Constrain Modified Gravity from Stochastic Gravitational Wave Background

2108.05344 **Q.Liang**, M.Trodden
2304.02640 **Q.Liang**, M-X.Lin, M.Trodden

Qiuyue Liang
Kavli IPMU (WPI), University of Tokyo

02/19/2024

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THE UNIVERSITY OF TOKYO

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- Brief review of nano Hertz Stochastic gravitational wave background
- Test Gravity in PTA

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- Parity-Violation signal in astrometry system

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- Discussion

Nano Hertz SGWB

- Astrophysical Source: Supermassive Black Hole Binary;
- Cosmological Source: Primordial Gravitational Wave; Phase transition;

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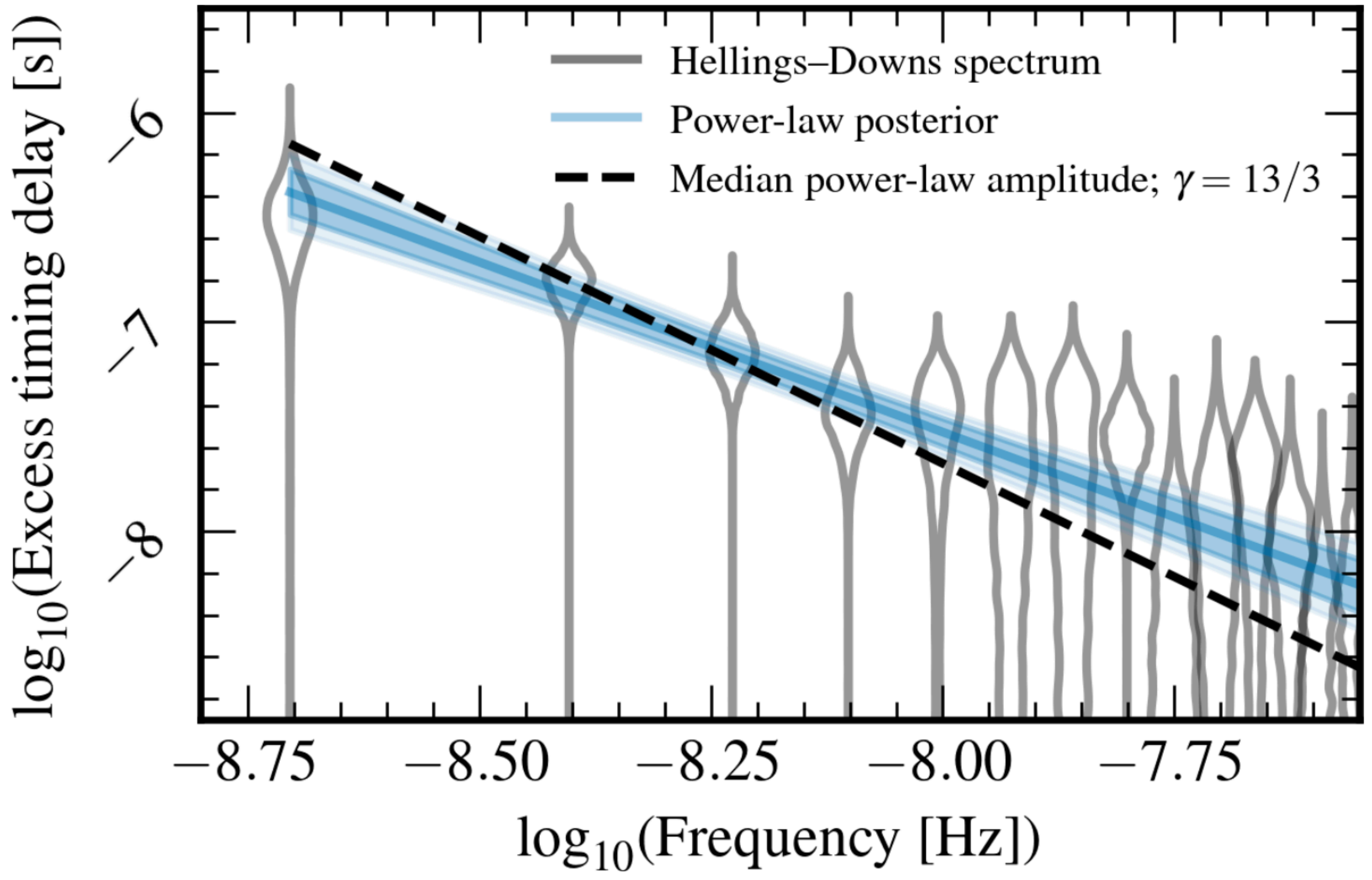
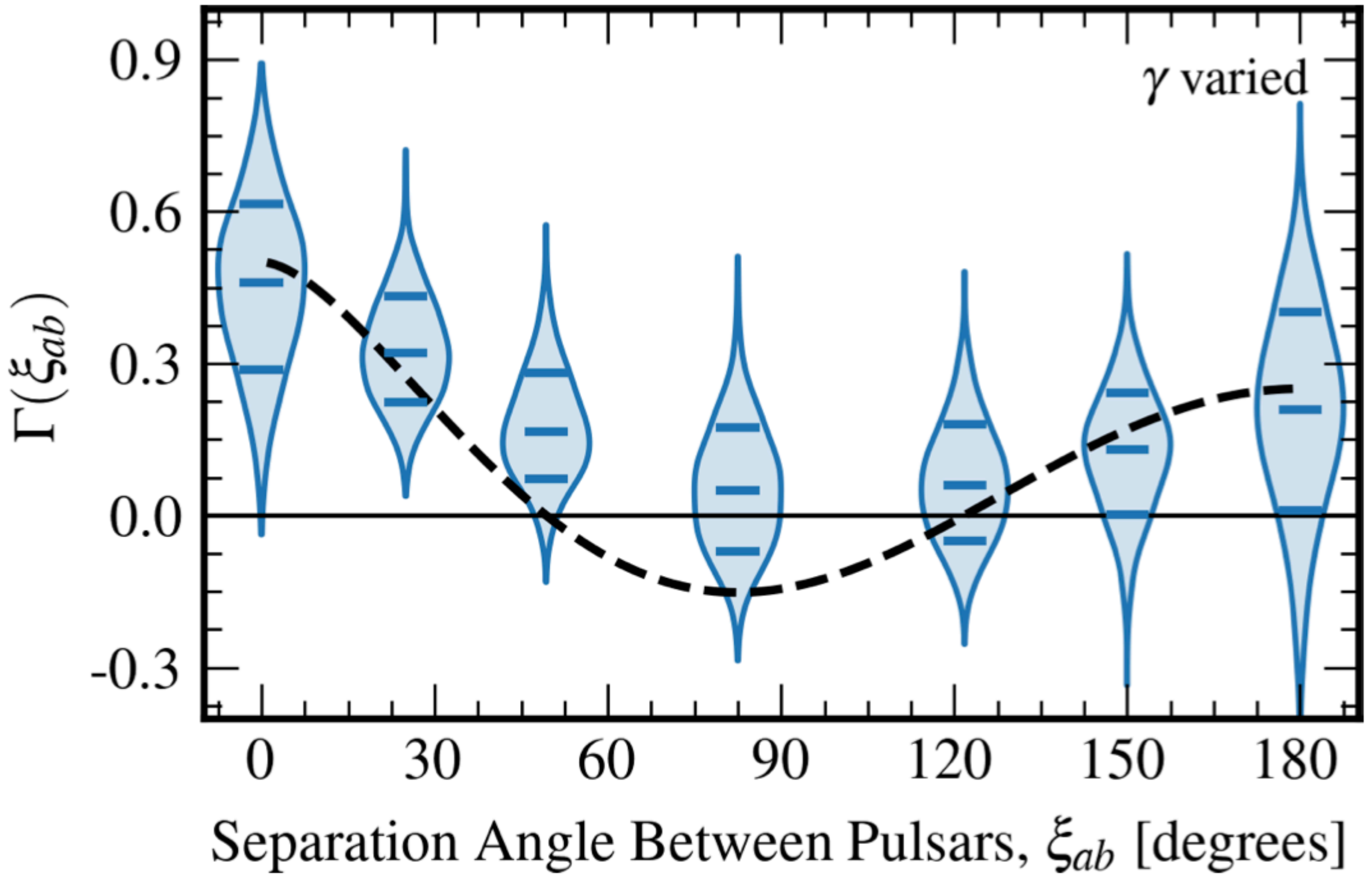
Nano Hertz SGWB

- Astrophysical Source: Supermassive Black Hole Binary;
- Cosmological Source: Primordial Gravitational Wave; Phase transition;
- PTA (pulsar timing array) collaboration claimed a detection in last July!

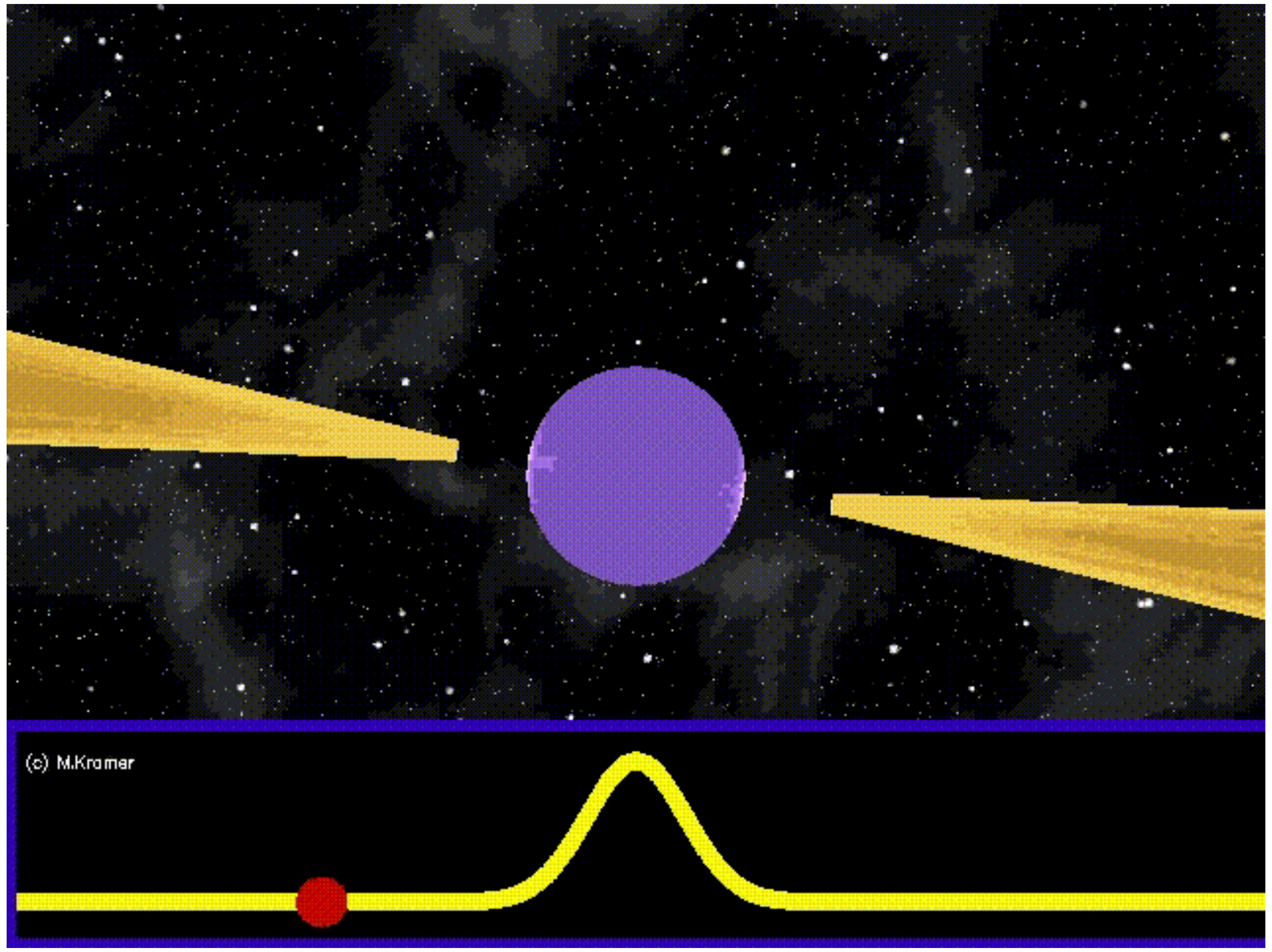


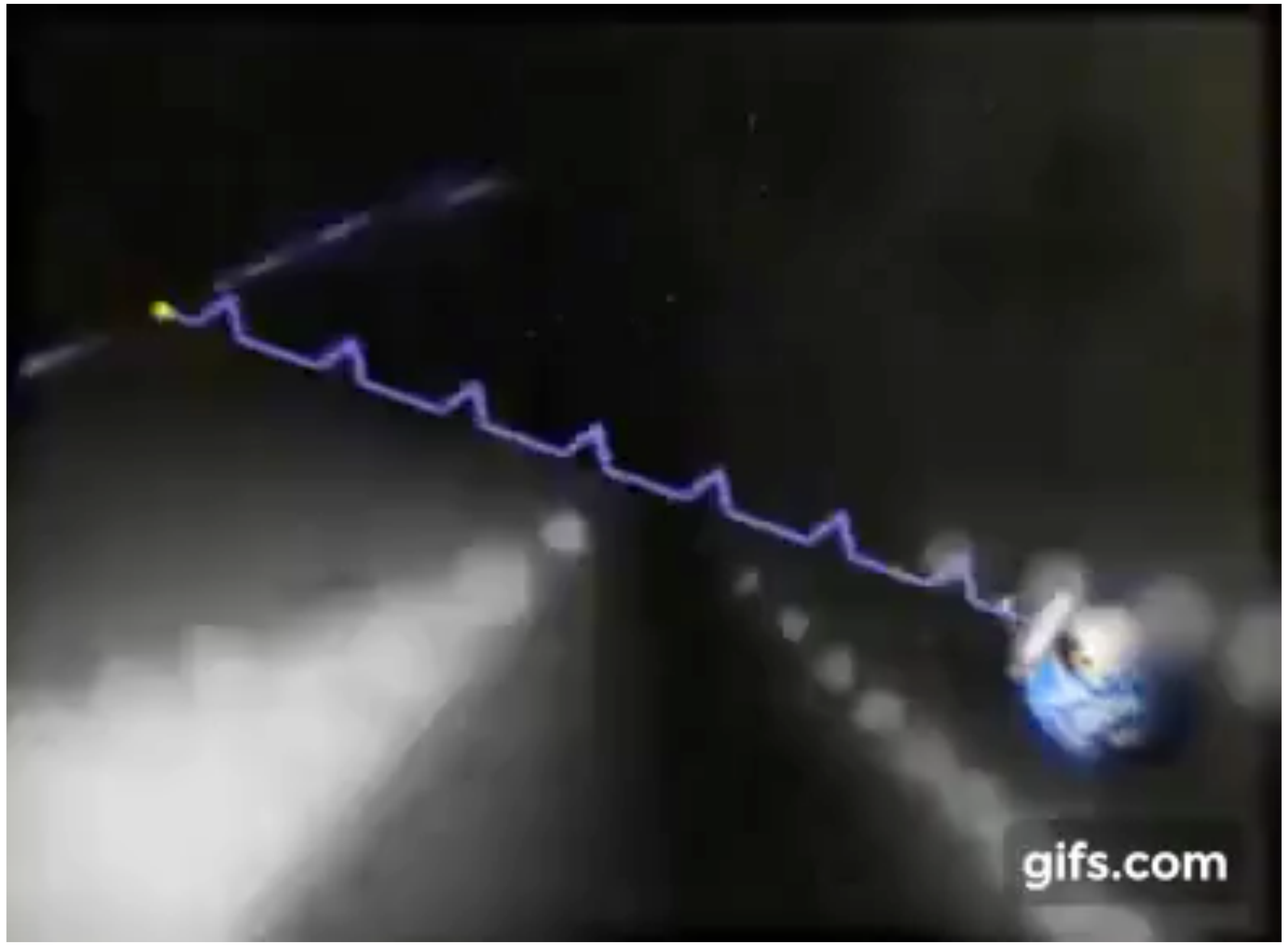


Power spectrum Angular correlation

Credit: NANOGrav 15 yr result

Pulsar and Gravitational wave





Pulsar Timing Array (PTA)

- A pulsar timing array (PTA) is a set of pulsars which is analyzed to search for correlated signatures in the pulse arrival times.

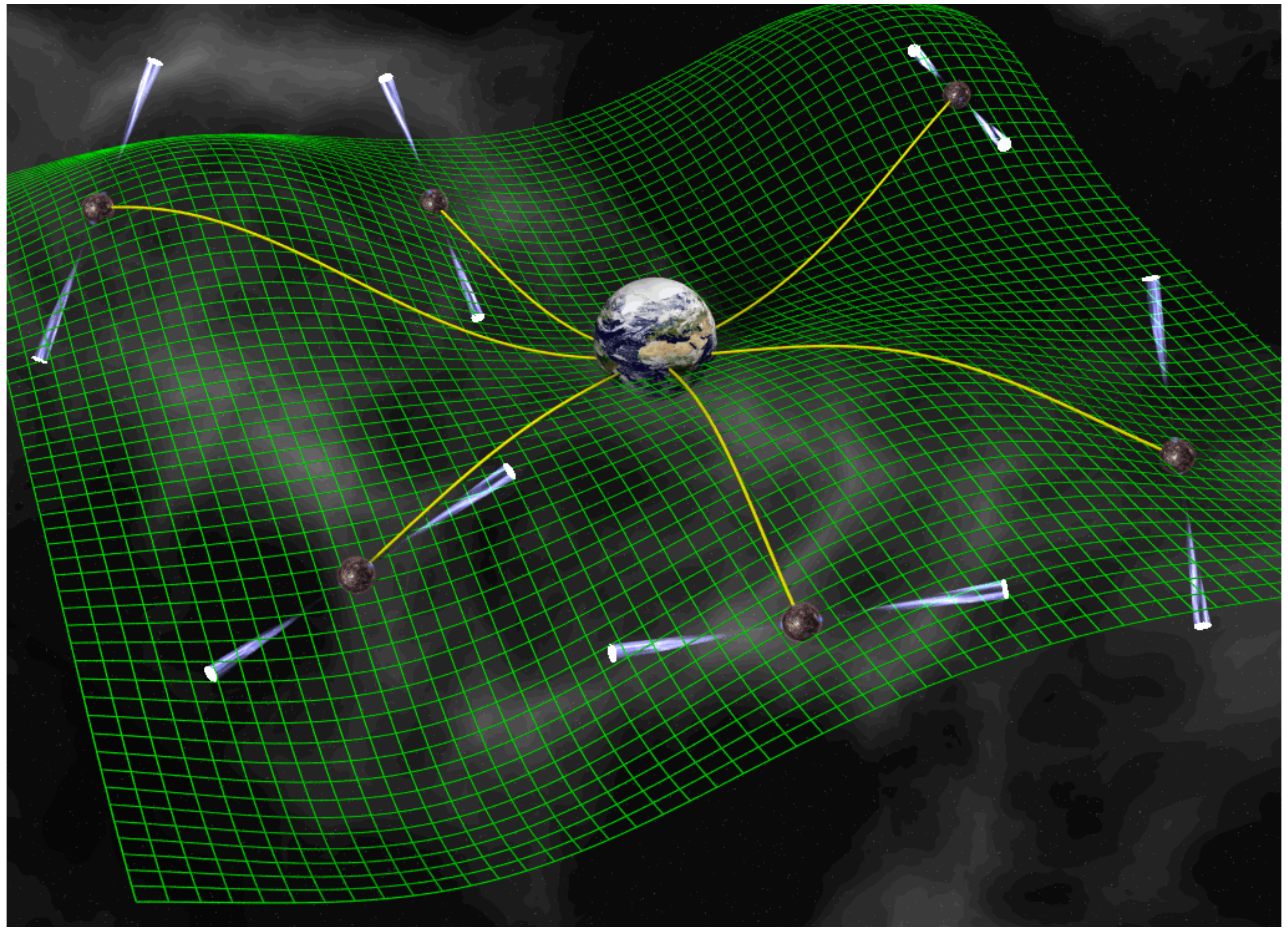
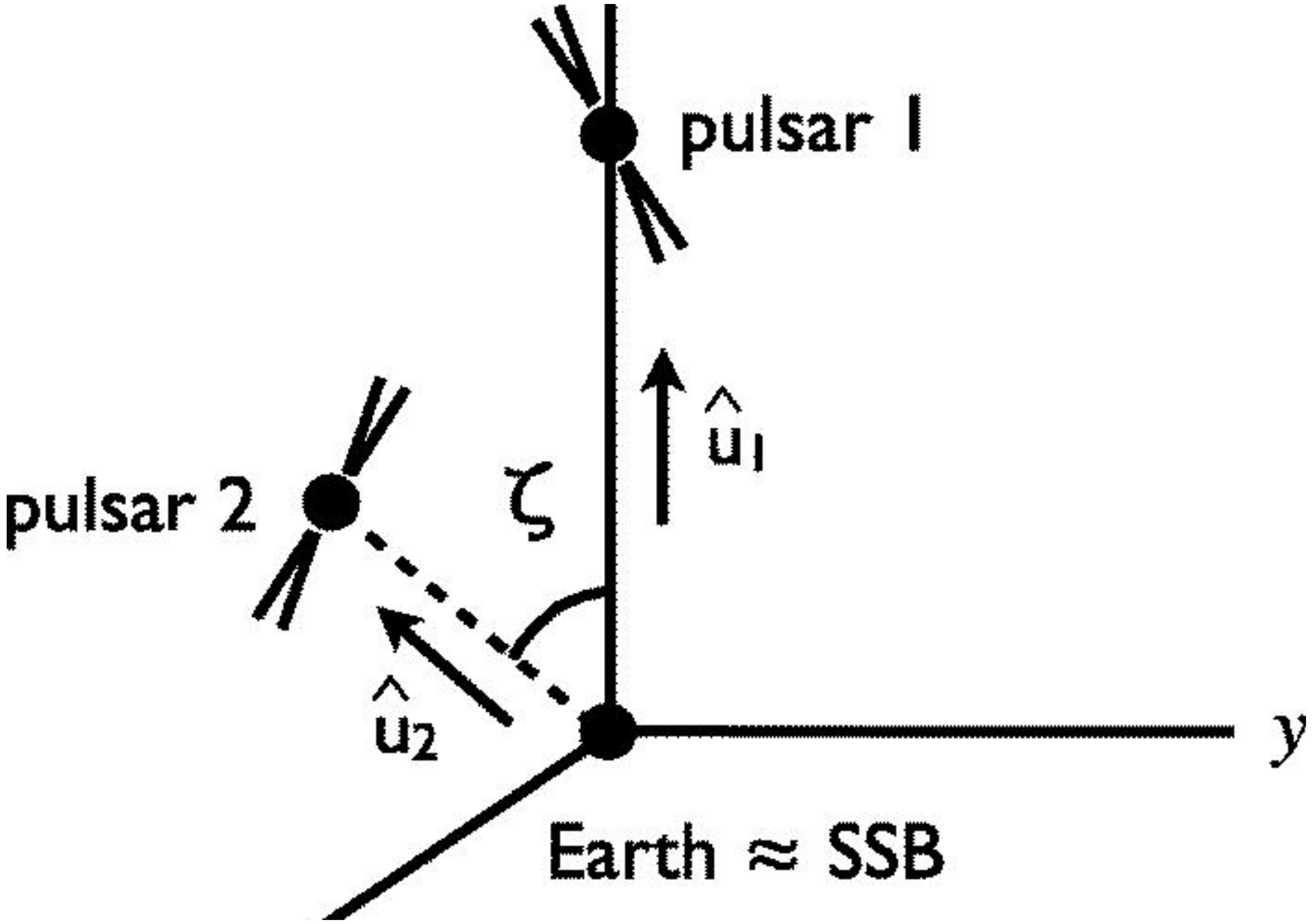


Image credit: David J. Champion



Fredrick A. Jenet, Joseph D. Romano

What can we learn from this SGWB?

- Astro: origin of supermassive black hole formation & population rate… 2401.04161 2312.06756 2306.17021,2306.16222 2305.05955
- Early universe: different inflation scenarios, primordial gravitational waves… 2212.05594 2311.03391 2311.02065 2311.00741
- Defect: cosmic string, phase transition,…2306.17205 2304.04793 2304.02636
- Beyond Standard Model physics: dark matter, baryon number violation, string compactification … 2402.03984 2306.05389 2305.11775 2304.10084 …
- Modified Gravity: 2304.02640 Q.Liang, M-X, Lin, M. Trodden 2108.05344 Q.Liang, M. Trodden

Credit: NANOGrav 15 yr result

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Power spectrum

- Modified Gravity: 2304.02640 Q.Liang, M-X, Lin, M. Trodden 2108.05344 Q.Liang, M. Trodden

Angular correlation

Credit: NANOGrav 15 yr result

How to obtain the Overlap reduction function?
How does modified gravity change the ORF?

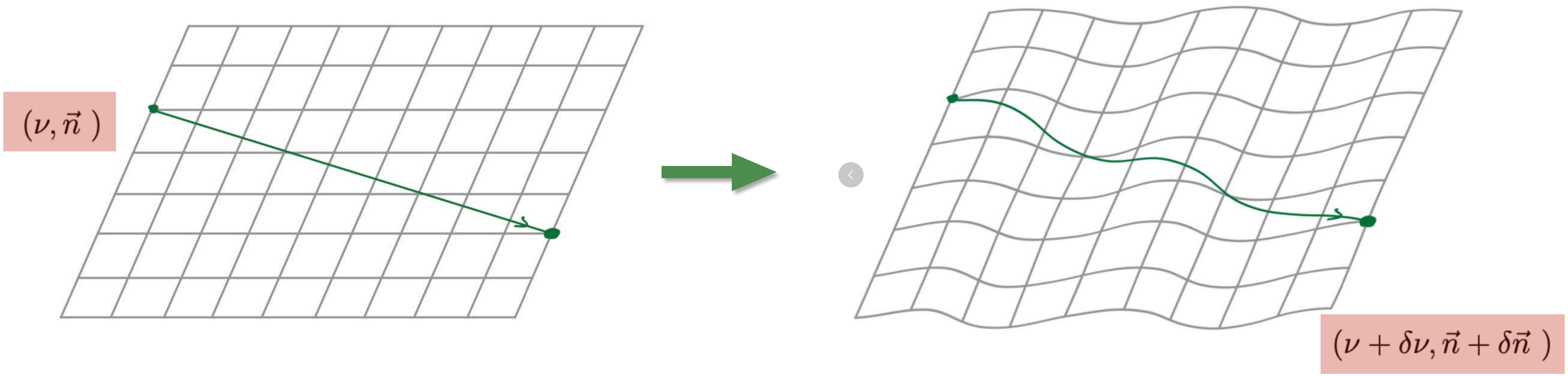
How to obtain the Overlap reduction function?

How does modified gravity change the ORF?

Using PTA to detect SGWB

$$R(t) \equiv \int_0^t dt' \left(\frac{\nu_0 - \nu(t')}{\nu_0}\right) \equiv \int_0^t dt'\ z(t')$$

$$\langle R(t, \hat{p}_1) R(t', \hat{p}_2) \rangle$$



Light path in flat space

Light path in perturbed space

Correlation function

- In GR, the metric perturbations only have two polarization modes, and we can express them as $h_{\mu\nu} = \sum_{A=+,\times} e^A_{\mu\nu} h_A$.
- For each mode, we define a receiving function to denote the influence on the redshift:

$$\tilde{z}(f, \hat{\Omega}) = \left(e^{-i2\pi f L(1+\hat{\Omega}\cdot\hat{p})} - 1\right) \sum_A h_A(f, \hat{\Omega}) F^A(\hat{\Omega})$$

$$F^A(\hat{\Omega}) \equiv e^A_{ij}(\hat{\Omega}) \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega}\cdot\hat{p}}.$$

- If we take the long wavelength limit, the receiving function would take the form as LIGO/VIRGO system

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Correlation function

- One can separate the two-point correlation function in power spectrum $\Omega_{\rm gw}(|f|)$ and the overlap reduction function $\Gamma(\xi)$ if assume isotropic SGWB $\langle \tilde{z}_1^*(f)\tilde{z}_2(f')\rangle = \frac{3H_0^2}{32\pi^3}\frac{1}{\beta}\delta(f-f')|f|^{-3}\Omega_{\rm gw}(|f|)\Gamma(|f|),$

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- Power spectrum:

$$\Omega_{\rm gw}(|f|) \equiv (3M_{\rm P}^2 H_0^2)^{-1} d\rho_{\rm gw}/d\ln f$$

Correlation function

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- Power spectrum:

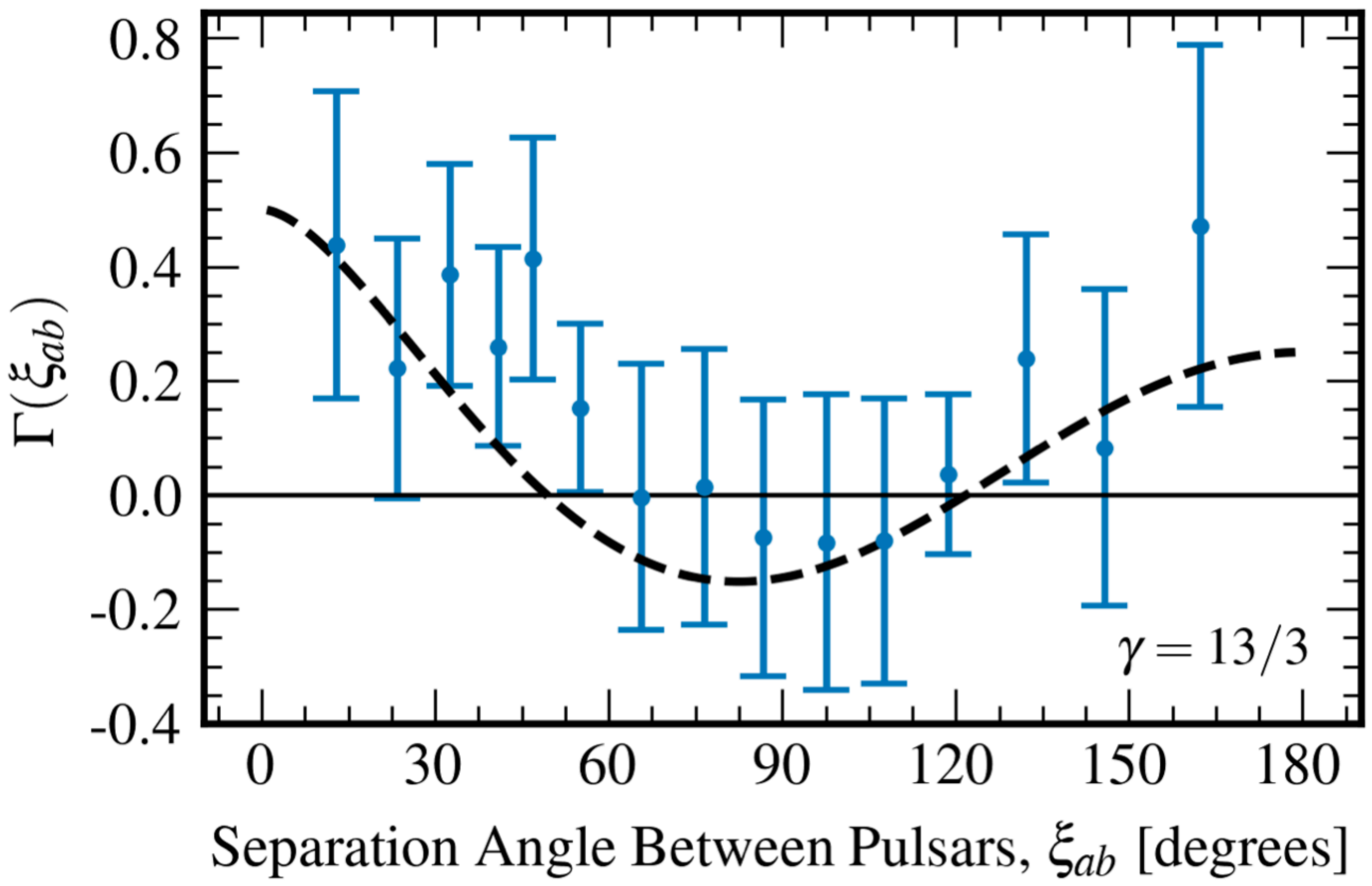
$$\Omega_{\rm gw}(|f|) \equiv (3M_{\rm P}^2 H_0^2)^{-1} d\rho_{\rm gw}/d\ln f$$

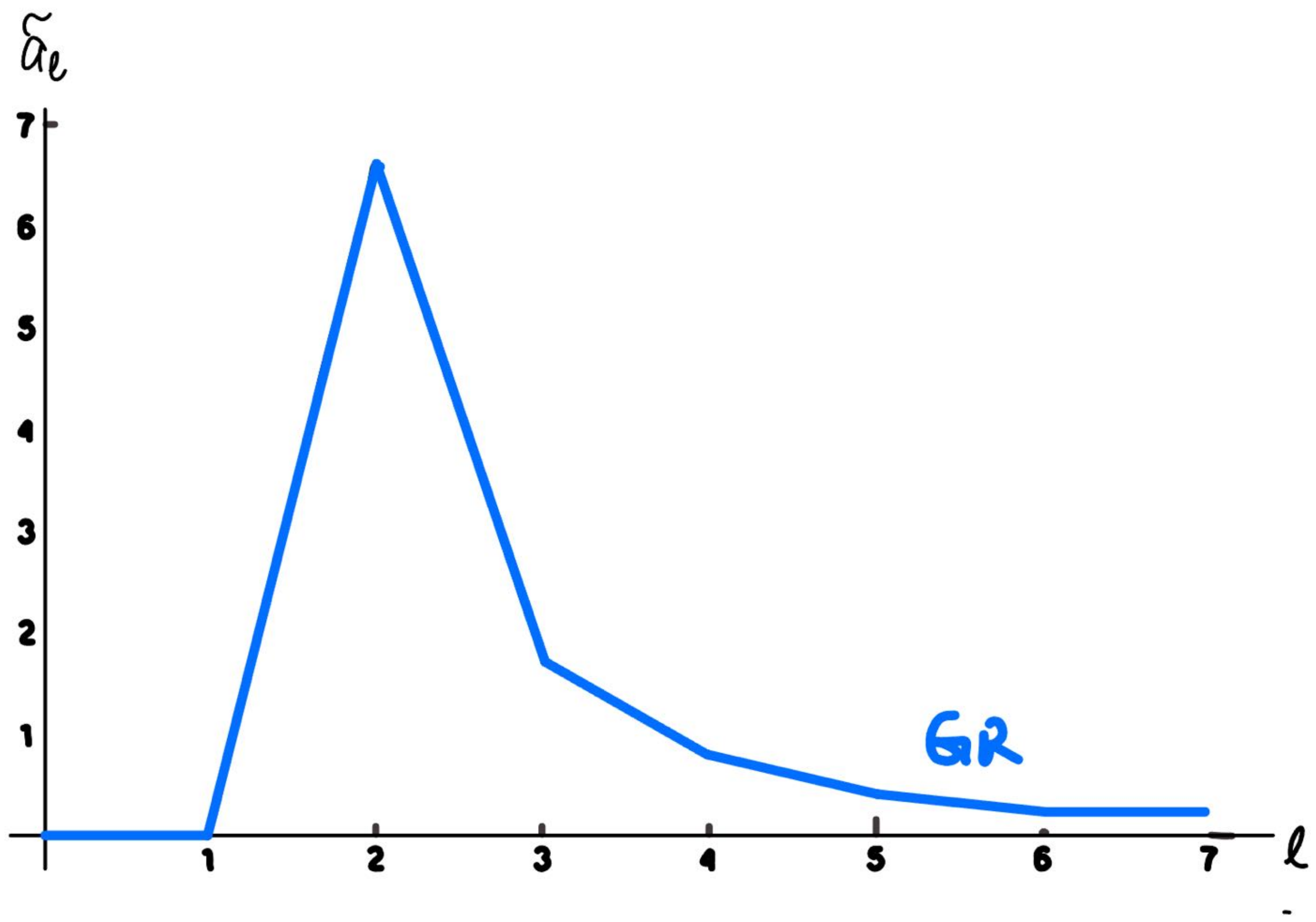
- Overlap reduction function (angular correlation):

$$\Gamma(|f|) = \beta \sum_A \int_{S^2} d\hat{\Omega} \left(e^{i2\pi f L_1(1+\hat{\Omega}\cdot\hat{p}_1)} - 1\right) \times \left(e^{-i2\pi f L_2(1+\hat{\Omega}\cdot\hat{p}_2)} - 1\right) F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega}),$$

Overlap reduction function

- First detection (3 sigma) of Hellings-Downs curve! (NANOGrav 15 yrs)





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How to obtain the Overlap reduction function?

How does modified gravity change the ORF?

How to obtain the Overlap reduction function?

How does modified gravity change the ORF?

- Extra polarizations
- Dispersion relation

- Massive gravity Phys.Rev.D 104 (2021) 8, 084052 Q.Liang, M.Trodden
- Dispersion relation 2304.02640 Q.Liang,M-X Lin, M.Trodden

Massive Gravity

- Linear Action:

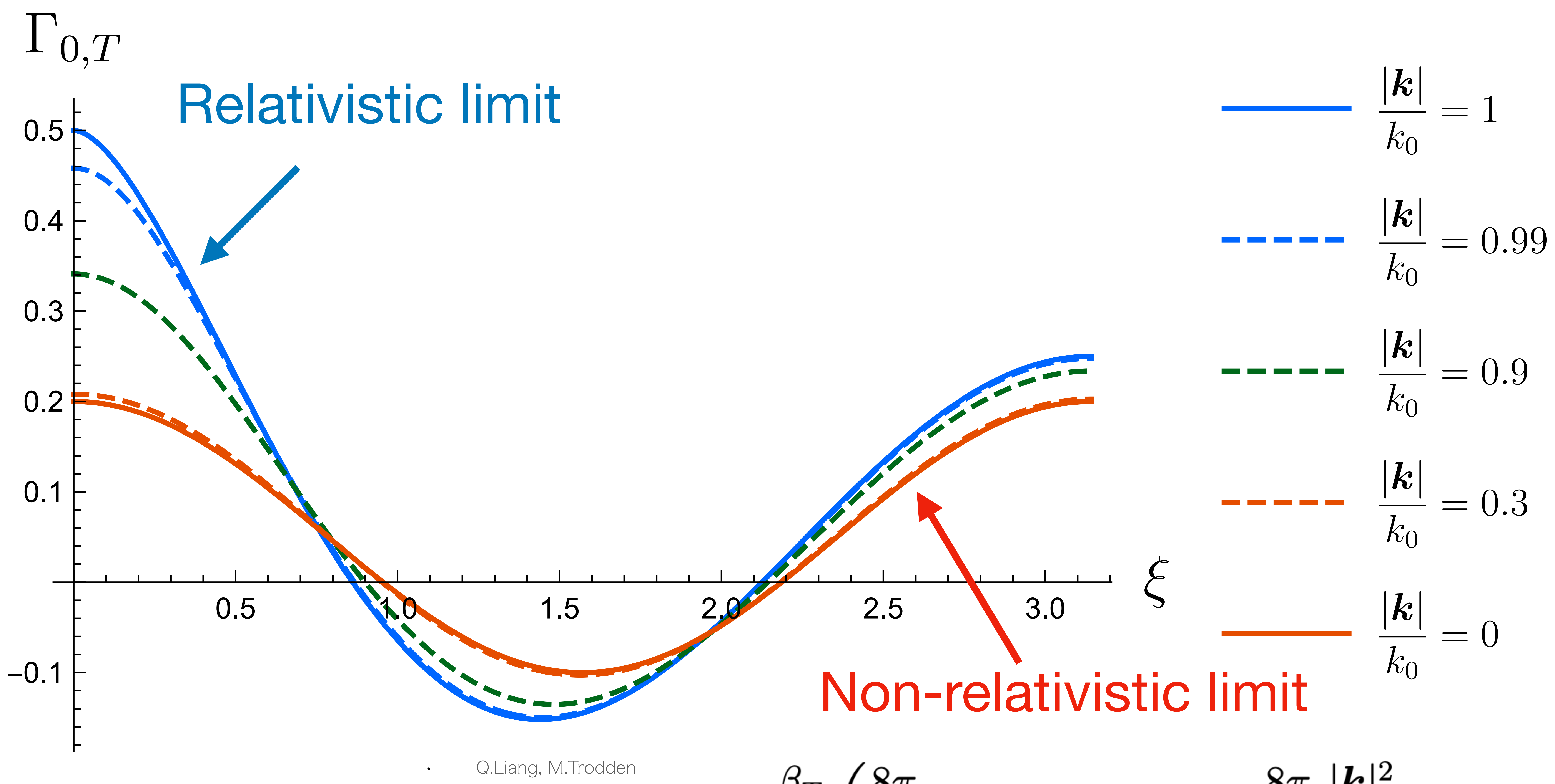
$$S = \int d^4x \left[\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} - \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} + \partial_\mu h^{\mu\nu}\partial_\nu h - \frac{1}{2}\partial_\lambda h\partial^\lambda h + \frac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

- **5 Polarization modes:** 2 tensor modes + 2 vector + 1 scalar mode

- **Modified dispersion relation:** phase velocity larger than speed of light (inversely related to the group velocity)

$$h_{\mu\nu}(x) = \frac{1}{2\pi}\int d^4k \frac{2\delta(|\boldsymbol{k}|^2 - (k_0^2 - m^2))}{|\boldsymbol{k}|} e^{ikx} h_{\mu\nu}(k) = \int_{-\infty}^{\infty} df \int_{\text{sky}} d^2\hat{\boldsymbol{\Omega}}\, e^{i2\pi f\left(t - \frac{|\boldsymbol{k}|}{k_0}\hat{\boldsymbol{\Omega}}\cdot\boldsymbol{x}\right)} h_{\mu\nu}\left(f, \frac{|\boldsymbol{k}|}{k_0}\hat{\boldsymbol{\Omega}}\right)$$

Tensor

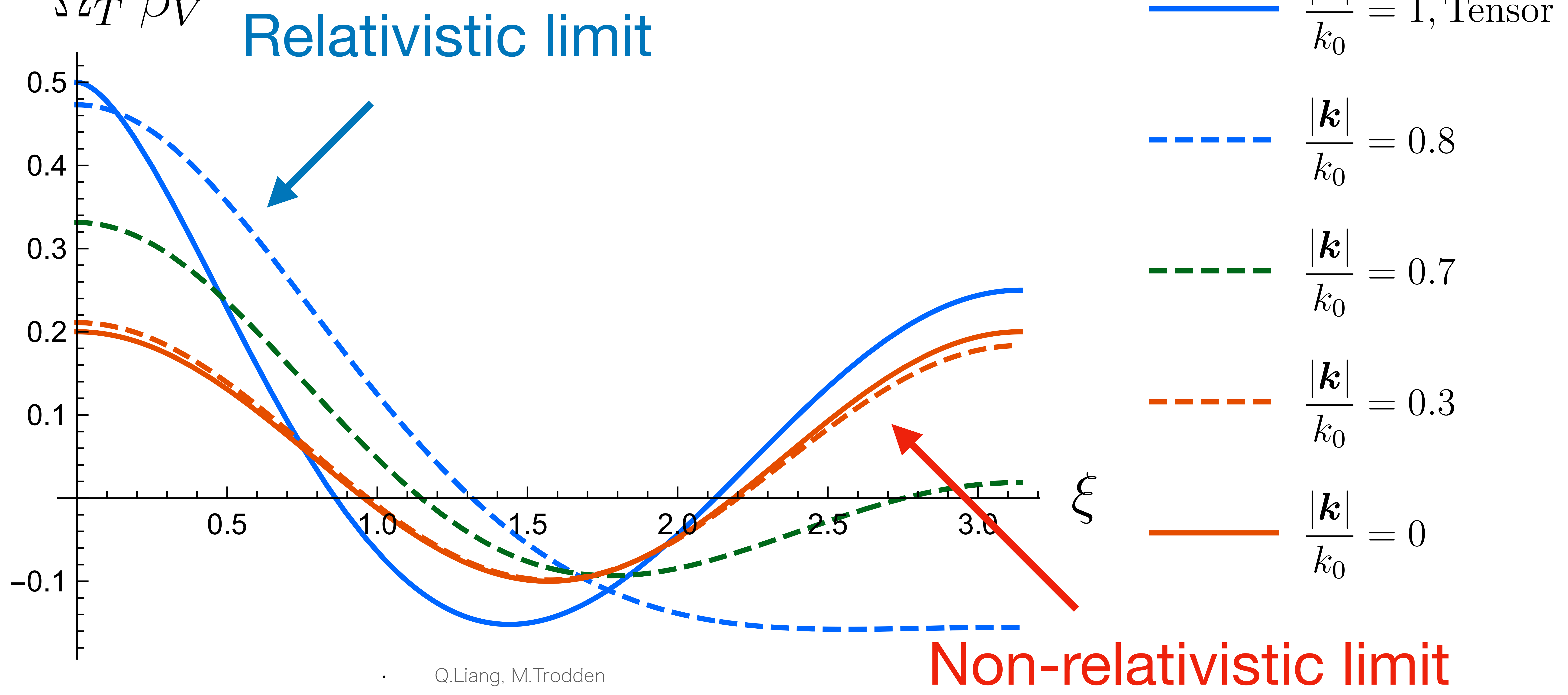


$$\Gamma_{0,T} = \frac{\beta_T}{4}\left(\frac{8\pi}{15}\left(-1+3\cos\xi^2\right) + \frac{8\pi}{105}\frac{|\boldsymbol{k}|^2}{k_0^2}\left(-2-3\cos\xi+6\cos\xi^2+5\cos\xi^3\right)\right)$$

Vector

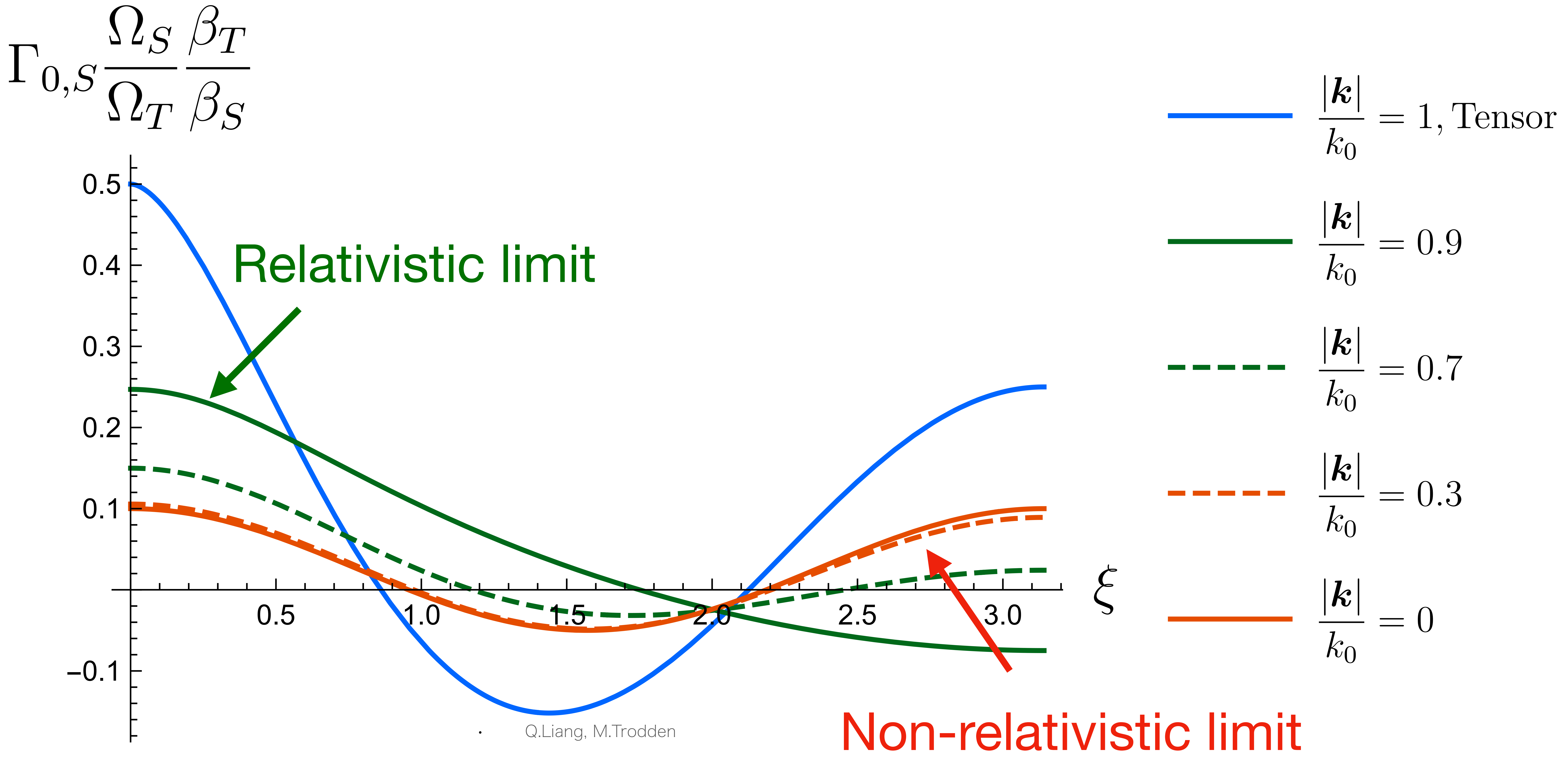
$$\Gamma_{0,V} = \frac{\beta_V}{4}\frac{k_0^2}{2m^2}\frac{8\pi}{3}\cos\xi$$

$\Gamma_{0,V}\dfrac{\Omega_V}{\Omega_T}\dfrac{\beta_T}{\beta_V}$



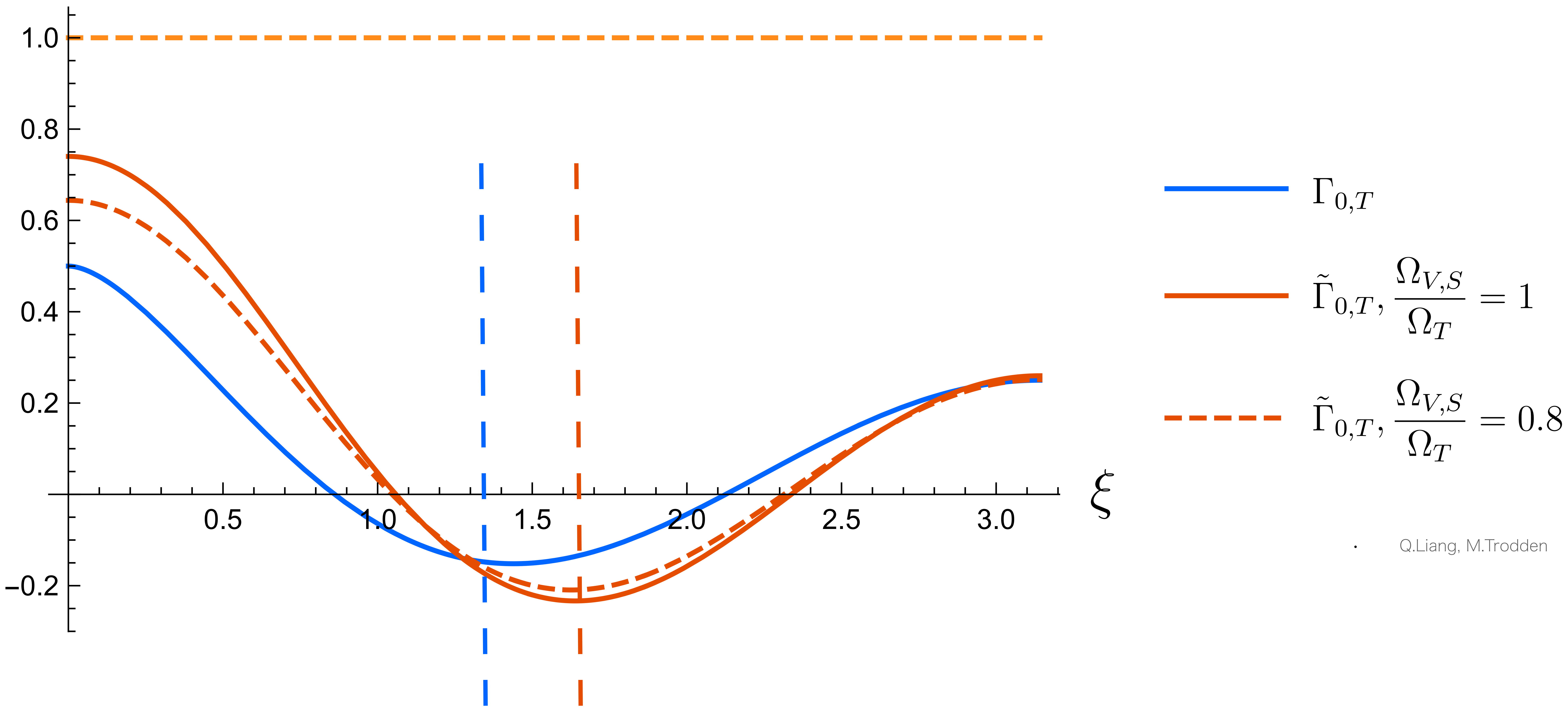
$$\Gamma_{0,V} = \frac{\beta_V}{4}\frac{8\pi}{15}(-1+3\cos^2\xi)$$

Scalar



$$\Gamma_{0,S} = \frac{\beta_S}{4}\frac{4\pi}{15}(-1 + 3\cos^2\xi)$$

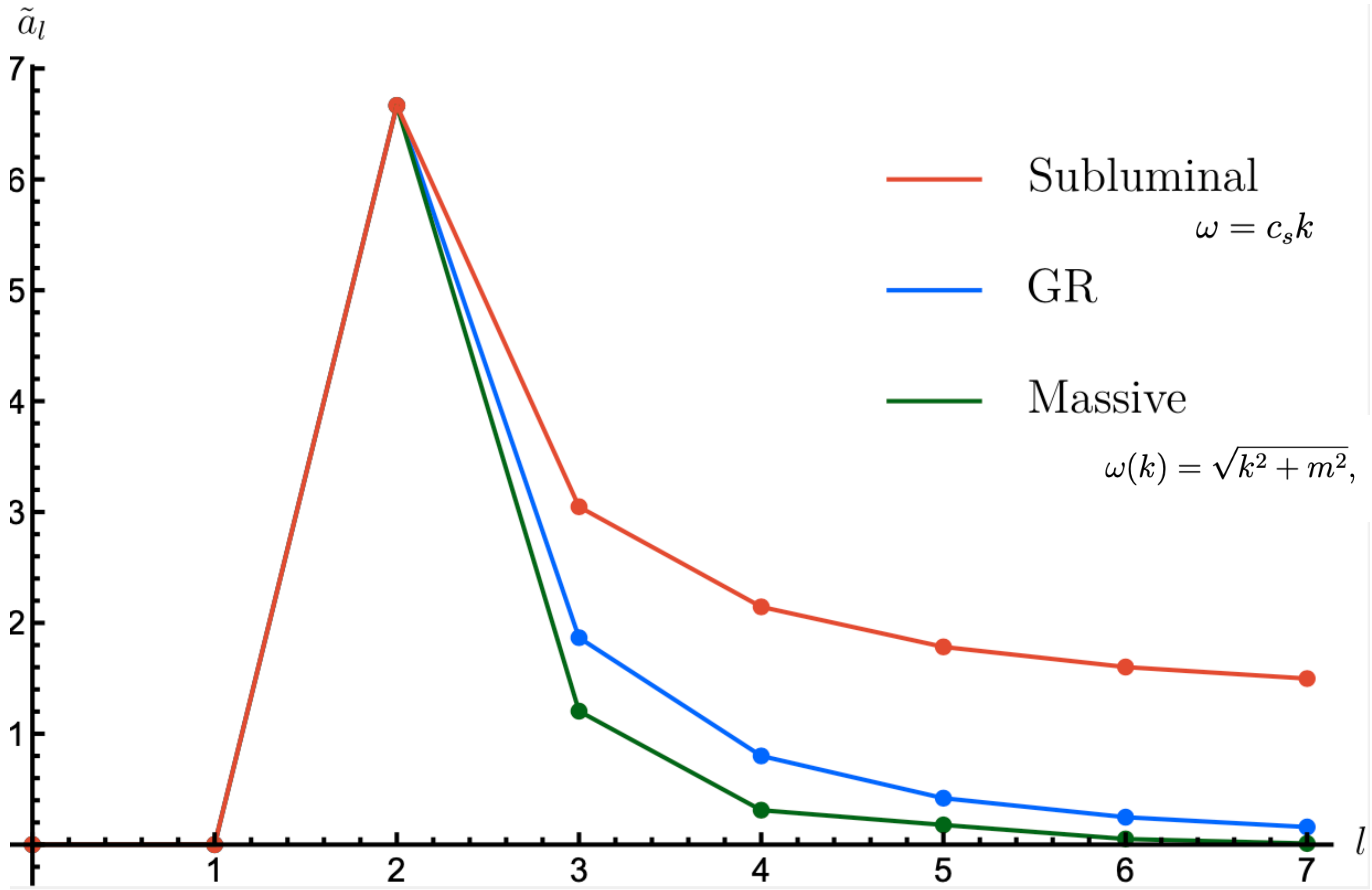
Combined effective overlap reduction function



Notice the shift of the minimum angle! This might be a distinguishable feature!

The minimum angle

- In GR, the minimal angle is determined by the coefficients of harmonic analysis
- If we only have the quadrupole, then the minimal angle is 90 degree.
- The coefficients of the higher multipole modes will make the minimal angle less than 90 degree.
- When the coefficients of higher modes got enhanced or suppressed in modified theories, the minimal angle will shift.



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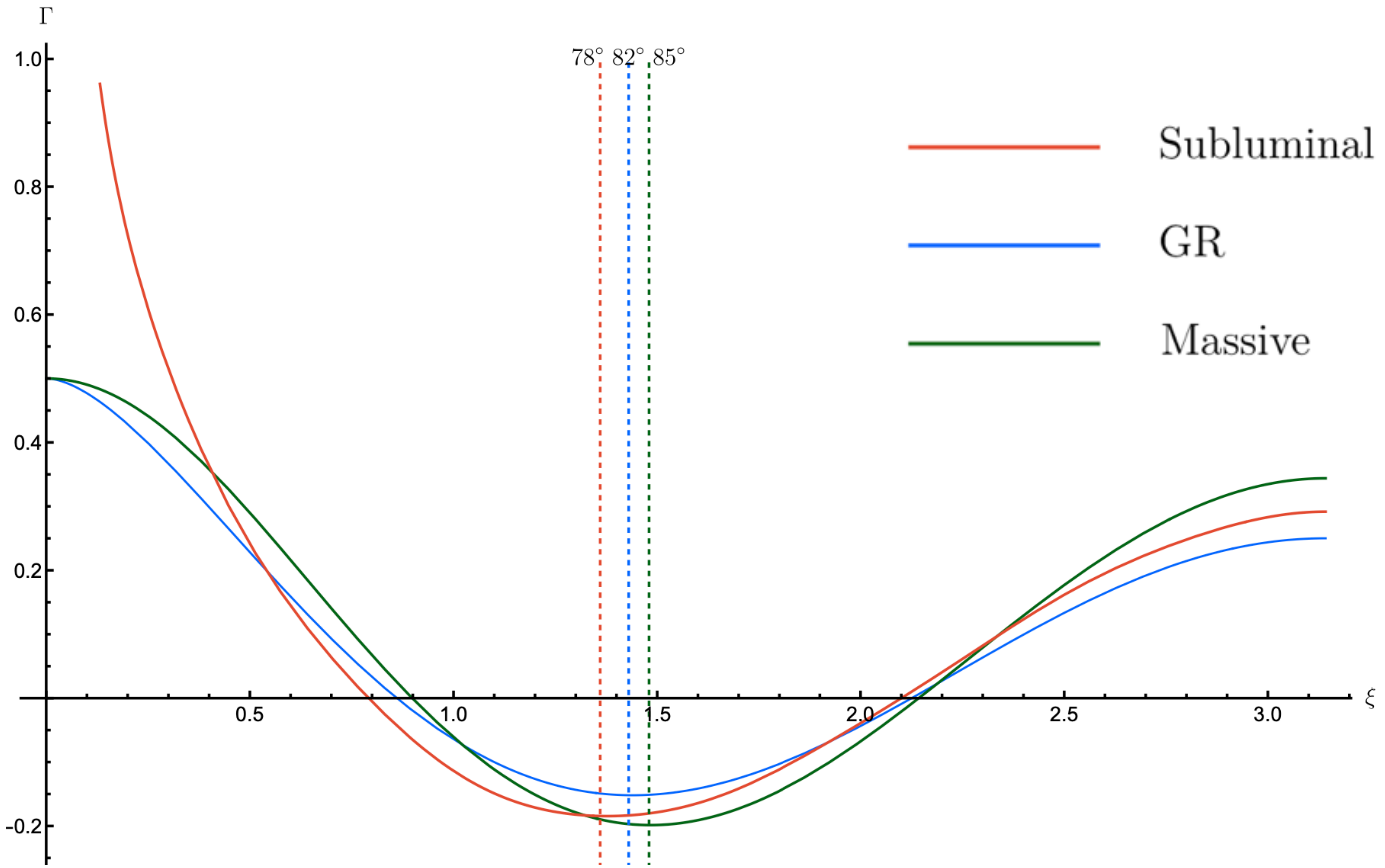
Modification of dispersion relation



$\omega = c_s k$

$v_{ph} < 1$ To left

$v_{ph} > 1$ To right

$\omega(k) = \sqrt{k^2 + m^2},$

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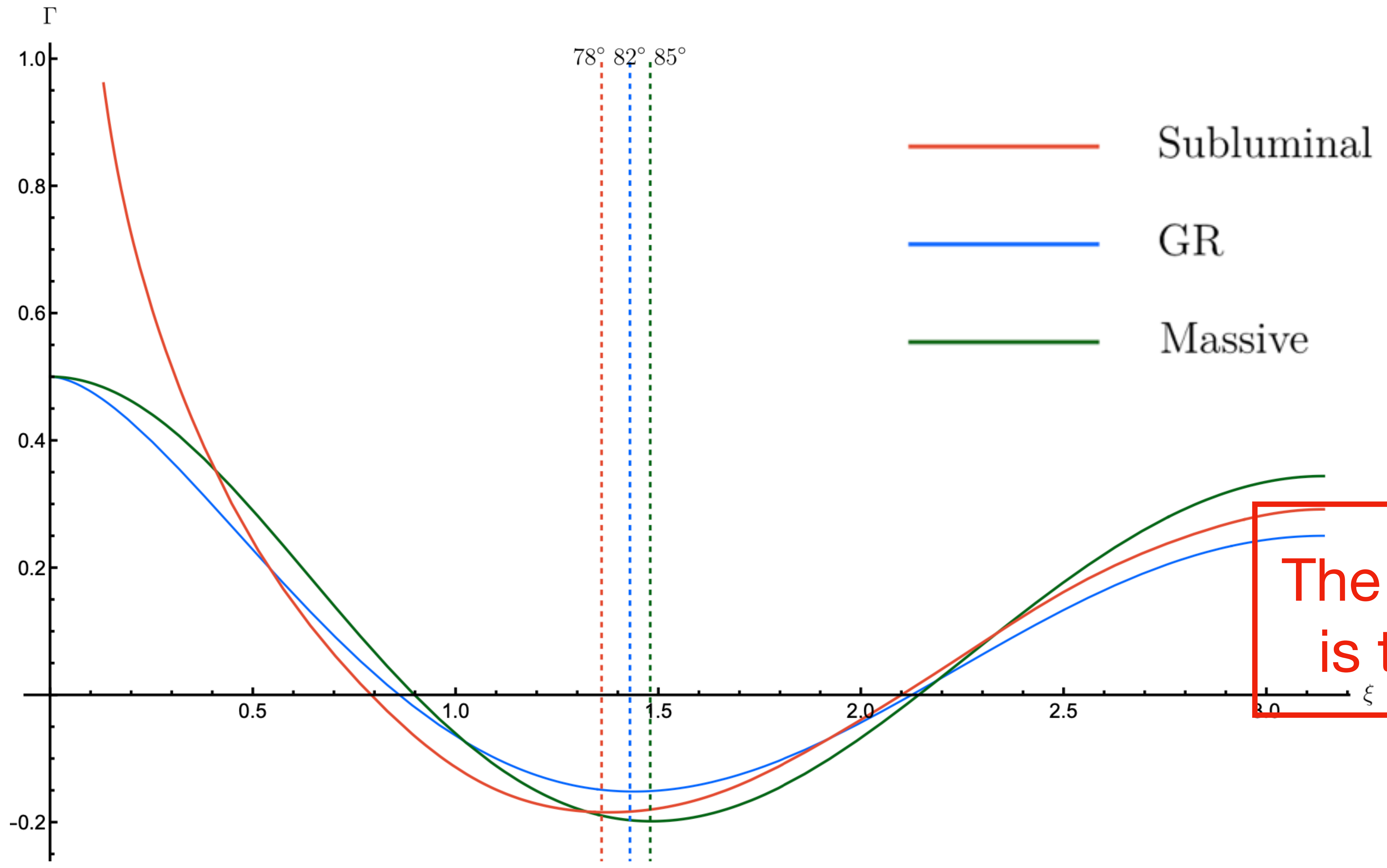
Modify the dispersion relation with plane wave assumption



$\omega = c_s k$

$v_{ph} < 1$ To left

$v_{ph} > 1$ To right

$\omega(k) = \sqrt{k^2 + m^2}$

The important parameter is the phase velocity!!!

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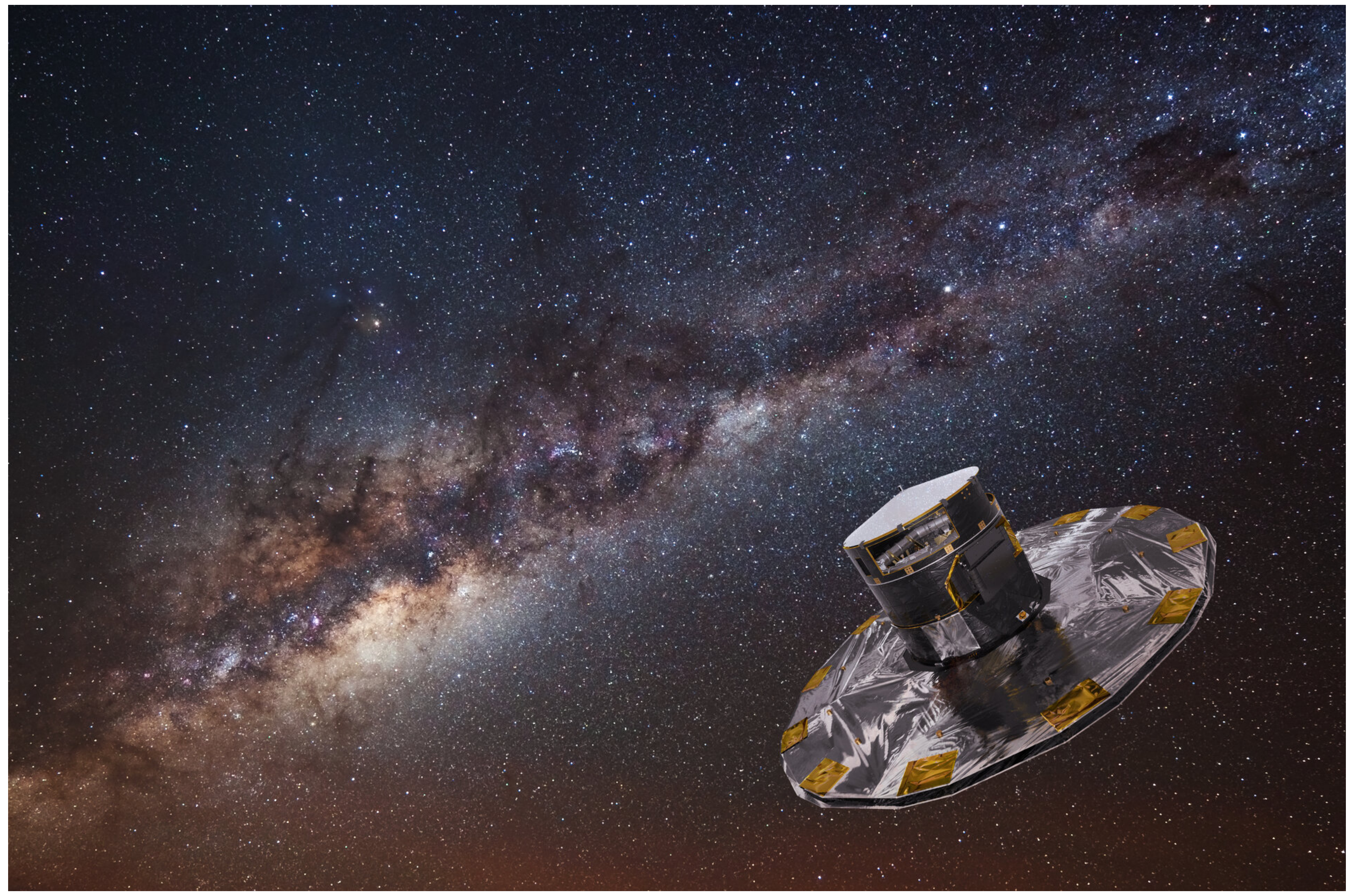
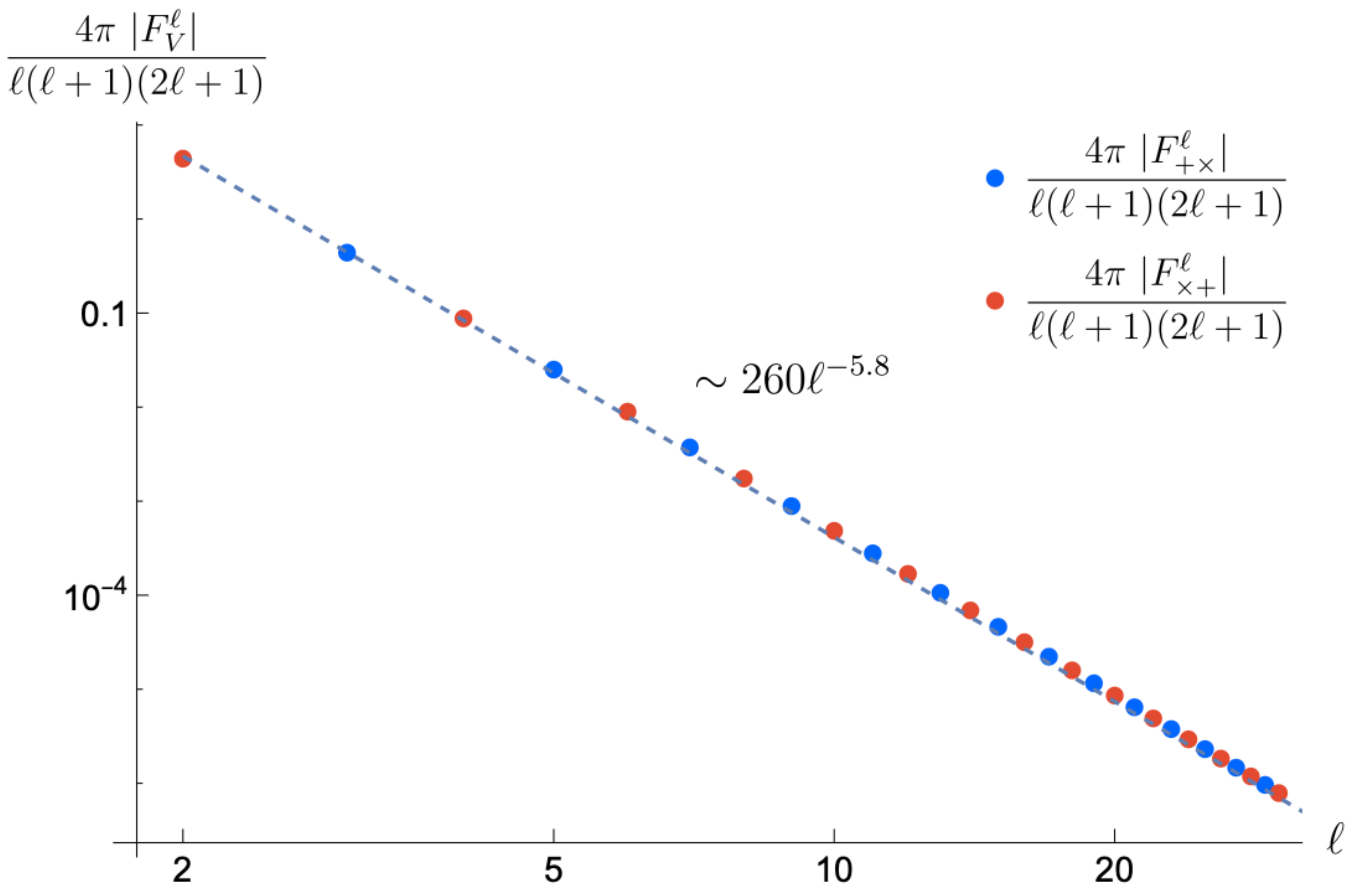
Astrometry can detect parity violation signal via EB correlation!!



$$\left\langle \delta n_{E\ell m}(t)\delta n_{B\ell' m'}(t)^* \right\rangle = \frac{\delta_{\ell\ell'}\delta_{mm'}}{\ell(\ell+1)}\frac{4\pi}{2\ell+1}\left(\mathcal{A}_{+\times}F^{\ell}_{+\times}+\mathcal{A}_{\times+}F^{\ell}_{\times+}\right)$$



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Conclusion and Discussion

- We compute the overlap reduction function of modified gravity theory in PTA system
- For some parameter space, it’s possible to distinguish massive gravity in future PTA data release.
- Stochastic gravitational wave background, including PTA detection and future astrometry detection, will provide a novel test to gravity, and also constraint BSM.

Ongoing and future work

- Constrain the phase velocity with frequency dependent ORF; Liang, Obata, Sasaki, 2402.xxxx

- Wave packet vs plane wave assumption; Hu, Liang, Lin, Trodden, 240x.xxxx

- Constrain axion-like particle (fuzzy dark matter) with PTA; Eberhardt, Ferreira, Liang, 240x.xxxx

Thanks for your attention!

qiuyue.liang@ipmu.jp