“Gravity and Cosmology 2024” 19th Feb. 2024

Circular polarization of gravitational waves passing through an axion domain wall

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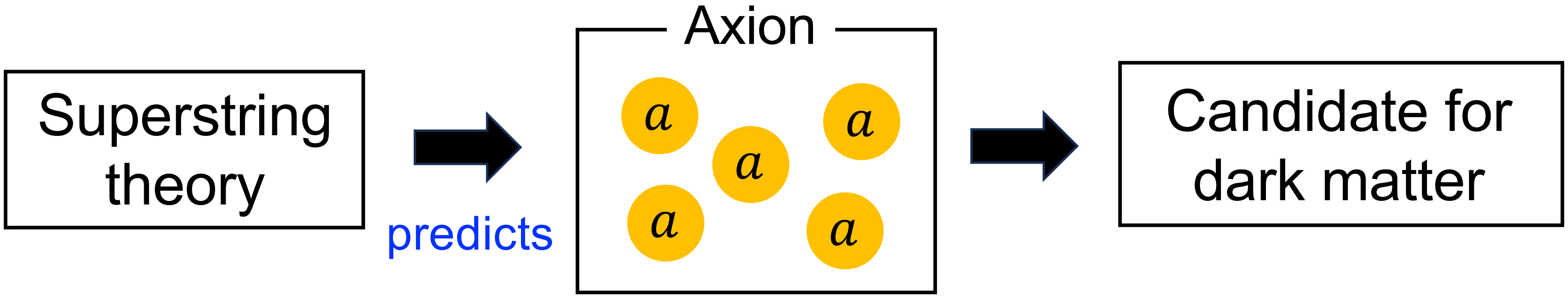
Based on PRD 108, 083525 (2023)

Axion and Chern-Simons gravity

Why is axion important?



It leads to verifying superstring theory and dark matter.

Chern-Simons(CS) gravity

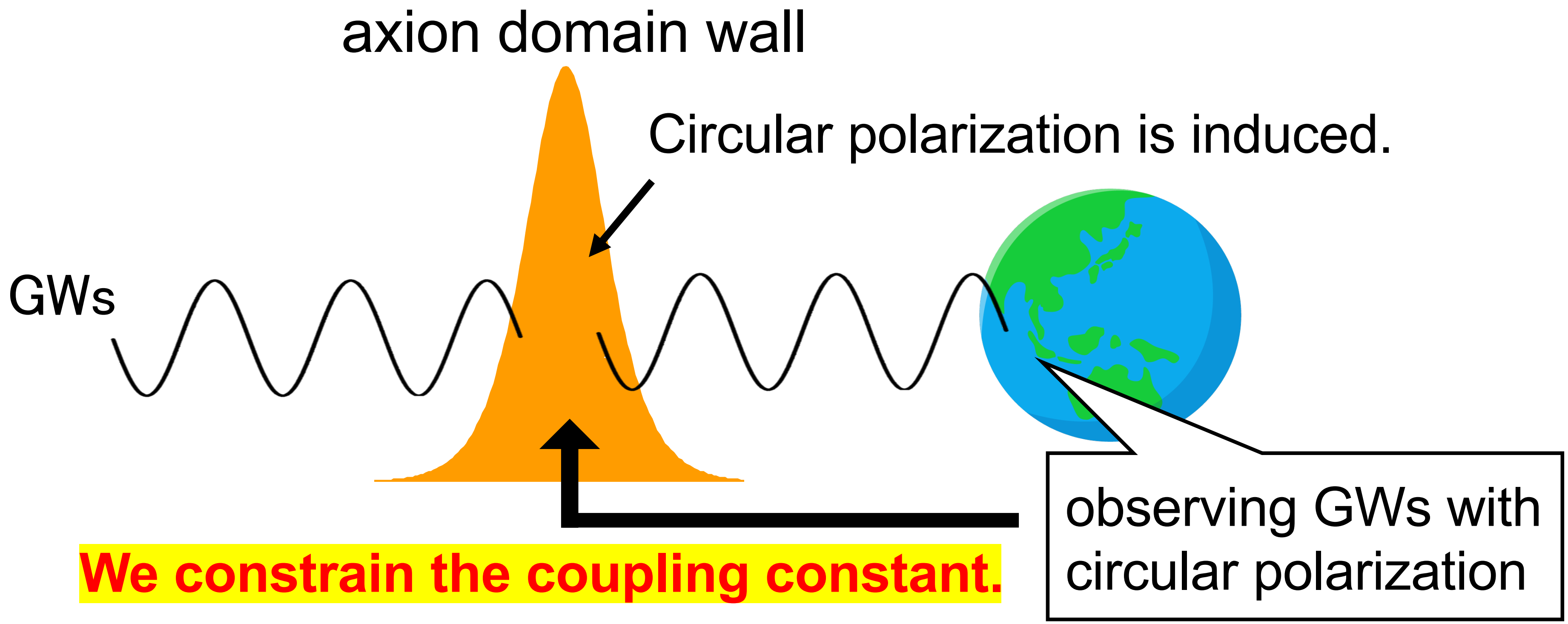
coupling constant: ℓ^2

axion: ϕ

$$S = \frac{M_{\mathrm{p}}^2}{2}\int d^4x\sqrt{-g}R + \underbrace{\frac{M_{\mathrm{p}}\ell^2}{8}\int d^4x\sqrt{-g}\,\phi\, R\tilde{R}}_{\text{CS term}} \quad \left[R\tilde{R} = \frac{1}{2}\varepsilon^{\rho\sigma\alpha\beta}R^{\mu\nu}{}_{\alpha\beta}R_{\nu\mu\rho\sigma}\right]$$

CS term induces the circular polarization in GWs.

Purpose of this study



What we investigated in this work

- Is the circular polarization large enough to be observable?
- How do we give the constraint on the coupling constant?

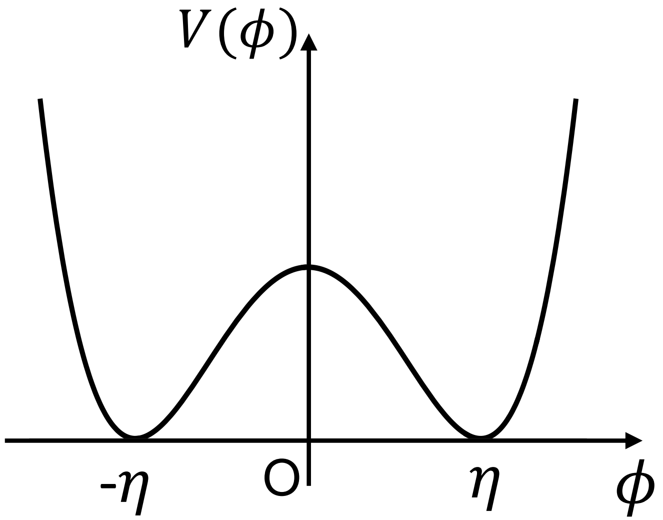
Axion domain wall

action

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

self-coupling constant

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2$$

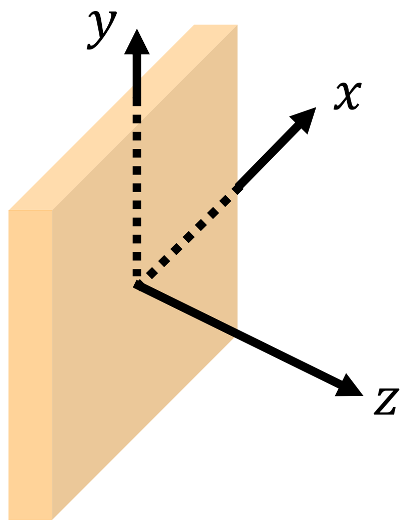


Assuming the domain wall is static and in (x, y)-plane gives

$$\frac{d^2\phi(z)}{dz^2} = \lambda(\phi(z)^2 - \eta^2)\phi(z)$$

The boundary condition: $\phi(\pm\infty) = \pm\eta$

The solution: $\phi(z) = \eta\tanh\left(\sqrt{\frac{\lambda}{2}}\eta z\right)$



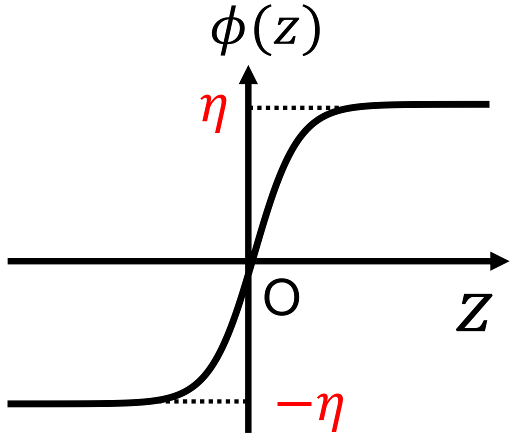
Axion domain wall

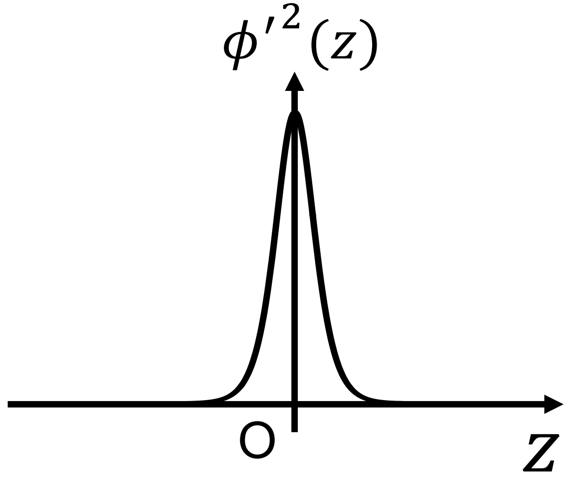
The domain wall solution

$$\phi(z) = \eta \tanh\left(\sqrt{\frac{\lambda}{2}}\eta z\right)$$

$$\phi'(z) = \eta^2 \sqrt{\frac{\lambda}{2}} \mathrm{sech}^2\left(\sqrt{\frac{\lambda}{2}}\eta z\right)$$

[prime: z - derivative]





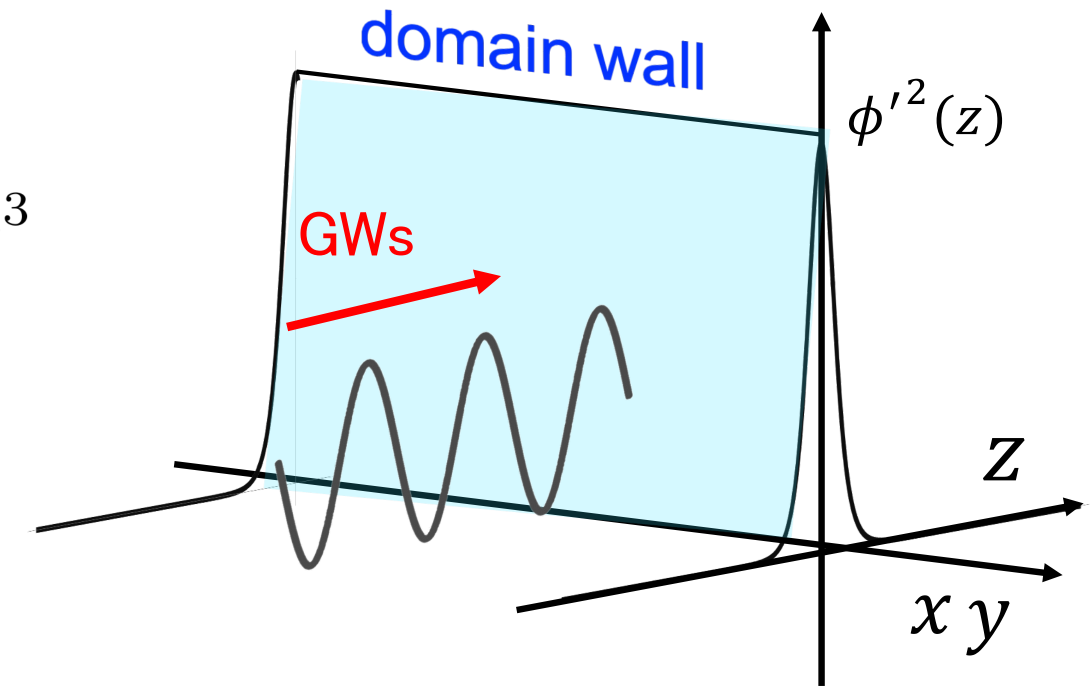
energy density

$$\sigma = \int dz \phi'^2(z) \;\propto \sqrt{\lambda}\eta^3$$

Constraint by CMB

$$\sigma < (0.93\ \mathrm{MeV})^3$$

A. Lazanu (2015)



Equation of motion for CS-GWs

metric

GWs

$$ds^2 = -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j$$

TT gauge $h^i{}_{j,i} = h^i{}_i = 0$

action

Coupling constant

$$S = \frac{M_\mathrm{p}^2}{2}\int d^4x\sqrt{-g}R + \frac{M_\mathrm{p}\ell^2}{8}\int d^4x\sqrt{-g}\,\phi\,R\tilde{R}$$

CS term

$$R\tilde{R} = \frac{1}{2}\varepsilon^{\rho\sigma\alpha\beta}R^{\mu\nu}{}_{\alpha\beta}R_{\nu\mu\rho\sigma}$$

$$\varepsilon^{0123} = \frac{1}{\sqrt{-g}}$$

$$= \frac{M_\mathrm{p}^2}{8}\int d^4x\left[\dot{h}^{ij}\dot{h}_{ij} - h^{ij,k}h_{ij,k} - \frac{\ell^2}{M_\mathrm{p}}\varepsilon^{izk}\partial_z\phi\left\{\ddot{h}^m{}_i\dot{h}_{km} + \dot{h}_{im,\ell}\left(h^\ell{}_k{}^{,m} - h^m{}_k{}^{,\ell}\right)\right\}\right]$$

EOM

$$\Box h^{ij} = \frac{\ell^2}{2M_\mathrm{p}}\varepsilon^{zik}\left[\phi'\Box + \phi''\partial_z\right]\dot{h}^j{}_k + \frac{\ell^2}{2M_\mathrm{p}}\varepsilon^{zjk}\left[\phi'\Box + \phi''\partial_z\right]\dot{h}^i{}_k$$

Equation of motion for CS-GWs

$$\Box h^{ij} = \frac{\ell^2}{2M_{\rm p}}\varepsilon^{zik}\left[\phi'\Box + \phi''\partial_z\right]\dot{h}^j{}_k + \frac{\ell^2}{2M_{\rm p}}\varepsilon^{zjk}\left[\phi'\Box + \phi''\partial_z\right]\dot{h}^i{}_k$$

$$h_{ij}(t,z) = h_R(t,z)e^{(R)}_{ij} + h_L(t,z)e^{(L)}_{ij}, \quad e^{(R)}_{ij} = \begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e^{(L)}_{ij} = \begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

polarization tensor

$$\Box h_{R/L}(t,z) = \pm i \frac{\ell^2}{M_{\rm p}}\left[\phi'\Box + \phi''\partial_z\right]\dot{h}_{R/L}(t,z)$$

Using Fourier modes

$$h_{R/L}(t,z) = H_{R/L}(z)e^{i\omega t}$$

ω : frequency of GWs

$$\left(1 \pm \frac{\omega\ell^2}{M_{\rm p}}\phi'\right)H''_{R/L} \pm \frac{\omega\ell^2}{M_{\rm p}}\phi'' H'_{R/L} + \omega^2\left(1 \pm \frac{\omega\ell^2}{M_{\rm p}}\phi'\right)H_{R/L} = 0$$

Equation of motion for CS-GWs

$$\Box h^{ij} = \frac{\ell^2}{2M_{\rm p}}\varepsilon^{zik}\left[\phi'\Box + \phi''\partial_z\right]\dot{h}^j{}_k + \frac{\ell^2}{2M_{\rm p}}\varepsilon^{zjk}\left[\phi'\Box + \phi''\partial_z\right]\dot{h}^i{}_k$$

$$h_{ij}(t,z) = h_R(t,z)e^{(R)}_{ij} + h_L(t,z)e^{(L)}_{ij}, \quad e^{(R)}_{ij} = \begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ & & \end{pmatrix} \quad e^{(L)}_{ij} = \begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ & 0 & 0 \end{pmatrix}$$

$\Box h_{R/L}$

Right-handed GWs H_R and left-handed GWs H_L follow different equations of motion!

➡ **Circular polarization**

$h_{R/L}(t,z)$ … $(z)e^{i\omega t}$ $\quad\omega$: frequency of GWs

$$\left(1 \pm \frac{\omega\ell^2}{M_{\rm p}}\phi'\right)H''_{R/L} \pm \frac{\omega\ell^2}{M_{\rm p}}\phi'' H'_{R/L} + \omega^2\left(1 \pm \frac{\omega\ell^2}{M_{\rm p}}\phi'\right)H_{R/L} = 0$$

How to solve the EOM

EOM

$$\left(1 \pm \frac{\omega\ell^2}{M_\mathrm{p}}\phi'\right) H''_{R/L} \pm \frac{\omega\ell^2}{M_\mathrm{p}}\phi'' H'_{R/L} + \omega^2 \left(1 \pm \frac{\omega\ell^2}{M_\mathrm{p}}\phi'\right) H_{R/L} = 0$$

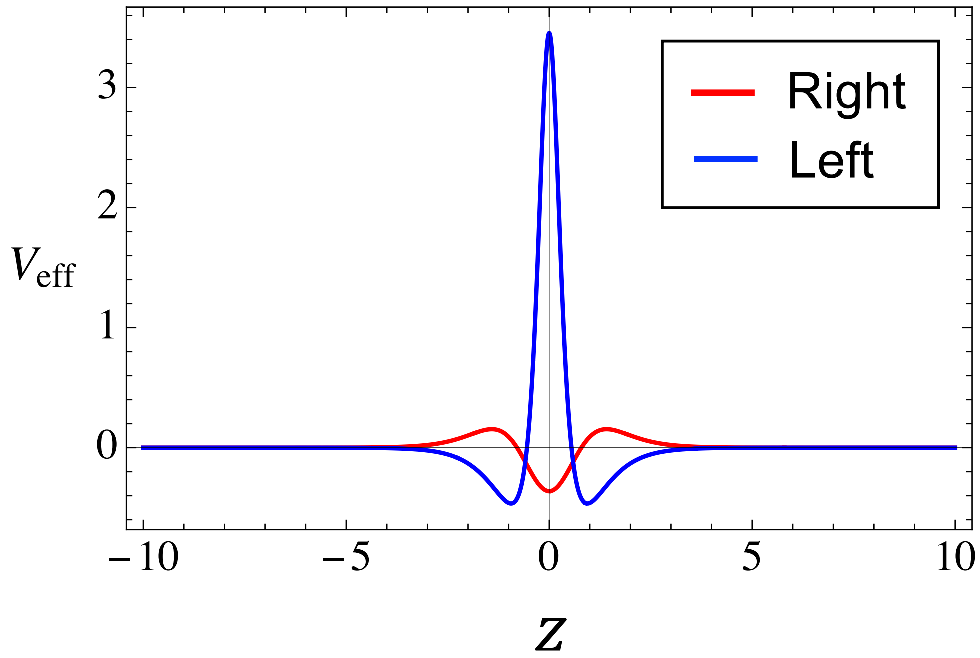
$$\tilde{H}_{R/L} = \sqrt{F(z)} H_{R/L} \qquad F(z) = 1 \pm \frac{\omega\ell^2}{M_\mathrm{p}}\phi'(z)$$

Schrödinger type Eq. :

$$\left[-\frac{d^2}{dz^2} + V_\mathrm{eff}(z)\right] \tilde{H}_{R/L} = \omega^2 \tilde{H}_{R/L}$$

$$V_\mathrm{eff} = -\frac{1}{4}\left(\frac{F'}{F}\right)^2 + \frac{1}{2}\frac{F''}{F}$$

Effective potential



How to solve the EOM

EOM

$$\left[-\frac{d^2}{dz^2}+V_{\rm eff}(z)\right]\tilde{H}_{R/L}=\omega^2\tilde{H}_{R/L}$$

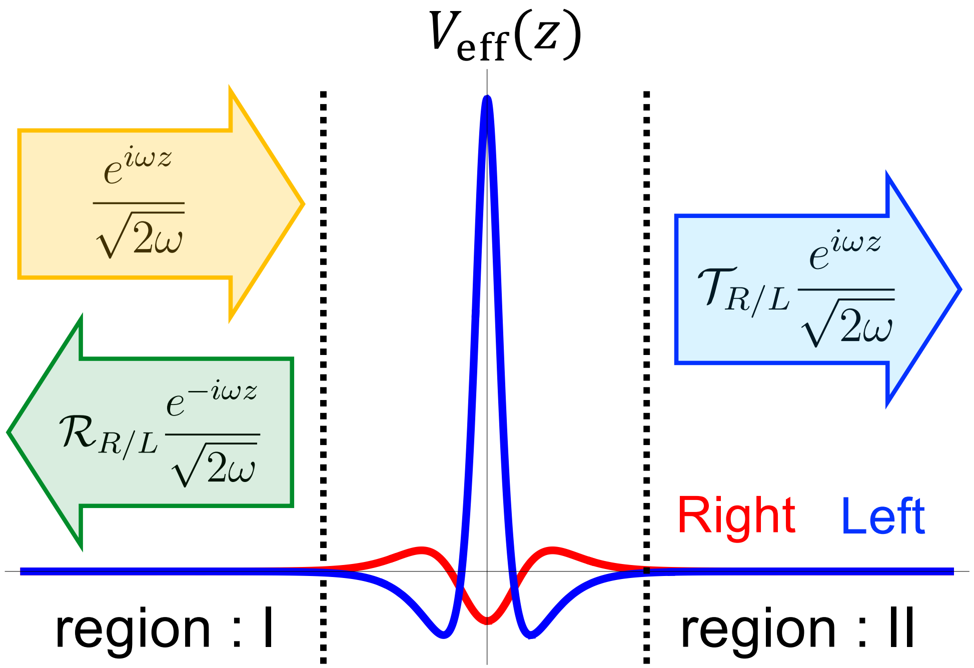
Scattering problem

region : I

$$\tilde{H}_{R/L}(z)=\frac{e^{i\omega z}}{\sqrt{2\omega}}+\mathcal{R}_{R/L}\frac{e^{-i\omega z}}{\sqrt{2\omega}}$$

region : II

$$\tilde{H}_{R/L}(z)=\mathcal{T}_{R/L}\frac{e^{i\omega z}}{\sqrt{2\omega}}$$



Degree of the circular polarization

$$\Pi=\frac{|\mathcal{T}_R|^2-|\mathcal{T}_L|^2}{|\mathcal{T}_R|^2+|\mathcal{T}_L|^2}$$

Just calculate $\mathcal{T}_{R/L}$ numerically as a scattering problem!

Result

$$\left(1 \pm \frac{\omega \ell^2}{M_\mathrm{p}} \phi'\right) H''_{R/L} \pm \frac{\omega \ell^2}{M_\mathrm{p}} \phi'' H'_{R/L} + \omega^2 \left(1 \pm \frac{\omega \ell^2}{M_\mathrm{p}} \phi'\right) H_{R/L} = 0$$

CS correction < 1

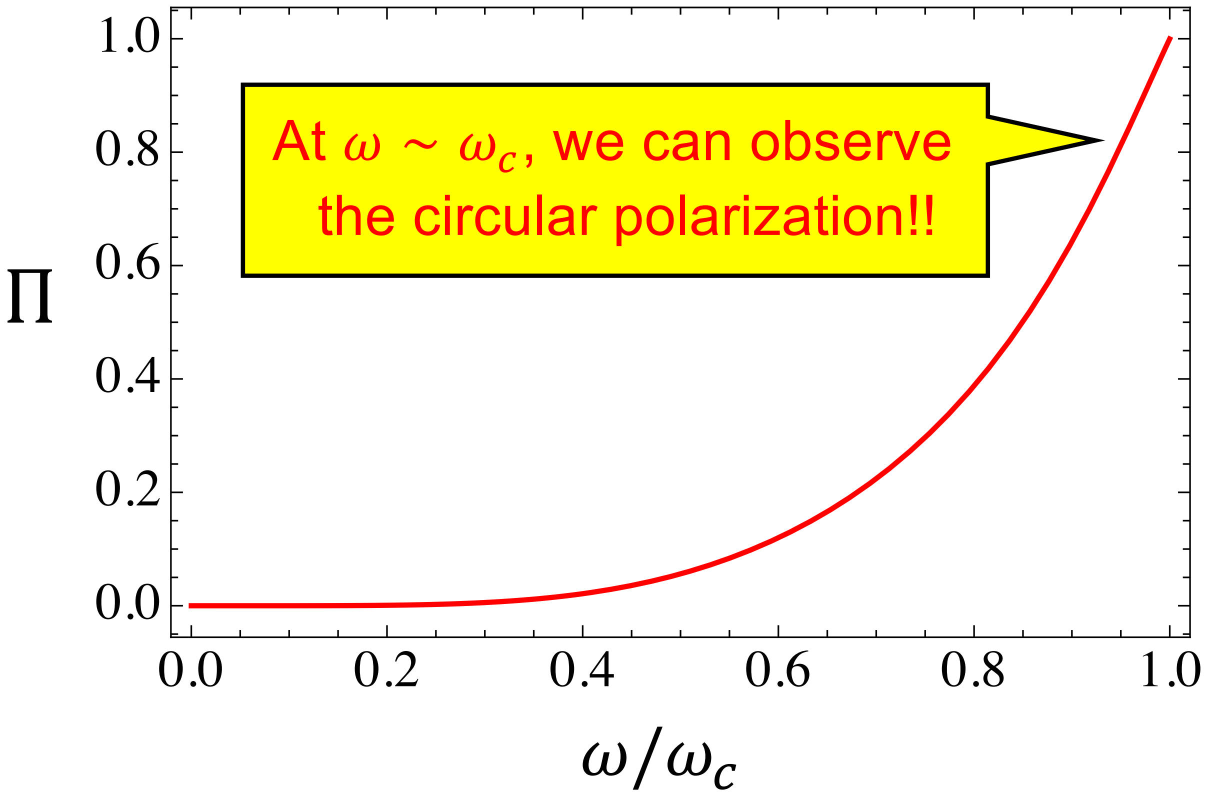
$$\frac{\omega \ell^2}{M_\mathrm{p}} \phi' = \frac{\omega \ell^2}{M_\mathrm{p}} \eta^2 \mathrm{sech}^2 (\eta z)$$

$$\leq \frac{\omega \ell^2}{M_\mathrm{p}} \eta^2 < 1$$

critical frequency

$$\omega_c \equiv \frac{M_\mathrm{p}}{\ell^2 \eta^2}$$

The degree of circular polarization



Constraint on the coupling constant

Constraint on ω and ℓ

the observation by Gravity Probe B

$\ell < 10^8$ km

Y. Ali-Haimoud, Y. Chen (2011)

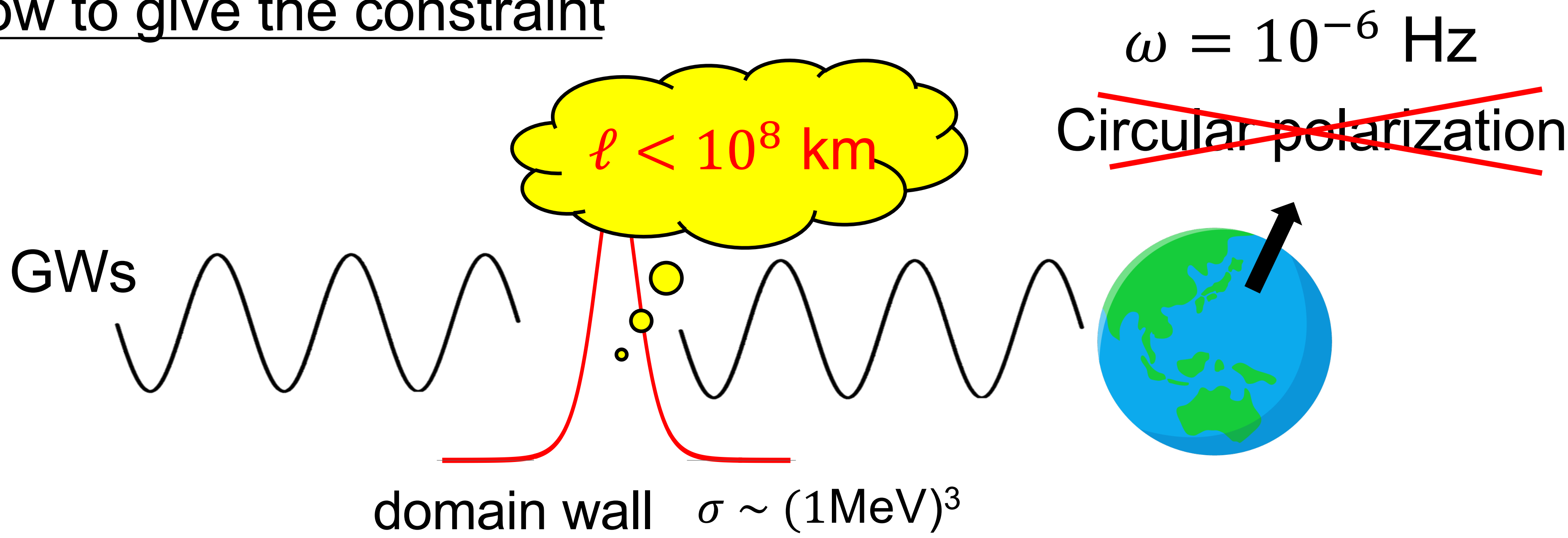
$$\frac{\omega \ell^2}{M_p}\eta^2 < 1 \Rightarrow \omega \ell^2 < 10^{15}\ (\mathrm{eV})^{-1}$$

$M_p = 10^{18}$ GeV

$\sigma \sim \eta^3 = (1\mathrm{MeV})^3$ ← CMB

$\ell \sim 10^8$ km $\longrightarrow$ $\omega_c \sim 10^{-6}$ Hz

How to give the constraint



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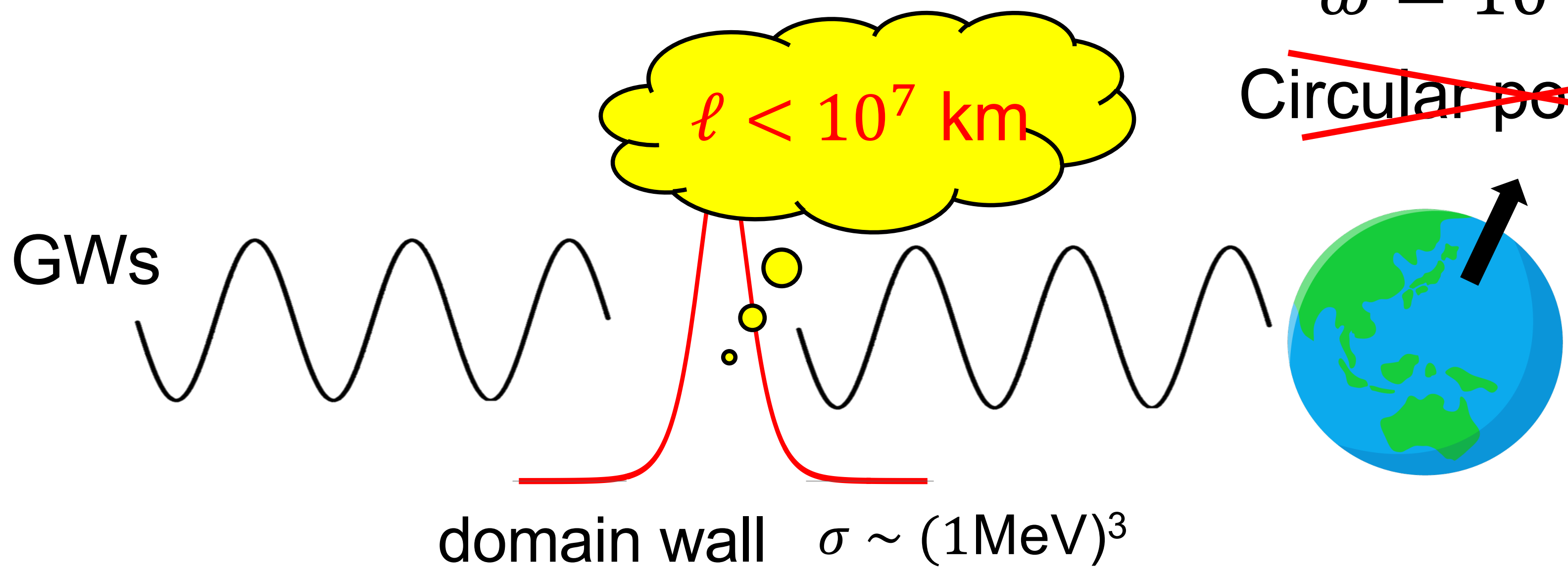
$\ell \sim 10^8$ km ⟶ $\omega_c \sim 10^{-6}$ Hz

How to give the constraint

$\omega = 10^{-4}$ Hz

~~Circular polarization~~



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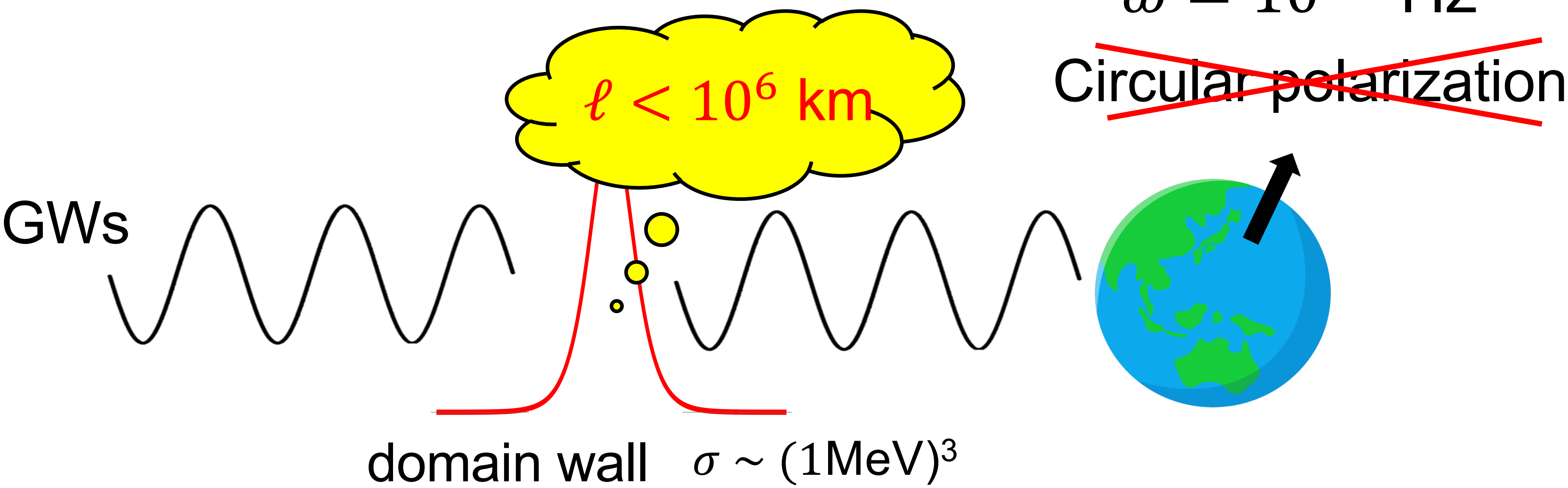

Summary

- The degree of the circular polarization becomes large enough to be observed at near the critical frequency.

$$\omega_c \equiv \frac{M_{\rm p}}{\ell^2\eta^2}$$

M_p : Plank mass

η : Energy of the domain wall

ℓ : coupling constant between axion and gravity

- Observing the circular polarization with increasing frequency, we can obtain constraints on the coupling constant ℓ.

Future Work

Enabling the observation of GWs across a broad frequency!

(Especially high frequency)