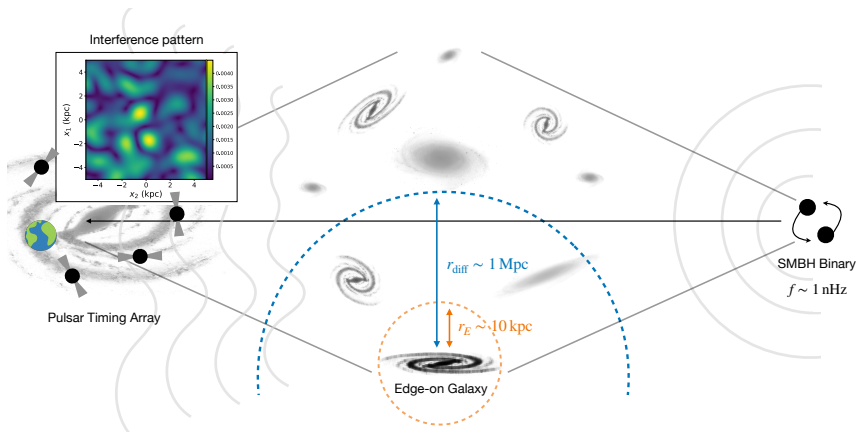


Resolving the Hubble tension through diffraction of gravitational waves

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Diffraction/Evanescent Gravitational Wave Lensing



Jow+ in prep

Gravitational Waves – Wave optics

- ▶ Coherent, distant source of radiation
- ▶ Interference effects under multi-path propagation
- ▶ Kirchoff-Fresnel path integral
- ▶ semi-classical concepts: stationary phase, Eikonal limit
- ▶ Witten 2010: generalized by Picard-Lefschetz theory
- ▶ Morse index, evanescent/complex images: measure time delays in weak lensing

Pulsar Timing Arrays (PTAs)

- ▶ initial GW evidence in 2023 (nanoGrav++)
- ▶ currently from a few dozen nearby PSRs
- ▶ full map of galaxy with SKA, orders of magnitude improvement in sensitivity, resolution
- ▶ Precision distances with Scintillometry
- ▶ Coherent imaging GW telescope with arcsecond resolution
- ▶ no “stochastic” regime: small number of SMBH near merger
- ▶ counterparts of supermassive BH binaries, precise redshifts

Diffractive (evanescent) imaging

- ▶ most galaxies too weak to form real gravitational lens images
- ▶ always form evanescent images through wave optics
- ▶ analogous to quantum mechanical tunnelling
- ▶ diffractive angle $\theta \sim \lambda/D$
- ▶ $\lambda \sim \text{pc}$, $D \sim 500\text{pc} \rightarrow \theta \sim 0.1^\circ$
- ▶ dominated by edge-on spirals!

Time delay H_0

- ▶ stack diffractive GW flux from all edge-on galaxies
- ▶ distance from angle-delay relation $\tau \sim \theta^2 L$
- ▶ Shapiro delay negligible off-axis
- ▶ sensitivity in SKA/DSA era

Waves, Particles, Geodesics

- ▶ Everything is a wave
- ▶ Short wavelength limit has classical particle interpretation: Eikonal
- ▶ Picard-Lefschetz: no waves, everything is a particle!
- ▶ evanescent lensing: complex eikonal lensing
- ▶ New tool for QM Cosmology, FRBs, pulsars
- ▶ alternative picture for quantization: quantum gravity?

Fermat's Principle

- ▶ light takes shortest path. Why?
- ▶ Huygen's principle: light takes all paths
- ▶ Kirchoff/Feynmann path integral
- ▶ stationary phase dominates
- ▶ classical equation of motion: extremal paths, independent of wavelength

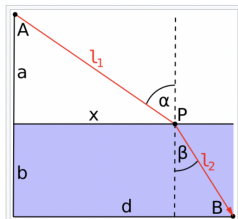


Fig. 1: Fermat's principle in the case of refraction of light at a flat surface between (say) air and water. Given an object-point A in the air, and an observation point B in the water, the refraction point P is that which

Huygen's Principle: Path Integral

- ▶ Kirchoff integral $A(\mu) = \int e^{iS(\theta,\mu)} d\theta$
- ▶ $S = \nu[(\theta - \mu)^2 + \Psi(\theta)]$
- ▶ Highly oscillatory integral, even for $\Psi = 0$
- ▶ Stationary phase points: $\partial_\theta S = 0$ leads to (complex) Eikonal images θ_i .
- ▶ flux/phase through curvature expansion (known as *steepest descent*): exact as $\nu \rightarrow \infty$
- ▶ Geometric limit considers only *Real* solutions θ_i and gives up phase information (length of trajectory)
- ▶ Geometric optics applicable at short wavelengths for extended sources (e.g. optical gravitational lensing of finite size sources, stars)
- ▶ roughly 1/4 of the degrees of freedom of Eikonal

Optics: Geometric, Eikonal, Wave, P-L

- ▶ Consider 1-D lens
- ▶ lensing potential $\Psi(\theta)$
- ▶ deflection Ψ'
- ▶ simplify for $D_{ds} = \infty$

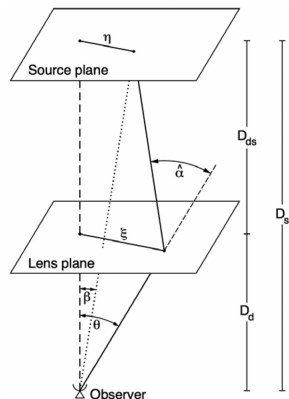
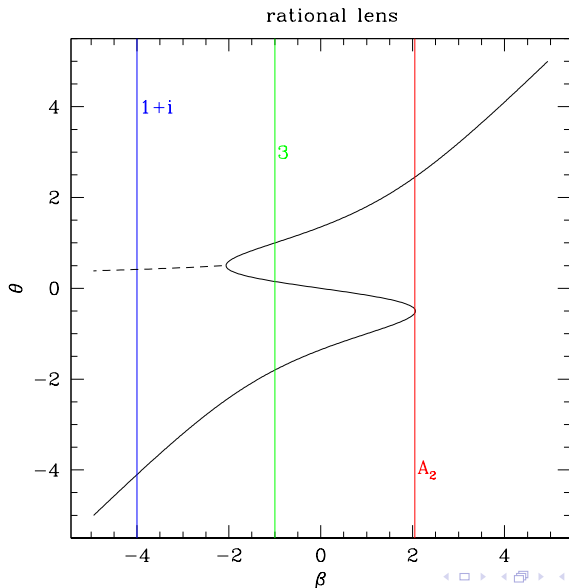


Figure 1. Geometry of a lensing event (reproduced from Schneider et al. 1992). See text for details.

Evanescent Images

- ▶ consider “rational lens” potential $\psi(\theta) = \alpha/(1 + \theta^2)$
- ▶ Geometric/eikonal images at $\psi' = \theta$
- ▶ 5 roots. 1 or 3 real roots, rest imaginary
- ▶ P-L: at most one imaginary image contributes!
- ▶ Resurgence theory to classify?
- ▶ Evanescent (imaginary) image can be brighter than unlensed real image

Rational 1-D lens

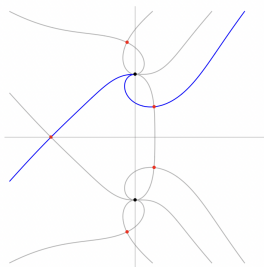


Picard-Lefschetz Theory

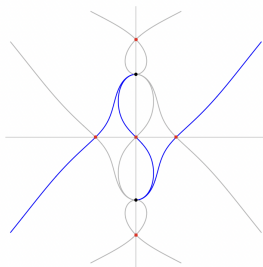
- ▶ descend integral along real line along Morse function $\text{Im}(S)$
- ▶ contour deforms into finite number of Thimbles of constant phase with maximum at saddle point (extrema $dS = 0$)
- ▶ correctly identifies relevant saddle points
- ▶ resolves numerical challenges of oscillatory integral
- ▶ complex analysis works in multiple variables
- ▶ elevates concept of “image” deep into wave optics
- ▶ multiple public implementations (Feldbrugge+, Jow+)

Picard-Lefschetz Theory

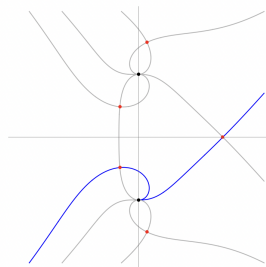
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(a) $\alpha = 2, \mu < -\mu_c$



(b) $\alpha = 2, -\mu_c < \mu < \mu_c$



(c) $\alpha = 2, \mu > \mu_c$

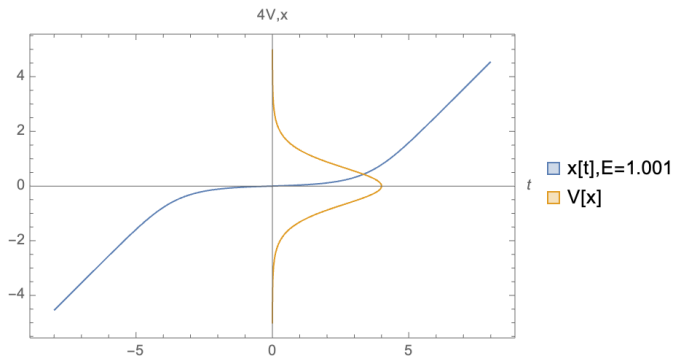
Feldbrugge+2019

Quantum Mechanics

- ▶ Feynmann path integral saddle points are similarly complex
- ▶ interprets tunneling as complex position path

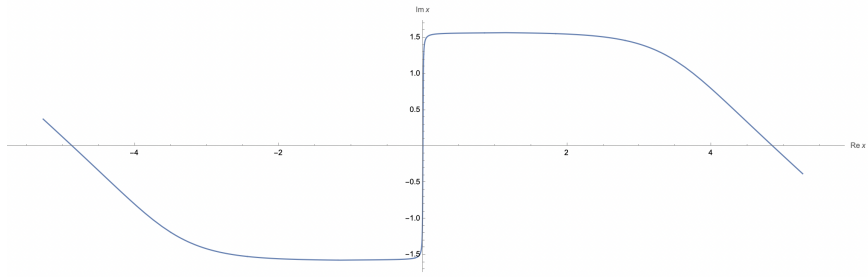
Rosen-Morse Potential

- ▶ $V(x) = \text{sech}^2(x)$
- ▶ exactly solvable: $\sinh[x(t)] = \sqrt{\frac{1-E}{E}} \cosh[\sqrt{E}(t + t_0)]$
- ▶ for $\lim_{E \rightarrow 1+\epsilon^2} = \sinh^{-1}[\epsilon \sinh(t)]$



Rosen-Morse Potential

- ▶ $V(x) = \text{sech}^2(x)$
- ▶ exactly solvable: $x(t) = -i \sin^{-1} \left[\sqrt{\frac{1-E}{E}} \sinh(\sqrt{E}t) \right]$
- ▶ for $\lim_{E \rightarrow 0} = -i \sin^{-1}(t)$



Reinterpretation of Quantum Mechanics: 2309.12420 w/Feldbrugge, Jow

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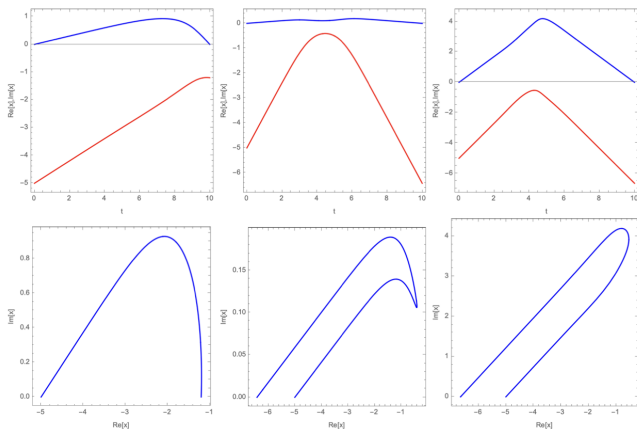


Figure 10: The complex paths as a function of time (top) and in the complex plane (bottom). *Left:* a classical path on the left blue arc. *Centre:* a classical path on the right blue arc. *Right:* a classical path on the green arc.

Discussion

- ▶ Eikonal effects applicable to compact radio sources, e.g. FRBs, pulsars
- ▶ full wave effect dominates for long wavelengths as Fresnel scale is bigger than Einstein radius
- ▶ microlensing down to planet size
- ▶ gravitational waves: LIGO, LISA, PTA
- ▶ with SKA PTA parameters and favourable source geometries, H_0 measurement at 0.1% possible.

Conclusions

- ▶ wave optics changes nature of astrophysical observables: Coherent FRB/pulsar/GW radiation one of the potentially most precise measurements in physics
- ▶ PTA weak diffractive lensing may give new tool for Hubble Constant tension
- ▶ evanescent lensing images understood through Picard-Lefschetz theory and complex/evanescent images
- ▶ at long wavelength, evanescent lensed images are unsuppressed