Imprints of cosmic magnetic fields on gravitational wave polarization

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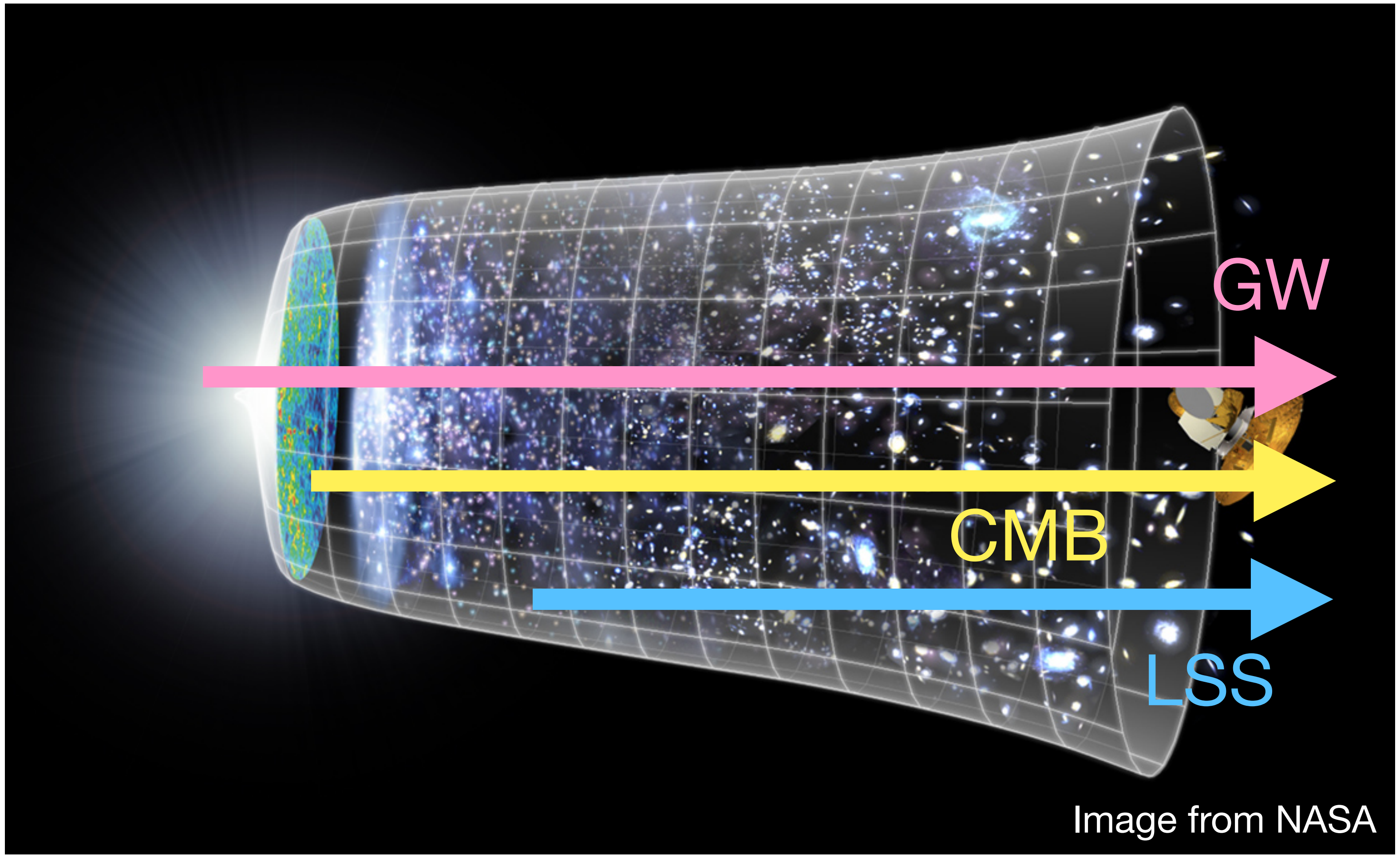
Introduction

Key elements in this talk

1. **Primordial gravitational waves** (PGWs) can carry information of the early universe to the present.



In particular, information on the source of PGWs (inflation, phase transition, …) would be encoded in the spectrum and polarization.

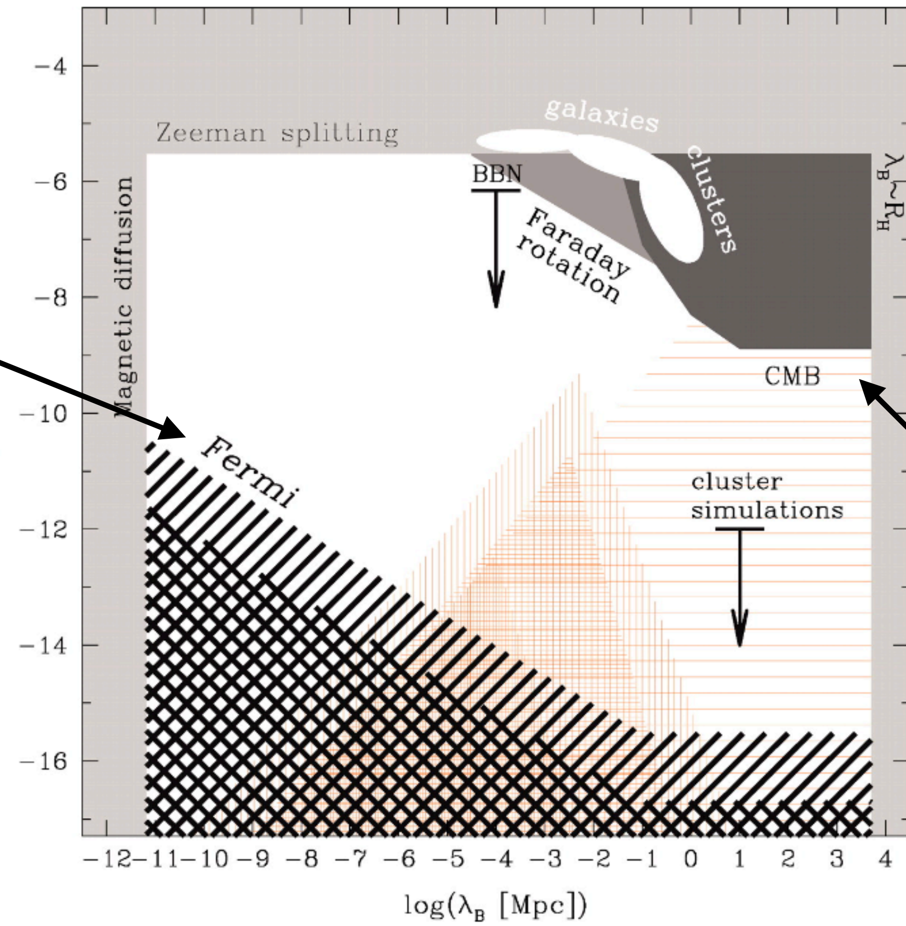
Introduction

Key elements in this talk

2. The presence of **cosmic magnetic fields** is suggested by blazar observations.

Fermi suggests lower limits.

Magnetic field magnitude



Correlation length

CMB, BBN, etc. provide upper limits.

[Neronov, Vovk 2010]

Given the presence of a magnetic field, gravitational waves are converted into photons [Gertsenshtein 1962], as we'll see in the next slide.

➡PGWs may be affected by conversion.

Graviton-Photon Conversion

[Gertsenshtein 1962]
[Raffelt, Stodolsky 1988]

- Action

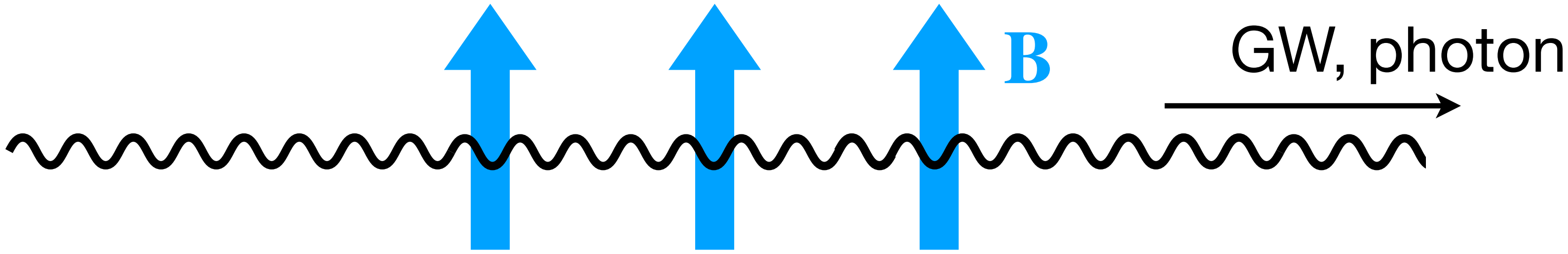
$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right]$$

- FLRW background + GWs

$$g_{\mu\nu} dx^\mu dx^\nu = a^2 \left[-d\eta^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j \right]$$

- Background magnetic field + propagating photons

$$F_{\mu\nu} = \bar{F}_{\mu\nu} + \partial_\mu A_\nu - \partial_\nu A_\mu$$



- Energy density of the background magnetic field decays with expansion.

$$\frac{1}{4} \bar{F}^{ij} \bar{F}_{ij} \propto \frac{1}{a^4} \quad \rightarrow \quad \bar{B}(a) = \frac{\bar{B}_0}{a^2}$$

← Present value

Graviton-Photon Conversion

- Equations of motion

GW: $\left(\partial_\eta^2 - \nabla^2 - \cancel{\frac{\partial_\eta^2 a}{a}}\right) y_{ij} = \kappa\, a\, [\epsilon_{ikl} \bar{B}_l(a)\, (\partial_j A_k - \partial_k A_j) + \epsilon_{jkl} \bar{B}_l(a)\, (\partial_i A_k - \partial_k A_i)]$

$[\, y_{ij} \equiv (2\kappa)^{-1}\, a\, h_{ij}\,]$

Photon: $(\partial_\eta^2 - \nabla^2) A_i = 2\, \kappa\, a\, \epsilon_{jkl} \bar{B}_l(a)\, \partial_k y_{ij}$

➡Mixing of GWs and photons via the magnetic field $\bar{B}(a)$ induces conversion.

- Move to Fourier space, and introduce a polarization basis (+ and ×).
- To see conversion, for convenience, we write

GW: $y_{+/\times}(\mathbf{k}, \eta) = \tilde{y}_{+/\times}(\mathbf{k}, \eta)\, e^{-ik\eta}$

Photon : $A_{+/\times}(\mathbf{k}, \eta) = \tilde{A}_{+/\times}(\mathbf{k}, \eta)\, e^{-ik\eta}$

Plane wave ($e^{-ik\eta}$)

Slowly varying amplitude due to mixing ($\tilde{A}_{+/\times}$)

➡We focus on the evolution of $\tilde{y}_{+/\times}(\mathbf{k}, \eta)$ and $\tilde{A}_{+/\times}(\mathbf{k}, \eta)$.

Graviton-Photon Conversion

- The evolution can be solved analytically.

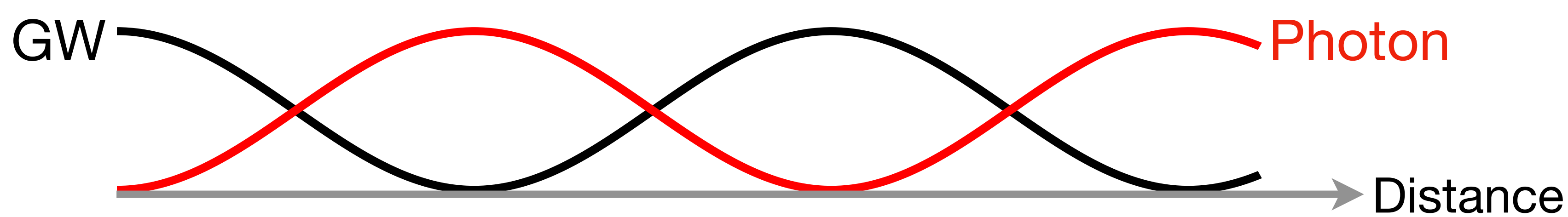
$$\tilde{y}_P(\mathbf{k},\eta) = \tilde{y}_P(\mathbf{k},\eta_\mathrm{i})\,\cos\left(\int_{\eta_\mathrm{i}}^{\eta} d\eta\,\Delta_M(a)\right) - i\,\tilde{A}_P(\mathbf{k},\eta_\mathrm{i})\,\sin\left(\int_{\eta_\mathrm{i}}^{\eta} d\eta\,\Delta_M(a)\right)$$

$$\tilde{A}_P(\mathbf{k},\eta) = \tilde{A}_P(\mathbf{k},\eta_\mathrm{i})\,\cos\left(\int_{\eta_\mathrm{i}}^{\eta} d\eta\,\Delta_M(a)\right) - i\,\tilde{y}_P(\mathbf{k},\eta_\mathrm{i})\,\sin\left(\int_{\eta_\mathrm{i}}^{\eta} d\eta\,\Delta_M(a)\right)$$

$$P = + / \times\,,\ \ \Delta_M(a) \equiv \frac{1}{\sqrt{2}\,M_\mathrm{Pl}}\frac{\bar{B}_0}{a}\,\sin\theta,\ \ \theta\text{: angle between } \mathbf{k} \text{ and } \mathbf{B}$$

<u>Points</u>

- Gravitational wave and photon intensities exhibit oscillatory behavior with respect to time (or propagation distance).



- The length scale of conversion: $\Delta_M^{-1}(a) \sim 10^9\,a$ Mpc $\left(\dfrac{10^{-9}\ \text{Gauss}}{\bar{B}_0 \sin\theta}\right)$
- Given a magnetic field, graviton-photon conversion occurs under minimal coupling without introducing additional interactions. Theoretically robust.

Conversion Length

- The length scale of conversion reads

$$\Delta_M^{-1}(a) \sim 10^9\, a\ \text{Mpc}\left(\frac{10^{-9}\ \text{Gauss}}{\bar{B}_0 \sin\theta}\right)$$

- For ordinary (Standard Model) photons,
  - From CMB, the magnetic field strength is constrained: $\bar{B}_0 \lesssim 10^{-9}$ Gauss
  - The onset of conversion should be the decoupling time: $z \sim 1100$

➡ $\Delta_M^{-1}(a) \gtrsim 10^3$ Gpc, far exceeding the present Hubble radius. Efficient conversion cannot be realized.
Indeed, the almost perfect blackbody spectrum of CMB indicates that graviton-SM photon conversion is inefficient.

Conversion Length

- The length scale of conversion reads

$$\Delta_M^{-1}(a) \sim 10^9\, a \text{ Mpc} \left(\frac{10^{-9} \text{ Gauss}}{\bar{B}_0 \sin\theta} \right)$$

- For dark photons decoupled from SM, the story may be different.
  - Magnetic field strength made of dark fields is less constrained. A bound comes from BBN, $\bar{B}_0 \lesssim 10^{-6}$ Gauss. [Kawasaki, Kusakabe 2012]
  - The onset of conversion may be much earlier.

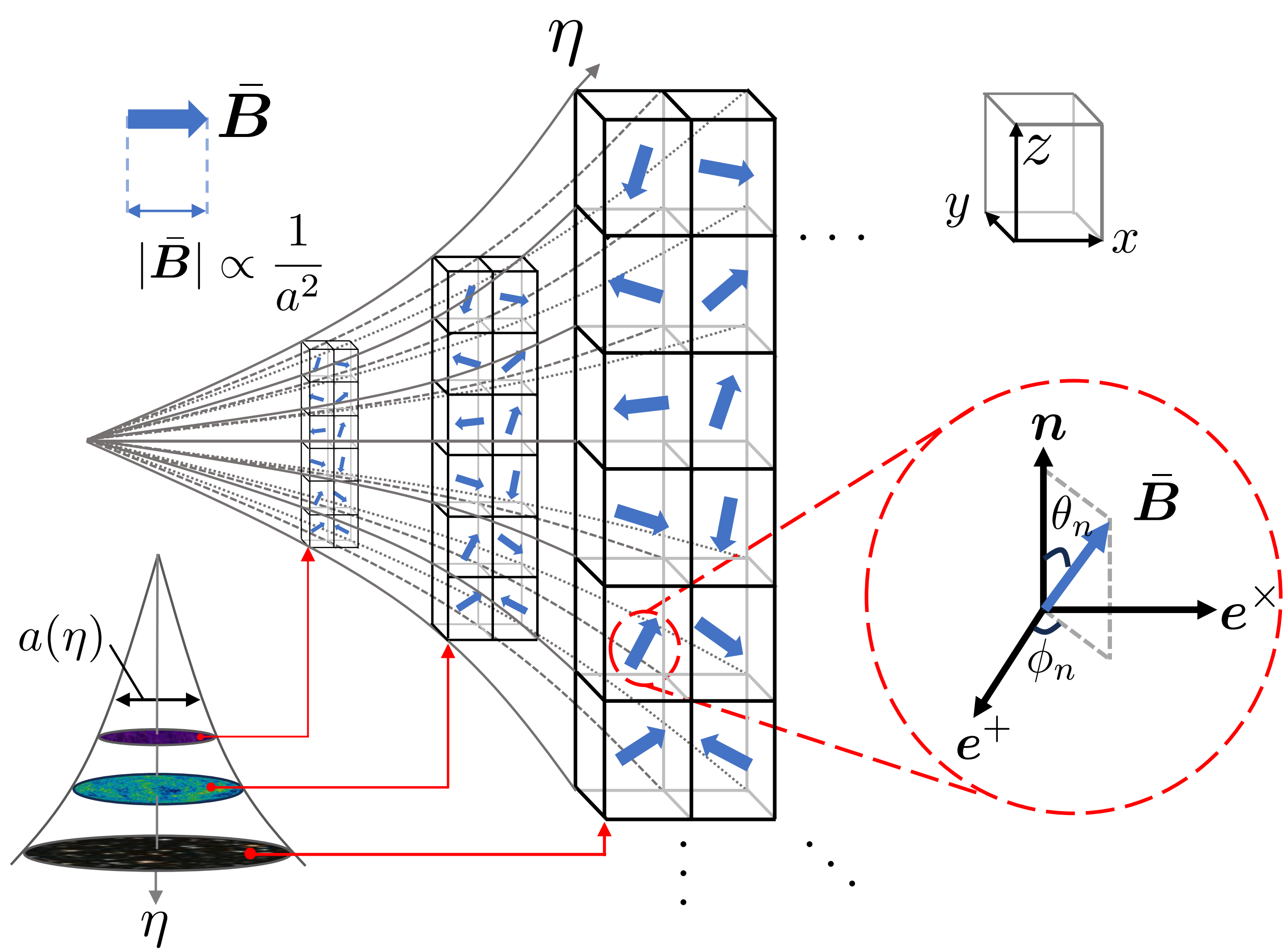
  ➡ $\Delta_M^{-1}(a)$ may be much shorter. Graviton-dark photon conversion may be efficient, and PGWs may be influenced.

- Several cosmological scenarios predict the generation of chiral PGWs.
  - Parity-violating gravity [Lue+ 1999]…
  - Axions with Chern-Simons coupling [Sorbo 2011]…

  ➡ Does graviton-dark photon conversion destroy these unique signatures?

Modeling of Magnetic Fields

- The cosmic magnetic field is supposed to have a finite correlation length. We model the structure of the magnetic field as a network of domains.



- Every domain is assumed to have equal comoving size ($\gtrsim$ Mpc in mind).
- Magnetic field is uniform within a single domain.
- The direction of the magnetic field is randomly chosen for each domain.

Sequence of Conversion

- We obtain a recurrence relation between the polarization components at the n-th domain and those at the (n-1)-th domain,

$$\begin{pmatrix} \tilde{y}_{+,n} \\ \tilde{y}_{\times,n} \\ \tilde{A}_{+,n} \\ \tilde{A}_{\times,n} \end{pmatrix} = \boldsymbol{\Phi}_n \cdot \boldsymbol{\Gamma}_n \cdot \begin{pmatrix} \tilde{y}_{+,n-1} \\ \tilde{y}_{\times,n-1} \\ \tilde{A}_{+,n-1} \\ \tilde{A}_{\times,n-1} \end{pmatrix}$$

- $\boldsymbol{\Gamma}_n$ is induced by the rotation of the magnetic field direction between domains.

$$\boldsymbol{\Gamma}_n \equiv \begin{pmatrix} \cos 2\gamma_n & \sin 2\gamma_n & 0 & 0 \\ -\sin 2\gamma_n & \cos 2\gamma_n & 0 & 0 \\ 0 & 0 & \cos \gamma_n & \sin \gamma_n \\ 0 & 0 & -\sin \gamma_n & \cos \gamma_n \end{pmatrix},$$ γ_n: Rotation angle of the magnetic field between domains

- $\boldsymbol{\Phi}_n$ is induced by mixing (conversion) in the magnetic field.

$$\boldsymbol{\Phi}_n \equiv \begin{pmatrix} \cos \Phi_n & 0 & -i \sin \Phi_n & 0 \\ 0 & \cos \Phi_n & 0 & -i \sin \Phi_n \\ -i \sin \Phi_n & 0 & \cos \Phi_n & 0 \\ 0 & -i \sin \Phi_n & 0 & \cos \Phi_n \end{pmatrix}, \quad \Phi_n \equiv \int_{\eta_{n-1}}^{\eta_n} d\eta \left(\frac{1}{\sqrt{2}\, M_{\mathrm{Pl}}} \right) \frac{\bar{B}_0}{a} \sin \theta_n$$

Intensity

- Using the recurrence relation, we can examine the statistics after passing through any number of magnetic domains.
- Definition of intensity

$$I_h(\mathbf{k},\eta) = |\tilde{y}_+(\mathbf{k},\eta)|^2 + |\tilde{y}_\times(\mathbf{k},\eta)|^2, \quad I_\gamma(\mathbf{k},\eta) = |\tilde{A}_+(\mathbf{k},\eta)|^2 + |\tilde{A}_\times(\mathbf{k},\eta)|^2$$

- By averaging over the magnetic field direction, we obtain

$$\langle I_h(\mathbf{k},\eta_n)\rangle = \frac{1+\Pi_n}{2} I_{h,0}(\mathbf{k}) + \frac{1-\Pi_n}{2} I_{\gamma,0}(\mathbf{k})$$

$$\langle I_\gamma(\mathbf{k},\eta_n)\rangle = \frac{1+\Pi_n}{2} I_{\gamma,0}(\mathbf{k}) + \frac{1-\Pi_n}{2} I_{h,0}(\mathbf{k})$$

$$\Pi_n \equiv \prod_{i=1}^{n} \langle \cos\left(2\int_{\eta_{i-1}}^{\eta_i} d\eta \left(\frac{1}{\sqrt{2}\,M_{\mathrm{Pl}}}\right)\frac{\bar{B}_0}{a}\sin\theta_i\right)\rangle$$

- Total intensity is conserved: $\langle I_h(\mathbf{k},\eta) + I_\gamma(\mathbf{k},\eta)\rangle = I_{h,0}(\mathbf{k}) + I_{\gamma,0}(\mathbf{k})$.
- In the large mixing limit, equipartition is realized:

$$\langle I_h(\mathbf{k},\eta)\rangle = \langle I_\gamma(\mathbf{k},\eta)\rangle = \frac{I_{h,0}(\mathbf{k}) + I_{\gamma,0}(\mathbf{k})}{2}$$

Linear Polarization

$$Q_h(\mathbf{k},\eta) = |\tilde{y}_+(\mathbf{k},\eta)|^2 - |\tilde{y}_\times(\mathbf{k},\eta)|^2$$

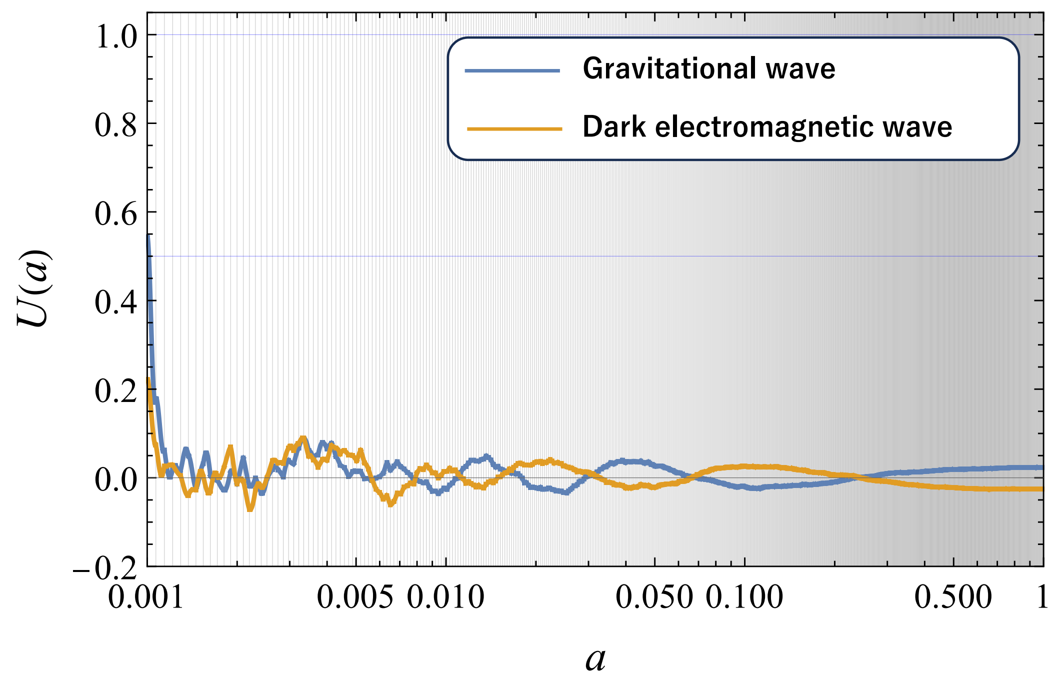
$$Q_\gamma(\mathbf{k},\eta) = |\tilde{A}_+(\mathbf{k},\eta)|^2 - |\tilde{A}_\times(\mathbf{k},\eta)|^2$$

$$U_h(\mathbf{k},\eta) = \tilde{y}^*_\times(\mathbf{k},\eta)\,\tilde{y}_+(\mathbf{k},\eta) + \tilde{y}^*_+(\mathbf{k},\eta)\,\tilde{y}_\times(\mathbf{k},\eta)$$

$$U_\gamma(\mathbf{k},\eta) = \tilde{A}^*_\times(\mathbf{k},\eta)\,\tilde{A}_+(\mathbf{k},\eta) + \tilde{A}^*_+(\mathbf{k},\eta)\,\tilde{A}_\times(\mathbf{k},\eta)$$

- Expectation values of the linear polarization vanish due to the randomness of the magnetic field direction:

$$\langle Q_h(\mathbf{k},\eta)\rangle = \langle Q_\gamma(\mathbf{k},\eta)\rangle = \langle U_h(\mathbf{k},\eta)\rangle = \langle U_\gamma(\mathbf{k},\eta)\rangle = 0.$$



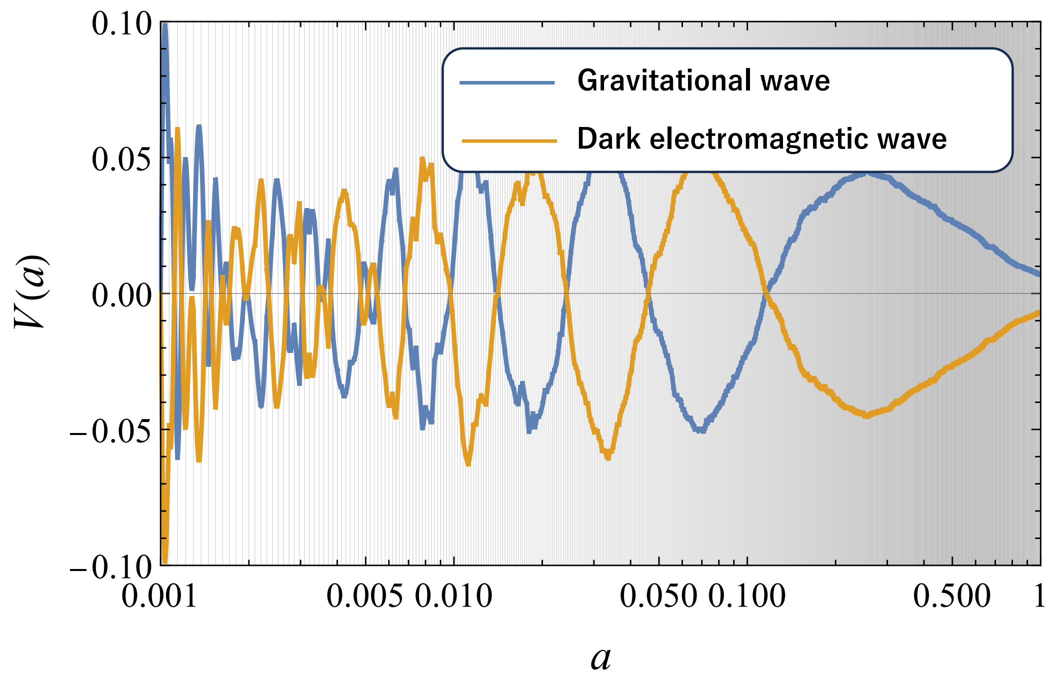
Circular Polarization

$$V_h(\mathbf{k},\eta) = i\left[\tilde{y}^*_\times(\mathbf{k},\eta)\,\tilde{y}_+(\mathbf{k},\eta) - \tilde{y}^*_+(\mathbf{k},\eta)\,\tilde{y}_\times(\mathbf{k},\eta)\right]$$

$$V_\gamma(\mathbf{k},\eta) = i\left[\tilde{A}^*_\times(\mathbf{k},\eta)\,\tilde{A}_+(\mathbf{k},\eta) - \tilde{A}^*_+(\mathbf{k},\eta)\,\tilde{A}_\times(\mathbf{k},\eta)\right]$$

- Total value is conserved: $\langle V_h(\mathbf{k},\eta) + V_\gamma(\mathbf{k},\eta)\rangle = V_{h,0}(\mathbf{k}) + V_{\gamma,0}(\mathbf{k})$.
  Indeed, there exists a "chiral" symmetry in the system because the + and × sectors are completely decoupled.
- As with intensity, equipartition is realized in the large mixing limit:

$$\langle V_h(\mathbf{k},\eta)\rangle = \langle V_\gamma(\mathbf{k},\eta)\rangle = \frac{V_{h,0}(\mathbf{k}) + V_{\gamma,0}(\mathbf{k})}{2}$$



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As a particular case, setting $I_{\gamma,0}(\mathbf{k}) = V_{\gamma,0}(\mathbf{k}) = 0$,

$$\frac{\langle V_h(\mathbf{k},\eta)\rangle}{\langle I_h(\mathbf{k},\eta)\rangle} = \frac{V_{h,0}(\mathbf{k})}{I_{h,0}(\mathbf{k})}.$$

That is, the expectation value of circular polarization (in units of intensity) is conserved. In this sense, the cosmological information imprinted as chiral GWs at generation is preserved.

Variance

- So far, we found that expectation value $\langle V_h \rangle / \langle I_h \rangle$ is preserved under conversion.
- Traces of conversion will appear as variances of intensity and polarization.
  Indeed, as GWs pass through a series of magnetic field domains, nontrivial variances ($\langle I_h^2 \rangle$, $\langle V_h^2 \rangle$, …) are generally produced.
- We are trying to derive general relationships between variances generated by conversion.

- Recently, it has been pointed out that stochastic GWs from astrophysical sources (such as many binary BH systems) also have nontrivial variance of circular polarization $\langle V^2 \rangle$, although the expectation value is zero $\langle V \rangle = 0$. [Dall'Armi+ 2023]
  These contributions must be discriminated.

Summary

- Graviton-photon conversion provides a valuable means of probing cosmic magnetic fields.
- For the Standard Model photons, the effect on PGWs seems tiny.
- For dark photons, conversion with GWs might occur from the very early universe.
  - After sufficient conversion, the intensity and circular polarization are equally distributed between GWs and dark photons.
  - Expectation value of linear polarization vanishes by averaging over various magnetic field directions.
  - Expectation value of circular polarization (in units of intensity) is conserved. In this sense, cosmological information imprinted as chiral GWs (from parity-violating gravity, axions, …) is preserved.
- A sequence of conversion generally produces nontrivial variances of the intensity and polarizations ($\langle I_h^2 \rangle$, $\langle V_h^2 \rangle$, …). These can be unique signatures of dark magnetic fields.