Entropy Bounds for Rotating AdS Black Holes

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Based on Amo-Frassino-Hennigar, *Phys. Rev. Lett*. 131, 241401 (2023) [2307.03011].

Extended BH Thermodynamics

Sekiwa (2006)
Kastor, Ray, Trashen (2009)
Cvetic, Gibbons, Kubznak, Pope (2011)

Smarr formula in AdS：$\frac{D-3}{D-2}M = TS + \sum_i \Omega^i J^i - \frac{2}{D-2}PV + \frac{D-3}{D-2}\sum_j \Phi^j Q^j$

absent in asymptotically flat

First law：$\delta M = T\delta S + \sum_i^N \Omega^i \delta J^i + V\delta P + \sum_j \Phi^j \delta Q^j$

where $P := -\frac{\Lambda}{8\pi}$ $V := -\frac{1}{2}\int_{\mathcal{H}} dS_{\alpha\beta}\omega^{\alpha\beta} \left(= \left(\frac{\partial M}{\partial P}\right)_{A,J^i,Q^j}\right)$ $\omega^{\alpha\beta}$:Killing potential

Extended BH thermodynamics is natural framework for AdS BH.

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Extended BH thermodynamics is natural framework for AdS BH.

Reverse Isoperimetric Inequality

cf.) Isoperimetric Inequality states $\mathcal{R} \leq 1$.

【Reverse Isoperimetric Inequality (Cvetic *et al.*, 2011)】

In GR, for stationary AdS BHs ($D \geq 4$), it was conjectured that

$$\mathcal{R} := \left(\frac{V}{\mathcal{V}_0}\right)^{1/(D-1)} \left(\frac{\mathcal{A}_0}{A}\right)^{1/(D-2)} \geq 1 .$$

A :BH area, V: thermodynamic volume , $\mathcal{A}_0$:area of unit sphere, $\mathcal{V}_0$:volume of unit ball

Checked for several solutions:

- Kerr-AdS ($D \geq 4$)
- Charged static AdS ($D = 4,5,7$)
- Kerr-Newman-AdS ($D = 4$)
- Charged rotating BH gauged SUGRA ($D = 4,5$)
- Charged rotating AdS-C ($D = 4$)
- Thin Black Ring ($D \geq 5$)

Another view of Reverse Isoperimetric Inequality:

$$A(V) \leq A_{\mathrm{Schw}}(V)$$

Analogy to Penrose Inequality

*All inequalities here are conjectures.

Penrose Inequality

in flat

$$A\left(M\right) \leq A_{\mathrm{Sch}}\left(M\right) := 4\pi\left(2M\right)^2$$

↓ Refine with angular momentum

$$A\left(M, J\right) \leq A_{\mathrm{Kerr}}\left(M, J\right)$$

Reverse Isoperimetric Inequality

in AdS

$$A\left(V\right) \leq A_{\mathrm{Sch}}\left(V\right)$$

↓ Refine with angular momentum

Refined Reverse Isoperimetric Inequality

$$A(M, J_i, V) \leq A_{\mathrm{Kerr}}(M, J_i, V)$$

Let us check our conjecture with AdS BH solutions!

1. Introduction & Conjecture
2. **Check with solutions**
3. Conclusion

Example 1 : Kerr-Newman-AdS ($D = 4$)

Kerr-Newman-AdS : $36\pi M^2V^2 - M^2A^3 - 64\pi^3J^4 = 16\pi^2Q^2J^2A$
≥ 0

Kerr-AdS : $36\pi M^2V^2 - M^2A^3 - 64\pi^3J^4 = 0$

$A_{\mathrm{KN}}(M, J, V) \leq A_{\mathrm{Kerr}}(M, J, V)$ holds.

Example 2 , 3

- ✓ **Pairwise-Equal Charge gauged supergravity BH** ($D = 4$, **with rotation & charge**)
- ✓ **AdS-C** ($D = 4$, **with rotation & charge**)

Both of above solutions : $36\pi M^2V^2 - M^2A^3 - 64\pi^3J^4 \geq 0$

Kerr-AdS : $36\pi M^2V^2 - M^2A^3 - 64\pi^3J^4 = 0$

$A(M, J, V) \leq A_{\text{Kerr}}(M, J, V)$ holds.

Other examples

- **AdS Thin Black Ring** $(D \geq 5)$
  - Checked for all parameters **analytically**
- **Charged rotating BH in gauged SUGRA** $(D = 5)$
  - Could not check for all parameter analytically
  - Checked for 10^6 parameters
- **Charged Rotating BH in gauged SUGRA** $(D = 7)$
  - Calculated thermodynamic volume **for the first time**
  - Could not check for all parameters analytically
  - 200 parameters

Summary

- Conjectured that area (entropy) is maximum for Kerr-AdS with fixed M, J, V.
- Checked for several examples, all of which supported our conjecture.

Future work

- Towards flat spacetimes: ex.) $V \geq \frac{A^{(2-\varepsilon)/(D-2)}}{(D-1)(D-2)} \left[(D-2)A^2 + 64\pi^2 \sum_i J_i^2 \right] \left[\omega_{D-2} \prod_j \left(A^2 + 64\pi^2 J_j^2 \right) \right]^{-1/(D-2)}$
- Towards $D = 3$