A narrow-band parameterization for the stochastic gravitational wave background

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arxiv: 2402.02415

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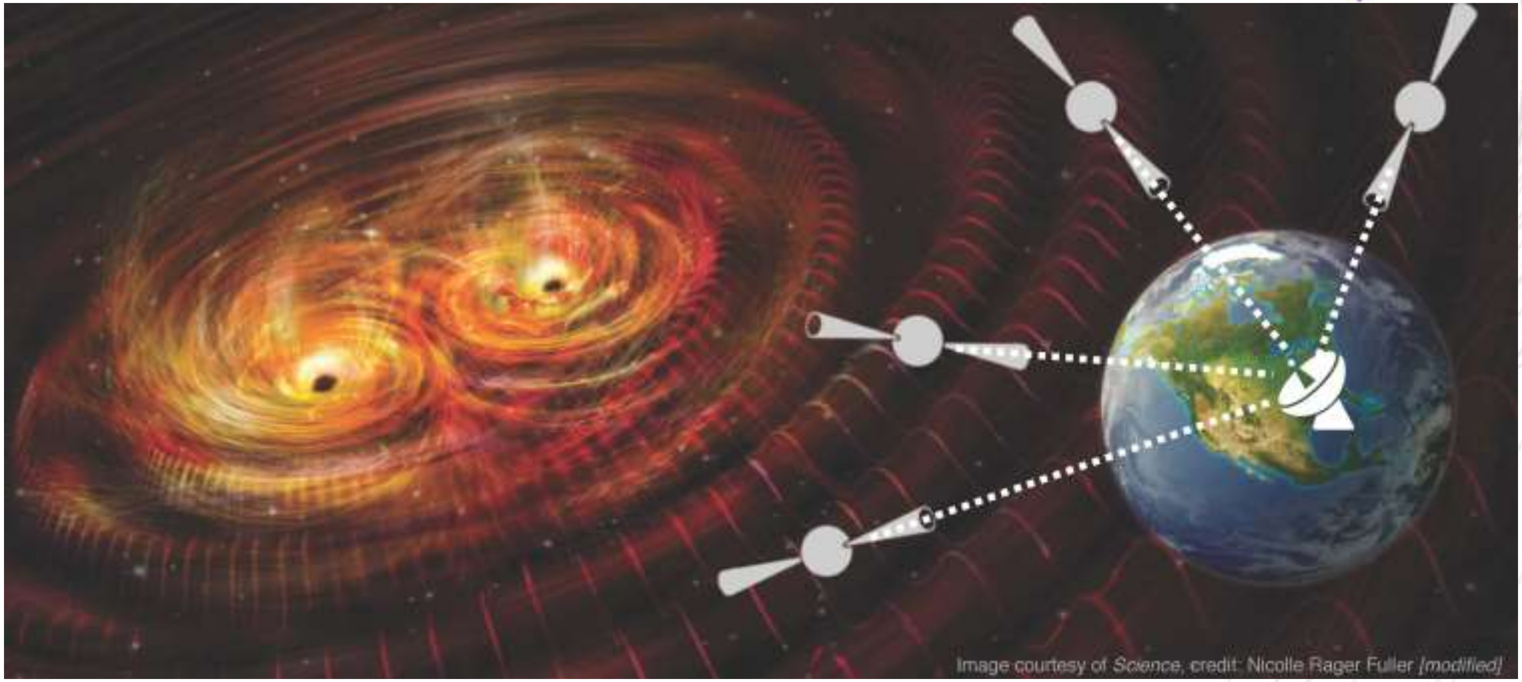
Contents

PGWs

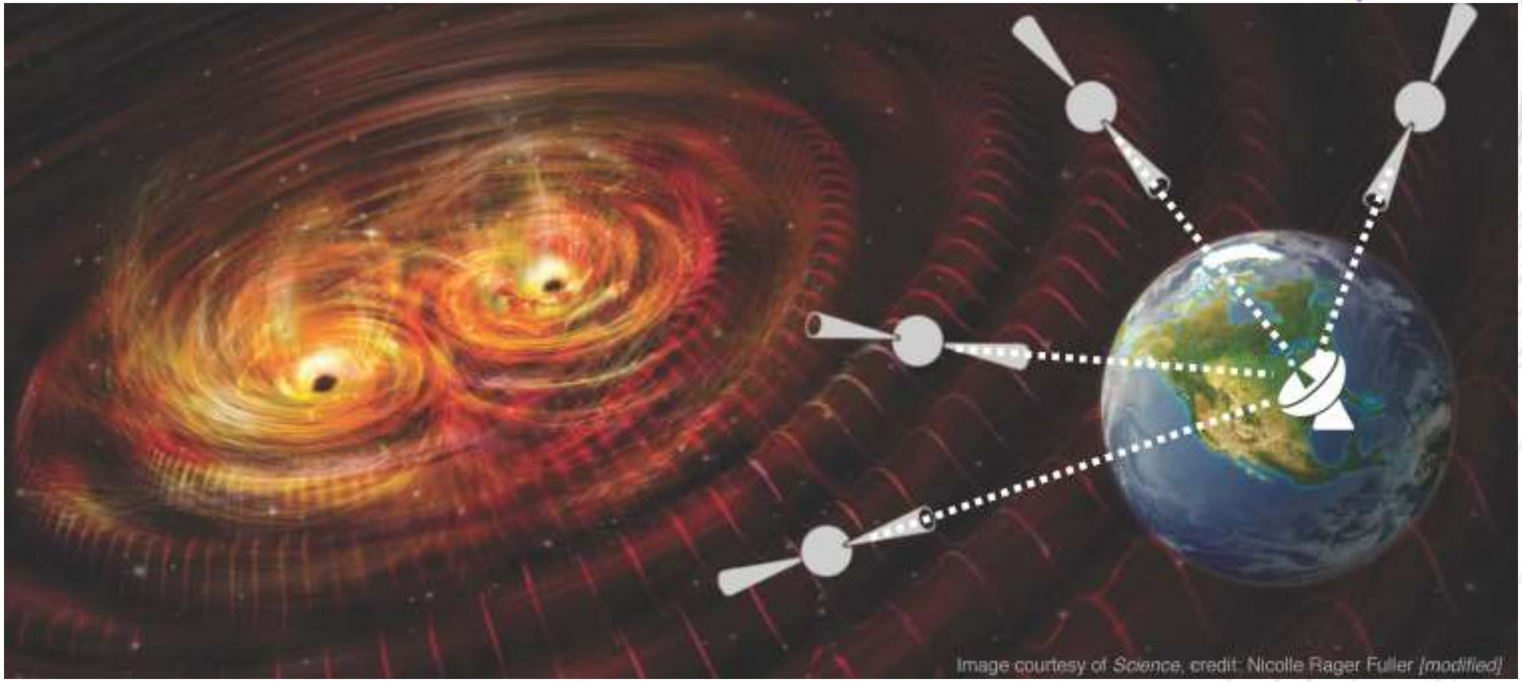
- The recent discovery of the evidence from Pulsar Timing Arrays supporting the existence of a stochastic gravitational wave background (SGWB).

NANOGrav, Astrophys. J. Lett. (2023)



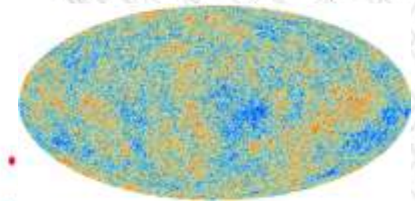
PGWs

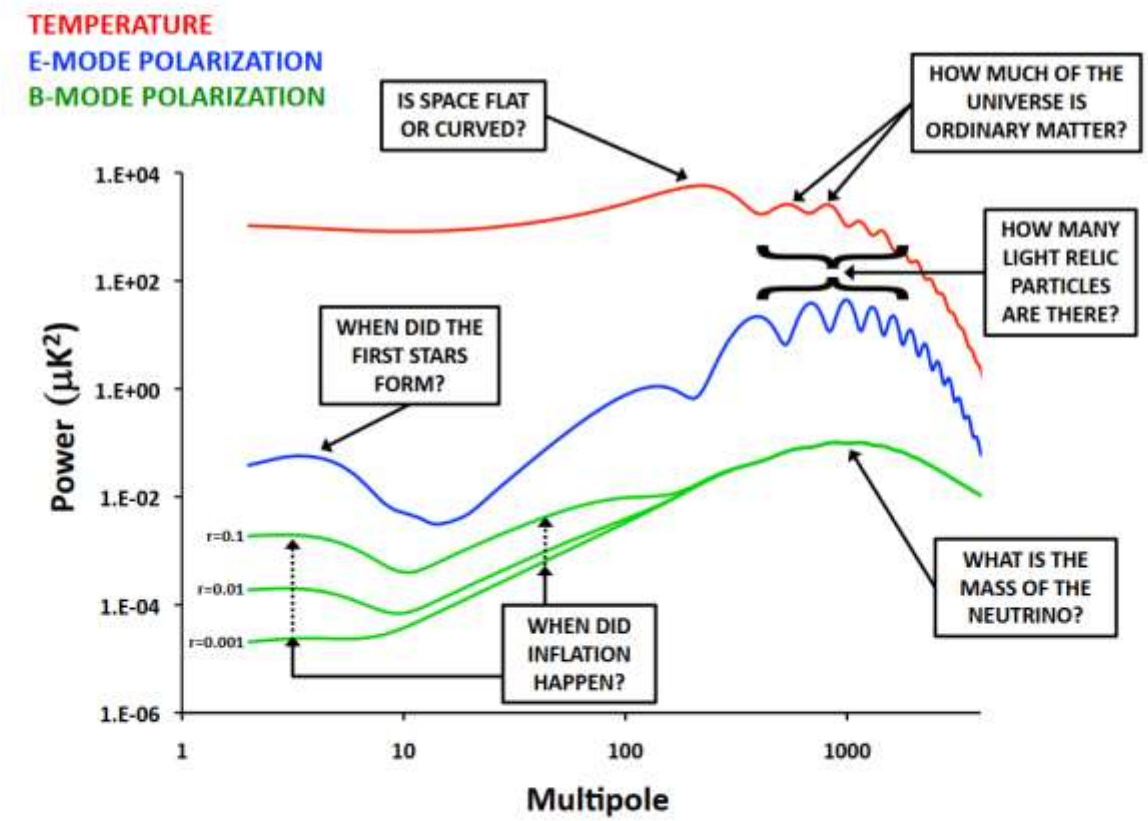
- The SGWB arises from a multitude of random, independent events. Many of these events trace back to the primordial era of the universe, produced as primordial gravitational waves (PGWs).



CMB B-mode

- Cosmic microwave background (CMB) polarization B-mode could also be used to constrain or detect primordial gravitational waves (PGWs).





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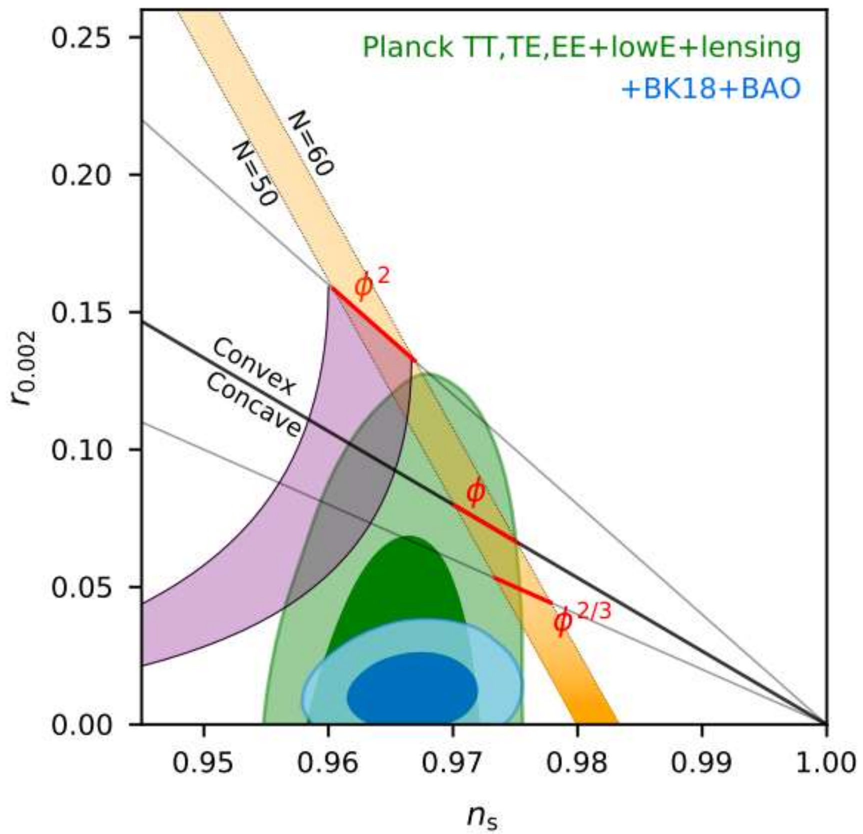
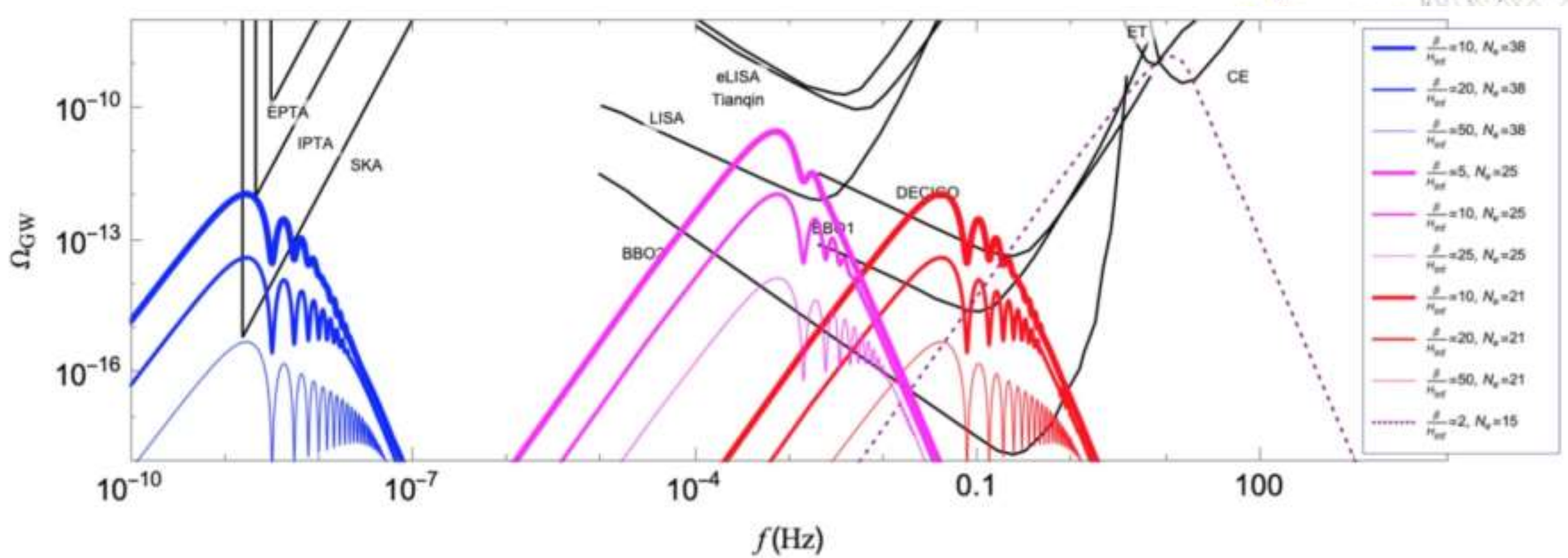


BICEP and Keck Collaborations, Phys.Rev.Lett. 127 (2021)

CMB B-mode

- CMB itself indicates a nearly scale-invariant primordial scalar power spectrum, while many GW production such as preheating, phase transition, topological effects and usually induce extra scalar perturbation as well.



Xingang Chen, Yi Wang, JCAP (2010)

Non-Conventional PGWs

- The tensor-to-scalar ratio sets a robust lower bound, known as the Lyth bound, on the inflation field excursion $\Delta\varphi$.

D. H. Lyth, Phys. Rev. Lett. (1997)

$$\Delta\phi \gtrsim \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2} M_{\rm Pl}$$

- The striking implication of this bound is that a measurement of r at the level of its current upper bound would imply super-Planckian field excursions, which would pose a theoretical challenge for model building.

$$\Delta\phi < M_{\rm Pl}$$

Non-Conventional PGWs

- Previous study put forward a novel mechanism for enhancing the primordial GWs compared to their production from vacuum fluctuations, hence beating the Lyth bound.

Yi-Fu Cai, Misao Sasaki, et al.. Phys. Rev. Lett. (2021)

$$\ddot{\delta\chi}_k + 3H\dot{\delta\chi}_k + \frac{k^2}{a^2}\delta\chi_k = \frac{\sqrt{2\epsilon_\chi}}{M_{\rm Pl}}\left[\ddot{\phi}\delta\phi_k + \mathcal{S}_k\right],$$

$$\ddot{\delta\phi}_k + 3H\dot{\delta\phi}_k + \left(\frac{k^2}{a^2} + \mathcal{M}_{\rm eff}^2\right)\delta\phi_k = 0\ ,$$

$$\mathcal{S}_k = \int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\left\{\frac{\boldsymbol{p}\cdot\boldsymbol{k}}{k^2}\left[\frac{(\boldsymbol{p}-\boldsymbol{k})^2}{a^2} + \mathcal{M}_{\rm eff}^2\right] - \frac{\boldsymbol{p}\cdot(\boldsymbol{k}-\boldsymbol{p})}{2a^2}\right\}\delta\phi_{|\boldsymbol{p}|}\delta\phi_{|\boldsymbol{k}-\boldsymbol{p}|}\ .$$

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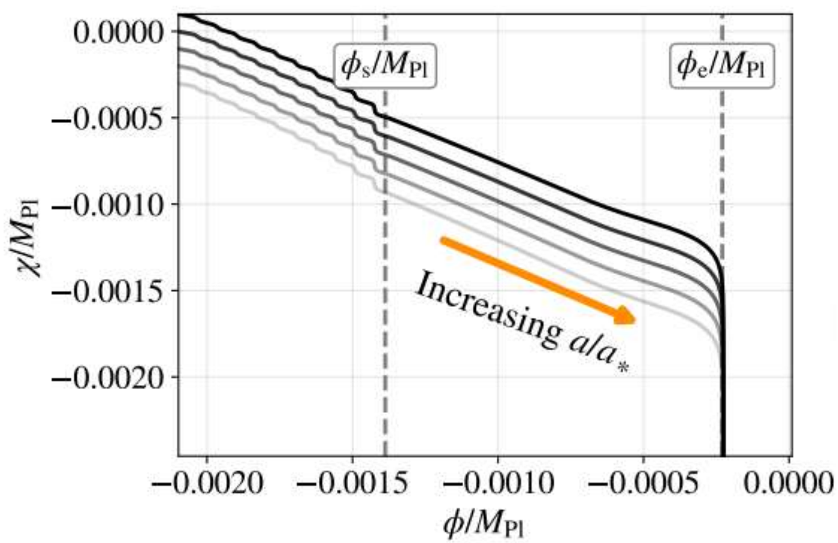
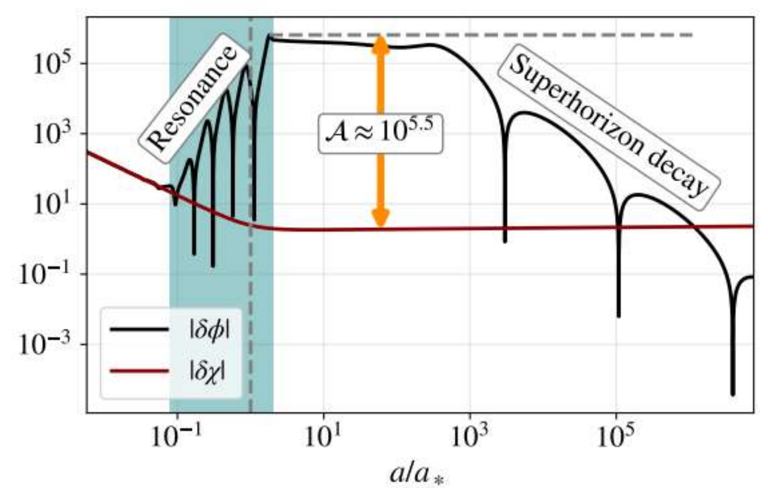
Yi-Fu Cai, Misao Sasaki, et al.. Phys. Rev. Lett. (2021)





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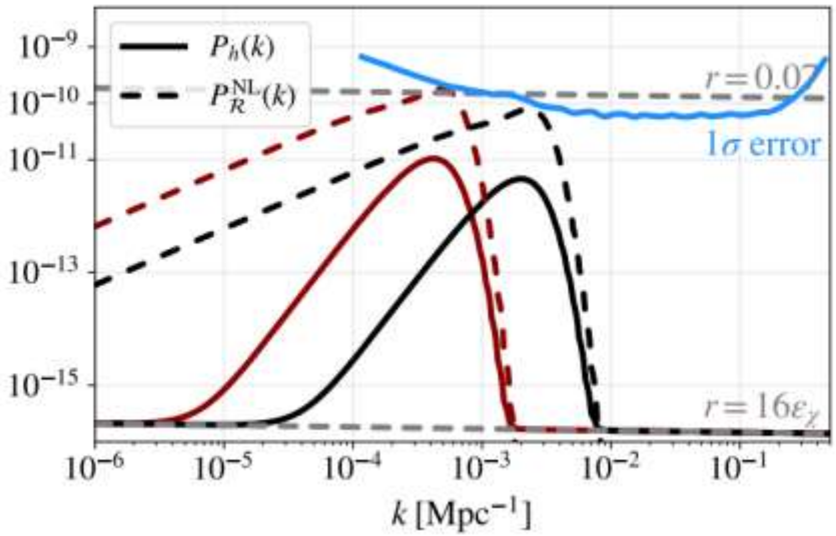
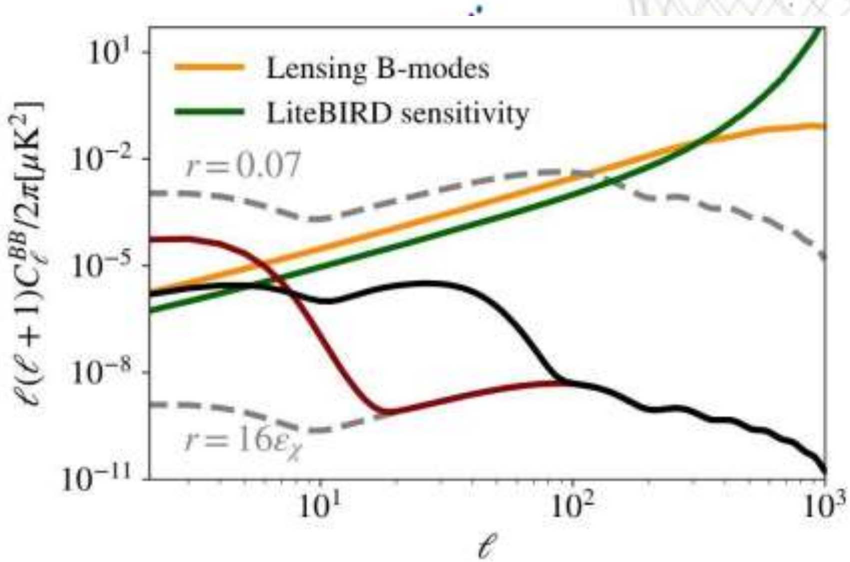


The Amplification Effect of Sound Resonance Mechanism on PGWs



Resonant Amplified Signal Prediction on BB Modes of the CMB

Goal

- Putting observational constraints on the resonance inflation model by its stochastic gravitational wave background (SGWB) production, from cosmic microwave background (CMB) B-mode anisotropy.

$$\Delta_t^2(k,\tau_{\text{end}}) = \frac{4}{\pi^4 M_p^4} k^3 \int_0^\infty dp p^6 \int_{-1}^1 d\cos\theta \sin^4\theta \times \left| \int_{\tau_0}^{\tau_{\text{end}}} d\tau_1 g_k(\tau_{\text{end}},\tau_1) \left(\delta\phi_p(\tau_1)\delta\phi_{|\boldsymbol{k}-\boldsymbol{p}|}(\tau_1) + \delta\chi_p(\tau_1)\delta\chi_{|\boldsymbol{k}-\boldsymbol{p}|}(\tau_1)\right) \right|^2$$

Zihan Zhou, Jie Jiang, Yi-Fu Cai, Misao Sasaki, and Shi Pi, Phys. Rev. D (2020)

Challenge

- High degree of freedom renders it impossible to constrain the model parameters directly.

Contents

Why we need a new parameterization?

- Models predict similar observational signals. Parameterizations use minimum degree of freedom to extract model-independent information.

- Commonly used parameterizations:
  - Broken power-law: e.g. GW from phase transitions.

$$f_{\rm BPL}(k) = A\frac{\alpha+\beta}{\beta(k/k_*)^{-\alpha}+\alpha(k/k_*)^{\beta}}$$

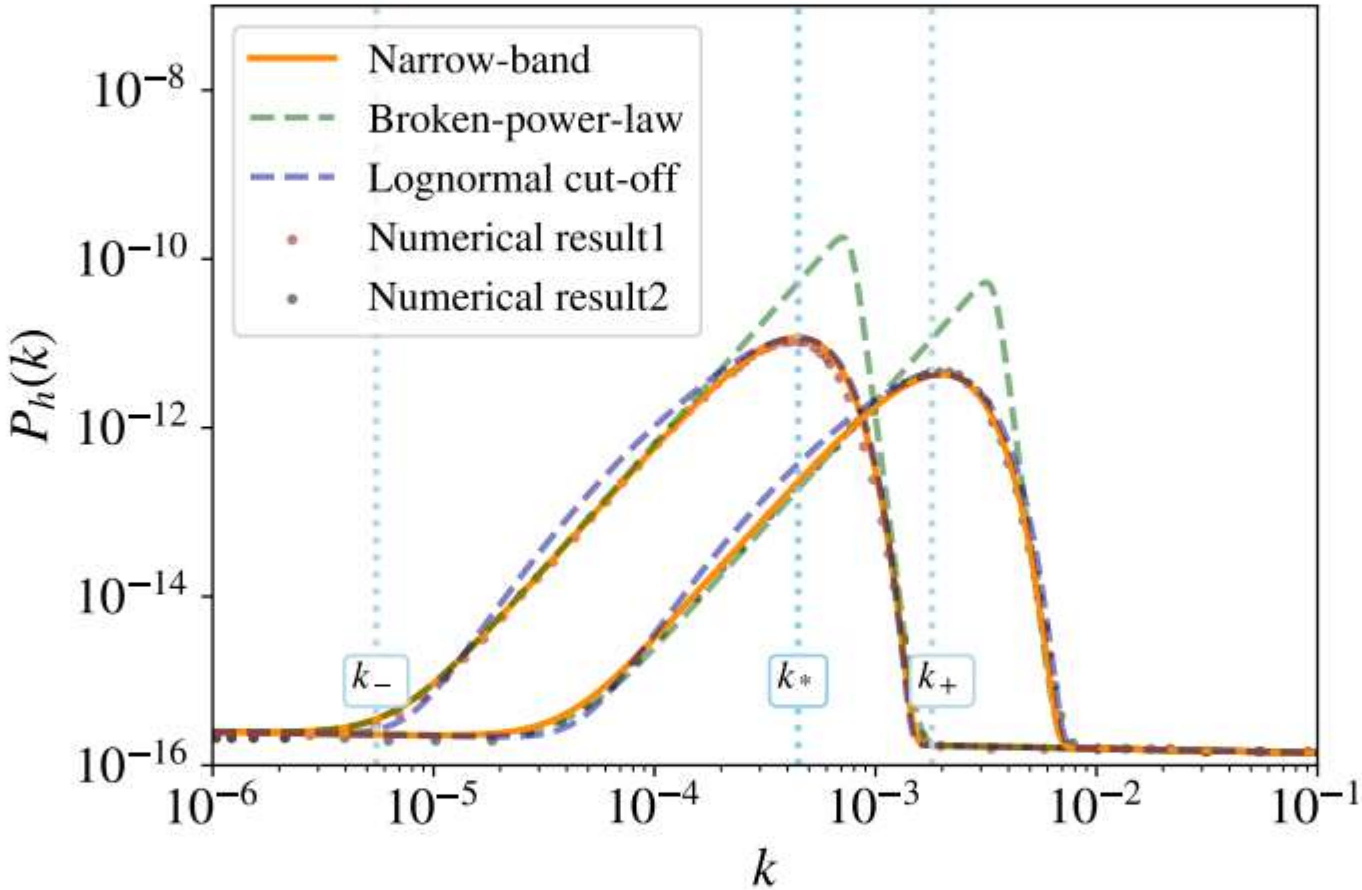
  - Log-normal with UV cutoff: Gaussian distribution in logarithm axis.

$$f_{LNC}(k) = h_R \exp\left(-\frac{\log^2\left(\frac{k}{k_*}\exp\left(\left(\frac{k}{k_*}\right)^g - 1\right)\right)}{2\Delta^2}\right)$$

- Both are insufficient for the resonance model.

Advantages over traditional parameterizations

- Goodness for fit



Comparison between parameterizations

Advantages over traditional parameterizations

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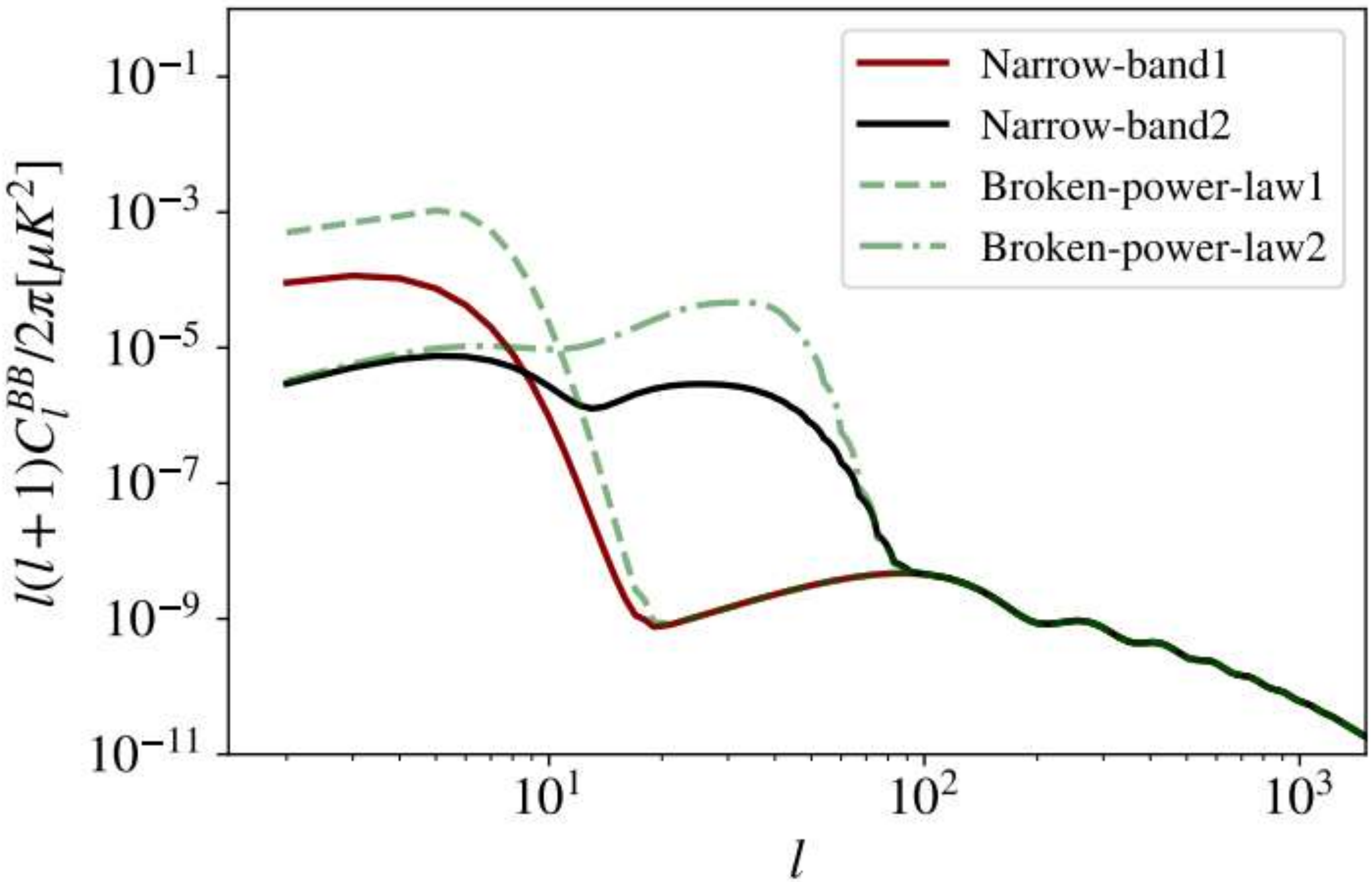


Comparison between parameterizations

Construction

- Ultraviolet (UV) limit: exponential cut-off
- Infrared (IR) limit: $\propto k^3$
  - A universal infrared limit of SGWB spectra for a class of GW sources. A physical understanding of it comes from causality.

    Rong-Gen Cai, Shi Pi, Misao Sasaki, Phys. Rev. D (2020)
  - We extend it from radiation dominated era to inflationary era.

$$\ln f(k) \sim 3 \ln k - \exp(g \ln k)$$

$$f(k) = h \exp\left(3 \ln(k/k_*) + (1 - \exp(g \ln(k/k_*))) \frac{3}{g}\right)$$

$$\Delta_t^2 = (1 + f(k)) A_s r \left(\frac{k}{k_{\text{pivot}}}\right)^{n_t}$$

Construction

(I) k is smaller compared to all the scales associated with the source term, such as $k(\eta_1 - \eta_2) \ll 1$ and $|\mathbf{k} - \mathbf{p}| \approx |-\mathbf{p}|$, where p is a integrated wavenumber index of the source and η_1, η_2 are two moment when source still exist.

(II) The energy-mometum tensor should possess a comparably general form.

$$T_{ab}(\tau, \mathbf{k}) = v_a(\tau, \mathbf{k}) v_b(\tau, \mathbf{k}) + \sum_I \partial_a \phi_I(\tau, \mathbf{k}) \partial_b \phi_I(\tau, \mathbf{k}) \qquad (1)$$

$$I = 1, 2, \cdots, \text{ as different scalar fields .}$$

(III) The integral over wavenumber for computing Ω_{GW} after taking $k \to 0$ should be finite. Namely,

$$0 < \int d\ell \left[(2\mathcal{P}_v + 3\mathcal{P}_w)^2 + 5\mathcal{P}_w^2 + 4 \sum_I \mathcal{P}_\phi^2 \right] < \infty , \qquad (2)$$

where,

$$\begin{aligned} \langle v^a(\boldsymbol{\ell}, \tau_1) v^{c*}(\boldsymbol{q}, \tau_2) \rangle &= \delta^{(3)}(\boldsymbol{\ell} - \boldsymbol{q}) \frac{2\pi^2}{\ell^3} \times \ell^2 \left[\mathcal{P}_w(\tau_1, \tau_2, l) \pi^{ac}(\boldsymbol{\ell}) + \mathcal{P}_v(\tau_1, \tau_2, l) \hat{\ell}^a \hat{\ell}^c \right] , \\ \langle \phi_I(\boldsymbol{\ell}, \tau_1) \phi_J^*(\boldsymbol{q}, \tau_2) \rangle &= \delta_{IJ} \delta^{(3)}(\boldsymbol{\ell} - \boldsymbol{q}) \frac{2\pi^2}{\ell^3} \mathcal{P}_{\phi_I}(\tau_1, \tau_2, \ell) , \end{aligned} \qquad (3)$$

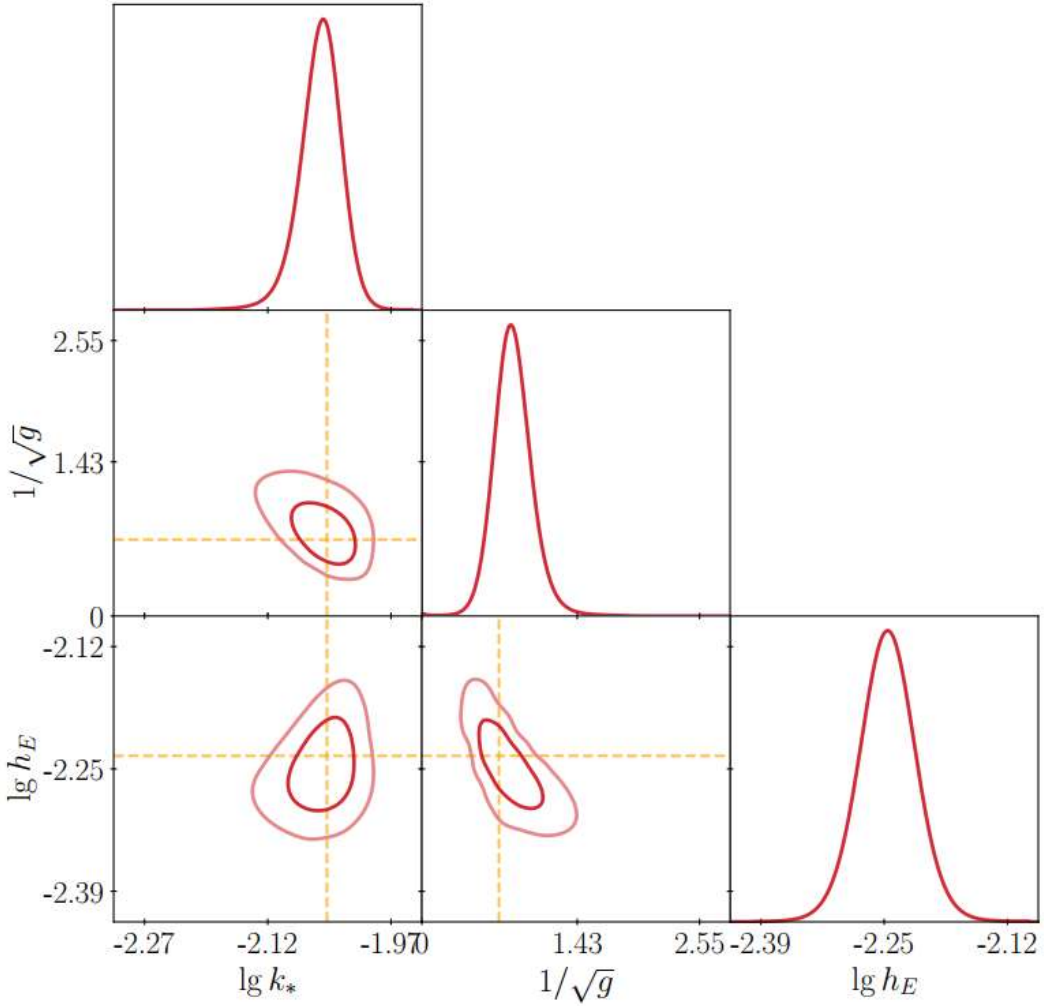
in which $\pi^{ac}(\ell) = \delta^{ab} - \hat{\ell}^a \hat{\ell}^c$ and $\mathcal{P}_w, \mathcal{P}_v$ are respectively longitudinal and perpendicular part of the power spectrum of $\langle vv \rangle$, while $\mathcal{P}_{\phi_I}$ represents the power spectrum of the scalar field noted by I.We have assumed the two-point function between different scalar fields should be zero.

(IV) Modes of interest reenter the Hubble horizon during the radiation-dominated era to produce GW (or GW is produced during inflationary era).

Rong-Gen Cai, Shi Pi, Misao Sasaki, Phys. Rev. D (2020)

Advantages over traditional parameterizations

- Suitability for Monte Carlo Markov Chain process



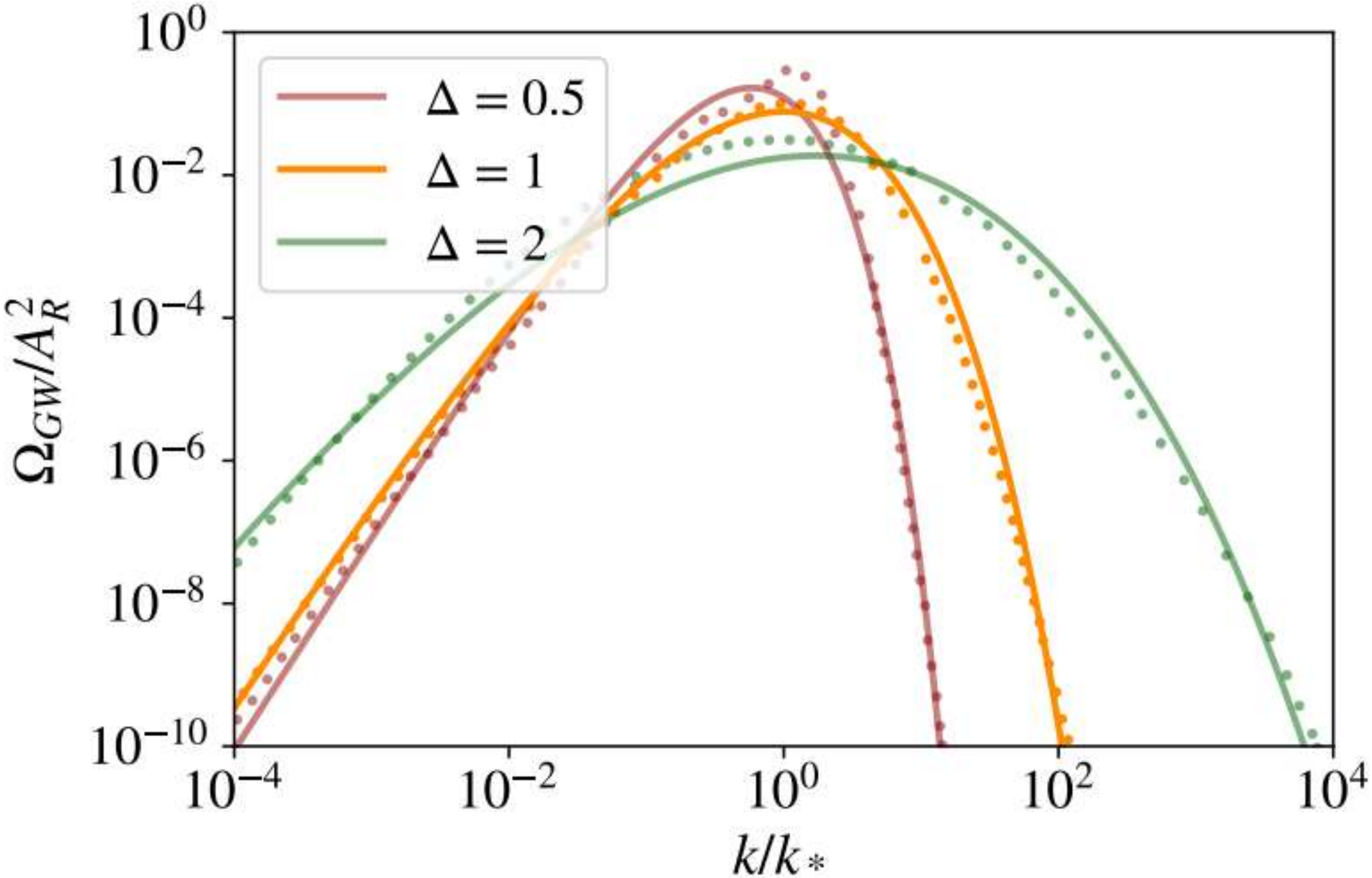
Narrow-band parameterization: Extension to a wider class of SGWB production

- When GW source is transient over time while spiky in wavenumber spectrum, IR k^3 scaling is easier to achieve.

- When the GW production is narrow-band, broken power-law produce unnatural spike, while exponential cut-off fluently transit the behavior.

Narrow-band parameterization: Extension to a wider class of SGWB production

- Example: scalar induced gravitational waves.



Contents

- Background
- Parameterization
- Summary and Outlook

Summary and Outlook

- In the era of precision cosmology, model-independent approaches such as parameterization method remain crucial for qualitatively understanding the relationship between the theoretical model and its observational features.

- In light of the non-perturbative resonance effects that may occur during inflation, we introduce a parametrization for the power spectrum of the stochastic gravitational wave background (SGWB) characterized by narrow-band amplification.

- One of the extensions of the parameterization refers to a different slope on IR scaling.

- Another extension concerns amendments for cosmological probes other than CMB, such as pulsar timing array.

Thanks!