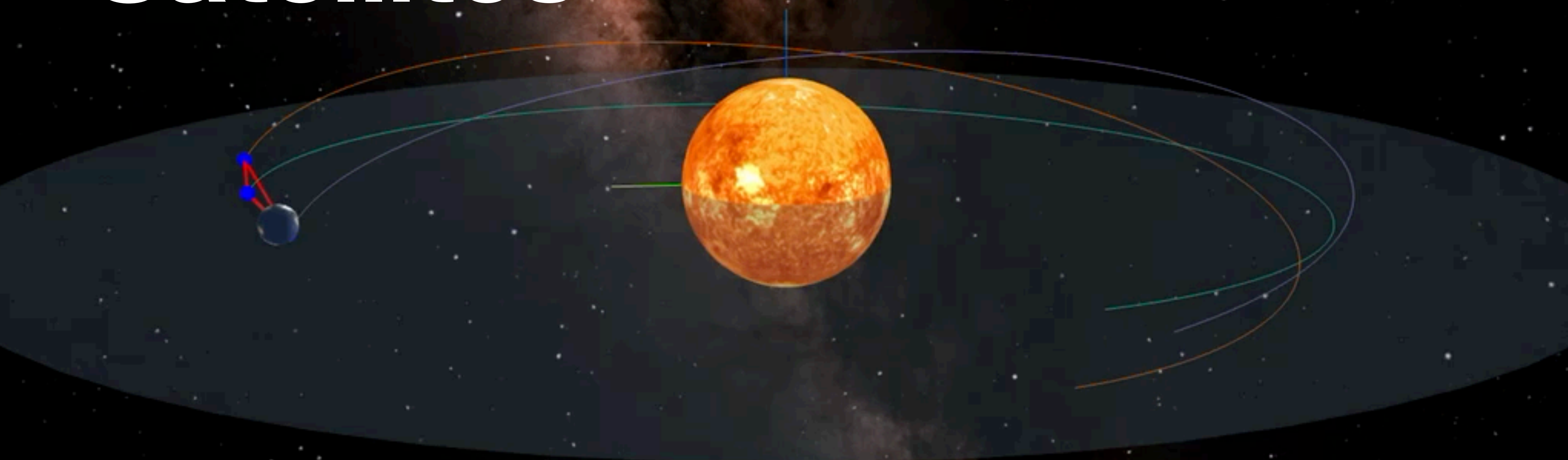
Non-Linear Dynamics Simulation for LISA Satellites



Ricardo Waibel, ITP Heidelberg, 19.02.2024

The LISA mission

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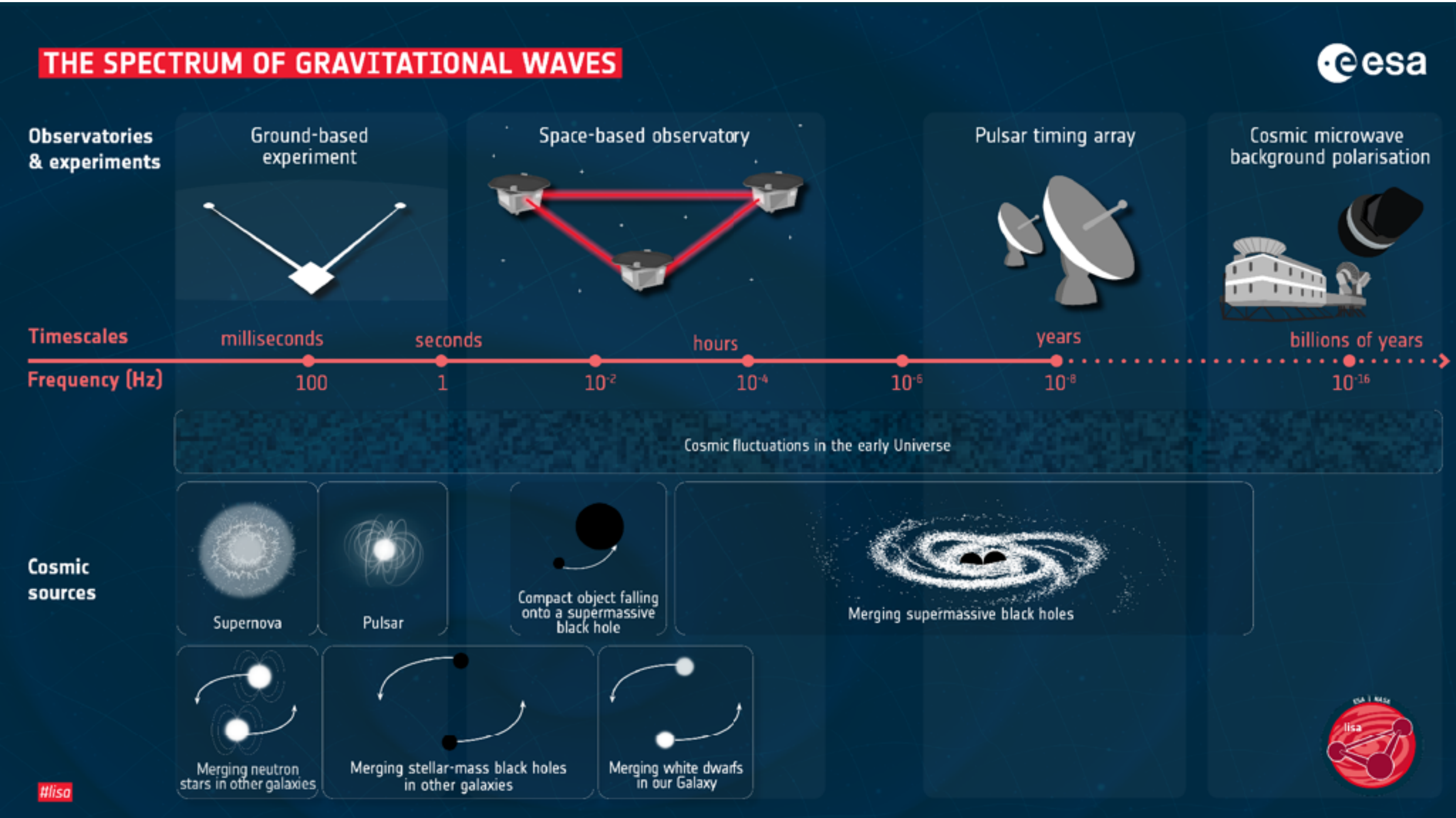

from: www.esa.int

The LISA mission

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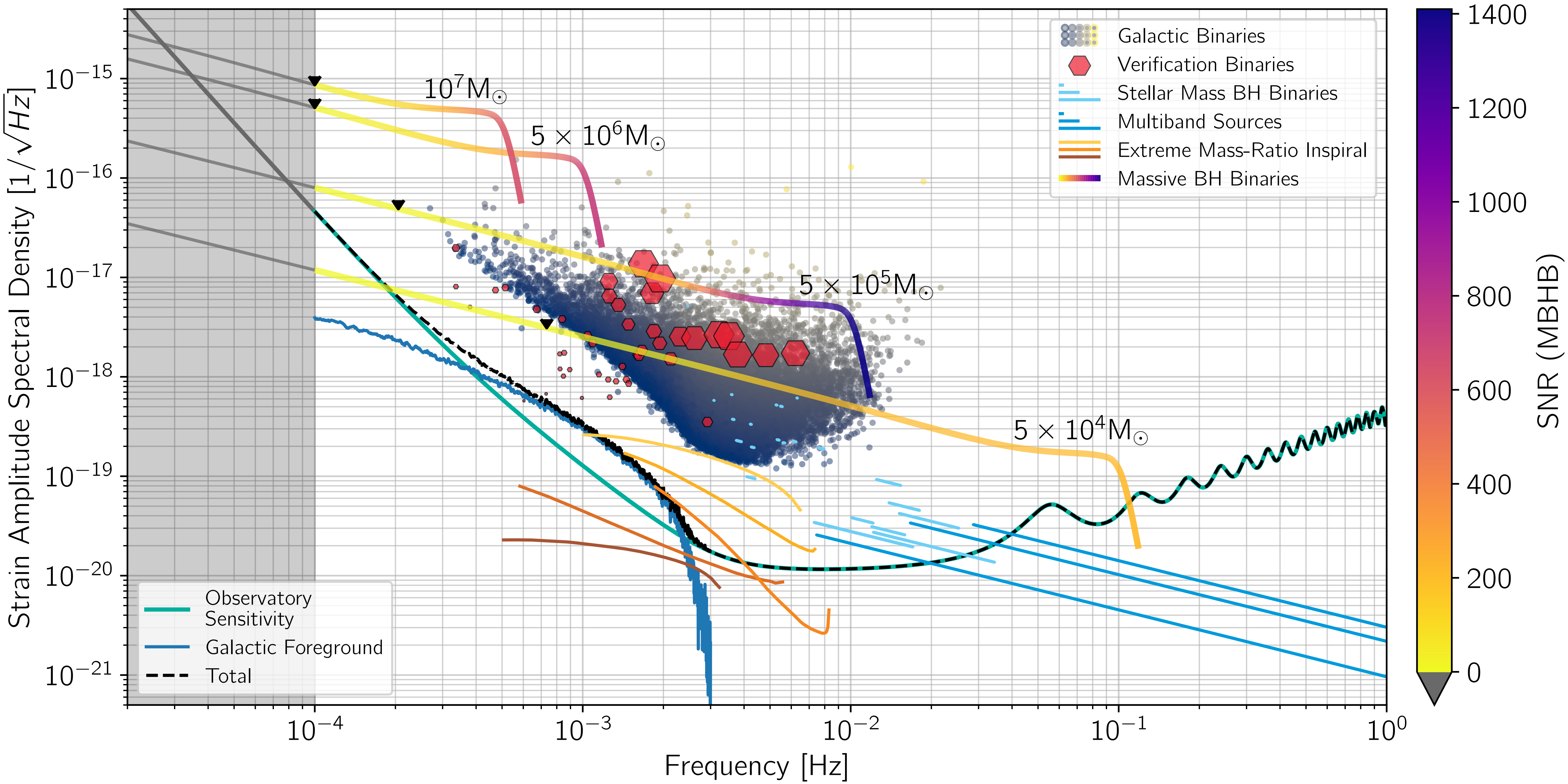
- LISA = Laser Interferometer Space Antenna
- Mission led by ESA, collaboration with NASA and international team of scientists
- Builds on success of LISA Pathfinder mission
- Launch of the three satellites planned for 2035, duration of 4.5 + 6 years
- Formal mission adoption 25th of January, 2024. This means industry contracts will be chosen over the next year and work on instruments will begin in 2025

The LISA mission



from arXiv:2402.07571

The LISA mission

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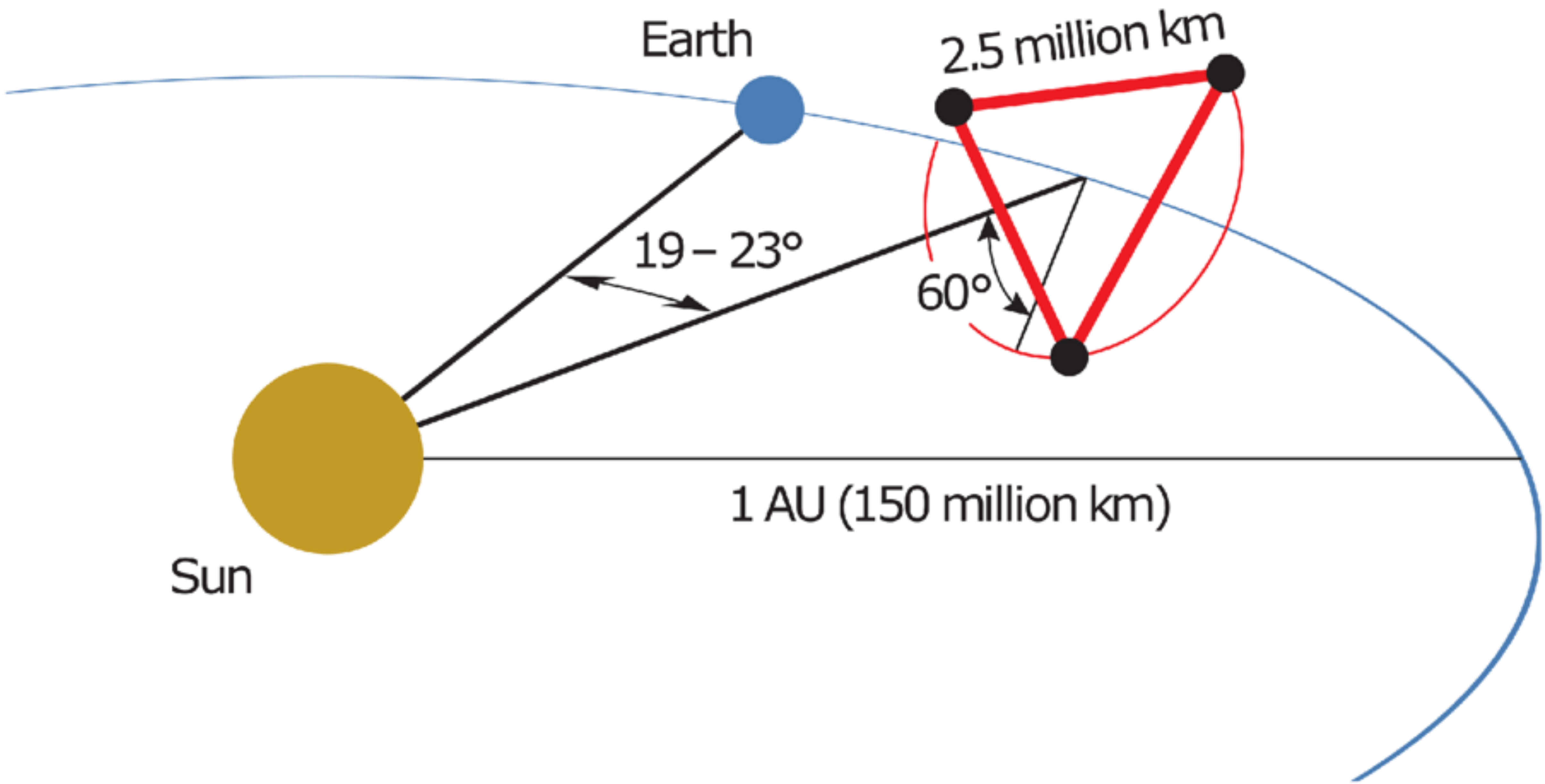

from arXiv:1702.00786

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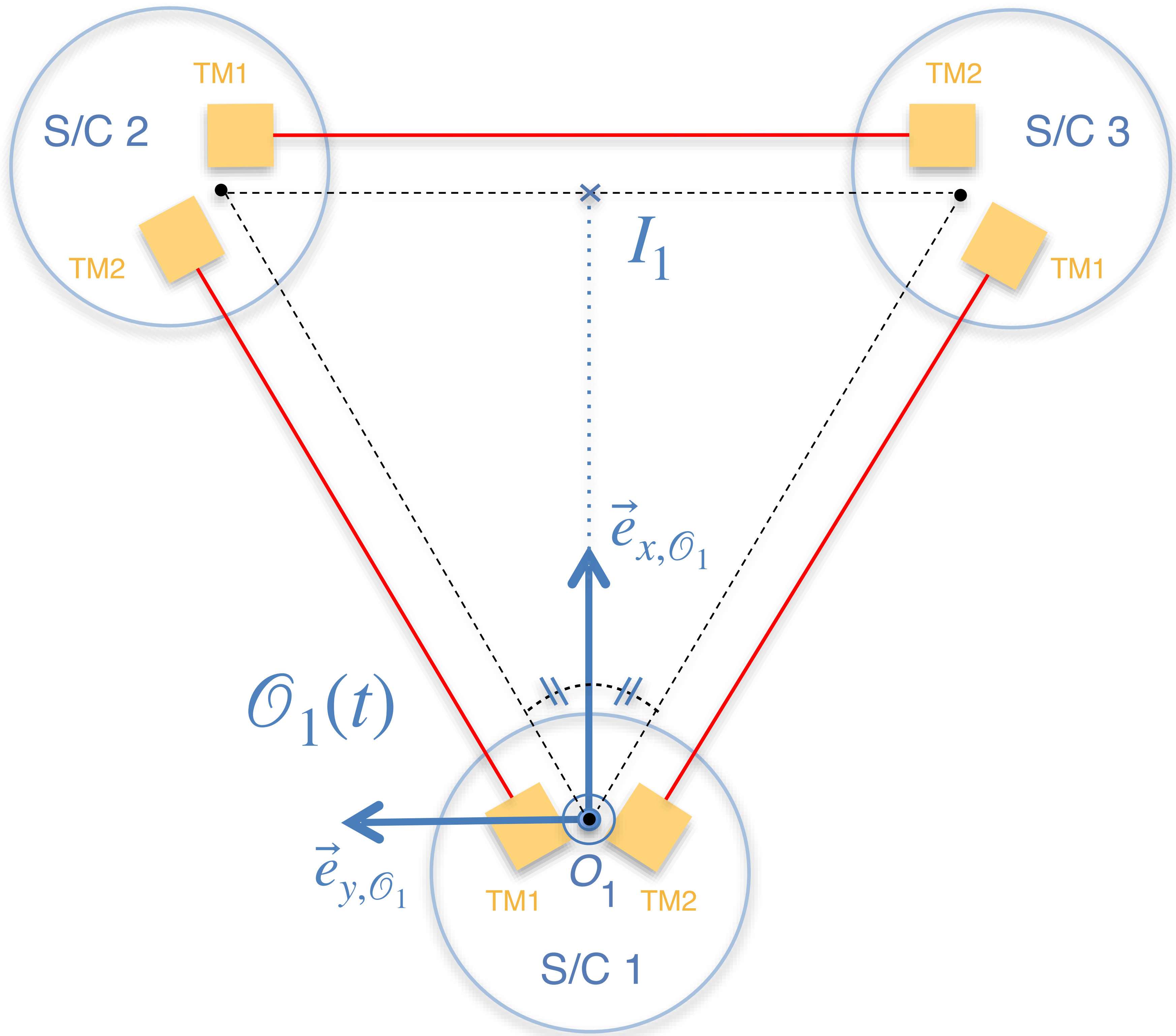

from arXiv:2306.17262

The LISA Satellites

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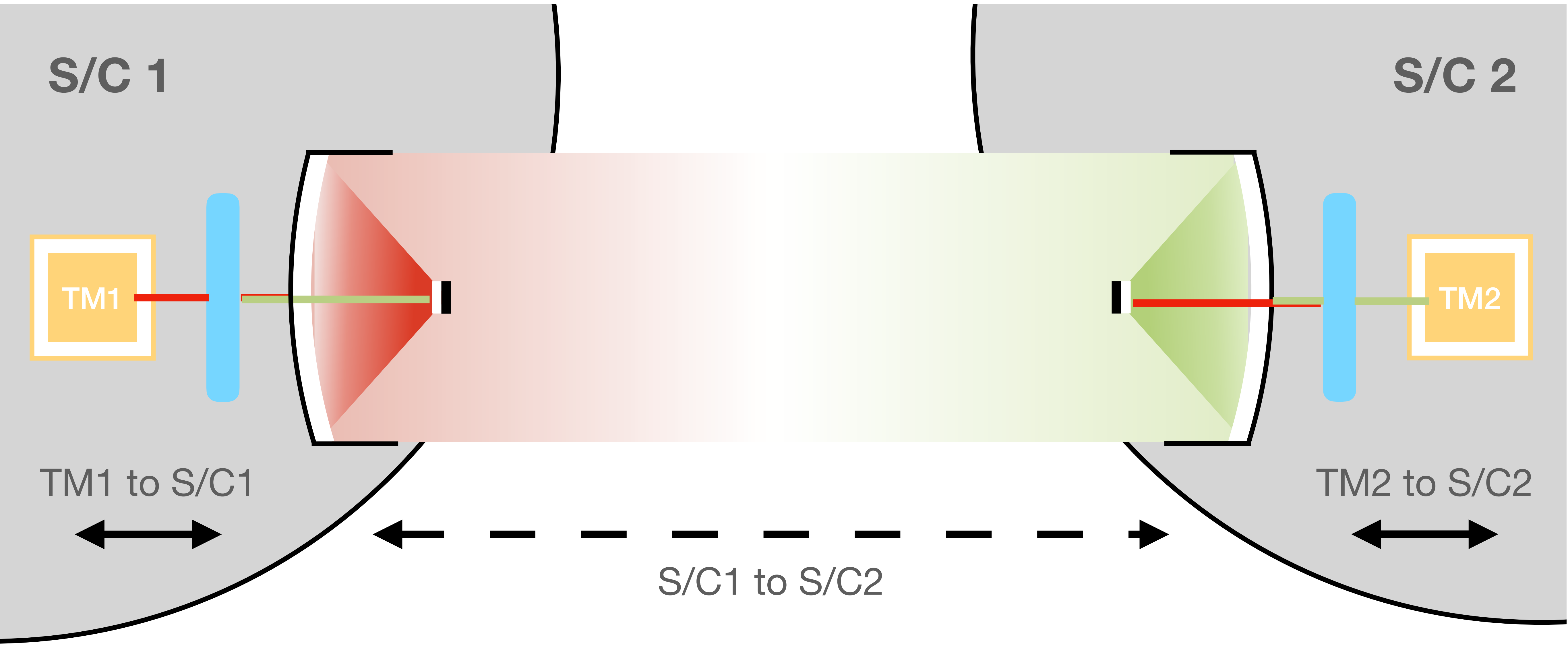

from arXiv:2306.17262

LISA Dynamics

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PHYSICAL REVIEW D **108**, 122007 (2023)

LISA dynamics and control: Closed-loop simulation and numerical demonstration of time delay interferometry

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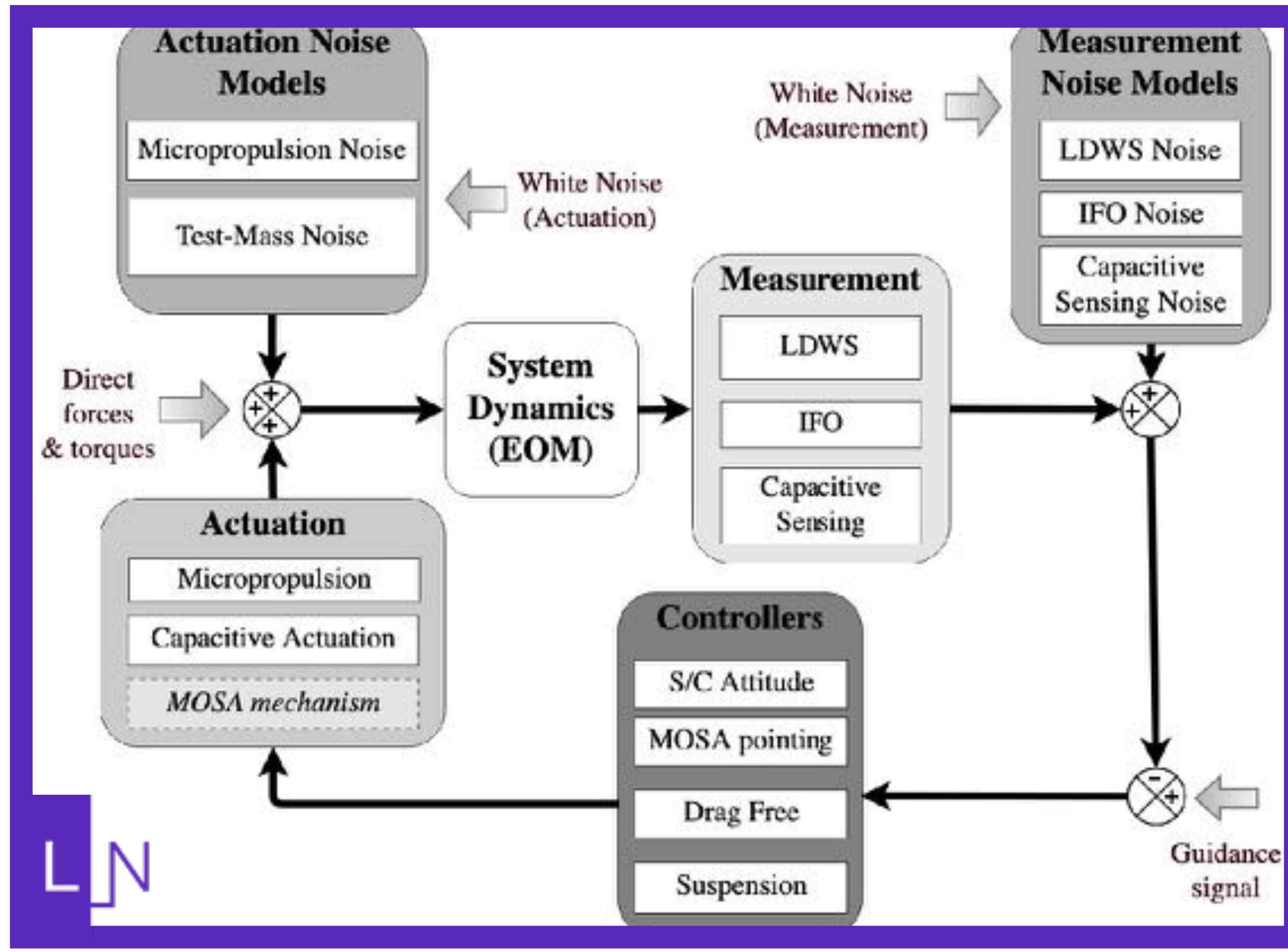
The Laser Interferometer Space Antenna (LISA), space-based gravitational wave observatory involves a complex multidimensional closed-loop dynamical system. Its instrument performance is expected to be less efficiently isolated from platform motion than was its simpler technological demonstrator, LISA Pathfinder. It is of crucial importance to understand and model LISA dynamical behavior accurately to understand the propagation of dynamical excitations through the response of the instrument down to the interferometer data streams. More generally, simulation of the system allows for the preparation of the processing and interpretation of in-flight metrology data. In this work, we present a comprehensive mathematical modeling of the closed-loop system dynamics and its numerical implementation within the LISA Consortium simulation suite. We provide, for the first time, a full time-domain numerical demonstration

LISA Dynamics

- EOM at the core of the closed-loop.
- Add measurement and actuation block, and noise.
- Add control transfer functions: [Inchauspé et al. 2022]
- Out-of-loop input / output ports.



LISA Dynamics

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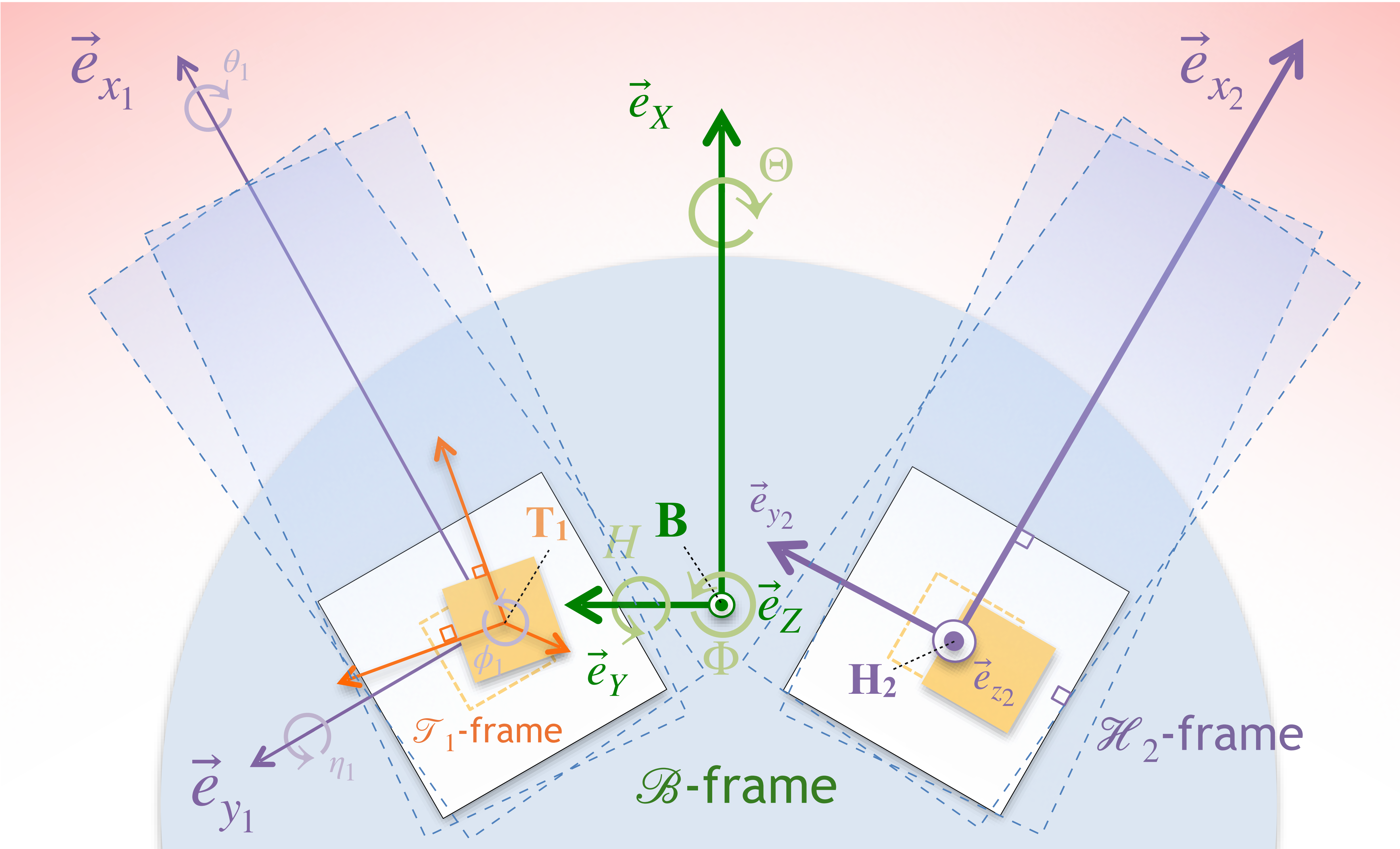

LISA Dynamics

Equations of Motion

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Test mass longitudinal displacement: $$\left.\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right|_{\mathcal{J}}[\vec{\boldsymbol{r}}_{T/J}] = \sum \frac{\vec{\boldsymbol{f}}_T}{m_T} \,,$$

Introducing relevant reference frames of observations

$$\left.\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right|_{\mathcal{H}}[\vec{\boldsymbol{r}}_{T/H}] + 2\vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}} \times \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\mathcal{H}}[\vec{\boldsymbol{r}}_{T/H}] + 2\vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}} \times \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\mathcal{H}}[\vec{\boldsymbol{r}}_{T/H}] + \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\mathcal{B}}[\vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}}] \times \vec{\boldsymbol{r}}_{T/H} + \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\mathcal{J}}[\vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}}] \times \vec{\boldsymbol{r}}_{T/H}$$

$$+ 2\vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}} \times (\vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}} \times \vec{\boldsymbol{r}}_{T/H}) + \vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}} \times (\vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}} \times \vec{\boldsymbol{r}}_{T/H}) + \vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}} \times (\vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}} \times \vec{\boldsymbol{r}}_{T/H}) + \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\mathcal{B}}[\vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}}] \times \vec{\boldsymbol{r}}_{H/P}$$

$$+ \vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}} \times (\vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}} \times \vec{\boldsymbol{r}}_{H/P}) + 2\vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}} \times (\vec{\boldsymbol{\omega}}_{\mathcal{H}/\mathcal{B}} \times \vec{\boldsymbol{r}}_{H/P}) + \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\mathcal{J}}[\vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}}] \times \vec{\boldsymbol{r}}_{H/B} + \vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}} \times (\vec{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{J}} \times \vec{\boldsymbol{r}}_{H/B})$$

$$= \sum \frac{\vec{\boldsymbol{f}}_T}{m_T} - \sum \frac{\vec{\boldsymbol{f}}_B}{m_B} \,.$$

LISA Dynamics

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EOM - Expression in Coordinate Systems

A couple of rules as example:

- Angular velocity expressed in body frames $\mathscr{B}$ and $\mathscr{T}$.
- Test mass position in housing frames $\mathscr{H}$.
- Dot operator understood as derivative w.r.t. reference frames of expression.

State-space representation:

$$\vec{\mathbf{X}} = \begin{bmatrix} \vec{\boldsymbol{\alpha}}_{\mathcal{B}/\mathcal{O}} & \omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}} & r^{\mathcal{H}_1}_{T_1/H_1} & \vec{\boldsymbol{\alpha}}_{\mathcal{T}_1/\mathcal{H}_1} & r^{\mathcal{H}_2}_{T_2/H_2} \\ \vec{\boldsymbol{\alpha}}_{\mathcal{T}_2/\mathcal{H}_2} & \dot{r}^{\mathcal{H}_1}_{T_1/H_1} & \omega^{\mathcal{T}_1}_{\mathcal{T}_1/\mathcal{H}_1} & \dot{r}^{\mathcal{H}_2}_{T_2/H_2} & \omega^{\mathcal{T}_2}_{\mathcal{T}_2/\mathcal{H}_2} \\ \delta\phi_{tel,1} & \delta\dot{\phi}_{tel,1} & \delta\phi_{tel,2} & \delta\dot{\phi}_{tel,2} & \end{bmatrix}$$

LISA Dynamics

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EOM - Expression in Coordinate Systems

Test mass longitudinal displacement:

$$
\begin{aligned}
&\ddot{r}^{\mathcal{H}}_{T/H} + 2\left[\omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} \dot{r}^{\mathcal{H}}_{T/H} + 2\left[T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}}\right]^{\times} \dot{r}^{\mathcal{H}}_{T/H} + 2\left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} \dot{r}^{\mathcal{H}}_{T/H} + \left[\dot{\omega}^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} r^{\mathcal{H}}_{T/H} \\
&+ \left[T^{\mathcal{H}}_{\mathcal{B}}\dot{\omega}^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}}\right]^{\times} r^{\mathcal{H}}_{T/H} + \left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\dot{\omega}^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} r^{\mathcal{H}}_{T/H} + 2\left[T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}}\right]^{\times} \left[\omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} r^{\mathcal{H}}_{T/H} \\
&+ 2\left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} \left[\omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} r^{\mathcal{H}}_{T/H} + \left[\omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} \left[\omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} r^{\mathcal{H}}_{T/H} \\
&+ \left[T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}}\right]^{\times} \left[T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}}\right]^{\times} r^{\mathcal{H}}_{T/H} + 2\left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} \left[T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}}\right]^{\times} r^{\mathcal{H}}_{T/H} + \left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} \left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} r^{\mathcal{H}}_{T/H} \\
&- \left[r^{\mathcal{H}}_{H/P}\right]^{\times} \dot{\omega}^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}} + \left[\omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} \left[\omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} r^{\mathcal{H}}_{H/P} + 2\left[\left[r^{\mathcal{H}}_{H/P}\right]^{\times} \omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}}\right]^{\times} T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}} - 2\left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} \left[r^{\mathcal{H}}_{H/P}\right]^{\times} \omega^{\mathcal{H}}_{\mathcal{H}/\mathcal{B}} \\
&- \left[r^{\mathcal{B}}_{H/B}\right]^{\times} T^{\mathcal{H}}_{\mathcal{B}}\dot{\omega}^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}} + \left[\left[T^{\mathcal{H}}_{\mathcal{B}}r^{\mathcal{B}}_{H/B}\right]^{\times} T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}}\right]^{\times} T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}} - 2\left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} \left[T^{\mathcal{H}}_{\mathcal{B}}r^{\mathcal{B}}_{H/B}\right]^{\times} T^{\mathcal{H}}_{\mathcal{B}}\omega^{\mathcal{B}}_{\mathcal{B}/\mathcal{O}} \\
&= \sum \frac{f^{\mathcal{H}}_{T}}{m_T} - \sum \frac{T^{\mathcal{H}}_{\mathcal{B}} f^{\mathcal{B}}_{B}}{m_B} - \left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\dot{\omega}^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} T^{\mathcal{H}}_{\mathcal{B}}r^{\mathcal{B}}_{H/B} - \left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} \left[T^{\mathcal{H}}_{\mathcal{B}}T^{\mathcal{B}}_{\mathcal{O}}T^{\mathcal{O}}_{\mathcal{J}}\omega^{\mathcal{J}}_{\mathcal{O}/\mathcal{J}}\right]^{\times} T^{\mathcal{H}}_{\mathcal{B}}r^{\mathcal{B}}_{H/B} \,.
\end{aligned}
$$

LISA Dynamics

State-Space Representation & Solving

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Linearized

- time-averaged
- linearized around target point

$$\frac{\mathrm{d}\,\vec{\mathbf{X}}}{\mathrm{d}\,t} = A\,\vec{\mathbf{X}}(t) + B\,\vec{\mathbf{u}}(t)$$

with constant A and B into the discrete

$$\vec{\mathbf{X}}\left(t^{(n+1)}\right) = A_{\text{disc.}}\vec{\mathbf{X}}\left(t^{(n)}\right) + B_{\text{disc.}}\vec{\mathbf{u}}\left(t^{(n)}\right)$$

$$A_{\text{disc.}} = \mathrm{e}^{A\,dt}$$

$$B_{\text{disc.}} = A^{-1}\left(\mathrm{e}^{A\,dt} - 1\right)B$$

Full Non-Linear

- solution of full system using numerical solver, e.g. Runge-Kutta 4

Solve $\frac{\mathrm{d}\,\vec{\mathbf{X}}}{\mathrm{d}\,t} = \vec{\mathbf{f}}(t;\vec{\mathbf{X}})$ with

$$\vec{\mathbf{X}}_{n+1} = \vec{\mathbf{X}}_n + \frac{dt}{6}\left(\vec{\mathbf{k}}_1 + 2\vec{\mathbf{k}}_2 + 2\vec{\mathbf{k}}_3 + \vec{\mathbf{k}}_4\right)$$

$$\vec{\mathbf{k}}_1 = \vec{\mathbf{f}}\left(t^{(n)};\vec{\mathbf{X}}_n\right)$$

$$\vec{\mathbf{k}}_2 = \vec{\mathbf{f}}\left(t^{(n)} + \frac{dt}{2};\vec{\mathbf{X}}_n + \frac{dt}{2}\vec{\mathbf{k}}_1\right)$$

$$\vec{\mathbf{k}}_3 = \vec{\mathbf{f}}\left(t^{(n)} + \frac{dt}{2};\vec{\mathbf{X}}_n + \frac{dt}{2}\vec{\mathbf{k}}_2\right)$$

$$\vec{\mathbf{k}}_4 = \vec{\mathbf{f}}\left(t^{(n)} + dt;\vec{\mathbf{X}}_n + dt\cdot\vec{\mathbf{k}}_3\right)$$

Simulation Experiments

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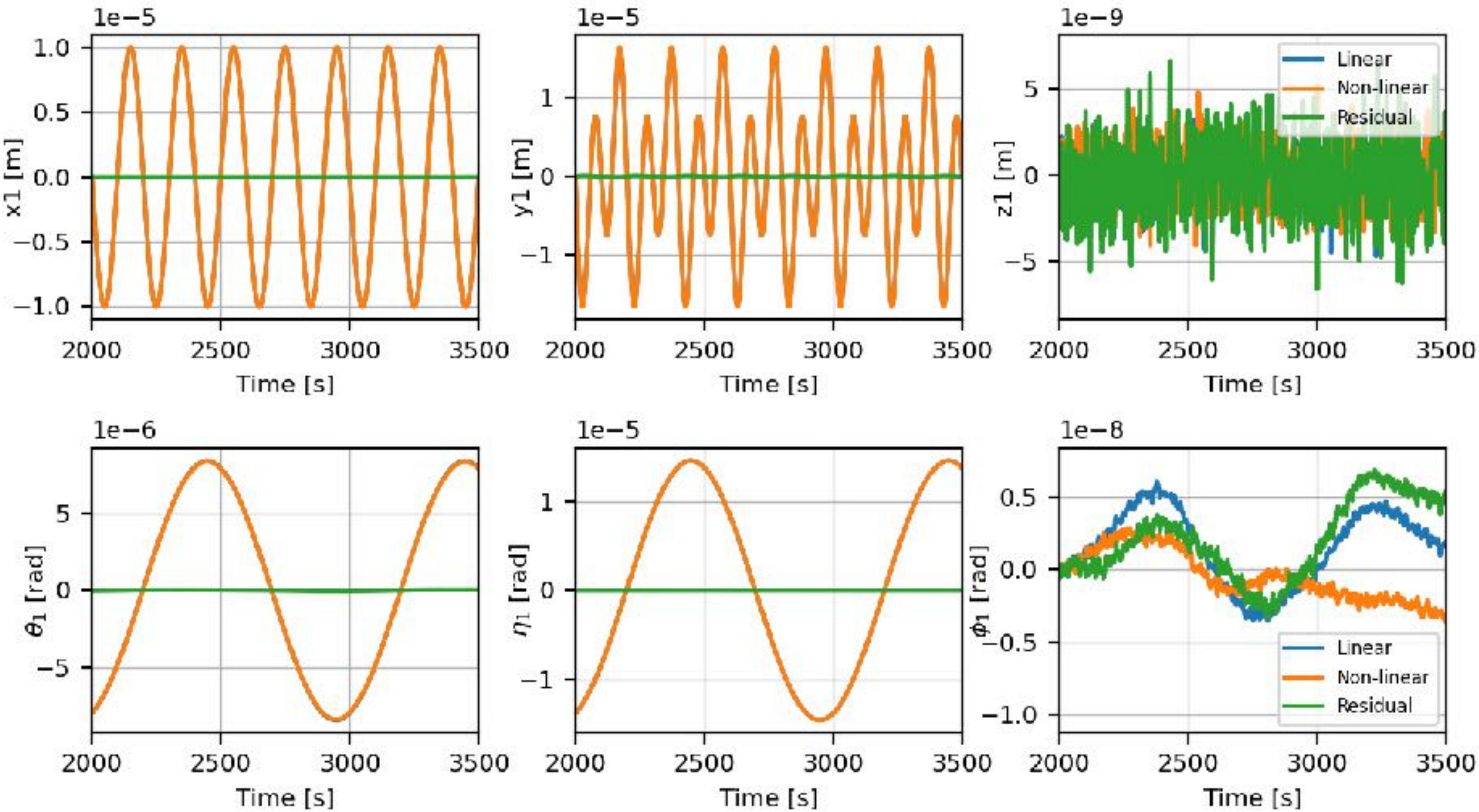
S/C and TM guidances



Guidance along H ($f = 1\,mHz$), x_1 ($f = 5\,mHz$) and x_2 ($f = 10\,mHz$)

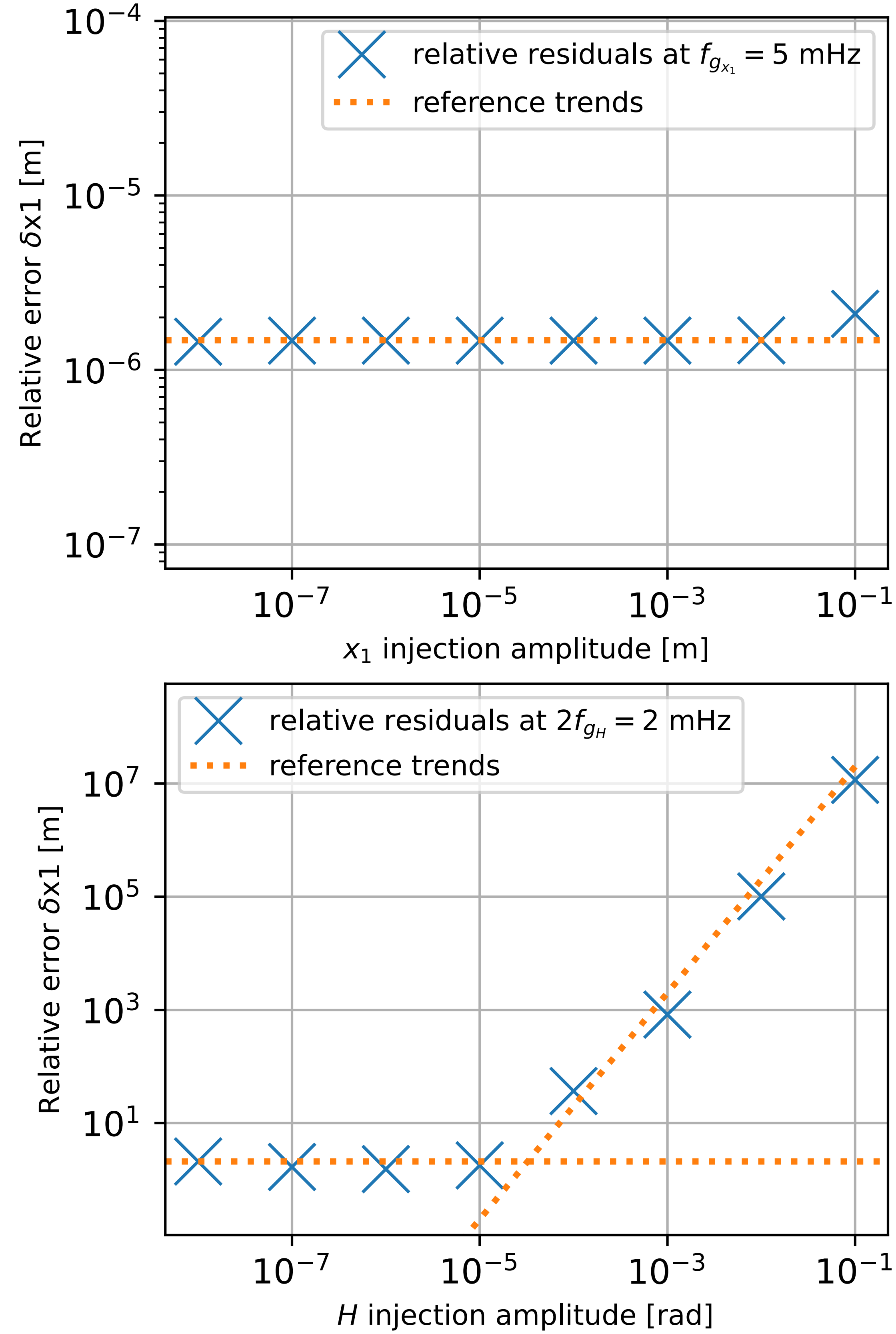
Simulation Experiments

Capturing Non-Linearities

$$\delta_{\text{res}} = \left| \frac{\left|\tilde{x}^{\text{nl}}(f_{\text{inj}})\right| - \left|\tilde{x}^{\text{l}}(f_{\text{inj}})\right|}{\left|\tilde{x}^{\text{nl}}(f_{\text{inj}})\right|} \right|$$

- Top: Linear discrepancy as linearized simulation cannot capture changing opening angle in MOSAs

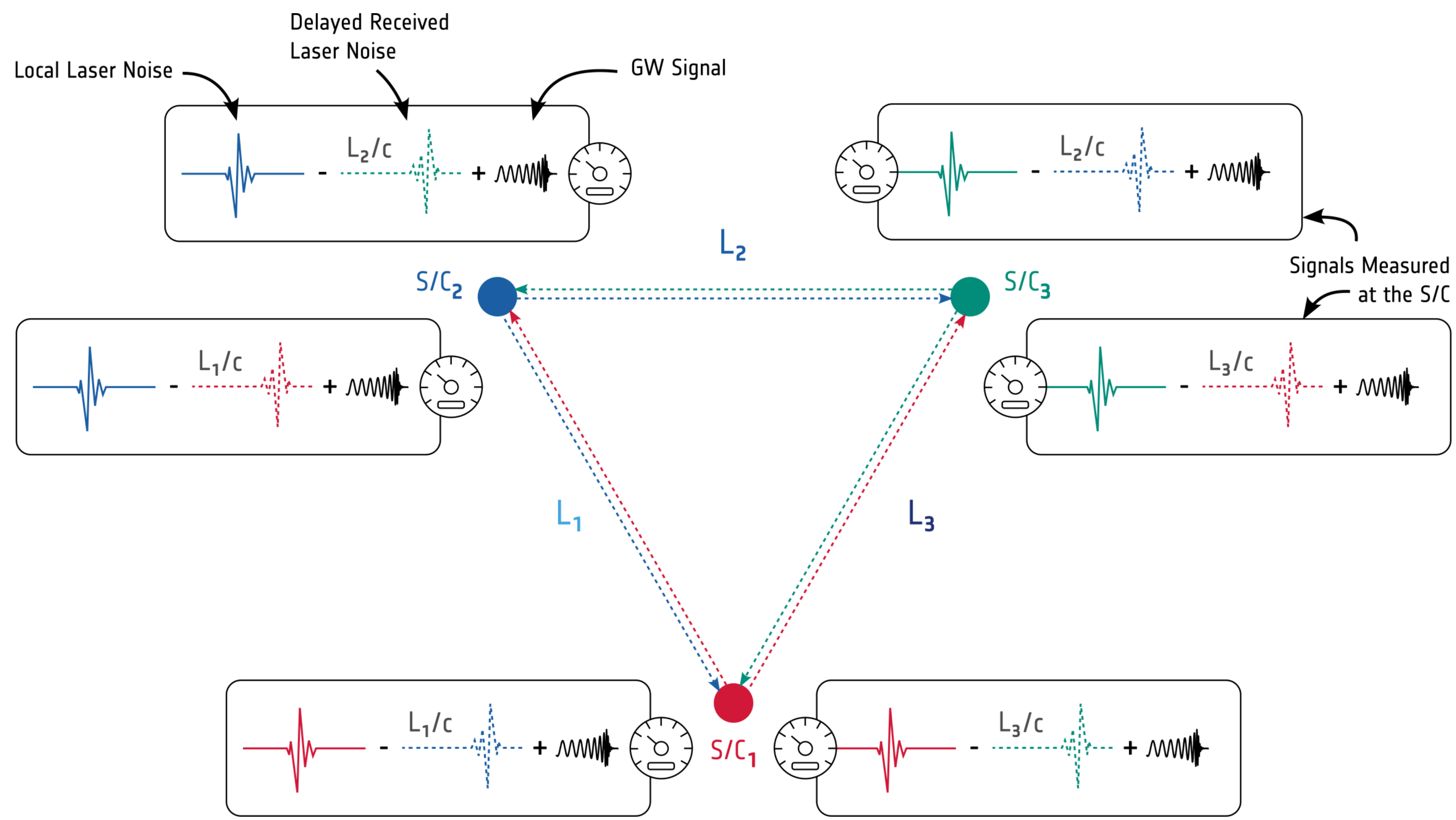
- Bottom: Response to rotational excitement is dominated by quadratic term above certain amplitude



Lasers in Space

Time-Delay Interferometry

- different arm lengths: laser noise does not cancel
- 6 independent phase-difference read-outs
- post-process signal by linearly combining them to cancel laser frequency noise



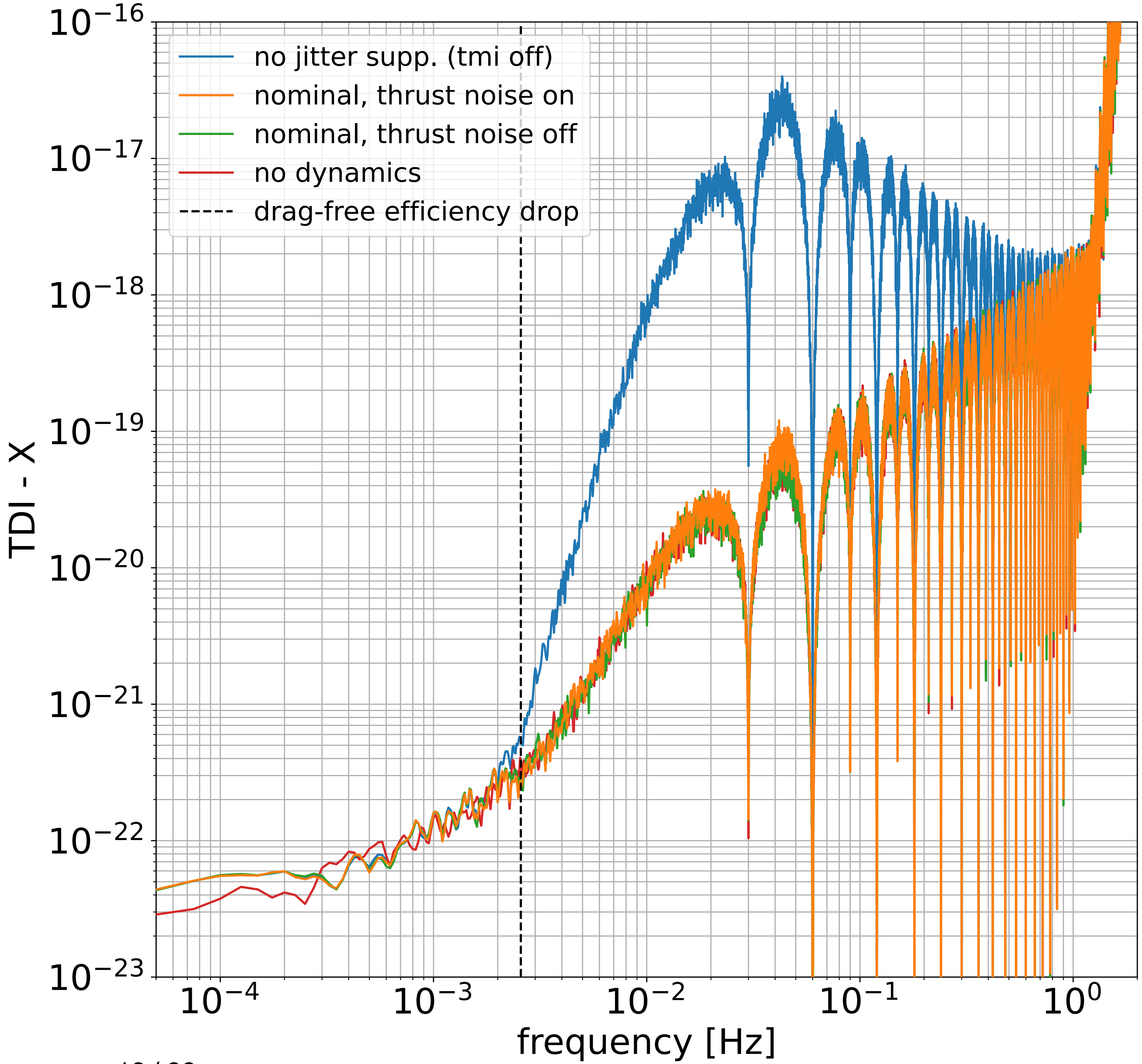
from arXiv:2402.07571

Time-Delay Interferometry, M. Tinto, S.V. Dhurandhar, Living Reviews in Relativity (2021)

Post-Processing with TDI

Can Jitter be suppressed?

- Dynamics on, full simulation
- Dynamics on, w/o thruster noise
- No dynamics (LISANode master branch)
- Dynamics on, TMI off (filled with zero's)



Conclusion

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Full, complete time-domain LISA Dynamics simulation achieved.

Next Steps:

- Study of tilt-to-length couplings and mitigation
- Quantitative study of propagation of dynamical artefacts (glitches, micro-meteoroides impact) on S/C down to TDI
- Test control strategies

Thank you!

Especially to Henri Inchauspé and
Lavinia Heisenberg!