Using Artificial Neural Networks to reconstruct cosmology

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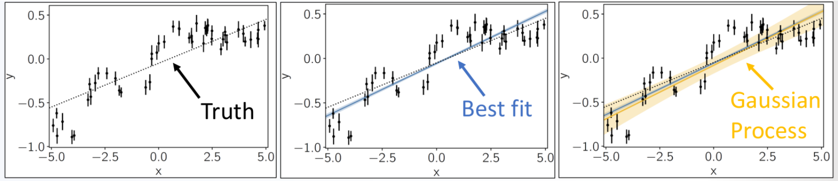
Take home message

- ANNs are a model independent tool that can help us reconstruct cosmological (and not only) parameters.
- We can use them to distinguish between the plethora of theories in the literature, based solely on the data without any physical or statistical assumption.

What are Gaussian processes?



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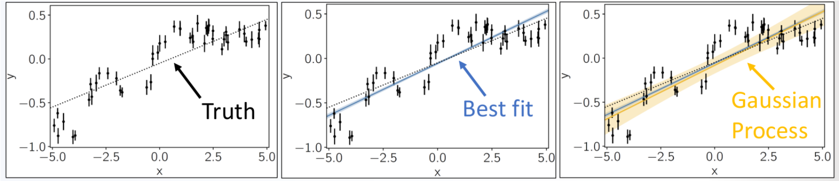


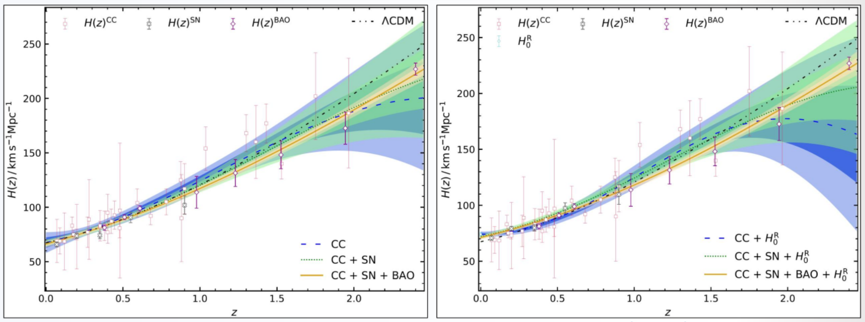
Definition: A GP is a stochastic (random) process where any finite subset is a **multivariant Gaussian distribution** with mean $\mu(x)$ and covariance $k(x, x')$.

Setting each $\mu(x)$ to zero, the **covariance function** can be used to learn the behavior that produced the data points.

Gaussian Process Regression

- The covariance function contains **non-physical hyperparameters** θ which define the distribution $k(\theta, x, x')$.
- Iterating over these values using Bayesian inference (or others) can produce better hyperparameters.
- The result is a **model independent reconstruction** (in physics) of the behabior of some parameter.
- This is superior to regular fitting because it is nonparametric and so **assumes no physical model** whatsoever.

Squared Exponential H_0 GP (GaPP code: Seikel et al. 2012)

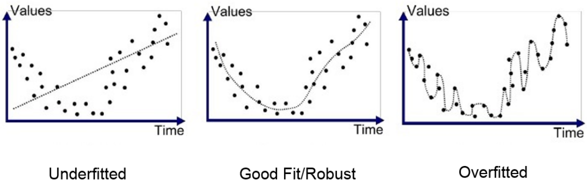


$H_0 = 67.539 \pm 4.772$km/s/Mpc
$H_0 = 67.001 \pm 1.653$km/s/Mpc
$H_0 = 66.197 \pm 1.464$km/s/Mpc

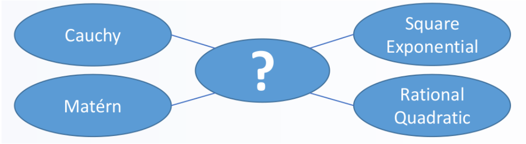
$H_0 = 73.782 \pm 1.374$km/s/Mpc
$H_0 = 72.022 \pm 1.076$km/s/Mpc
$H_0 = 71.180 \pm 1.025$km/s/Mpc

Open problems with GP reconstructions

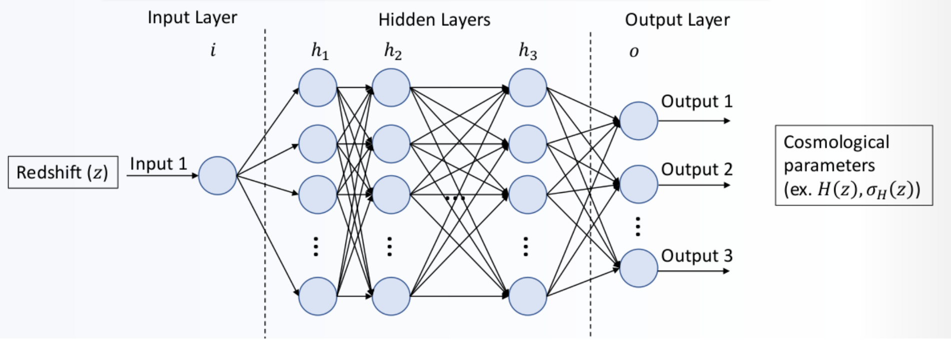
- **Overfitting**: GP is very prone to overfitting for small data sets, which is especially pronounced at the origin, i.e. Hubble constant



- **Kernel Selection Problem**: There is no natural kernel for cosmology

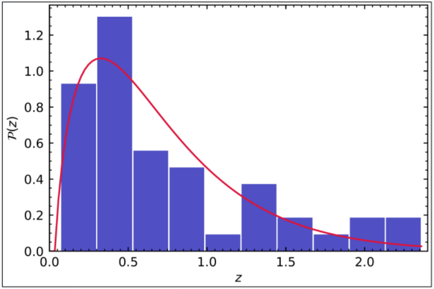


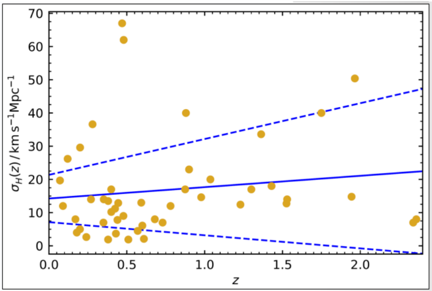
Artificial Neural Networks (ANN)



ReFANN code from Wang et al. (2020)

Training data for the ANN





$$P(z,\alpha,\lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} z^{\alpha-1} e^{-\lambda z}$$

Mean: $\sigma_H = 14.25 + 3.42z$
Upper error: $\sigma_H = 21.37 + 10.79z$
Lower error: $\sigma_H = 7.14 - 3.95z$

Designing the ANN

- **Risk**: Optimizes the **number of hidden layers and neurons** in an ANN

$$\mathrm{risk} = \sum_{i=1}^{N} (\mathrm{Bias}_i^2 + \mathrm{Variance}_i) = \sum_{i=1}^{N} \left([H_{\mathrm{obs}}(z_i) - H_{\mathrm{pred}}(z_i)]^2 + \sigma_H^2(z_i) \right)$$

- **Loss**: Balances the **number of iterations** a system needs to predict the observational data

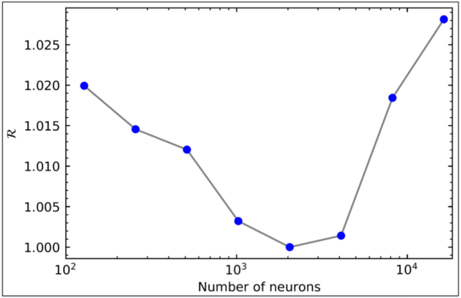
1. Least absolute deviation (**L1**)

$$L1 = \sum_{i=1}^{N} |H_{\mathrm{obs}}(z_i) - H_{\mathrm{pred}}(z_i)|$$

2. Smoothed L1 (**SL1**)
3. Mean Square Error (**MSE**)

$$\mathrm{MSE} = \frac{1}{N} \sum_{i=1}^{N} (H_{\mathrm{obs}}(z_i) - H_{\mathrm{pred}}(z_i))^2$$

Building the ANN



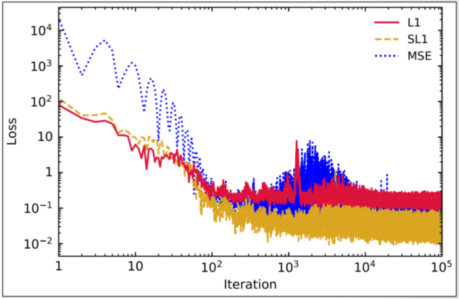


Figure: **Left:** Risk function for **one layer** (number of neurons 2^n, $n \in 7, ..., 14$), **Right:** Evolution of L1, SL1 and MSE loss functions

Using the ANN (KD, Levi Said et al. '21) (KD, Mukherjee et al. '23)

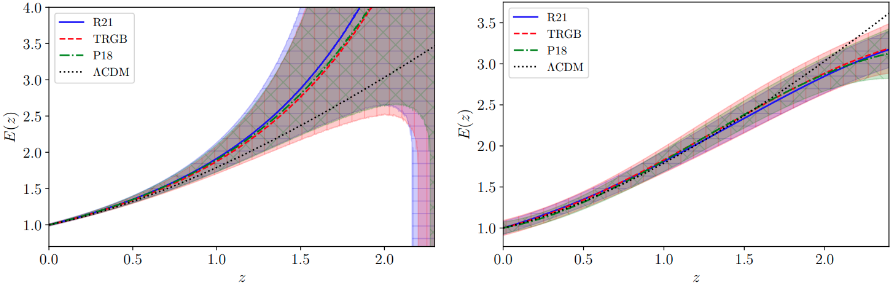


Figure: Reconstructed reduced Hubble parameter from the (i) Pantheon SN compilation (left) and (ii) combined CC+BAO Hubble data set (right), using ANNs.

Om diagnostics (Sahni, Shafieloo, Starobinsky '08) (Shafieloo, Clarkson '10)

Distinguish ΛCDM from alternative dark energy and modified gravity models:

$$Om(z) = \frac{E^2(z) - 1}{(1+z)^3 - 1} .$$

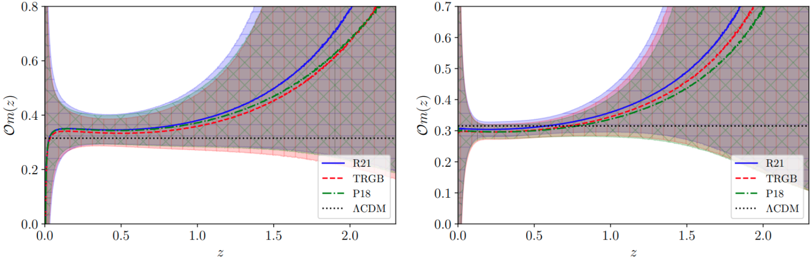


Figure: Reconstructed *Om* diagnostics using (i) ANNs (left) and (ii) GPs (right) from the Pantheon SN data for three different priors.

H0 diagnostics (Krishnan, Colgáin, Sheikh-Jabbari, Yang '20)

It is defined as

$$\mathbf{H0} = \frac{H(z)}{\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}},$$

and its non-constancy suggests evidence for new physics beyond ΛCDM.

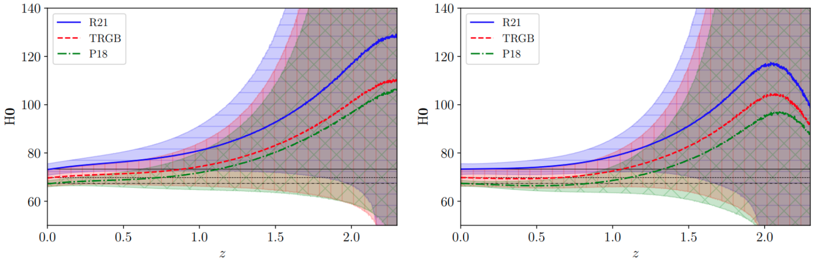


Figure: Reconstructed H_0 diagnostics using (i) ANNs (left) and (ii) GPs (right) from the Pantheon SN data for three different priors.

Constraining theories Arjona, Cardona, Nesseris '19

Example: Horndeski mapping:

$$G_2 = K(X),\ G_3 = G(X),\ G_4 = 1/2, \text{ and } G_5 = 0\,,$$

The action is given by:

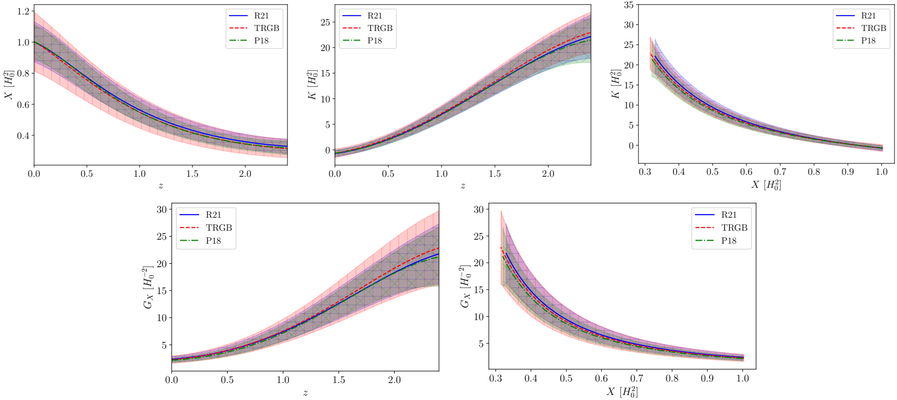
$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\frac{R}{2} - K(X) - G(X)\Box\phi\right) + S_{\mathrm{mat}}(\psi, g_{\mu\nu})\,.$$

Cosmological equations (flat FLRW):

$$K(X) = -3H_0^2\left(1-\Omega_{m0}\right) + \frac{\mathcal{J}\sqrt{2X}H^2(X)}{H_0^2\Omega_{m0}} - \frac{\mathcal{J}\sqrt{2X}\left(1-\Omega_{m0}\right)}{\Omega_{m0}}\,,$$

and

$$G_X(X) = -\frac{2\mathcal{J}H'(X)}{3H_0^2\Omega_{m0}}\,.$$



(KFD, Mukherjee, Levi Said, Mifsud '23)

We can also compute the DE EoS as

$$w_\phi = \frac{-K + \sqrt{2X}\dot{X}G_X}{K - 2X(K_X + 3\sqrt{2X}HG_X)}$$

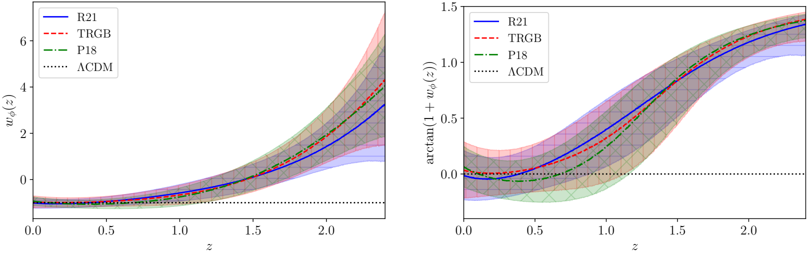


Figure: Plots for dark energy EoS $w_\phi(z)$ (left) and its compactified form $\arctan(1 + w_\phi(z))$ (right) considering R21, TRGB, and P18 H_0 priors. The shaded regions with '$-$', '$|$' and '$\times$' hatches represent the 1σ confidence levels for the above priors respectively.

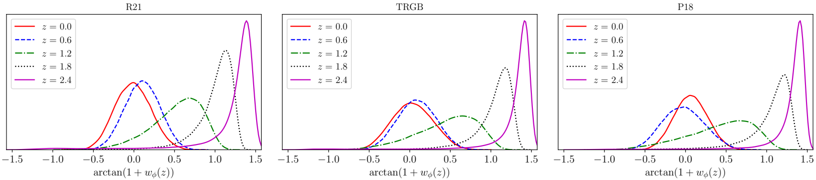


Figure: Plots showing the posteriors of probability distribution of the compactified dark energy EoS for the theory at some sample redshifts for the R21, TRGB, and P18 H_0 priors, respectively.

Conclusion and Prospects

- GP and ANN both have positive features in reconstructing cosmological data sets.
- However, ANN shows greater promise in that they rely on less rigid training data and can model more complex structures of data sets.

From now on, it would be interesting to

- forecast observations for experiments in progress that are about to publish their results,
- use the reconstructed Hubble parameter and its derivative to constrain or even eliminate more alternative cosmological models,
- consider observations related to the perturbative part of the theory, such as LSS or GWs, in the context of ANNs.

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