Pulsar Timing Residuals induced by wideband ultralight dark matter

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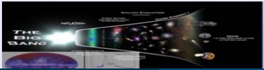
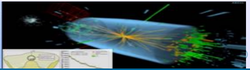
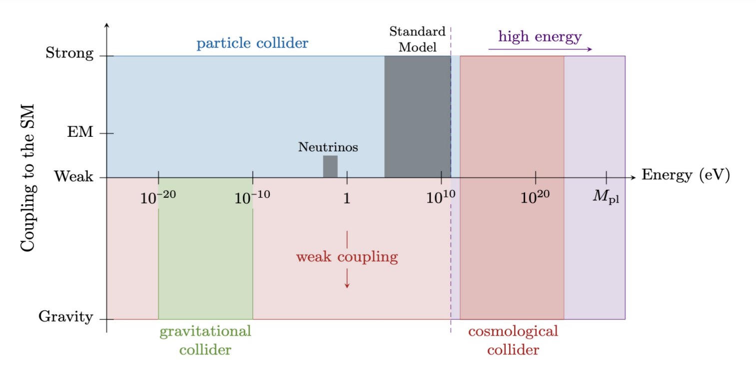
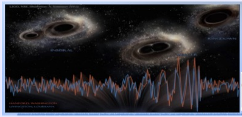


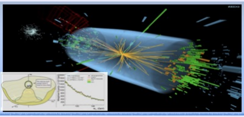



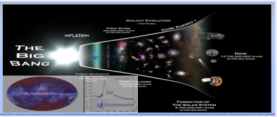
Motivation: new physics in ultra-low energy



[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]







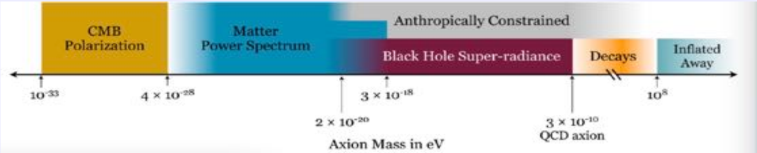
Axion-like Dark Matter

QCD axion

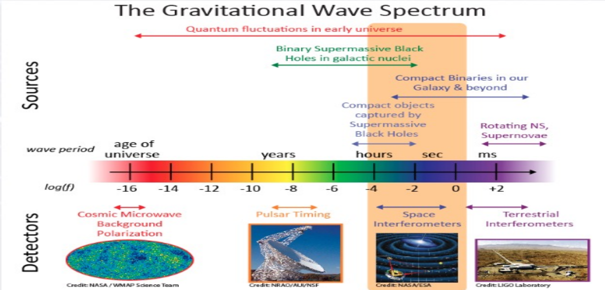
- $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} - \alpha_\theta \theta F^a_{\mu\nu} \tilde{F}_a^{\mu\nu} + \bar{\psi} \left(i\gamma^\mu D_\mu - m e^{i\theta' \gamma_5} \right) \psi$
- Strong CP problem: no CP violation in measurement
- Peccei–Quinn (1977): introduce new pseudoscalar
- Wilczek- Weinberg: relax the CP-violation parameter

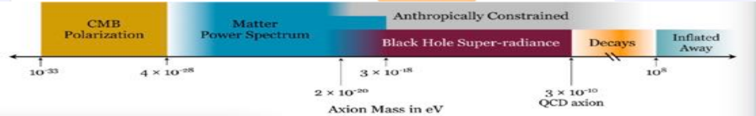
Axion-like: ultra light dark matter

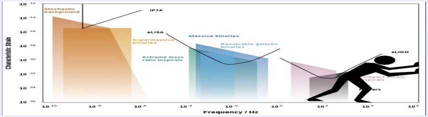
- e.g. Fuzzy Dark Matter, Dark photon dark matter



Spectrum of gravitational wave and axion mass







[cf. LISA/Ultra-High-Frequency Gravitational Waves Initiative]

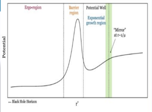
Ultralight dark matter and gravitational wave

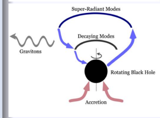
GW & EM signals from axion like DM

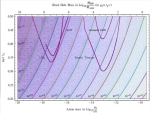
- Axion annihilation $\vartheta + \vartheta \to h$(Stochastic GW)
  Energy transition $\vartheta^{+} \to \vartheta^{-} + h$(Monochromic)
- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_\vartheta} \simeq \left(\frac{M_{BH}}{M_\odot}\right)\left(\frac{m_\vartheta}{10^{-10}\mathrm{eV}}\right)$
- Fast Radio Burst from Axion $\sim \vartheta F\tilde{F}$ $(\vartheta \to \gamma\gamma)$
- GW burst from Axion $\sim \vartheta R\tilde{R}$ $(\vartheta \to hh)$

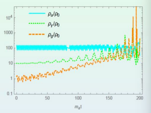
GW detection and Ultra-light DM

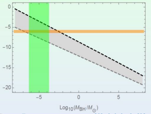
- Tabletop exp: QCD axion & GW burst ($\sim$ GHz)
- LISA & LVK: BH Superradiance ($\sim$ mHz - kHz)
- PTA & SKA : Ultra-light DM ($\sim$ nHz)
- LSS & CMB : DE & Modified gravity ($\sim$ nnHz)



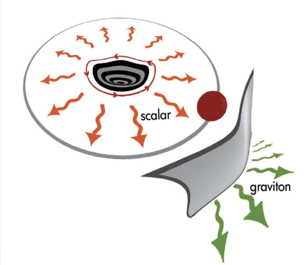


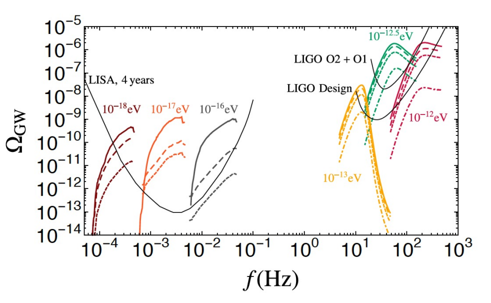






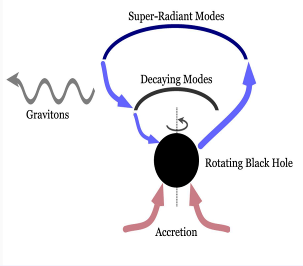
Axion annihilation and Stochastic GWs

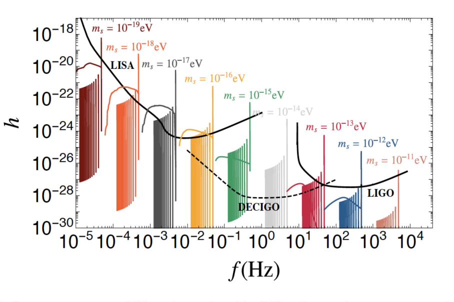




- Axion annihilation $\vartheta + \vartheta \to h$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Energy level transition and Monochromic GW





- Energy transition $\vartheta^+ \to \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Fast radio burst from axion decay $\vartheta \to \gamma\gamma$

Stimulated axion clouds around primordial black holes

- Coupling term $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}$
- $\left[-\partial_t^2 + \partial_z^2 \mp i\alpha_\gamma\dot{\vartheta}(t)\partial_z\right]A_\pm(t,z) = 0, \quad A_\pm = b_\pm(t)e^{ikz}.$
- In momentum space $\ddot{b}_\pm(t) + \left[k^2 \mp \alpha_\gamma k\dot{\vartheta}\right]b_\pm(t) = 0.$
- Amplification factor $e^{\Gamma_\gamma t_\gamma}$, $\Gamma_\gamma = \alpha_\gamma\vartheta_0\frac{m_\vartheta}{2}, \quad t_\gamma \simeq \frac{1}{m_\vartheta v_\vartheta c}.$

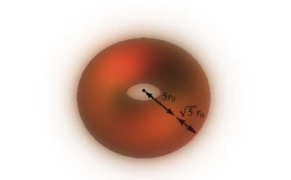
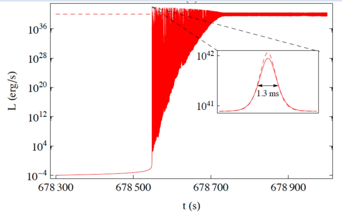


FIG. 1. Model of the superradiant 2p-axion cloud around a central Kerr black hole.

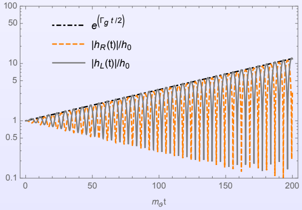


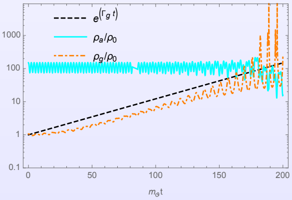
- [cf. Rosa-Kephart , PRL(2018)]

Fast GW burst from axion decay $\vartheta \to hh$

EOM of GW($I = L, R$)

- Coupling term $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}$,
- $\left[\ddot{h}_I(t) + k^2 h_I(t)\right]\left[1 - \varepsilon_I \kappa_4 \alpha_g k \dot{\vartheta}(t)\right] = \varepsilon_I \kappa_4 \alpha_g k \ddot{\vartheta}(t) \dot{h}_I(t)$.
- Factor $e^{\Gamma_g t_g}$, $\frac{\Gamma_g}{m_\vartheta} \sim \left(\frac{\kappa_4 \alpha_g}{1\text{eV}^{-3}}\right)\left(\frac{m_\vartheta}{10^{-9}\text{eV}}\right)^2\left(\frac{\vartheta_0}{10^9\text{GeV}}\right)$.

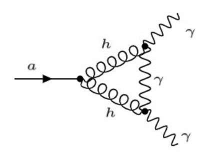


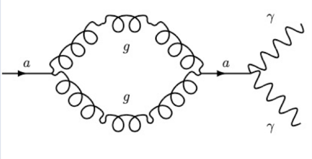


[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

Branch Ratio of EM & GW ($\vartheta \to \gamma\gamma$ & $\vartheta \to hh$)

- The triangle Feynman diagram: where the axion-photon coupling is generated from Chern-Simon gravity coupling.
- $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}.$



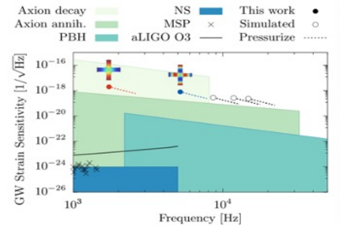


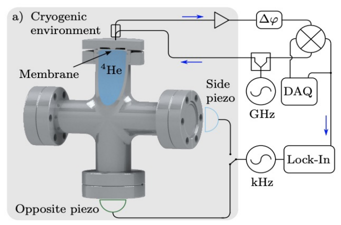
- The triangle diagram is divergent as $\alpha_\gamma \sim \alpha_g(\Lambda_{cs}/M_{pl})^4$, where Λ_{cs} is the cut-off for Chern-Simons theory.
- Two powers of M_{pl} from $h_{\mu\nu}T^{\mu\nu}$ coupling.

[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

Branch Ratio and GWs

- $\frac{\mathrm{Br}(\vartheta\to gg)}{\mathrm{Br}(\vartheta\to\gamma\gamma)} \simeq \frac{\alpha_g^2}{\alpha_\gamma^2} \simeq \left(\frac{M_{pl}}{\Lambda_{cs}}\right)^8$, (Power of FRB $P_{(\gamma)} \sim 10^{42}$ergs/s).
- High frequency $h_{(g)} \sim 10^{-26}\left(\frac{1\mathrm{GHz}}{\nu}\right)\left(\frac{P_{(g)}}{P_{(\gamma)}}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$
- Low freq. $h_{(g)} \sim 10^{-21}\left(\frac{10^{-2}\mathrm{Hz}}{\nu}\right)^{1/2}\left(\frac{M_{\mathrm{BH}}}{10^7 M_\odot}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$



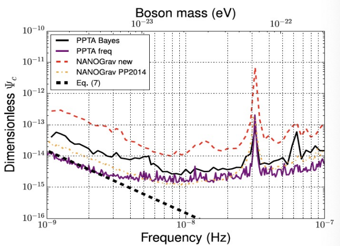


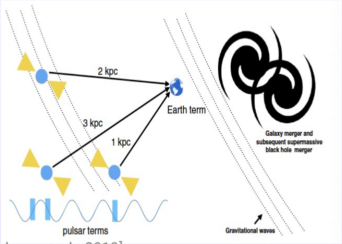
cf. PRD'21, S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps.
PRD'21, V. Vadakkumbatt et al, Prototype superfluid gravitational wave detector.

Pulsar timing residuals & Ultralight dark matter

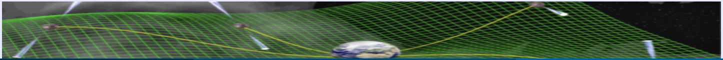
PTA & Ultra light dark matter

- Fuzzy like DM $f_c = \frac{m}{\pi} \simeq 4.8\,\text{nHz} \left(\frac{m}{10^{-23}\text{eV}}\right)$
- Coherence length: $\lambda_{\text{dB}} = \frac{2\pi\hbar}{mv} \simeq 4\text{kpc} \left(\frac{10^{-23}\text{eV}}{m}\right)\left(\frac{10^{-3}}{v}\right)$
- Pulsar timing residuals: $R_c(f) = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left(\frac{f_s}{f}\right)^{1/2}$

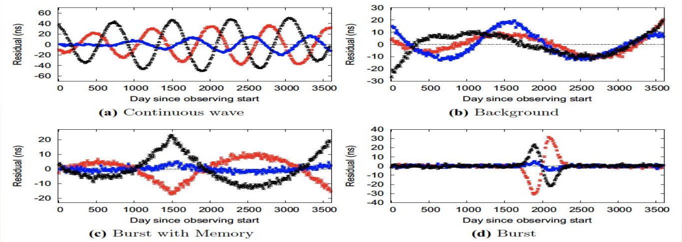


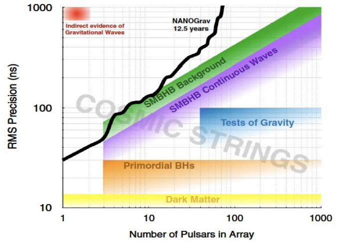


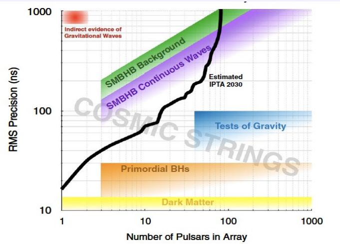
[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]



The timing residuals of nHz gravitational waves







[cf. Burke-Spolaor, *et al.*, “The astrophysics of nanohertz gravitational waves”]

Pulsar timing residual and fuzzy dark matter

DM oscillation induced time residual

- Metric: $ds^2 = -(1+2\Phi)\,\mathrm{d}t^2 + [(1-2\Psi)\,\delta_{ij} + h_{ij}]\,\mathrm{d}x^i\mathrm{d}x^j$.
- e.g. the scalar field $\phi(x,t) = \phi(x)\cos[mt + \theta_0(x)]$,
- Oscillating potential $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos[2(mt + \theta_0(x))]$
- Doppler effect: $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$.
- Timing residual in the pulse $R_\phi(t) = \int_0^t z_\phi(t')dt'$
- **Strain** $h_\phi = 2\sqrt{3}\,\Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2}\frac{\rho_\phi}{m^2} \simeq 5.2 \times 10^{-17}\alpha_0 \left(\frac{f_{yr}}{f}\right)^2$,
- **GW Timing residual** $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}}\frac{h_c(f)}{2\pi f}\left(\frac{f_s}{f}\right)^{1/2}$

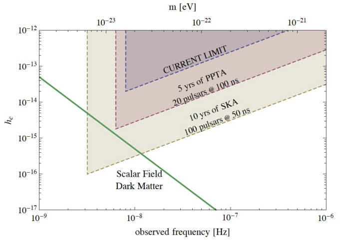
[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]

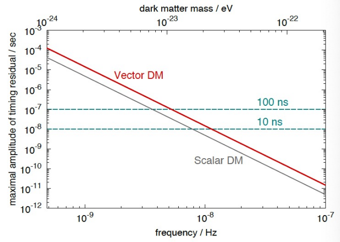
Pulsar timing constraints on ULDM

DM oscillation induced time residual

- Spin-0: massive scalar field $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]



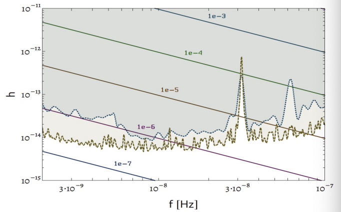


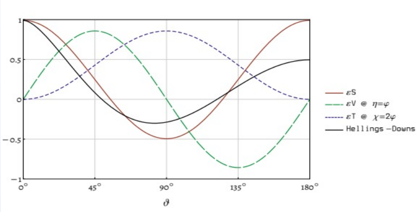
[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020)]

Pulsar timing constraints on spin-2 ULDM

spin-2 ultralight fields

- Spin-2: massive tensor field(Fierz-Pauli): Bi-metric gravity,
$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 \left(M_{\mu\nu} M^{\mu\nu} - M^2 \right)$$
- The oscillating solution $M_{ij} = \mathcal{M} \cos\left[mt + \theta_2(x)\right] \varepsilon_{ij}$
- Effective metric perturbations: $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift $z(t) = \frac{\omega(t) - \omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$



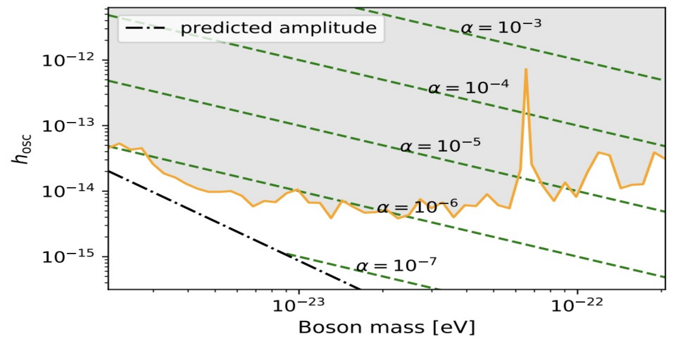


[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM", JCAP(2020)]

Gravitational effects & coupling effects

$$S=\frac{M_{\rm Pl}^2}{1+\alpha^2}\int d^4x\Big[\sqrt{|g|}R(g)+\alpha^2\sqrt{|f|}R(f)-2\frac{\alpha^2M_{\rm Pl}^2}{1+\alpha^2}\sqrt{|g|}V(g,f;\beta_n)\Big]+\int d^4x\sqrt{|g|}\mathcal{L}_{\rm m}(g,\Psi)$$

$$S^{(2)}=\int d^4x\sqrt{|\bar{g}|}\Big[\mathcal{L}^{(2)}_{\rm GR}(\mathcal{G})+\mathcal{L}^{(2)}_{\rm FP}(M)+-\frac{1}{M_{\rm Pl}}(\mathcal{G}_{\mu\nu}-\alpha M_{\mu\nu})T^{\mu\nu}(\Psi)\Big],$$



[cf. Y. M. Wu, Z. C. Chen, Q. G. Huang, JCAP **09**, 021 (2023)

"Pulsar timing residual induced by ultralight tensor dark matter,"]

Angular correlation & Hellings-Downs curves

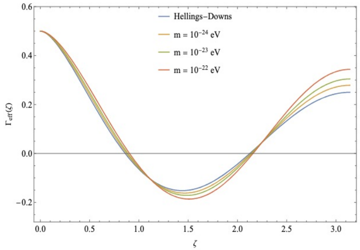


FIG. 2: Effective cross-correlation curves with $\alpha = 10^{-6}$ and mass ranging from 10^{-24} to 10^{-22}. It can be seen that in this range, the deformation of spin-2 dark matter on the Hellings-Downs curve is relatively small.

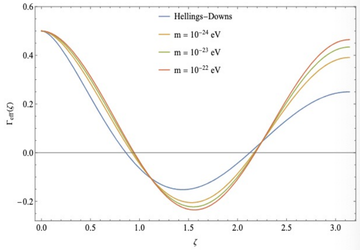


FIG. 3: Effective cross-correlation curves with $\alpha = 10^{-5.5}$. The deformation is very strong in this range, suggesting that if the coupling constant α is above this magnitude, existing ultralight mass spin-2 dark matter would have considerable effects on the deformation of the Hellings-Downs curve at corresponding frequency.

$$S = \frac{M_{\rm Pl}^2}{1+\alpha^2}\int d^4x\Big[\sqrt{|g|}R(g) + \alpha^2\sqrt{|f|}R(f) - 2\frac{\alpha^2 M_{\rm Pl}^2}{1+\alpha^2}\sqrt{|g|}V(g,f;\beta_n)\Big] + \int d^4x\sqrt{|g|}\mathcal{L}_{\rm m}(g,\Psi)$$

$$S^{(2)} = \int d^4x\sqrt{|\bar{g}|}\Big[\mathcal{L}^{(2)}_{\rm GR}(\mathcal{G}) + \mathcal{L}^{(2)}_{\rm FP}(M) + -\frac{1}{M_{\rm Pl}}(\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu})T^{\mu\nu}(\Psi)\Big]\ ,$$

[cf. R.G.Cai, J.R.Zhang, Y.L. Zhang, arXiv: 2402.03984,

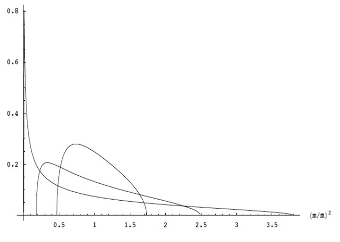
Angular correlation and deformed Hellings-Downs curve by spin-2 ultralight dark matter]

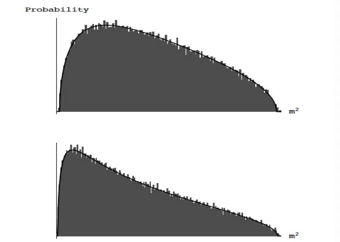
Multi fields: Marcenko-Pastur distribution

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2-m_-^2)(m_+^2-m^2)}}{2\pi\beta m_0^2 m^2}$,
- Energy density: $\rho_\phi \equiv \int dm\tilde{\rho}(m) = \int dm\frac{1}{2}m^2\tilde{\phi}(m)^2 P(m)$.
- Convenient choice: $\tilde{\rho}(m) \simeq \rho_\phi P(m)$, $\int dmP(m) = 1$.

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices,"(1967)]





[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)
Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009)]

Soft gravitons in Massive gravity

GW & ULDM
Ultralight DM
Axion-like DM
Resonance
PTA and DM
spin-0 DM
spin-2 DM
Summary

Massive Gravity from Extra Dimensions

- DGP braneworld $\rho_{DGP}(m^2) \sim \frac{m_0}{\pi m} \frac{1}{m^2+m_0^2}$
- flat extra dimension $\rho_{KK}(m^2) \sim \delta\left(m^2 - (nm_0)^2\right)$

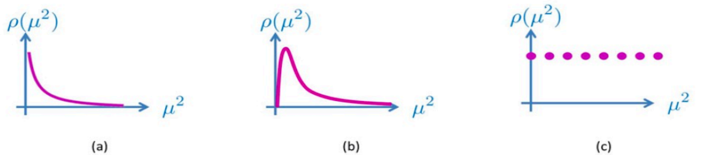


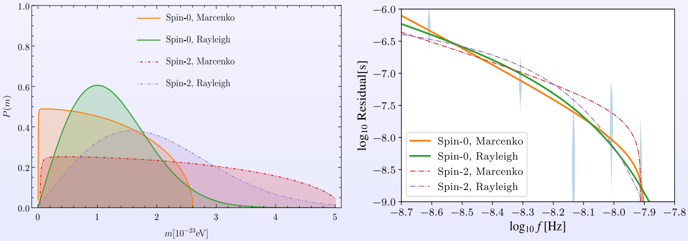
Figure 1: Spectral representation of different models. (a) DGP (b) higher-dimensional cascading gravity and (c) multi-gravity. Bi-gravity is the special case of multi-gravity with one massless mode and one massive mode. Massive gravity is the special case where only one massive mode couples to the rest of the standard model and the other modes decouple. (a) and (b) are models of soft massive gravity where the graviton mass can be thought of as a resonance.

[cf. C de Rham, Massive Gravity, Living Rev. Relativity, 17, (2014), 7]

Phenomenological fitting results

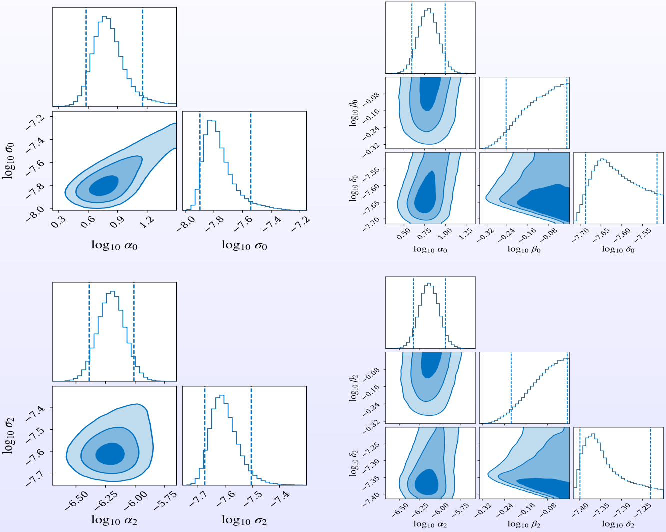
Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Rayleigh distribution: $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$.



[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Corner Figures of Bayesian Fitting



Our phenomenological fitting results

The effective strain

- $h_c^{\phi}(f) = \frac{\alpha_0}{M_{pl}^2} \frac{\sqrt{3}\rho_{DM}}{4\pi f} P(\pi f)$
- $h_c^{M}(f) = \frac{\alpha_2}{M_{pl}} \frac{m\mathcal{M}P(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}} \frac{2\sqrt{\rho_M}}{\sqrt{5}} P(2\pi f).$

	Parameters	spin-0	spin-1	spin-2
Marcenko	α_i	$5.9^{+1.9}_{-1.3}$	$\sim 3\alpha_0$	$7.6^{+2.2}_{-1.7} \times 10^{-7}$
Marcenko	$m_-^i/(10^{-23}\text{eV})$	$2.9^{+3.6}_{-0.3} \times 10^{-3}$	$\sim \delta_0(1-\sqrt{\beta_0})$	$6.3^{+6.0}_{-1.7} \times 10^{-3}$
Marcenko	$m_+^i/(10^{-23}\text{eV})$	$2.61^{+0.21}_{-0.01}$	$\sim \delta_0(1+\sqrt{\beta_0})$	$5.08^{+0.02}_{-0.01}$
Rayleigh	α_i	$5.6^{+3.8}_{-1.0}$	$\sim 3\alpha_0$	$6.1^{+2.1}_{-1.3} \times 10^{-7}$
Rayleigh	$\sigma_i/(10^{-23}\text{eV})$	$1.0^{+0.4}_{-0.1}$	$\sim \sigma_0$	$1.6^{+0.3}_{-0.1}$

[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Summary and Outlook

GW detection & Ultra-light DM with multi band

- Tabletop exp: GW burst & QCD axion ($\sim$ GHz)
- LVK & LISA: BH Superradiance ($\sim$ mHz - kHz)
- PTA & SKA : Fuzzy dark matter ($\sim$ nHz)
- LSS & CMB : DE & Modified gravity ($\sim$ nnHz)

Pulsar timing residuals & ULDM with spin 0,1,2

- **Wideband mass spectrum** extension of ULDM
- Spin-2 ultralight dark matter & Bi-metric gravity
- **Deformed Hellings-Downs** & Angular correlation
- Search/constraint for ULDM with PTAs & SKA

Thanks a lot for your attention!