Massive Neutrino Self-Interactions and the Hubble Tension

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Talk at YITP

This Talk is based on ...

- **Shouvik Roy Choudhury**, Steen Hannestad, Thomas Tram, "*Updated constraints on massive neutrino self-interactions from cosmology in light of the H_0 tension*," **arXiv: 2012.07519 (JCAP 03 (2021) 084)**.

Part 1

- **Part 1: Related to Hubble Tension.**

Introduction: Bayesian Statistics

- Bayes' Theorem: $P(B)P(A|B) = P(B|A)P(A)$, where A and B are different events.
- Notations: $D \equiv$ data, $\theta \equiv \{\theta_1, \theta_2, ...\theta_n\} \equiv$ parameters, $M \equiv$ model.
- **For a particular model M**,

$$P(\theta|D, M)P(D|M) = P(D|\theta, M)P(\theta|M) \tag{1}$$

- **Posterior** $\times$ **Evidence** $=$ **Likelihood** $\times$ **Prior.**
- **Normalization:** $\int P(\theta|D, M)d\theta = 1$.
- **Evidence**:

$$Z_M \equiv P(D|M) = \int P(D|\theta, M)P(\theta|M)d\theta. \tag{2}$$

Introduction: Bayesian Statistics

- **In cosmological parameter estimation**, we are usually interested in the posterior probability distribution, $\mathcal{P}_M(\theta) \equiv P(\theta|D, M)$, given the likelihood $\mathcal{L}_M(\theta) \equiv P(D|\theta, M)$, and priors $\pi_M(\theta) \equiv P(\theta|M)$.
- If we are interested in Bayesian model comparison, **Evidence is the most important quantity.**
- Let us apply Bayes' theorem again,

$$P(M|D)P(D) = P(D|M)P(M) \equiv Z_M P(M) \tag{3}$$

- If we have two models M_1 and M_2,

$$\frac{P(M_1|D)}{P(M2|D)} = \frac{P(M_1)}{P(M_2)} \frac{Z_{M_1}}{Z_{M_2}} \tag{4}$$

- Typically, models are assigned the same prior preference, $P(M_1) = P(M_2)$.

Introduction: Bayesian Statistics

- Thus we have,

$$\frac{P(M_1|D)}{P(M_2|D)} = \frac{Z_{M_1}}{Z_{M_2}} \equiv B, \tag{5}$$

where B is called the Bayes' factor.

- ln B $\simeq$ 0.77 (1σ), 3 (2σ), 5.9 (3σ), 9.7 (4σ), 14.37 (5σ).

Introduction

- Einstein's field equations of classical General Relativity state that:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{6}$$

- The universe is homogeneous and isotropic on large scales $\rightarrow$ Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{7}$$

- Assuming $T_{\mu\nu} = \mathrm{diag}(\rho, P, P, P)$ (corresponding to a perfect fluid with energy density ρ and pressure P) $\rightarrow$ Friedmann equations:

$$H(a)^2 = \frac{8\pi G}{3}\rho(a) - \frac{K}{a^2} \tag{8}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P), \tag{9}$$

with the dot denoting a time derivative.

Introduction

- The contribution to the energy density $\rho(a)$ comes from various sources: photons (γ), massive neutrinos (ν), baryons(b), dark matter (c), dark energy (DE). Introducing the redshift as $z = 1/a - 1$, we can write,

$$\rho(z) = \rho_\gamma(z) + \rho_c(z) + \rho_b(z) + \rho_{\rm DE}(z) + \rho_\nu(z). \tag{10}$$

- The Equation of State (EoS) w_i of a particular component of the universe (except curvature) is defined as $P_i = w_i \rho_i$.

$$\rho_i(z) \propto (1+z)^{3(1+w_i)}. \tag{11}$$

- In general, we use the subscript 0 to denote quantities evaluated at the present time.

$$\Omega_i = \frac{\rho_{i,0}}{\rho_{cr,0}}, \qquad \rho_{\rm cr,0} = \frac{3H_0^2}{8\pi G}. \tag{12}$$

for $i \equiv \gamma, \nu, b, c$, DE. We also define $\Omega_{\rm k} = -K/H_0^2$.

Introduction

- Since photons always behave as radiation, $w_\gamma = 1/3$, whereas for CDM and baryons behave as matter for most of the evolution of the universe and thus one can take $w_c = w_b = 0$.
- For DE, we for now allow for an arbitrary but constant EoS, i.e. $w_{DE} = w$. If dark energy is described by a cosmological constant, Λ, then $w = -1$, and in that case we shall denote $\Omega_{\rm DE}$ as Ω_Λ.

$$H(z)^2 = H_0^2 \Bigg[\Omega_\gamma (1+z)^4 + (\Omega_c + \Omega_b)(1+z)^3 + \\ + \Omega_{\rm DE}(1+z)^{3(1+w)} + \Omega_{\rm k}(1+z)^2 + \frac{\rho_\nu(z)}{\rho_{\rm cr,0}} \Bigg] . \quad (13)$$

Introduction: Very Brief Thermal History

- **Neutrino Decoupling:** $T \sim 1$ MeV. Weak interaction rate becomes less than universal expansion rate. Not instantaneous.
- **Electron-positron annihilation:** $T \sim 0.5$ MeV. Slightly heats up the neutrinos which haven't fully decoupled. Mostly heats up the photons.
- **Big Bang Nucleosynthesis:** $T \sim 100$ keV.
- **Matter-radiation equality:** $T \sim 0.75$ eV.
- **Recombination:** $T \sim 0.3$ eV.
- **Photon decoupling:** $T \sim 0.26$ eV.
- **Drag epoch:** Baryons are dragged along with photons. Continues up to $T \sim 0.20$ eV.
- **Reionization:** Ends the dark ages. When the first stars form, the ensuing UV radiation reionizes neutral Hydrogen in the intergalactic medium. $T \sim 5$ meV.
- **Matter-Dark energy equality:** $T \sim 0.75$ meV.
- **Today:** $T \sim 0.24$ meV.

Introducing Neutrinos

- Active neutrinos have three mass eigenstates (ν_1, ν_2, and ν_3) which are quantum superpositions of the 3 flavour eigenstates (ν_e, ν_μ, and ν_τ). The sum of the mass of the neutrino mass eigenstates, is the quantity,

$$\sum m_\nu \equiv m_1 + m_2 + m_3, \tag{14}$$

  where m_i is the mass of the i^{th} neutrino mass eigenstate.
- Tightest bounds on $\sum m_\nu$ come from cosmology.
- We use the approximation, $m_i = \sum m_\nu / 3$ for all i.
- The radiation density in the early universe can be written as,

$$\rho_r = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\rm eff} \right] \rho_\gamma \tag{15}$$

  $N_{\rm eff}$ is the effective number of relativistic degrees of freedom.

The ΛCDM parametrization

- The **ΛCDM** model parametrization is given by:

$$\boldsymbol{\theta = \{\Omega_c h^2, \Omega_{\rm b} h^2, 100\theta_{MC}, \tau, \ln(10^{10} A_s), n_s\}.} \tag{16}$$

- $\omega_c \equiv \Omega_c h^2$ and $\omega_b \equiv \Omega_b h^2$ are the present-day physical CDM and baryon densities respectively.
- θ_{MC} is the parameter for **angular size of the sound horizon**, i.e. ratio between the sound horizon r_s^* and the angular diameter distance D_A^* at photon decoupling.
- τ is the optical depth to reionization. $\tau = \int_0^{z_{re}} n_e \sigma_T dl$ where n_e is free electron number density, σ_T is the Thomson scattering cross-section.
- n_s and A_s are the power-law spectral index and amplitude of the primordial scalar perturbations, respectively, at the pivot scale of $k_* = 0.05$ h Mpc^{-1}, i.e. the primordial power spectrum $P(k) = A_s (k/k_*)^{n_s - 1}$.

The sound horizon at last scattering

- The comoving sound horizon at the CMB last scattering is

$$r_s^* = \int_{z_*}^{\infty} \frac{c_s(z)dz}{H(z)} \tag{17}$$

- r_s^{drag} is the comoving sound horizon at the end of drag epoch, which is slightly higher (around 2%) than r_s^*
- The angular diameter distance to the last scattering surface is

$$D_A^* = \int_0^{z_*} \frac{dz}{H(z)} \tag{18}$$

- $\theta_{MC} = r_s^*/D_A^* \simeq \pi/\Delta l$, where Δl is the peak spacing in CMB temperature power spectrum.
- Remember, in ΛCDM (+massive neutrinos):

$$H(z)^2 = \left[\omega_\gamma(1+z)^4 + (\omega_c + \omega_b)(1+z)^3 + \omega_\Lambda + \frac{\rho_\nu(z)}{\rho_{\mathrm{cr},0}}\right]. \tag{19}$$

The Hubble Tension

Value from Planck 2018 in ΛCDM : $H_0 = 67.36 \pm 0.54$ km/s/Mpc
Value from calibrated type Ia Supernovae in the local universe: H_0=73.04 ± 1.04 km/s/Mpc (SH0ES 2022)

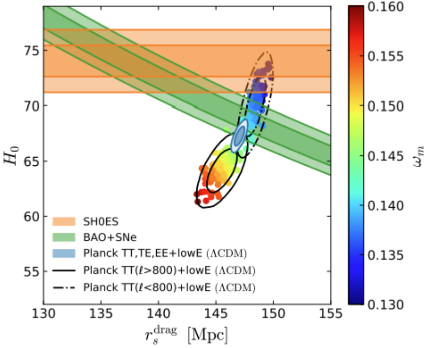


Figure: Depiction of the H0 tension

Lloyd Knox, Marius Millea, arXiv: 1908.03663 (Phys. Rev. D 101, 043533 (2020)

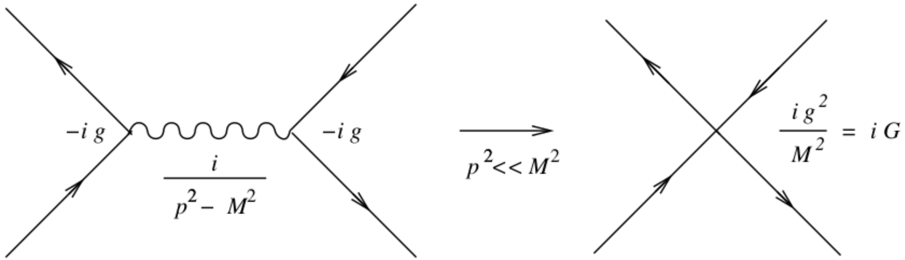
Extra light relics in the early universe

- $100\theta_{MC} = 1.04109 \pm 0.00030$ (68%, Planck 18 TT,TE,EE+lowE). This is a measurement with 0.03%. θ_{MC} is the most well-constrained parameter in all of cosmology.
- Theoretical value of $N_{\text{eff}}^{SM} = 3.0440 \pm 0.00024$ assuming standard model of particle physics.
- Extra $\Delta N_{\text{eff}} \simeq 1$ can increase $H(z)$ in the early universe, which will decrease r_s^* enough to solve the Hubble tension.
- But in ΛCDM+N_{eff} model: $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ (95%, Planck 2018 TT,TE,EE+lowE+lensing+BAO)
- **Simple light relics are not enough to solve the 5σ Hubble tension.**

Neutrino Self-interactions mediated by a heavy scalar

- In this paper we have updated the constraints from cosmology on flavour universal neutrino self-interactions mediated by a heavy scalar ($m_\phi \geq 1$ keV), in the effective 4-fermion interaction limit (CMB temperature is far lower than the keV range).
- Simplified universal interaction: $\mathcal{L}_{\text{int}} \sim g_{ij}\bar{\nu_i}\nu_j\Phi$, with $g_{ij} = g\delta_{ij}$.
- The effective self-coupling, $G_{\text{eff}} = g^2/m_\Phi^2$, with $G_{\text{eff}} > G_F$ (Fermi constant), so that they remain interacting with each other even after decoupling from the photons at $T \sim 1$ MeV.
- The self-interaction rate per particle $\Gamma = n\langle\sigma v\rangle \sim G_{\text{eff}}^2 T_\nu^5$, where $n \propto T_\nu^3$ is the number density of neutrinos. Neutrinos don't free-stream until $\Gamma < H$.
- Introducing this kind of interaction had shown potential in solving the Hubble tension in previous works in the very strong interaction range ($G_{\text{eff}} \sim 10^9 G_F$) using older data.

Feynman Diagram



$\mathrm{M} \equiv m_\Phi$

The Cosmological Model of interest

- Cosmological model: $\mathbf{\Lambda CDM + \log_{10}\left[G_{eff} MeV^2\right] + N_{eff}} + \sum \boldsymbol{m_\nu}$.
- Kreisch et. al., Phys. Rev. D 101, 123505 (2020) (arXiv: 1902.00534) found the 68% bounds:
  $\mathbf{\log_{10}\left[G_{eff} MeV^2\right] = -1.41^{+0.20}_{-0.066}}$ (strong self-interactions),
  $\boldsymbol{H_0} = \mathbf{71.1 \pm 2.2\ km/s/Mpc}$,
  $\boldsymbol{N_{\mathrm{eff}}} = \mathbf{3.80 \pm 0.45}$,
  $\sum \boldsymbol{m_\nu} = \mathbf{0.39^{+0.16}_{-0.20}\ eV}$
  with **Planck 2015 low-l and high-l TT+lensing** combined with **BAO**, with similar goodness of fit to the data as **ΛCDM**.
- In this model, $\boldsymbol{N_{\mathrm{eff}}}$ and $\boldsymbol{H_0}$ are **positively correlated** → Solution to the Hubble tension came from high $\boldsymbol{N_{\mathrm{eff}}} \simeq \mathbf{4}$ values.
- Planck polarization data was not used for main conclusions.

The Cosmological Model of Interest

Image Credit: Kreisch et. al., Phys. Rev. D 101, 123505 (2020), arXiv: 1902.00534

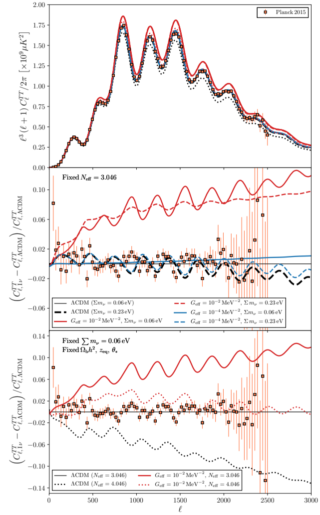


Figure: Degeneracy of of $\boldsymbol{G_{\rm eff}}$ with $\boldsymbol{N_{\rm eff}}$ and $\sum \boldsymbol{m_\nu}$ in the CMB TT spectrum.

The Cosmological Model of interest

- **With the public release of the Planck 2018 likelihoods, we thought it is timely to test the model again.**
- We made runs which incorporated the full prior range of $\mathbf{log_{10}}\left[\mathbf{G_{eff}}\mathrm{MeV}^{\mathbf{2}}\right]$, i.e. $\mathbf{-5.5 \to -0.1}$.
- We also run the non-interacting case ($\mathbf{NI\nu}$: $\boldsymbol{G_{\mathrm{eff}} = 0}$), the moderately interacting case $\mathbf{MI\nu}$ ($\mathbf{log_{10}\left[G_{eff}MeV^2\right] \lesssim -2}$), and the strongly interacting case ($\mathbf{SI\nu}$) ($\mathbf{log_{10}\left[G_{eff}MeV^2\right] \gtrsim -2}$) separately.
- We sample the parameter space using the nested sampling technique. We use the publicly available **PolyChord** extension of **CosmoMC**, called **CosmoChord**.
- Use of the nested-sampling package PolyChord enables us to calculate evidences accurately, and properly sample this parameter space of **bimodal posterior distributions**.
- We modify the **CAMB** code to incorporate the neutrino self-interactions in the perturbation equations.

Collisional Boltzmann Equations

The perturbed metric in the Synchronous gauge:

$$ds^2 = a^2(\tau)\{-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\}\,. \tag{20}$$

The scalar mode of h_{ij} can be Fourier expanded as:

$$h_{ij}(\vec{x},\tau) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \left\{\hat{k}_i\hat{k}_j h(\vec{k},\tau) + (\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij})\, 6\eta(\vec{k},\tau)\right\}\,, \quad \vec{k} = k\hat{k}\,. \tag{21}$$

Here h and η are the metric perturbations, defined from the perturbed space-time metric in synchronous gauge.
The Boltzmann equation can generically be written as

$$L[f] = \frac{Df}{D\tau} = C[f], \tag{22}$$

where $L[f]$ is the Liouville operator. The collision operator on the right-hand side describes any possible collisional interactions.

Collisional Boltzmann Equations

One can then write the distribution function as

$$f(x^i, q, n_j, \tau) = f_0(q)[1 + \Psi(x^i, q, n_j, \tau)], \qquad (23)$$

where $f_0(q)$ is the unperturbed distribution function.
In synchronous gauge the Boltzmann equation can be written as an evolution equation for Ψ in k-space

$$\frac{1}{f_0} L[f] = \frac{\partial \Psi}{\partial \tau} + i \frac{q}{\epsilon} \mu \Psi + \frac{d \ln f_0}{d \ln q} \left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2} \mu^2 \right] = \frac{1}{f_0} C[f], \qquad (24)$$

where $\mu \equiv n^j \hat{k}_j$ and $\epsilon = (q^2 + a^2 m^2)^{1/2}$.

Collisional Boltzmann Equations

The perturbation is then expanded as

$$\Psi = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l P_l(\mu). \tag{25}$$

$$\begin{aligned}
\dot{\Psi}_0 &= -\frac{qk}{\epsilon}\Psi_1 + \frac{1}{6}\dot{h}\frac{d\ln f_0}{d\ln q}\,, \\
\dot{\Psi}_1 &= \frac{qk}{3\epsilon}\left(\Psi_0 - 2\Psi_2\right)\,, \\
\dot{\Psi}_2 &= \frac{qk}{5\epsilon}\left(2\Psi_1 - 3\Psi_3\right) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q} + \boldsymbol{\alpha_2 \dot{\tau}_\nu \Psi_2}\,, \\
\dot{\Psi}_l &= \frac{qk}{(2l+1)\epsilon}\left[l\Psi_{l-1} - (l+1)\Psi_{l+1}\right] + \boldsymbol{\alpha_\ell \dot{\tau}_\nu \Psi_l}\,, \quad l \geq 3\,.
\end{aligned} \tag{26}$$

where $\dot{\tau}_\nu \equiv -aG_{\text{eff}}^2 T_\nu^5$ is the neutrino self-interaction opacity, and α_l ($l > 1$) are model dependent coefficients of order unity.

Plots from runs with full prior range of $\log_{10}[G_{\rm eff}\,{\rm MeV}^2]$

Main conclusions follow from the TTTEEE+lowE+EXT dataset (blue curve).

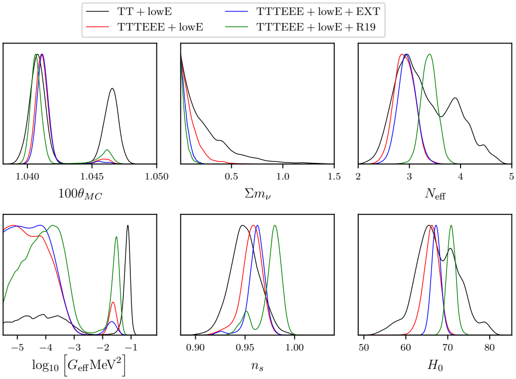
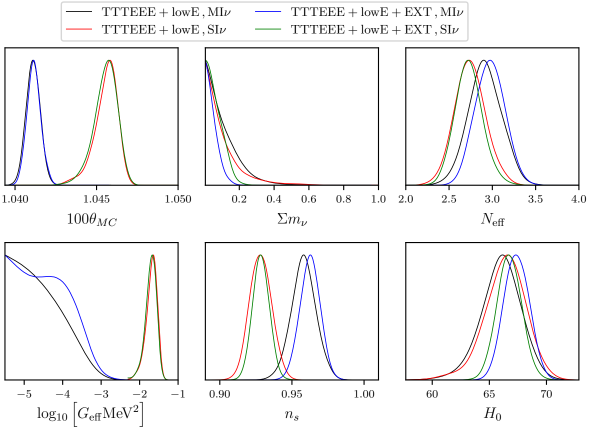


Figure: Here TTTEEE+lowE denotes the full Planck 2018 temperature and polarisation data. EXT denotes Planck 2018 lensing + BAO + RSD + SNeIa. R19 is the Gaussian prior of $H_0 = 74.03 \pm 1.42$ km/s/Mpc.

Roy Choudhury et al, arXiv 2012.07519 (JCAP 03 (2021) 084)

Mode separation: MIν and SIν plots shown separately



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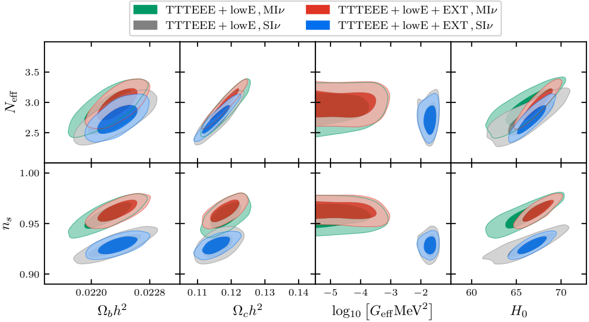

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Discussion

- $\log_{10}\left[\mathbf{G_{eff}MeV^2}\right]$ is degenerate with $\boldsymbol{\theta_{MC}}$ and $\boldsymbol{n_s}$. This allows for a bimodal posterior distribution, even with the latest full Planck data.
- With **TTTEEE+lowE+EXT** we found the following **95% bounds**, for the **SI$\boldsymbol{\nu}$**
  $\boldsymbol{H_0 = 66.7^{+2.2}_{-2.1}}$ **km/s/Mpc**
  $\boldsymbol{N_{\rm eff} = 2.73^{+0.34}_{-0.31}}$
  $\boldsymbol{\sum m_\nu < 0.15}$ **eV**.
- Even if one were to re-analyze the data with a fixed $\boldsymbol{N_{\rm eff} = 3.044}$ with massive neutrinos and strong interactions, one would very likely get $\boldsymbol{H_0}$ values in the ballpark of $\mathbf{69-70}$ **km/s/Mpc** (as can be seen from the plots above), which does not work as a solution to the Hubble tension, albeit reducing the tension slightly compared to vanilla **ΛCDM**.
- For the Non-interacting case ($\mathbf{NI\nu : \Lambda CDM + N_{eff} + \sum m_\nu}$), we find $\boldsymbol{H_0 = 67.3 \pm 2.2}$ **km/s/Mpc (95%)** $\rightarrow$ The strongly interacting model doesn't work better than this simple extension to **ΛCDM**.

EXT ≡ Planck 2018 lensing + BAO + RSD + SNeIa

Discussion

- Furthermore, **Neutrino self-interactions are also strongly constrained from particle physics experiments**, with the exception of flavour specific interaction among the $\boldsymbol{\tau}$-neutrinos.
- We find, $\mathbf{-2\,[\log\,(\mathcal{L}_{SI\nu}/\mathcal{L}_{NI\nu})] = 3.4}$ (approx. $\mathbf{\Delta\chi^2}$), and $\boldsymbol{Z_{SI\nu}/Z_{NI\nu} = 0.06}$ (evidence ratio), with **TTTEEE+lowE+EXT**.
- **Bayesian evidences and log likelihood values both disfavour very strong self-interactions** compared to $\mathbf{\Lambda CDM + N_{eff} + \sum m_\nu}$, i.e. the non-interacting scenario $\mathbf{NI\nu}$.
- **To conclude, with current data, the strong neutrino self-interaction model does not look like a promising solution to the current $\boldsymbol{H_0}$ discrepancy.**

Particle Physics Constraints

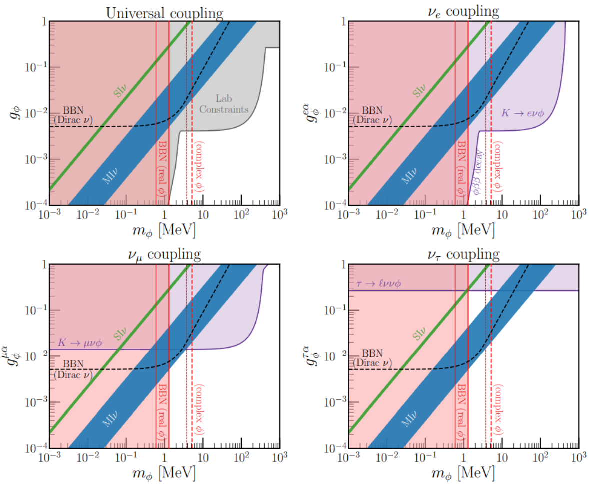

Figure: Constraints from particle physics

Nikita Blinov et al, arXiv 1905.02727 (Phys.Rev.Lett. 123 (2019) 19, 191102)

THE END

Thanks for listeninng! Questions are welcome!