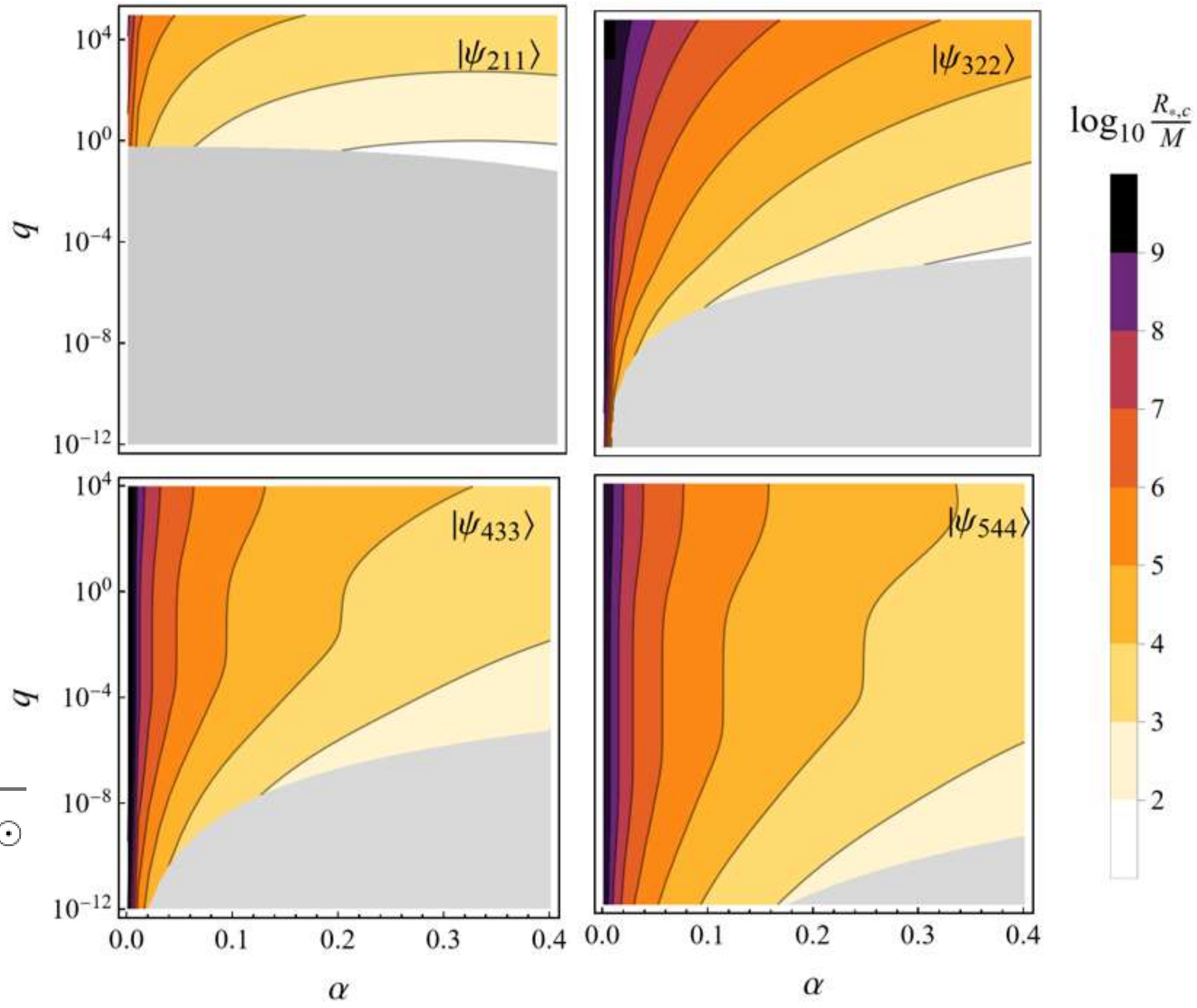
**Fine structure const:** $\alpha = GM\mu$

- The critical distance $R_{*,c}$ of $|\psi_{nlm}\rangle$ is defined as

$$\tilde{\Gamma}_{nlm}(R_{*,c}) = \Gamma_{nlm} + \Delta\Gamma_{nlm}(R_{*,c}) \equiv 0$$

- $R_{*,c}(nlm)$ is the distance below which ***no superradiance*** can happen

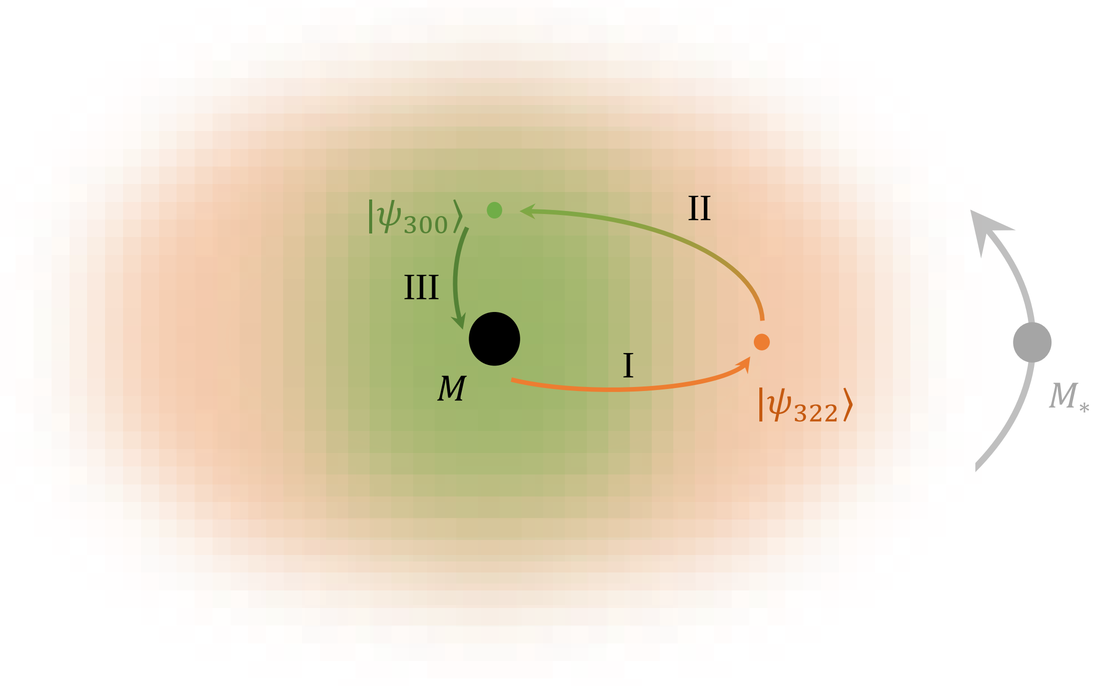
$$R_{*,c}(322) \simeq 10^6\ \mathrm{km} \left(\frac{\alpha}{0.1}\right)^{-23/6} \left(\frac{q}{0.2}\right)^{1/3} \frac{M}{10 M_\odot}$$




[Tong, Wang & Zhu, 2022]

# So … what?

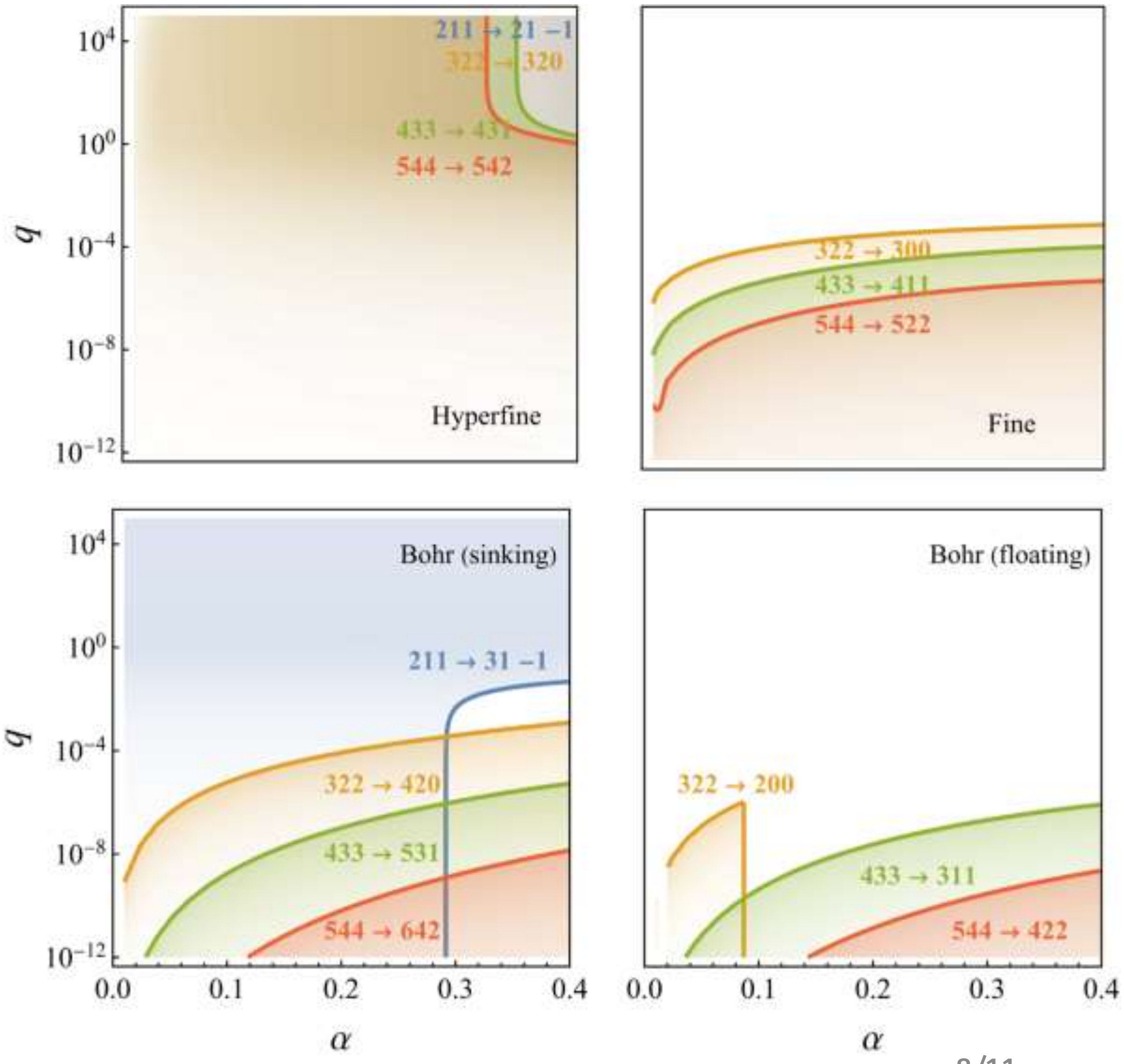
# Consequences of ST: Impacts on GCP




- **Successful GCP transition**

$$R_{*,r}(nlm \to n'l'm') > R_{*,c}(nlm)$$

[Tong, Wang & **Zhu**, 2022]



# ST backreaction: Orbital flow of EMRIs ( $q \ll 1$)

[Fan, Tong, Wang & **Zhu**, 2023]

- **General binary orbits:** $\{p(t), e(t), \iota(t)\} \cup \{S_c(t)\}$

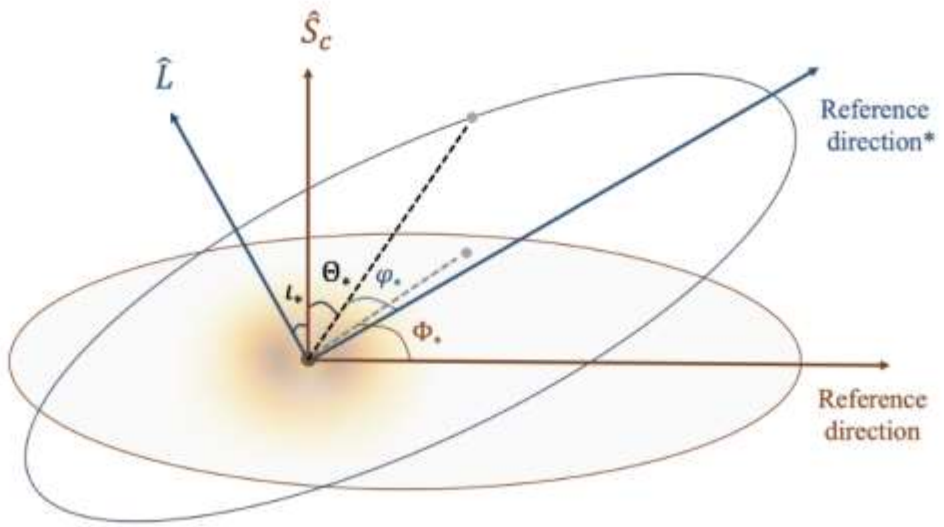
  - $S_c(t)$ → Cloud angular momentum
  - $\iota(t)$ → Inclination angle
  - $e(t)$ → Eccentricity
  - $p(t)$ → Semi-latus rectum

$$\frac{\mathrm{d}}{\mathrm{d}t}[L(t)\,\cos\iota_*(t)] = \tau_\mathrm{c} + \tau_\mathrm{bGW}\,\cos\iota_*(t)\;,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[L(t)\,\sin\iota_*(t)] = \tau_\mathrm{bGW}\,\sin\iota_*(t)\;.$$

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = P_\mathrm{c} + P_\mathrm{bGW}\;.$$

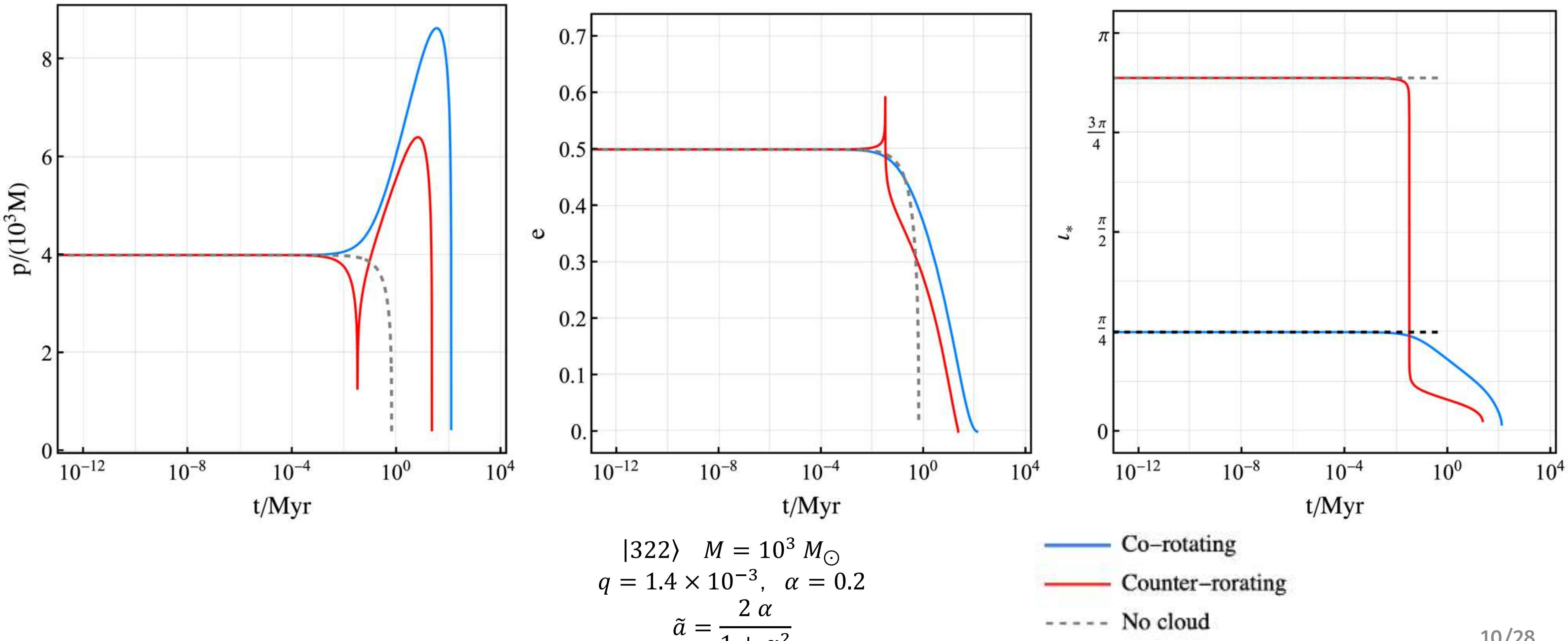
$$\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t} = \left(\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t}\right)_\mathrm{ST} + \left(\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t}\right)_\mathrm{cGW}$$



# ST backreaction: Orbital flow of EMRIs

[Fan, Tong, Wang & **Zhu**, 2023]

- **Orbital evolution**




$|322\rangle \quad M = 10^3\, M_\odot$

$q = 1.4 \times 10^{-3}, \quad \alpha = 0.2$

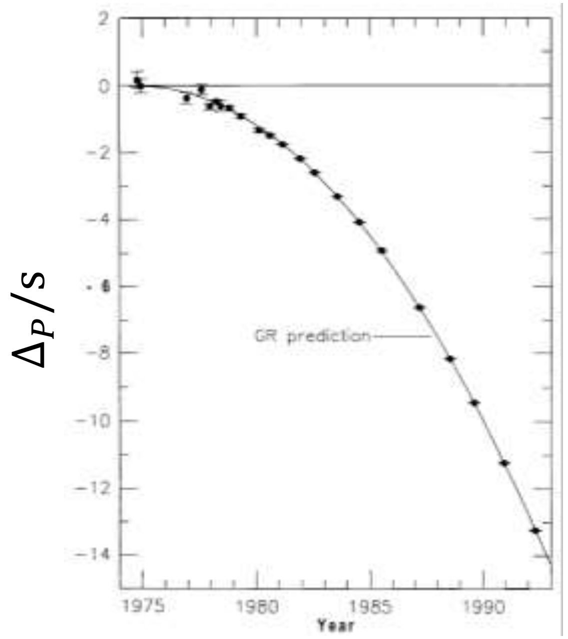
$\tilde{a} = \dfrac{2\,\alpha}{1+\alpha^2}$

# Consequences of ST: Pulsar Timing

- **Observable: Periastron time shift**

$$\Delta_P \equiv t - P(0)\int_0^t \frac{\mathrm{d}t'}{P(t')} \approx \frac{1}{2}\frac{\dot{P}}{P}t^2$$

**Rømer delay + pulse counting**

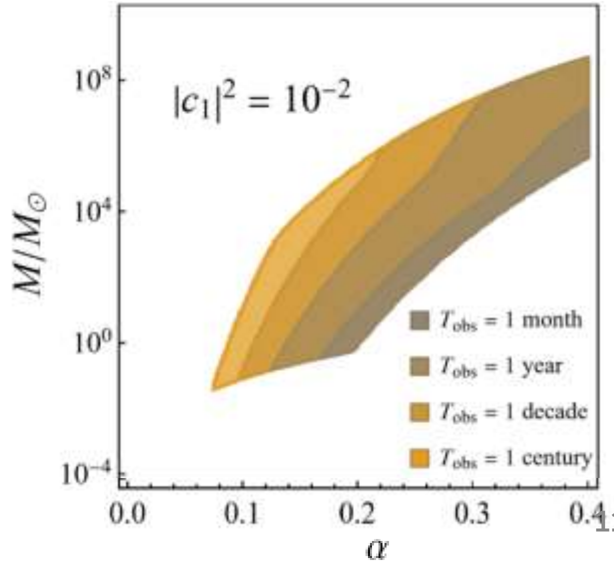



**[Hulse & Taylor 1975]**

**GR:** $$(\dot{P})_{GR} = -\frac{96}{5}(2\pi)^{8/3}\frac{q}{(1+q)^{1/3}}M^{5/3}P^{-5/3}$$

- **We expect a backreaction-induced deviation**

$$(\dot{P})_C = -3(2\pi)^{1/3}(1+q)^{-2/3}\frac{S_{c,0}\,m_1}{M^2}\frac{\mathrm{d}|c_1(t)|^2}{\mathrm{d}t}M^{1/3}P^{2/3}$$




[Tong, Wang & **Zhu**, 2022]

# Summary and outlook

- ✓ BH superradiance instability
- ✓ GA enjoys a rich phenomenology
- ✓ Yet a binary companion can destabilize the cloud
- ✓ This leads to ST at a critical distance
- ✓ ST poses tight constraints on possible GCP transitions
- ✓ Orbital backreactions observable from pulsar timing

- ☐ Alleviate the boson mass bound (To what extent)?
- ☐ High Spin? Fully relativistic treatment?
- ☐ Self gravity?

Thank you for listening!

# Backup slides

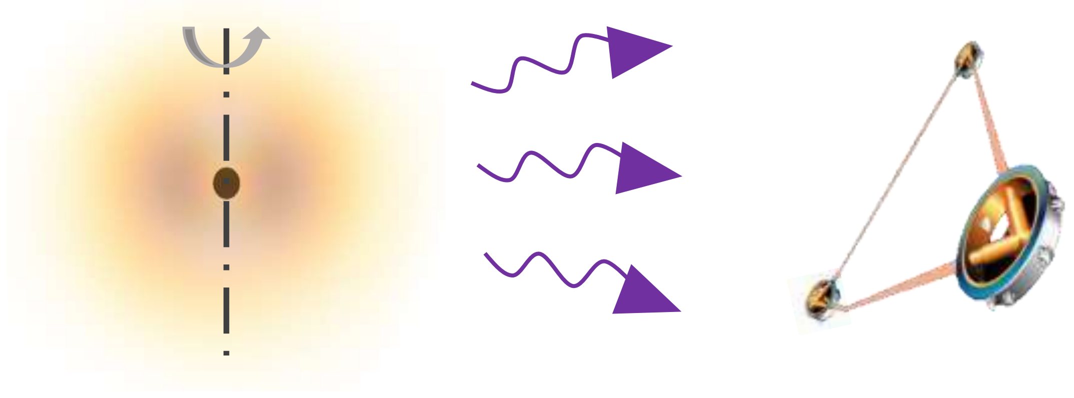
# Appendix: Pulsar Timing Accuracy

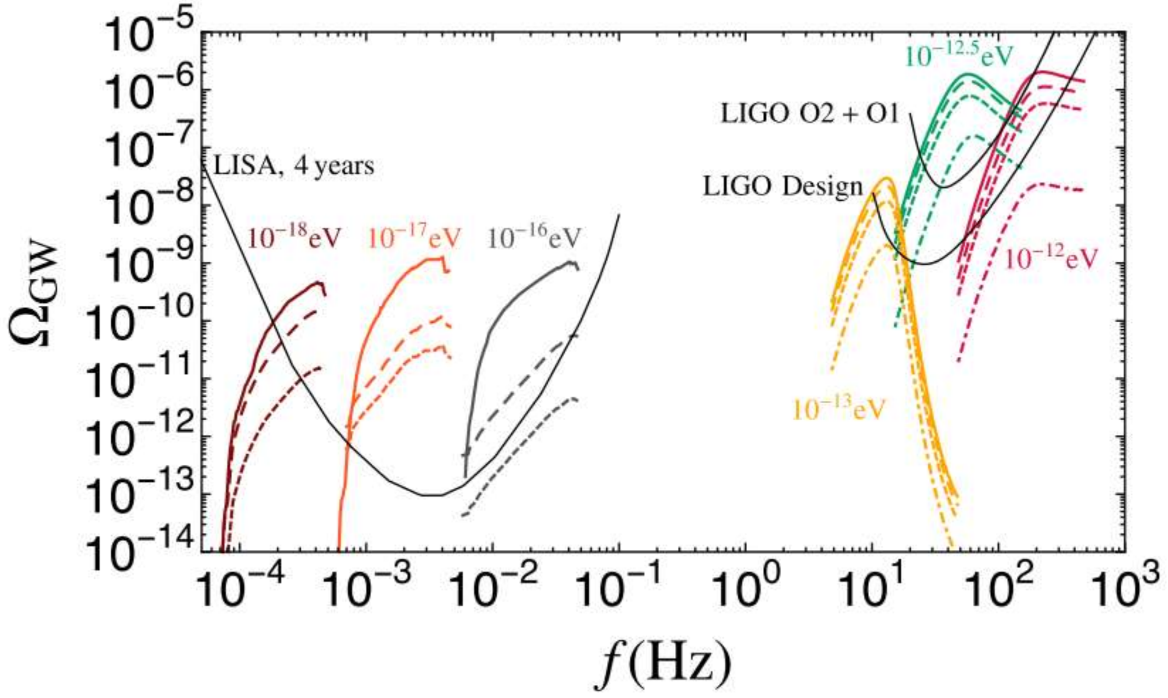
- **Suppose we observe the pulsar for $t_{obs}$ every day, and the pulse period $\tau$.**
- **We can measure $t_{obs}/P$ periods every day.**
- **The error for every single continuous measurement is $\tau/[\min(t_{obs}, t)/P]$.**
- **If we observe for $0 < t \leq T_{obs}$, where $T_{obs}$ is the longest observation time. Then the uncertainty for Periastron time shift is**

$$\sigma_{\Delta P} = \frac{1}{\sqrt{\left[\frac{t}{1day}\right]}} \frac{\tau}{\min(t_{obs}, t)\ /P}$$

# GA phenomenology in isolation

- **Near-monochromatic GW**



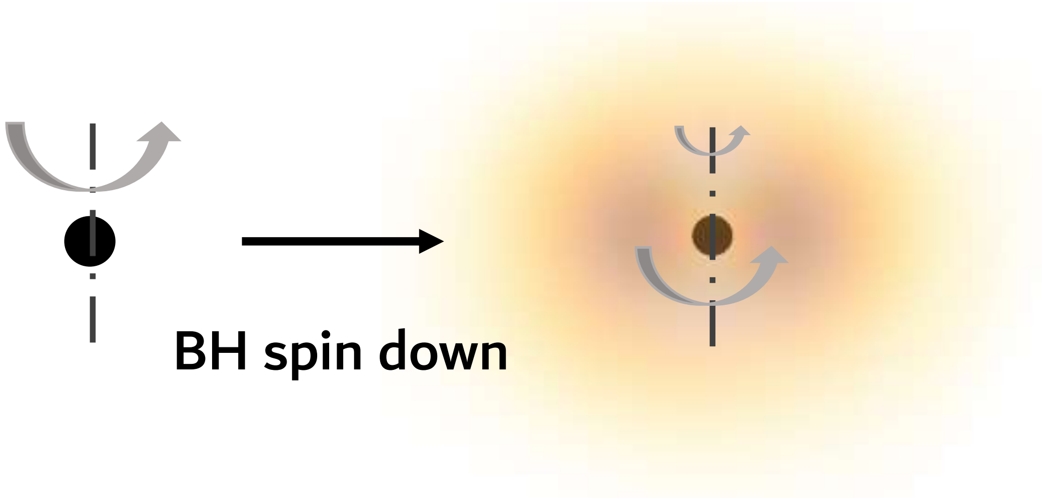


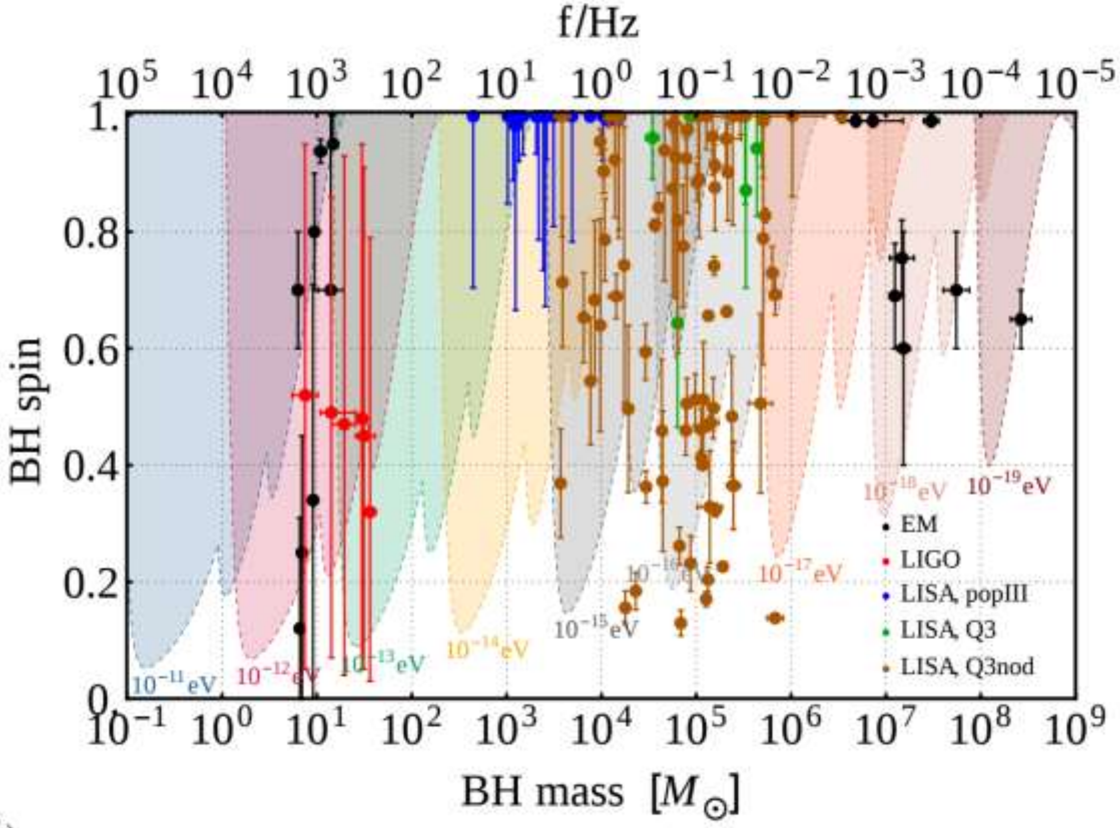

$$f_{\rm GW} \sim \omega_R/\pi \sim 5\,{\rm kHz}\left(\frac{\mu\hbar}{10^{-11}{\rm eV}}\right)$$

[Brito et al., 2017]

# GA phenomenology in isolation

- **Spin cutoff by superradiance**




[Brito et al., 2017]

$$m\Omega_H \downarrow > \omega \sim \mu \Rightarrow \frac{a}{M} = \frac{4m(M\omega)}{m^2+4(M\omega)^2} = \frac{4\alpha}{m} + \mathcal{O}(\alpha^3)$$

**Spin @ saturation**

# GA phenomenology in binaries

[Baumann et al, 2019]
[Baumann et al, 2020]
[Baumann et al, 2022]
[Ding, Tong & Wang, 2020]
[Tong, Wang & Zhu, 2021]

- **Atomic transitions a.k.a. “Gravitational Collider Physics” (GCP)**

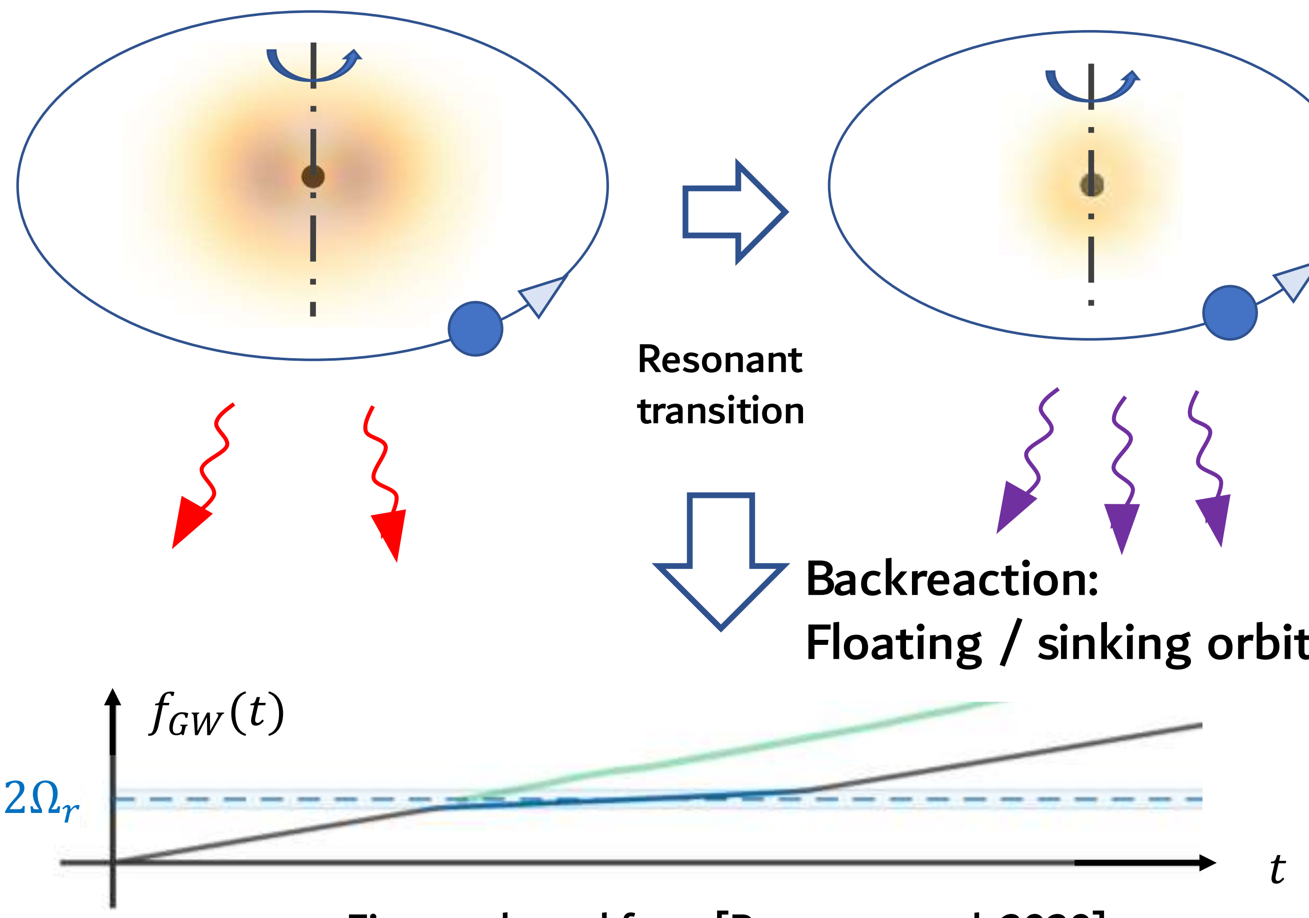
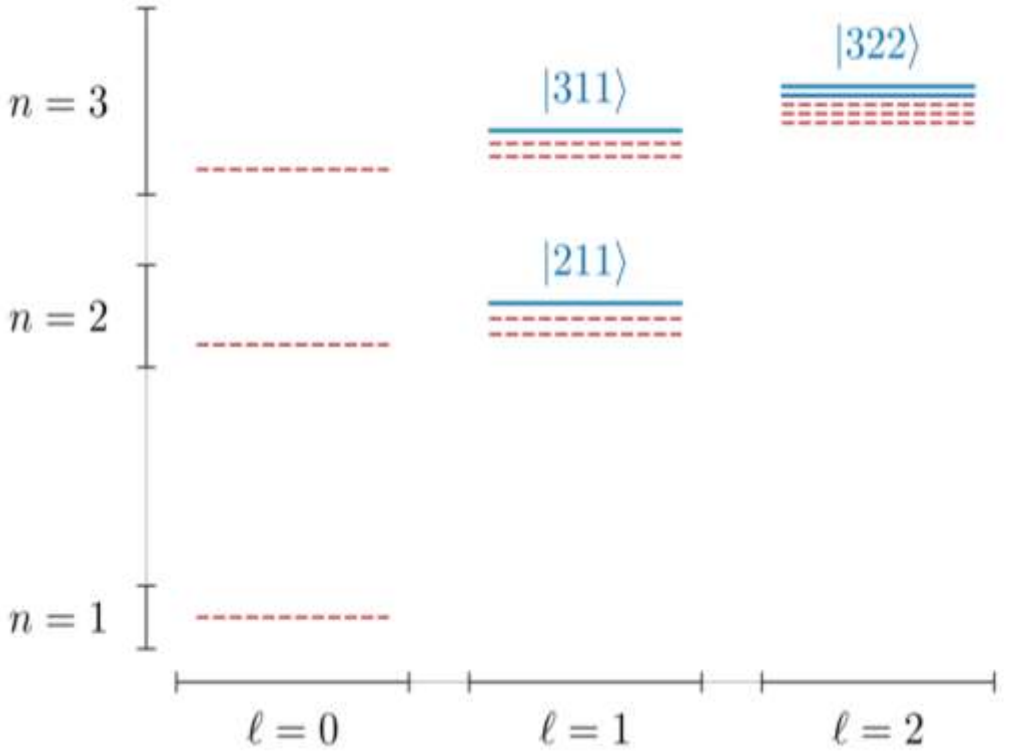
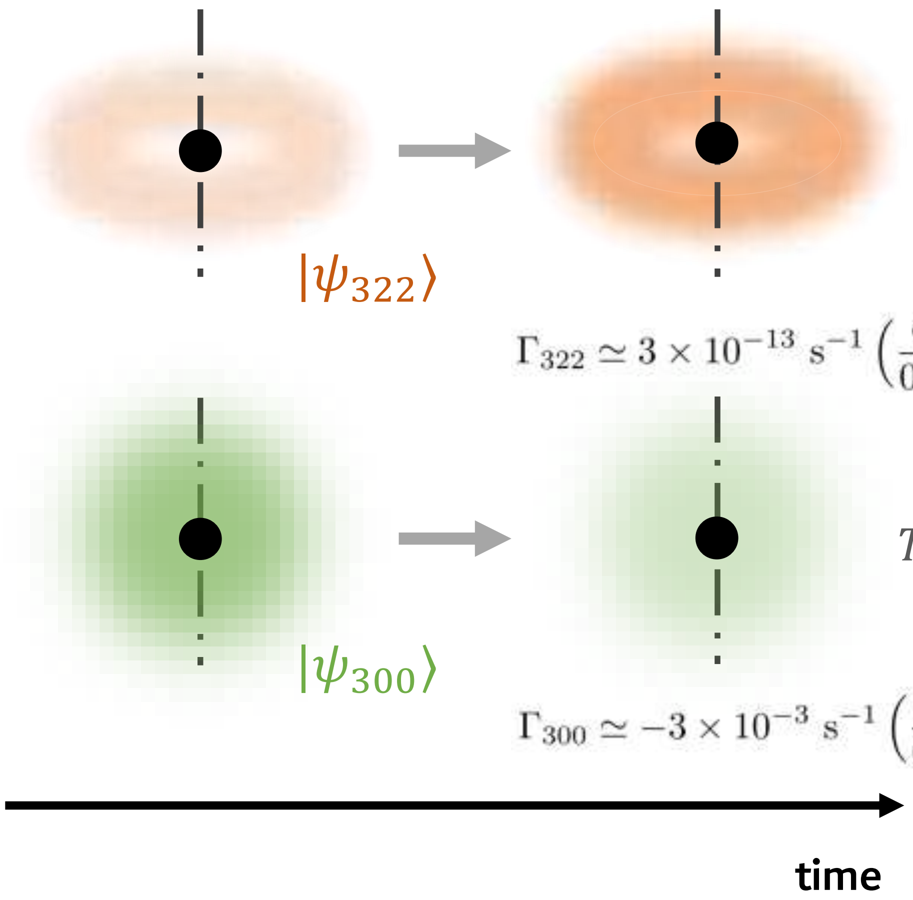



Figure adapted from [Baumann et al, 2020]



# GA in a nutshell


[Press & Teukolsky, 1972]
[Damour et al., 1976]
[Detweiler, 1980]
[Baumann et al, 2019, 2020]

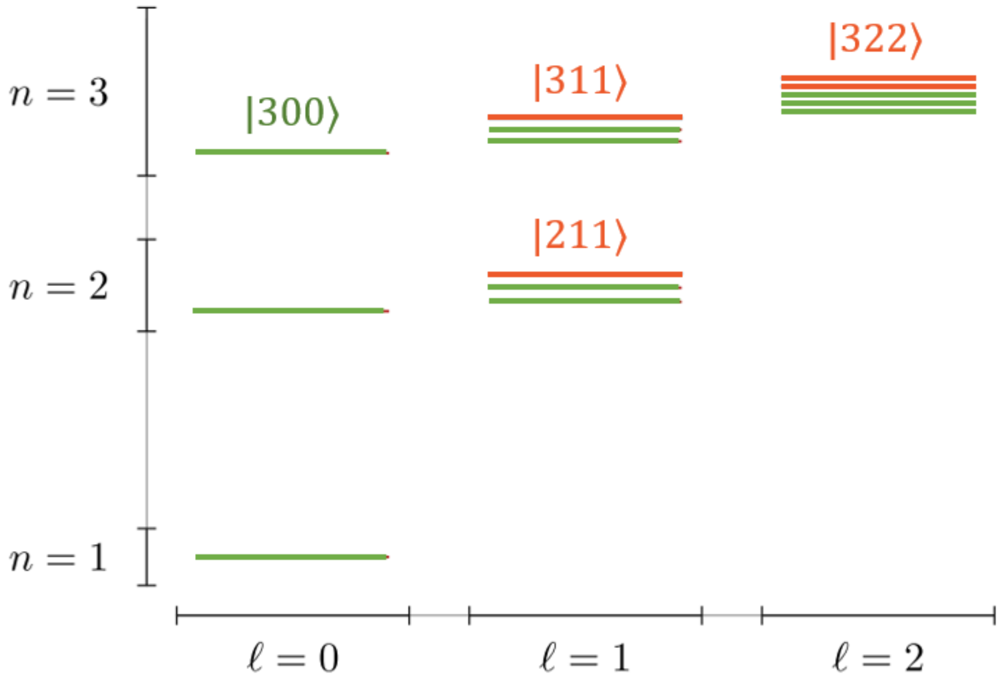




$$T^{(grow)}_{nlm} = \Gamma^{-1}_{nlm}$$

$$\Gamma_{322} \simeq 3 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{13} \left(\frac{M}{10 M_\odot}\right)^{-1}$$

$$T^{(absorb)}_{nlm} = \Gamma^{-1}_{nlm}$$

$$\Gamma_{300} \simeq -3 \times 10^{-3}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{5} \left(\frac{M}{10 M_\odot}\right)^{-1}$$



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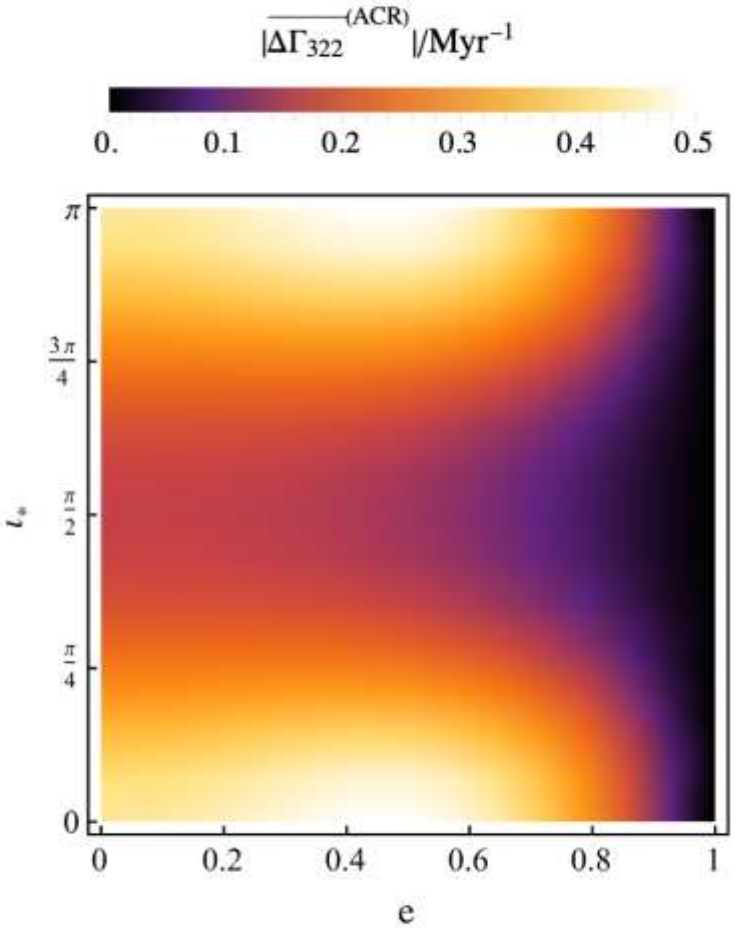
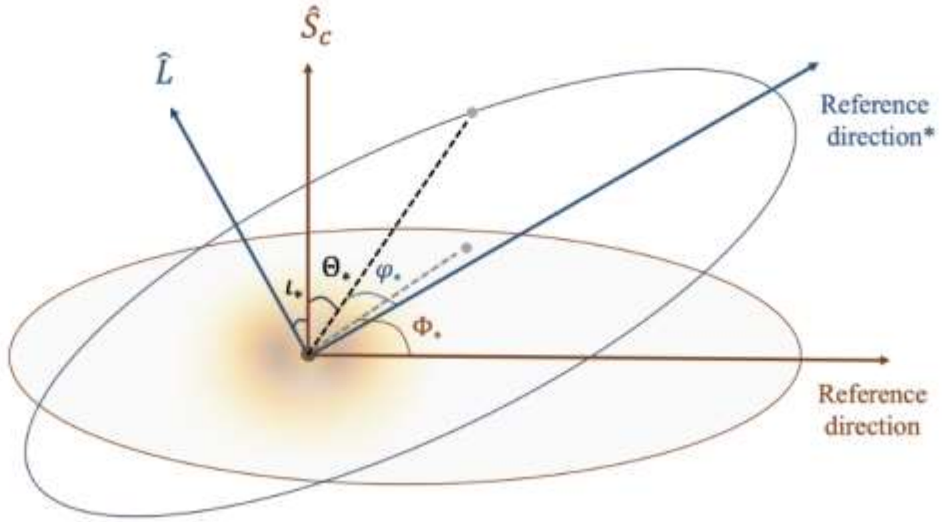
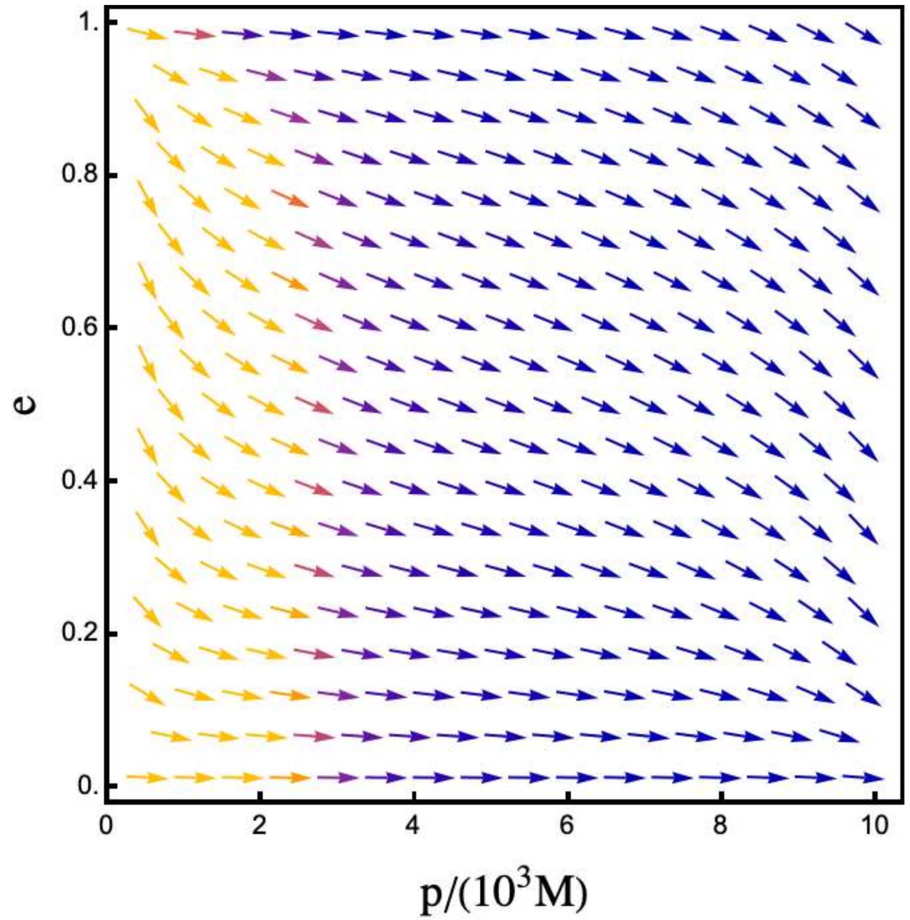


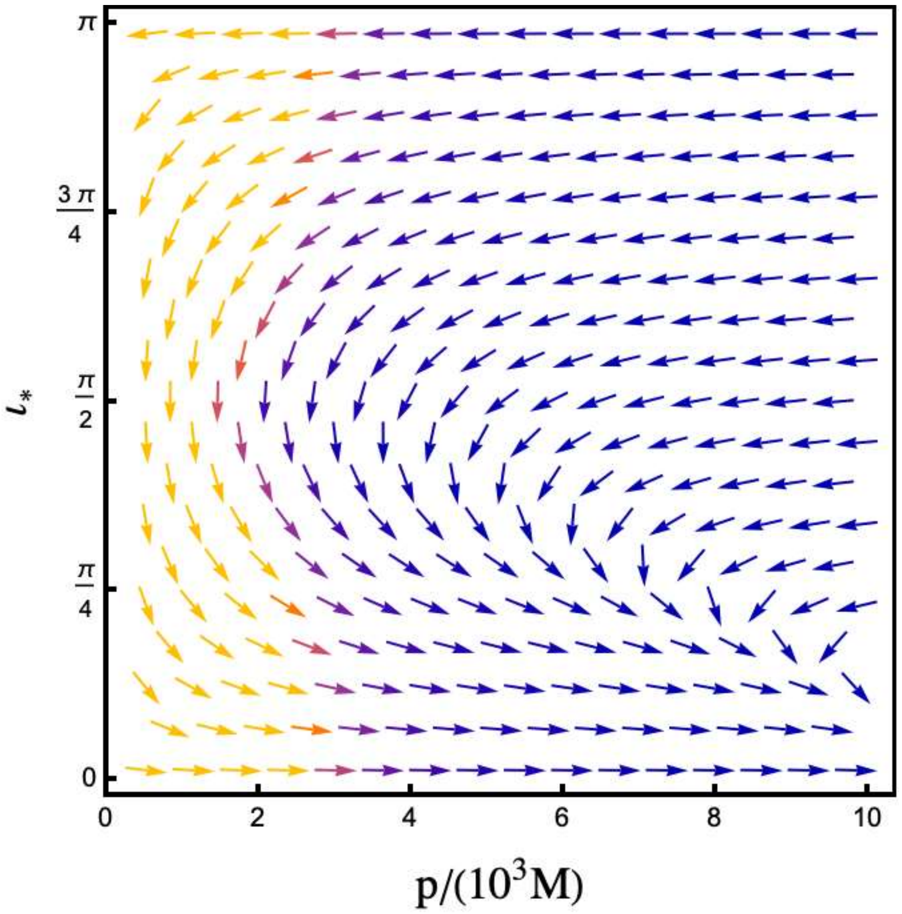



# ST backreaction: Orbital flow of EMRIs

[Fan, Tong, Wang & **Zhu**, 2023]

- **Flow of orbital parameters**

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