Anharmonic Effects on the Squeezing of Axion Perturbations

Collaborators:

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Introduction

✓ The history of the Universe undergoes a period of exponential expansion, **inflation**.

✓ Quantum fluctuations provide the seeds for structure formation.

✓ The CMB sky we see today is classical.

Quantum to classical transition

✓ First source of classicalization: **reheating**.

✓ Inflation itself provides an explanation to the "classicalization": **squeezing**.

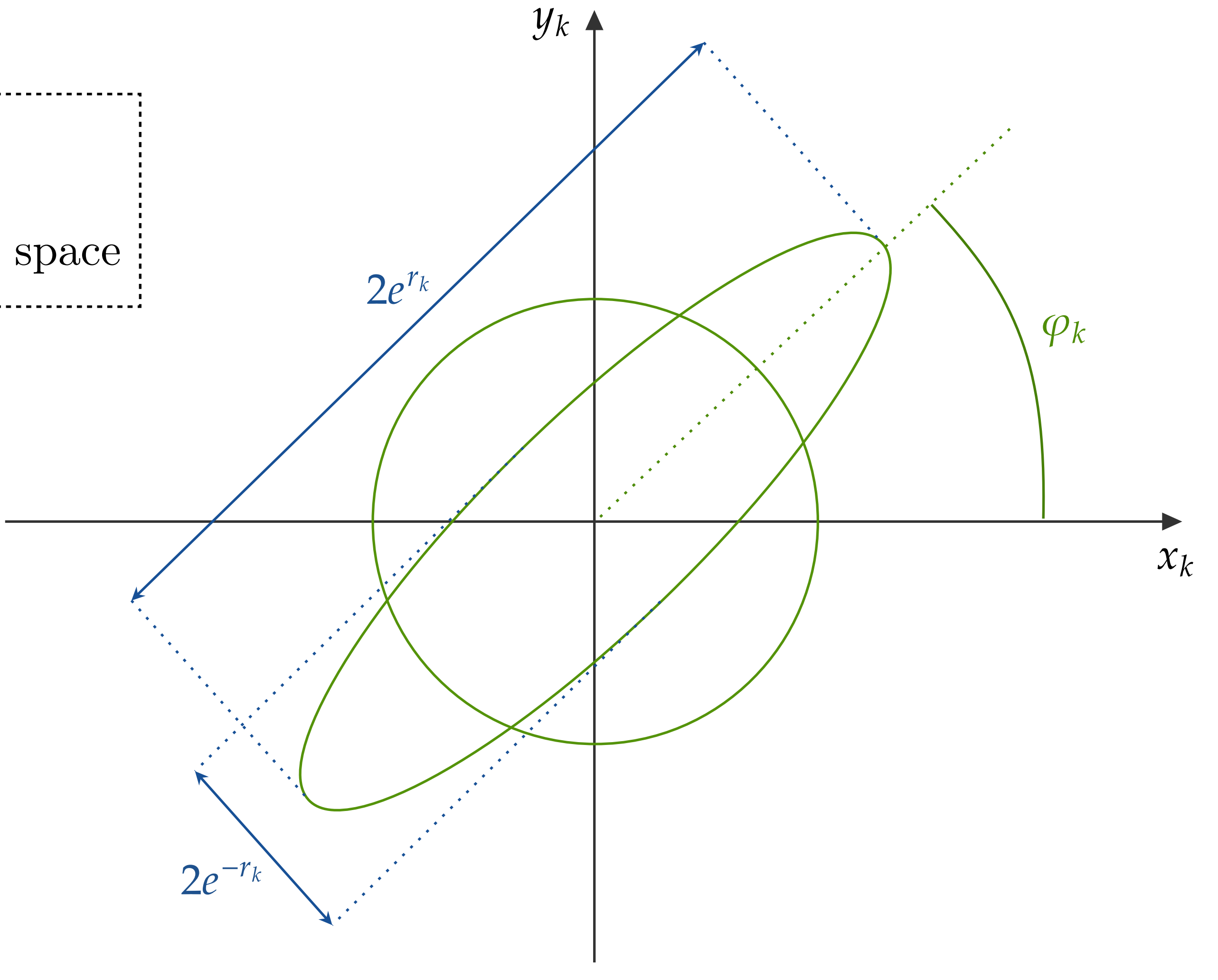
D.D. Polarski and A. A. Starobinsky *Class. Quant. Grav.* **13** (1996), 377-392
L. P. Grishchuk and Y. V. Sidoro *Phys. Rev. D* **42** (1990), 3413-3421

What is squeezing?

A **squeezed state** is a special quantum state for which **one variable** has an **arbitrarily small uncertainty**, while **its conjugate** counterpart has a **huge uncertainty**, correspondingly.

✓ r_k tells how much the ellipse is squeezed

✓ φ_k tells of which angle it is rotated in phase space



J.-T. Hsiang and B.-L. Hu *Universe* **8** (2022), no. 1 27, [arXiv:2112.04092]
J. Martin and V. Vennin *Phys. Rev. D* **93** (2016), no. 2 023505, [arXiv:1510.04038]

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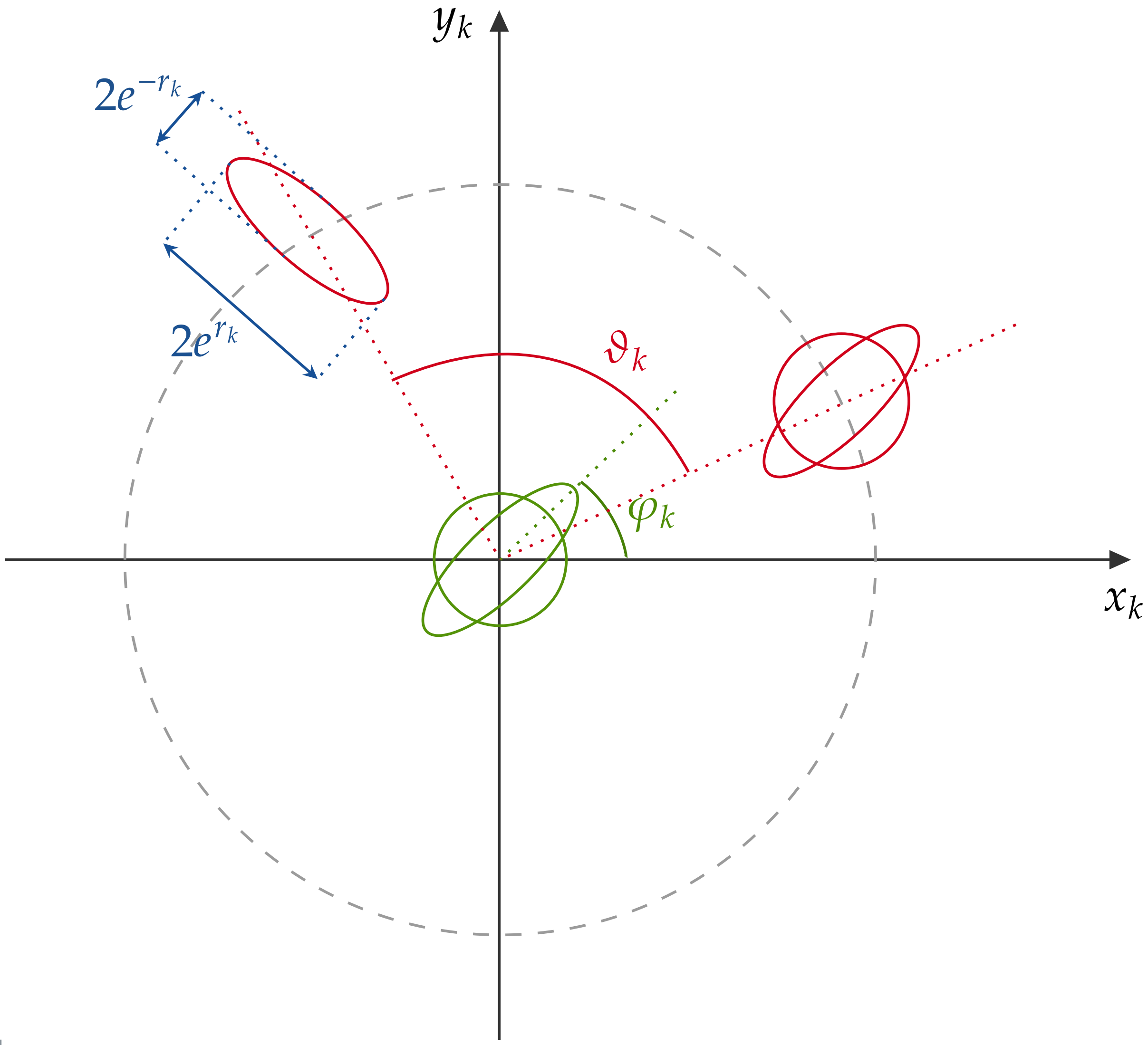
- ✓ φ_k tells of which angle it is rotated in phase space
- ✓ ϑ_k tells of which angle the whole state is rotated in phase space



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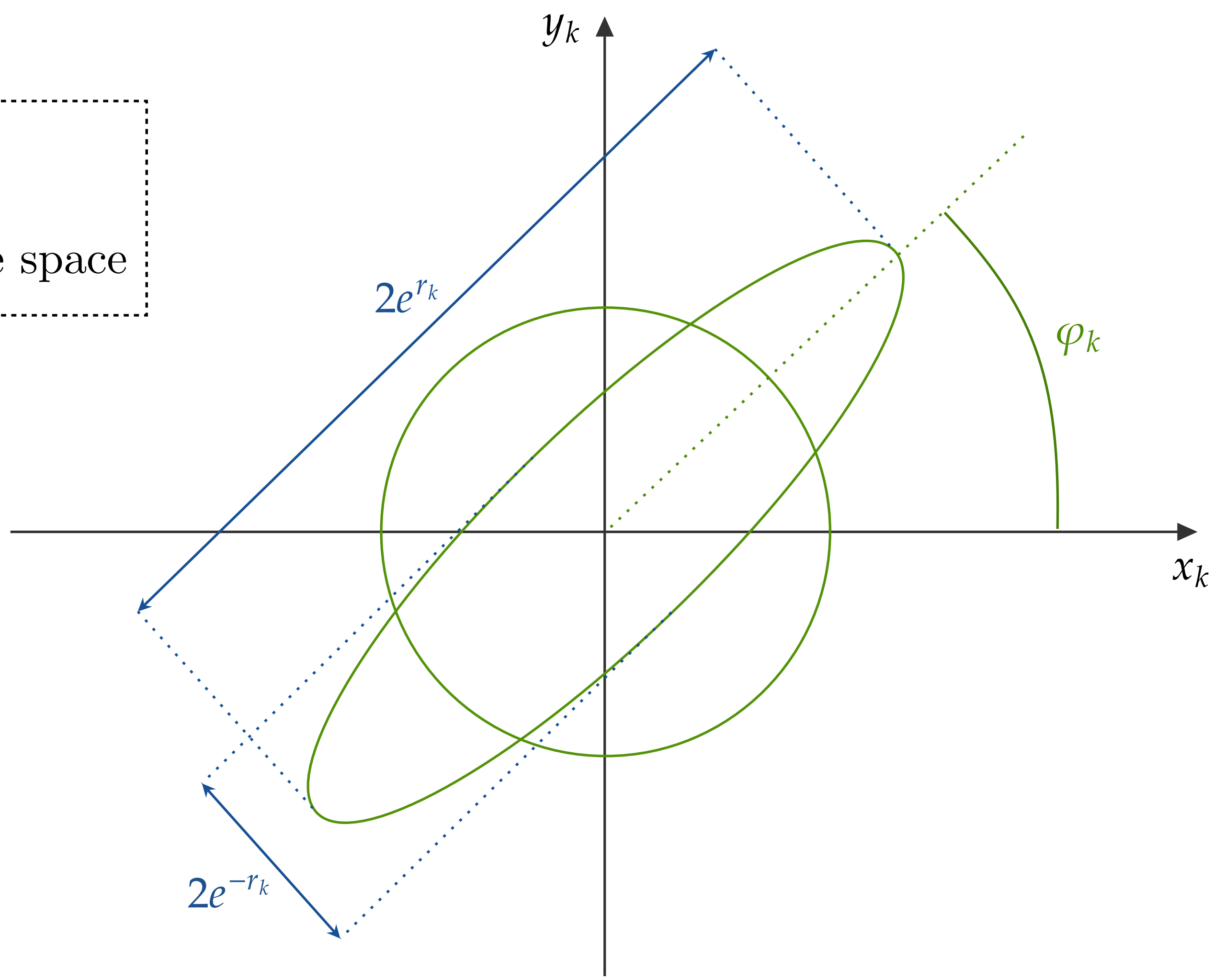
↓

Squeezed states are highly quantum mechanical states.



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Why squeezing is "classical"?

From an observational point of view, a squeezed state is indistinguishable from a classical phase-space distribution, if one considers a Gaussian state.

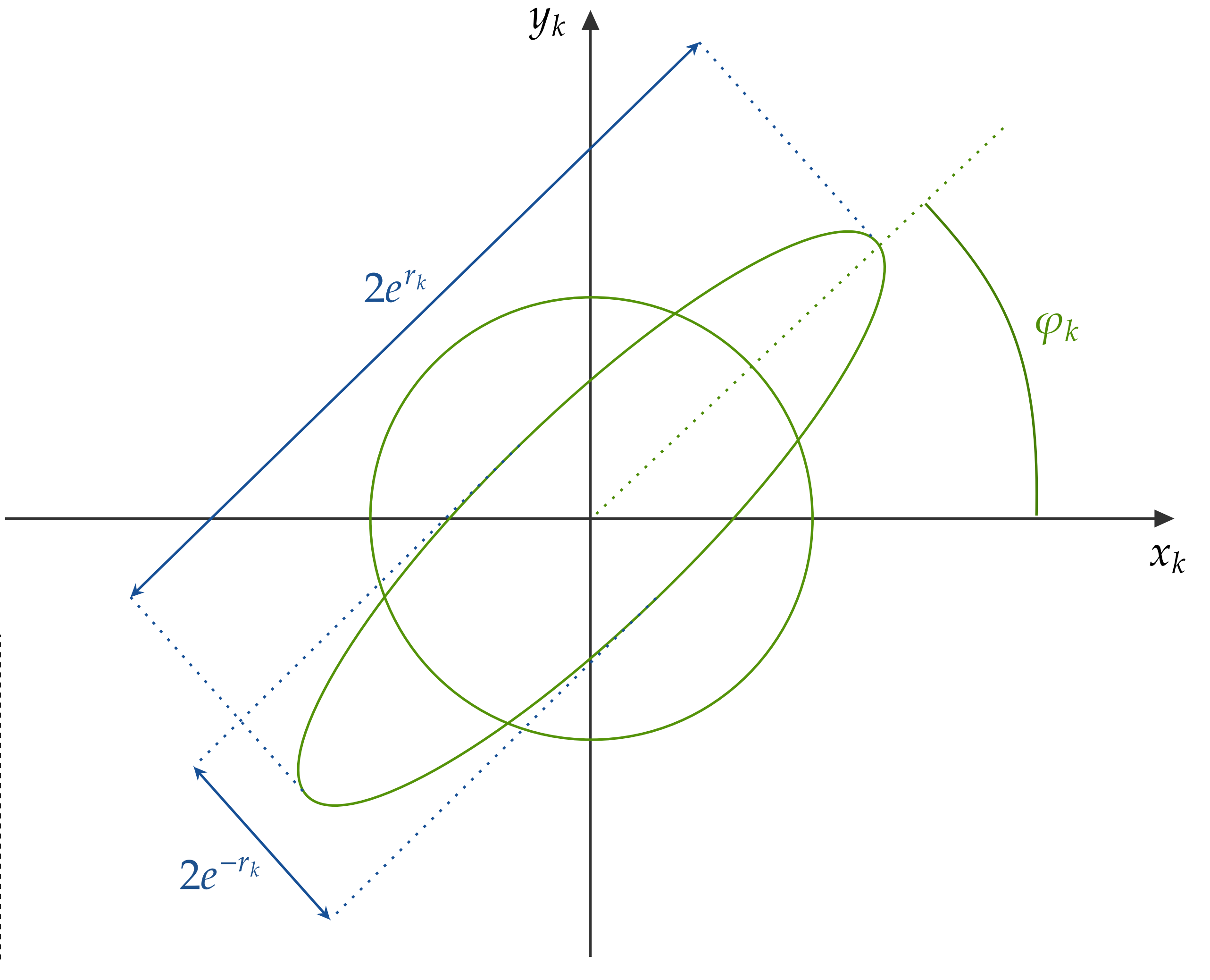
In an effective classical stochastic description, quantum averages are given by:

$$\langle \hat{O} \rangle = \langle O \rangle_{stoch}$$

$$\langle O \rangle_{stoch} = \int \tilde{O}(\mathbf{q}_i, \mathbf{p}_i) W(\mathbf{q}_i, \mathbf{p}_i) d\mathbf{q}_i d\mathbf{p}_i$$

Probability density function

1. W is positive everywhere
2. W obeys the classical equation of motion
3. The quantum average is given by the stochastic average



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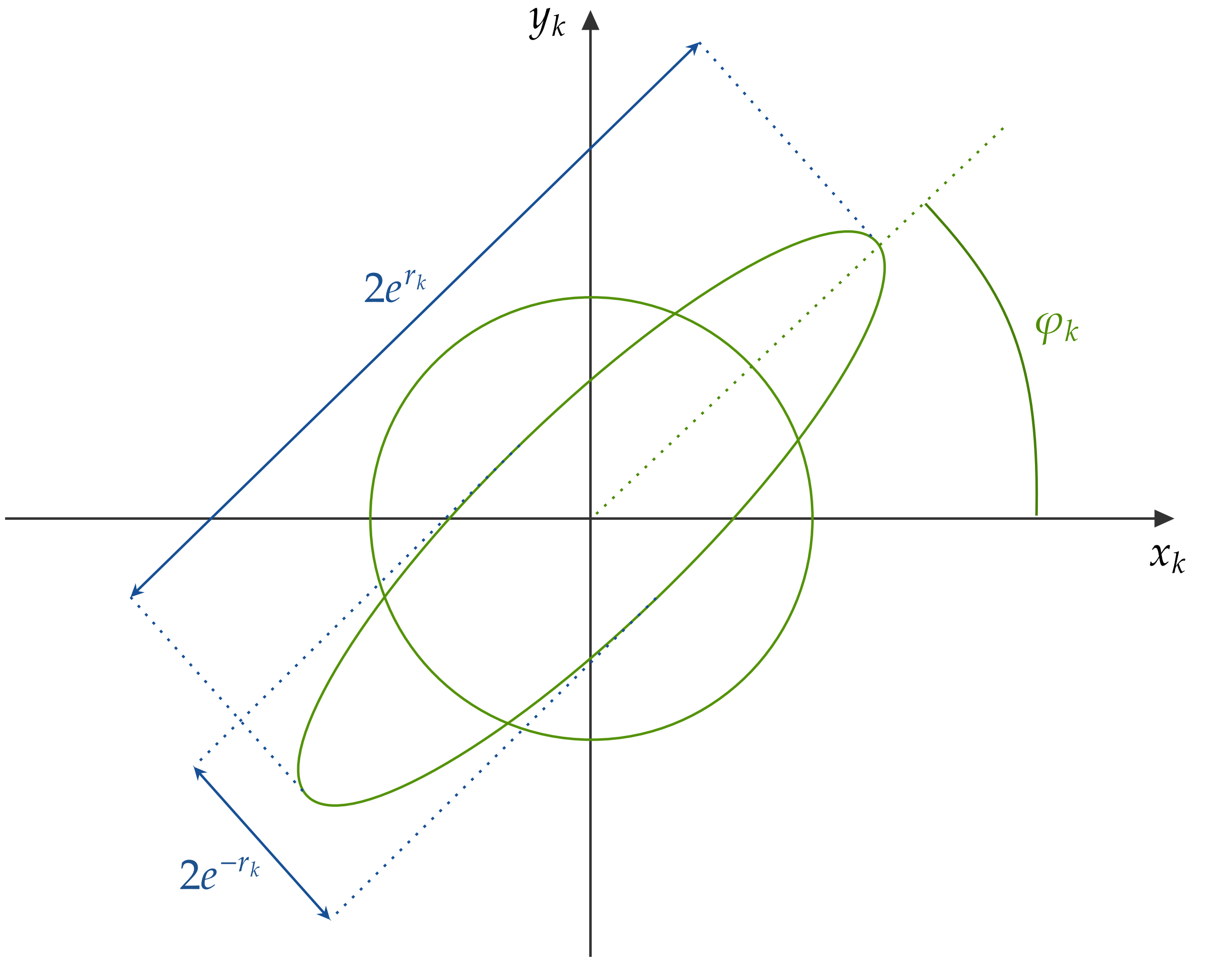
Probability density function

Wigner function = Weyl transform of the density operator



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For a Gaussian state:

$$W(R) = \frac{1}{\pi^2\sqrt{\det \gamma}} e^{-R^T\gamma^{-1}R} \qquad R = (\mathbf{q}_i, \mathbf{p}_i)$$

Covariance matrix (γ)

- ✓ As far as the two-point correlators are concerned, the quantum and stochastic descriptions cannot be observationally distinguished, independently on the amount of squeezing
- ✓ Higher order correlators depend on squeezing but in the large squeezing limit, the stochastic description gives accurate results for these correlators too.

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J. Martin and V. Vennin *Phys. Rev. D* **93** (2016), no. 2 023505, [arXiv:1510.04038]

Framework

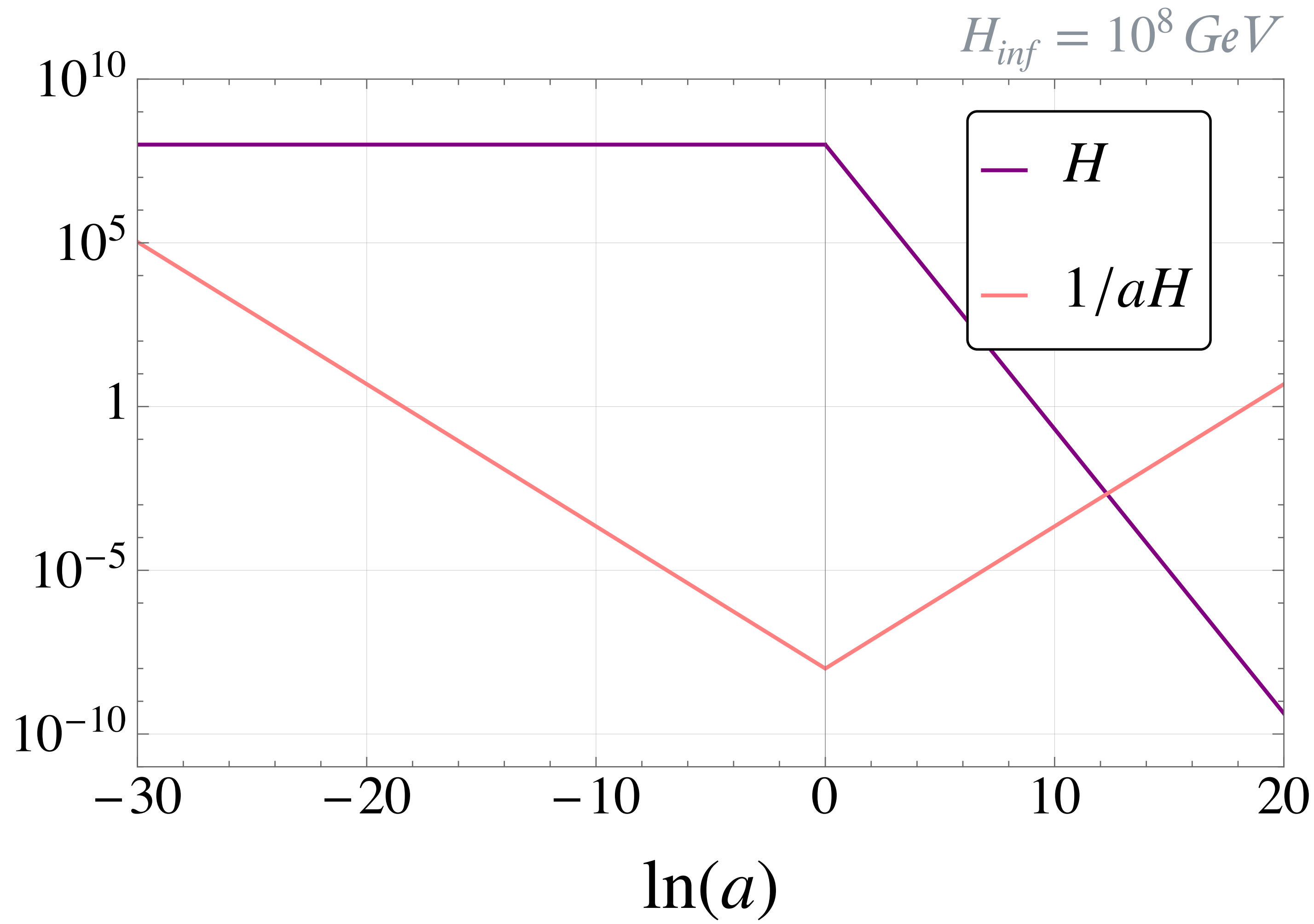
✓ De Sitter (DS) inflation followed by a Radiation Domination (RD) phase

✓ Axions produced via misalignment mechanism with $f > \boldsymbol{max}(T_{rh}, H_{inf})$

✓ Axion is a spectator field

✓ Instantaneous reheating

$$H = \begin{cases} H_{inf} & a < 0 \\ H_{inf}\, e^{-2\ln(a/a_{reh})} & a > 0 \end{cases}$$



J.J. L. J. Kuß and D. J. E. Marsh *Open J. Astrophys.* **4** (6, 2021) 2021, [arXiv: 2106.03528]

Framework

What about the axion potential?

$$V(\phi) = f^2\, m_\phi^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

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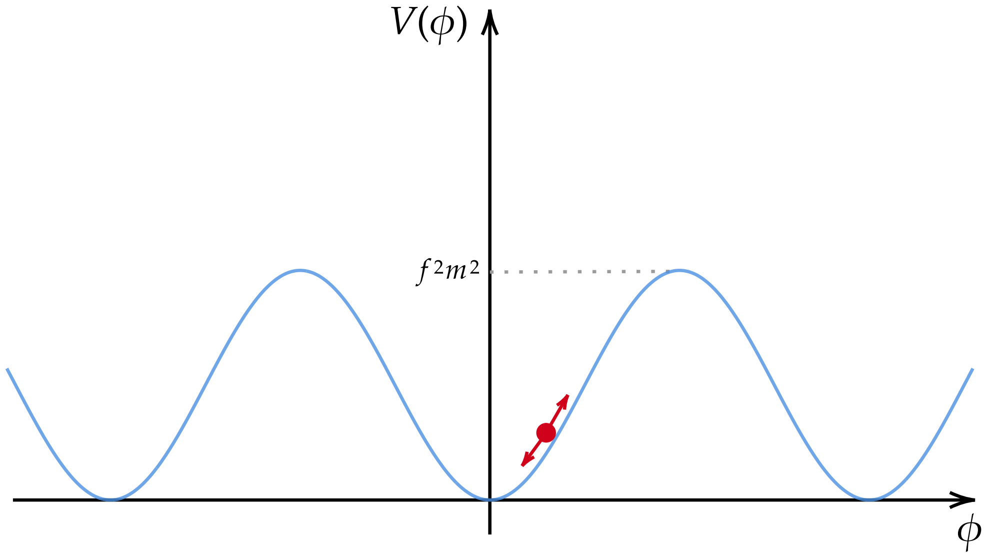
$$V(\phi) = f^2\, m_\phi^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

Taylor expanding: $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2$



$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + m_\phi^2 \bar{\phi} = 0$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left[\frac{k^2}{a^2} + m_\phi^2\right]\delta\phi = 0$$

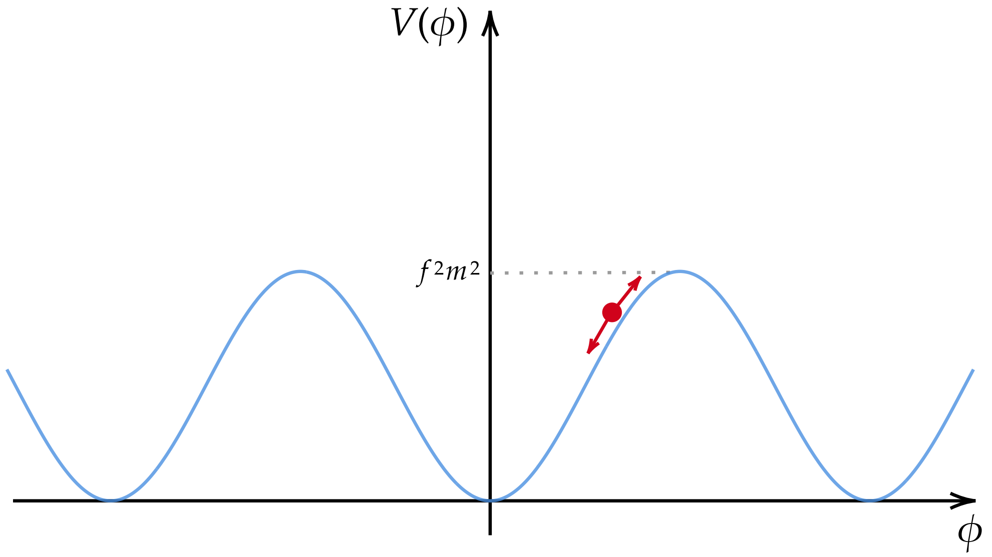
Framework

What about the axion potential?

$$V(\phi) = f^2 m_\phi^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

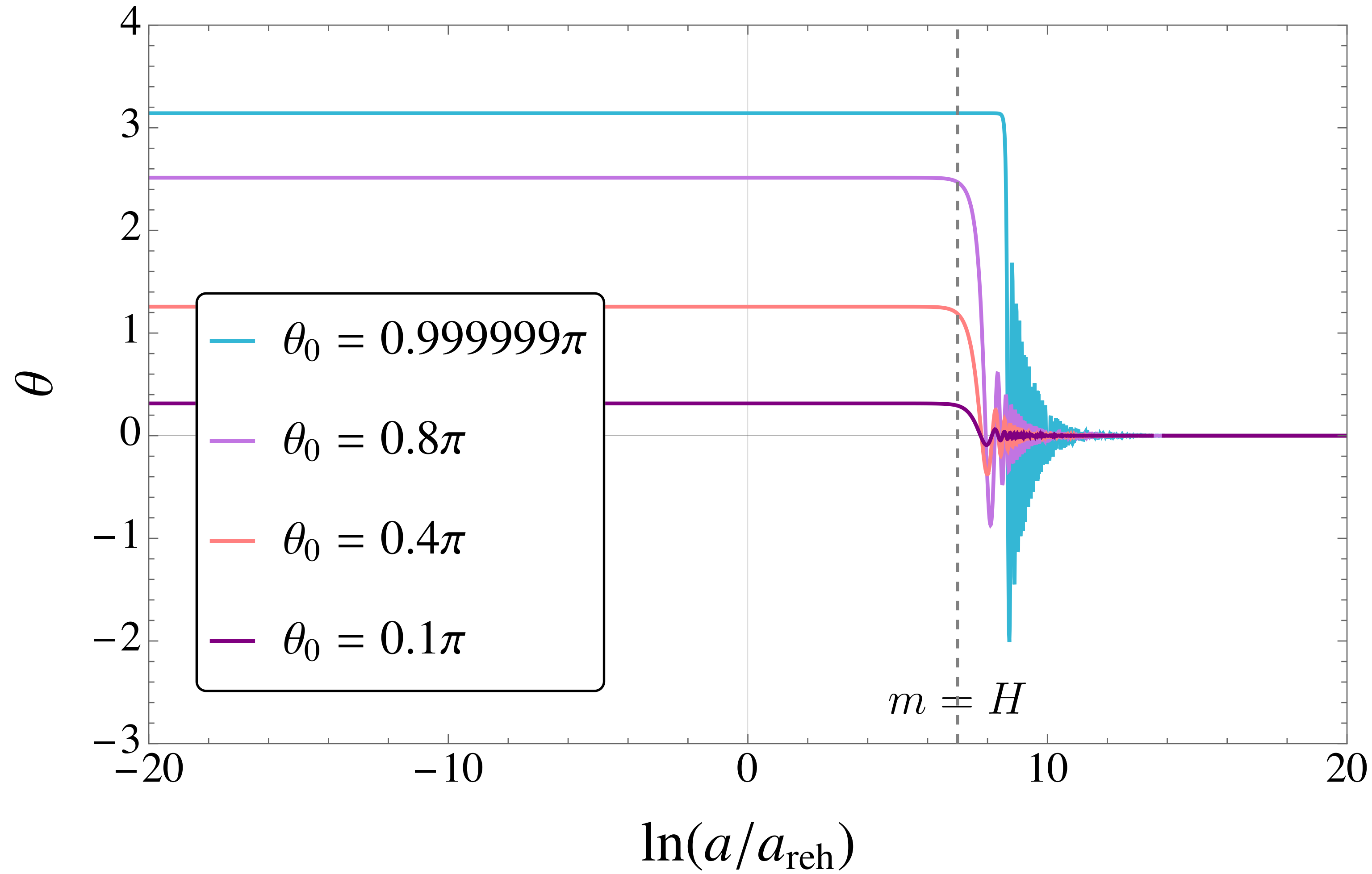
$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + f m_\phi^2 \sin\left(\frac{\bar{\phi}}{f}\right) = 0$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left[\frac{k^2}{a^2} + m_\phi^2 \cos\left(\frac{\bar{\phi}}{f}\right)\right]\delta\phi = 0$$



Background Field: Equation of Motion

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + f m_\phi^2 \sin\left(\frac{\bar{\phi}}{f}\right) = 0 \quad \xrightarrow{\theta = \frac{\bar{\phi}}{f}} \quad \ddot{\theta} + 3H\dot{\theta} + m_\phi^2 \sin\theta = 0$$



$m = 10^2\,GeV$
$H_{inf} = 10^8\,GeV$

The **action** to consider is:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \cos\left(\frac{\bar{\phi}}{f}\right) \phi^2 \right]$$

$$= \int d^3x d\tau\, a^2 \left[\frac{1}{2}\phi'^2 - \frac{1}{2}(\partial_i \phi)^2 - \frac{1}{2} m_\phi^2\, a^2 \cos\left(\frac{\bar{\phi}}{f}\right) \phi^2 \right]$$

Define:

$$\chi(\tau) = a(\tau)\phi(\tau)$$

We can compute the corresponding **Hamiltonian** (in Fourier space):

$$\mathscr{H} = \frac{1}{2(2\pi)^3} \int d^3k \left[p_{\mathbf{k}} p_{\mathbf{k}}^* + \left(k^2 + m_{eff}^2 a^2 - \frac{a''}{a} \right) \chi_{\mathbf{k}} \chi_{\mathbf{k}}^* \right]$$

$$\hat{p}_{\mathbf{k}} = \hat{\chi}'_{\mathbf{k}}$$

$$m_{eff}^2 = m_\phi^2 \cos\left(\frac{\bar{\phi}}{f}\right)$$

Axion Perturbations

We quantize the fields introducing **time-dependent ladder operators**:

$$\chi_{\mathbf{k}} = \frac{1}{\sqrt{2|\omega_k|}}\Big(a_{\mathbf{k}}(\tau) + a^{\dagger}_{-\mathbf{k}}(\tau)\Big)$$

$$p_{\mathbf{k}} = -i\sqrt{\frac{|\omega_k|}{2}}\Big(a_{\mathbf{k}}(\tau) - a^{\dagger}_{-\mathbf{k}}(\tau)\Big)$$

$$\boxed{\omega_k^2 = k^2 + m_{eff}^2 - \frac{a''}{a}}$$

Respecting canonical commutation relations:

$$\Big[\chi_{\mathbf{k}}(\tau), p^{\dagger}_{\mathbf{k}'}(\tau)\Big] = i\delta^{(3)}(\mathbf{k}-\mathbf{k}')\,, \qquad \Big[a_{\mathbf{k}}(\tau), a^{\dagger}_{\mathbf{k}'}(\tau)\Big] = \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

Time-dependent ladder operators are linked with time-independent ladder operators via **Bogoliubov transformation**:

$$\begin{cases} a_{\mathbf{k}}(\tau) = \alpha_k(\tau)\, a_{\mathbf{k}}(\tau_0) + \beta_k(\tau) a^{\dagger}_{-\mathbf{k}}(\tau_0) \\ a^{\dagger}_{-\mathbf{k}}(\tau) = \tilde{\alpha}_k(\tau)\, a^{\dagger}_{-\mathbf{k}}(\tau_0) + \tilde{\beta}_k(\tau) a_{\mathbf{k}}(\tau_0) \end{cases}$$

The fields $\chi_{\mathbf{k}}$ and $p_{\mathbf{k}}$ can be written alternatively in terms of the **time-independent ladder operators** directly:

$$\chi_{\mathbf{k}} = u_k(\tau)\, a^0_{\mathbf{k}} + u^*_k(\tau)\, a^{0\,\dagger}_{-\mathbf{k}}$$

$$p_{\mathbf{k}} = u'_k(\tau)\, a^0_{\mathbf{k}} + u^{*\,\prime}_k(\tau)\, a^{0\,\dagger}_{-\mathbf{k}}$$

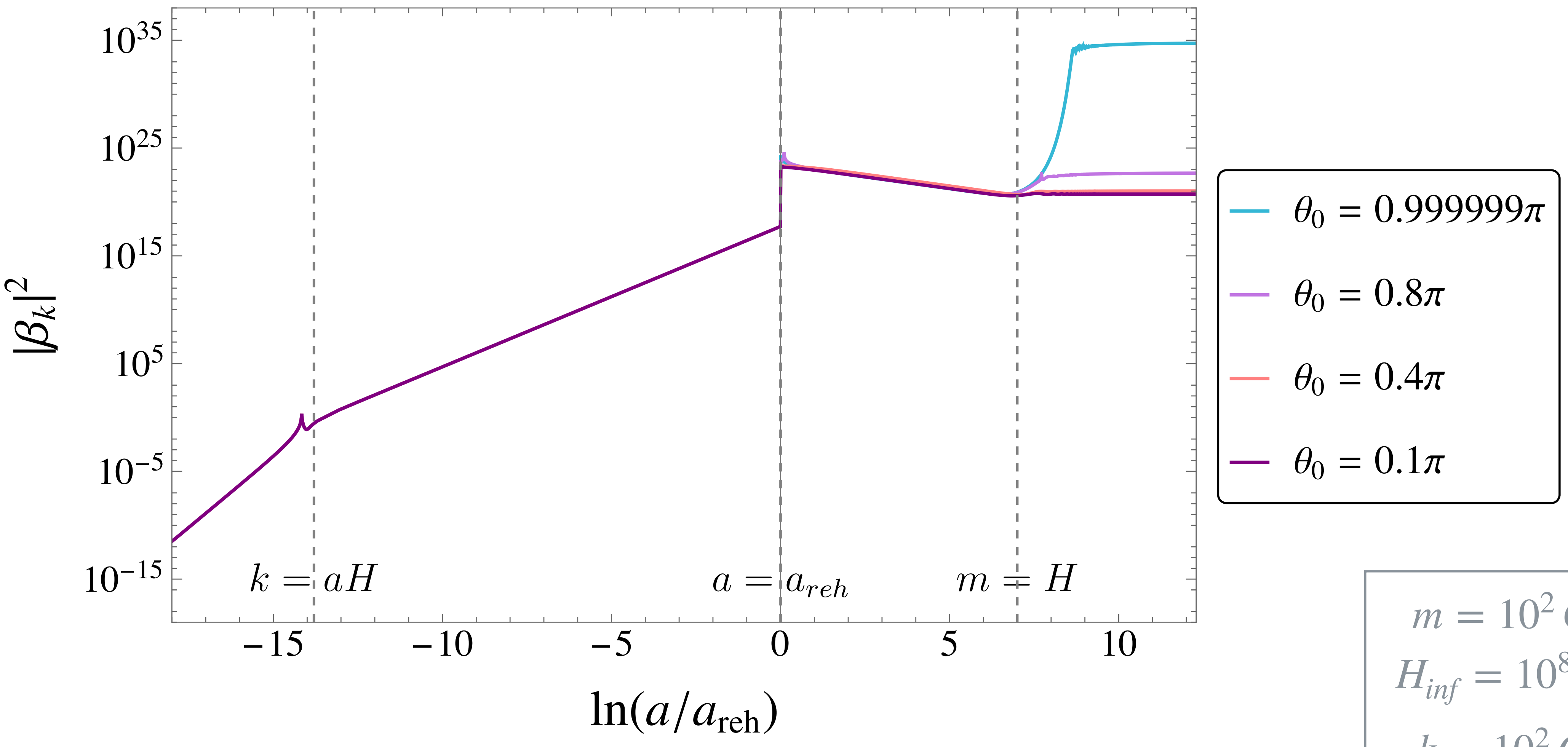
Comparing:

$$\boxed{\begin{aligned} \alpha_k &= \sqrt{\frac{|\omega_k|}{2}}\, u_k(\tau) - \frac{i}{\sqrt{2\,|\omega_k|}}\, u'_k(\tau) \\ \beta_k &= \sqrt{\frac{|\omega_k|}{2}}\, u^*_k(\tau) - \frac{i}{\sqrt{2\,|\omega_k|}}\, u^{*\,\prime}_k(\tau) \end{aligned}}$$

$$\longrightarrow \qquad |\beta_k|^2 = \frac{|\omega_k|}{2}\,|u_k|^2 + \frac{1}{2\,|\omega_k|}\,|u'_k|^2 - \frac{1}{2}$$

Analysis of the Beta Coefficient

$$|\beta_k|^2 = \frac{|\omega_k|}{2}|u_k|^2 + \frac{1}{2|\omega_k|}|u_k'|^2 - \frac{1}{2}$$



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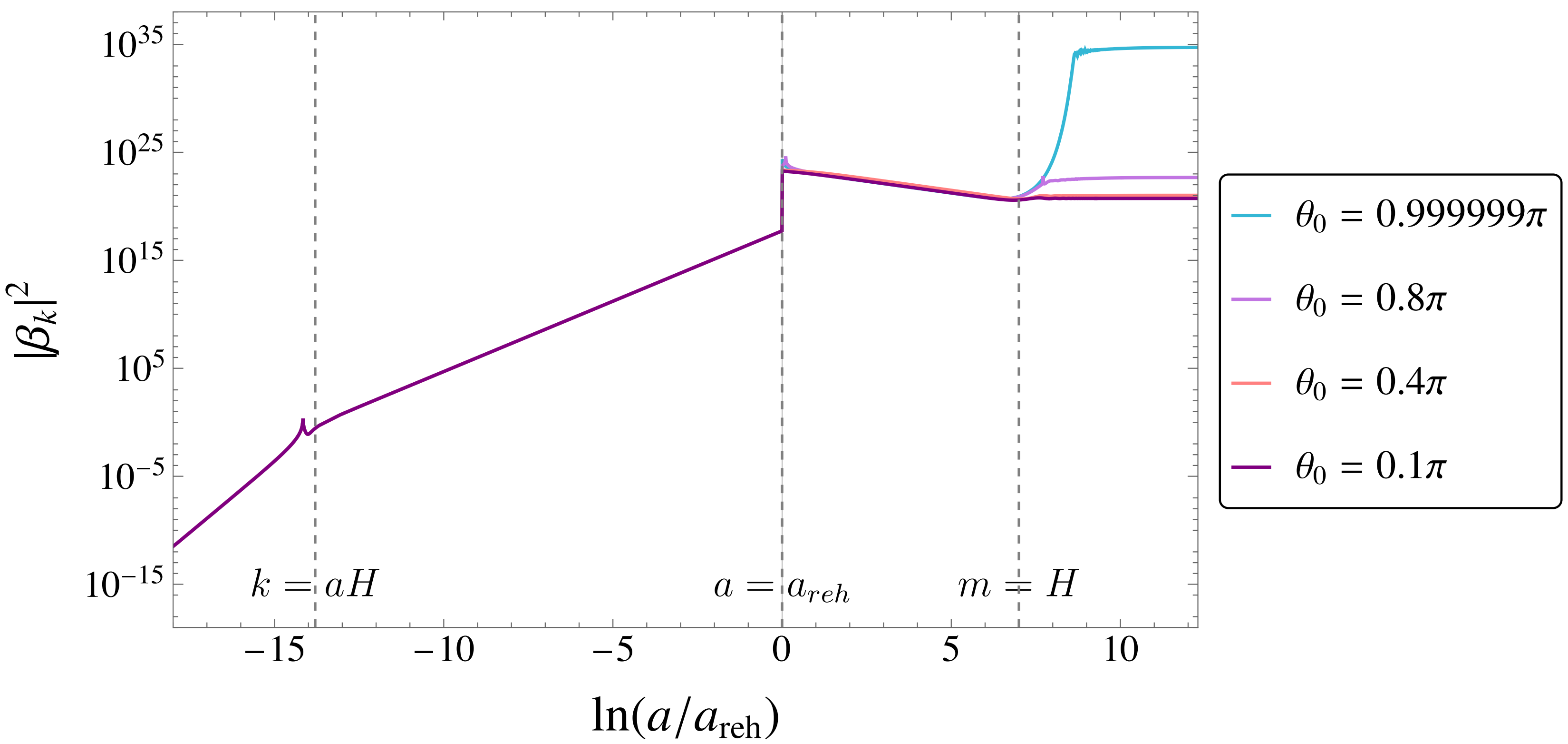


$m = 10^2\, GeV$
$H_{inf} = 10^8\, GeV$
$k = 10^2\, GeV$

✓ $|\beta_k|^2$ approaches zero in the far past and a constant after the onset of the oscillations

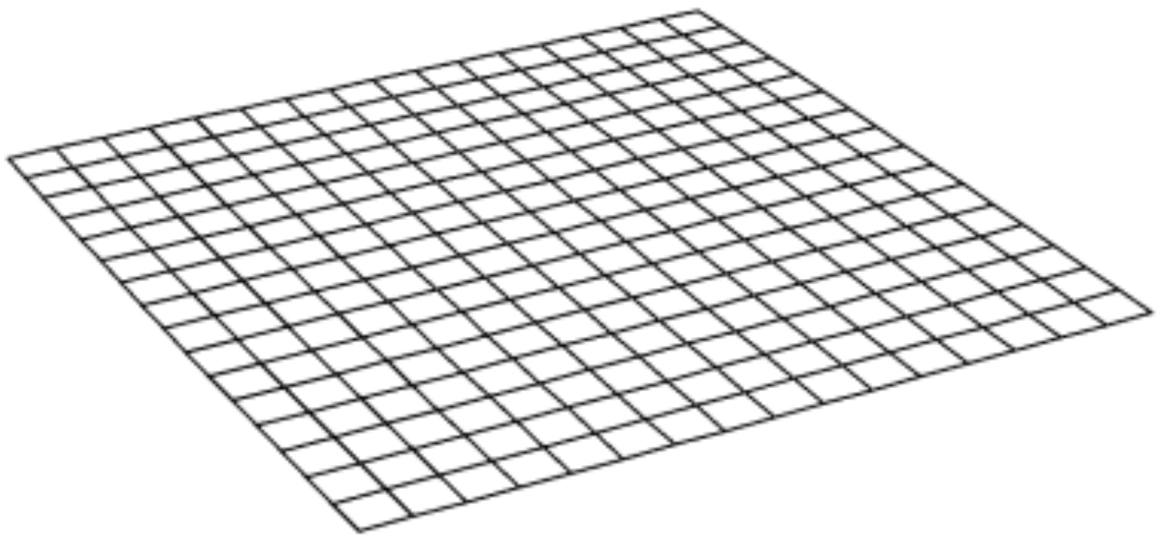
✓ The behavior of $|\beta_k|^2$ changes with the initial misalignment angle

✓ The spikes, as well as the discontinuous jump at reheating, can be understood in terms of the frequency ω_k

Particle Creation in Curved Spacetime

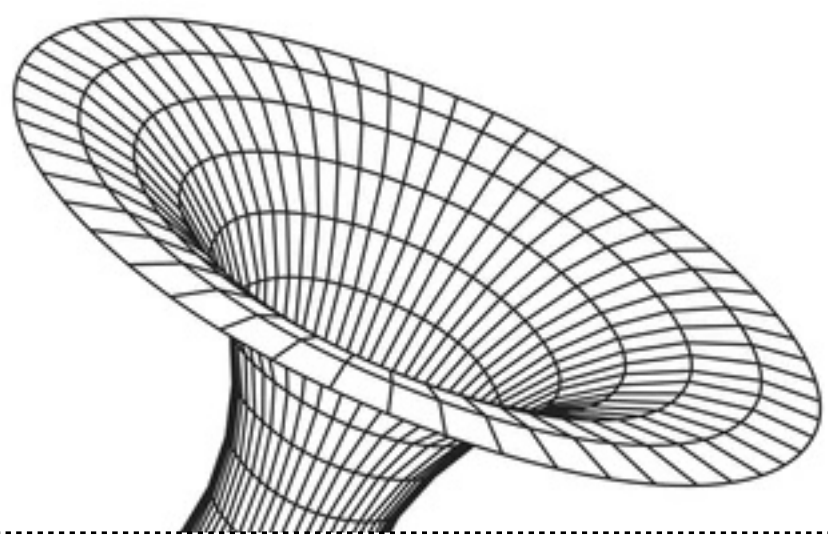
Cosmological framework: the instantaneous vacuum defined by the time-dependent ladder operators $(a_{\mathbf{k}}(\eta), a^{\dagger}_{\mathbf{k}}(\eta))$ is filled with particles associated with the initial time-independent operators $(a^{0}_{\mathbf{k}}, a^{0\,\dagger}_{\mathbf{k}})$.

What is the correct choice for the initial ladder operators?



In **Minkowski** spacetime there is a **unique choice for the vacuum state**.

On an arbitrary spacetime, there are in general **no** isometries that allow to define **uniquely the vacuum state**.



These ambiguities can be solved assuming **Minkowski in the asymptotic past and future**.

$$a_{\mathbf{k}}(\eta) \xrightarrow[\eta\to-\infty]{} a_{\mathbf{k}}^{in}\,, \qquad\qquad a_{\mathbf{k}}(\eta) \xrightarrow[\eta\to+\infty]{} a_{\mathbf{k}}^{out}$$

Linked via time-independent Bogoliubov coefficients A_k and B_k .

Time-dependent Bogoliubov coefficients are their late time limit:

$$\alpha_k(\eta) \xrightarrow[\eta\to+\infty]{} A_k\,, \qquad\qquad \beta_k(\eta) \xrightarrow[\eta\to+\infty]{} B_k$$

When the background felt by the fields can be approximated as **constant in time**?

Adiabaticity condition

Adiabaticity Condition

The adiabaticity condition is defined as:

$$\left|\frac{\omega_k'}{\omega_k^2}\right|, \left|\frac{\omega_k''}{\omega_k^3}\right| \ll 1$$

If the adiabaticity condition holds: $$u(\tau) = \frac{A_k}{\sqrt{2k}}\, e^{+i\int^\tau \omega_k(\tau')d\tau'} + \frac{B_k}{\sqrt{2k}}\, e^{-i\int^\tau \omega_k(\tau')d\tau'}$$

It can be proved that in radiation domination:

$$\frac{\omega_k'}{\omega_k^2} \to \begin{cases} \dfrac{a^3 H m^2}{k^3} & k \gg a\, m \\ \dfrac{H}{m} & k \ll a\, m \end{cases} \qquad \frac{\omega_k''}{\omega_k^3} \to \begin{cases} \dfrac{m^2 a^4 H^2}{k^4} & k \gg a\, m \\ \dfrac{k^2 H^2}{a^2 m^4} & k \ll a\, m \end{cases}$$

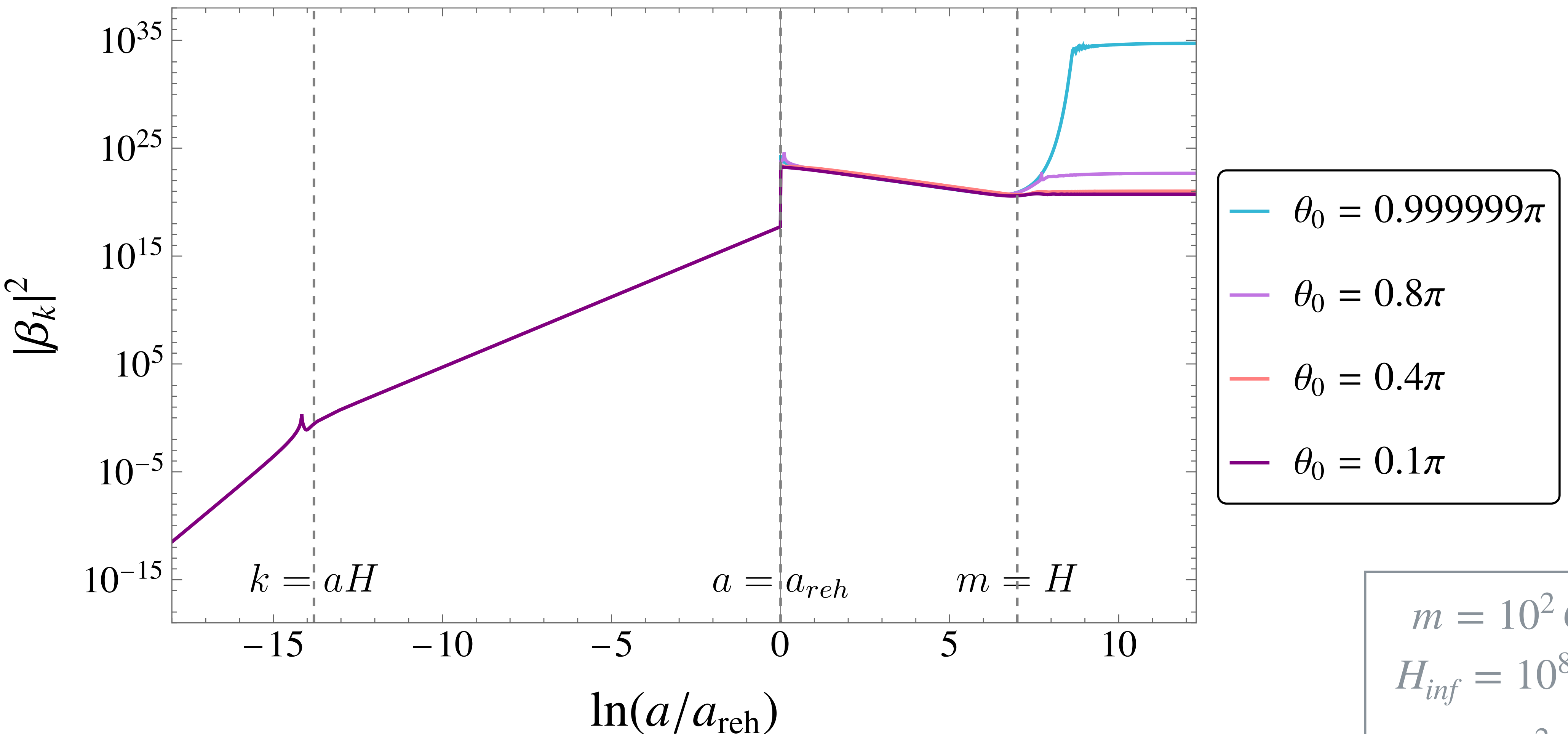
We restrict ourselves to wave modes that are outside the horizon at the onset of the oscillations, i.e. $k < a_{osc} m$.

Hence when $a > a_{osc}$ and $H < m$, the adiabaticity conditions are given by the second lines, which are smaller than unity.

Analysis of the Beta Coefficient

$$|\beta_k|^2 = \frac{|\omega_k|}{2}|u_k|^2 + \frac{1}{2|\omega_k|}|u_k'|^2 - \frac{1}{2}$$

$$\langle\delta\rho\rangle \simeq \frac{\langle\mathscr{H}\rangle}{a^4V} = \frac{m}{a^3}\int\frac{d^3k}{(2\pi)^3}\left(|\beta_k|^2 + \frac{1}{2}\right)$$



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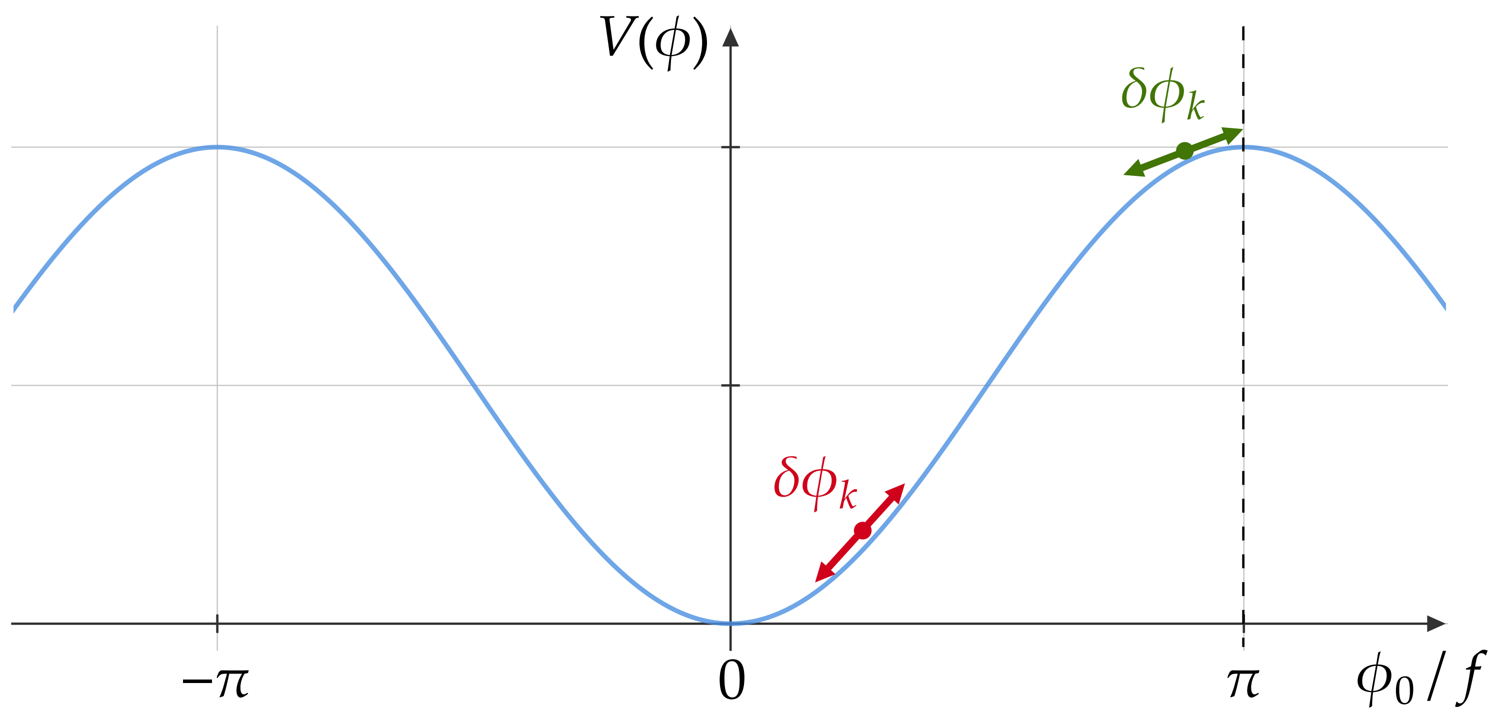


✓ The rolling down of the field is delayed increasing the initial field value.

✓ If we consider the field perturbation near the minimum, the delay in the onset of the oscillation is too tiny to affect the evolution in time of $\delta\phi_k$.

✓ Close to the hilltop even a small difference in the initial position leads to a huge delay. Hence patches of the Universe that differ by a tiny variation of the initial misalignment angle will start to oscillate at very different times, sourcing huge fluctuations.

Squeezing Parameters

The Bogoliubov coefficients can be parameterised by the **squeezing parameters**:

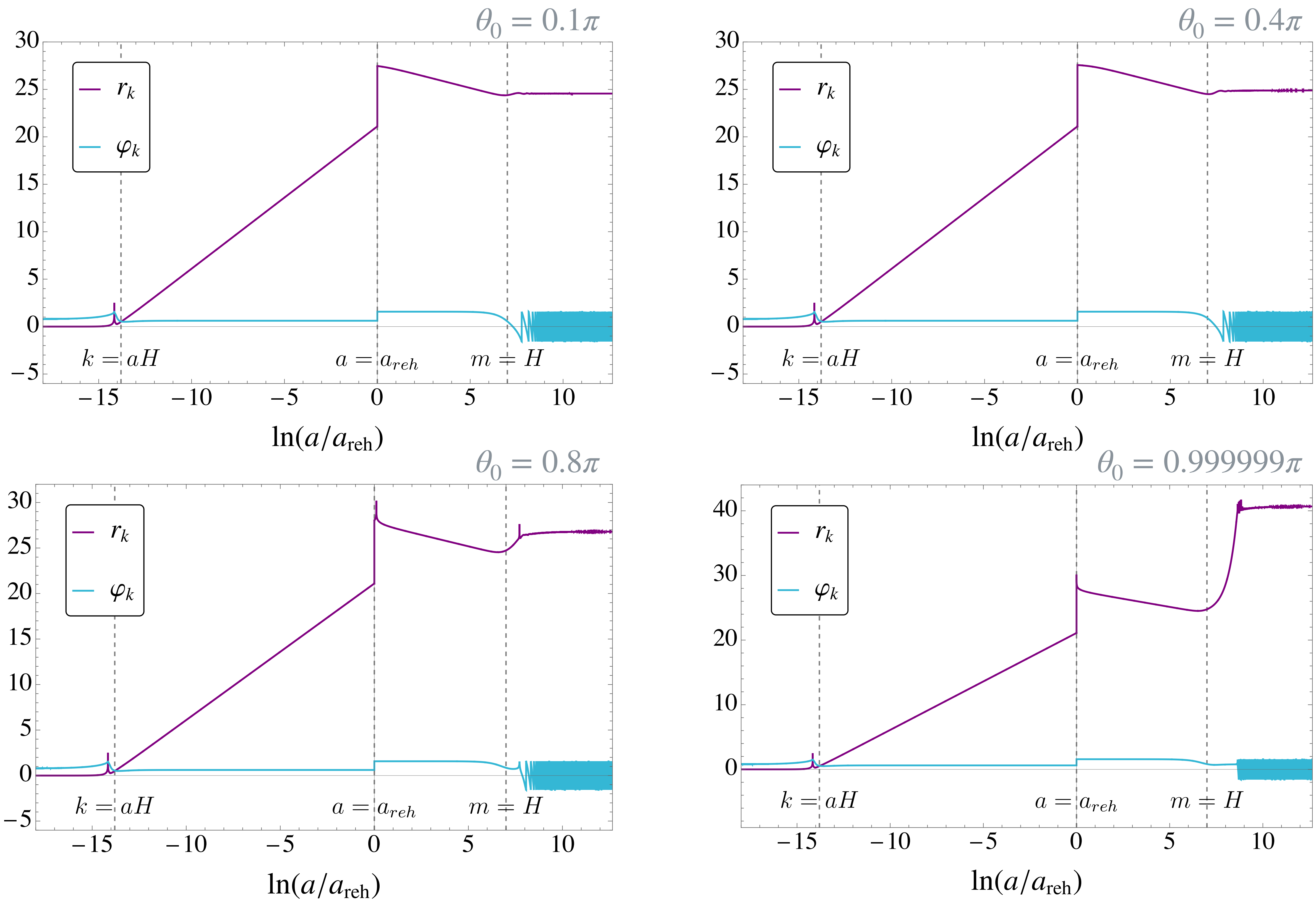
$$\begin{cases} \alpha_k(\tau) = e^{-i\vartheta_k(\tau)} \cosh r_k(\tau) \\ \beta_k(\tau) = e^{i\left[\vartheta_k(\tau) + 2\varphi_k(\tau)\right]} \sinh r_k(\tau) \end{cases}$$

Inverting these relations:

$$r_k = \sinh^{-1} |\beta_k| \,, \qquad \vartheta_k = -\arg\left(\alpha_k\right) \,, \qquad \varphi_k = \frac{1}{2}\arg\left(\alpha_k \beta_k\right)$$

These parameters are linked to the squeezing introduced before.

Analysis of the Squeezing Parameters



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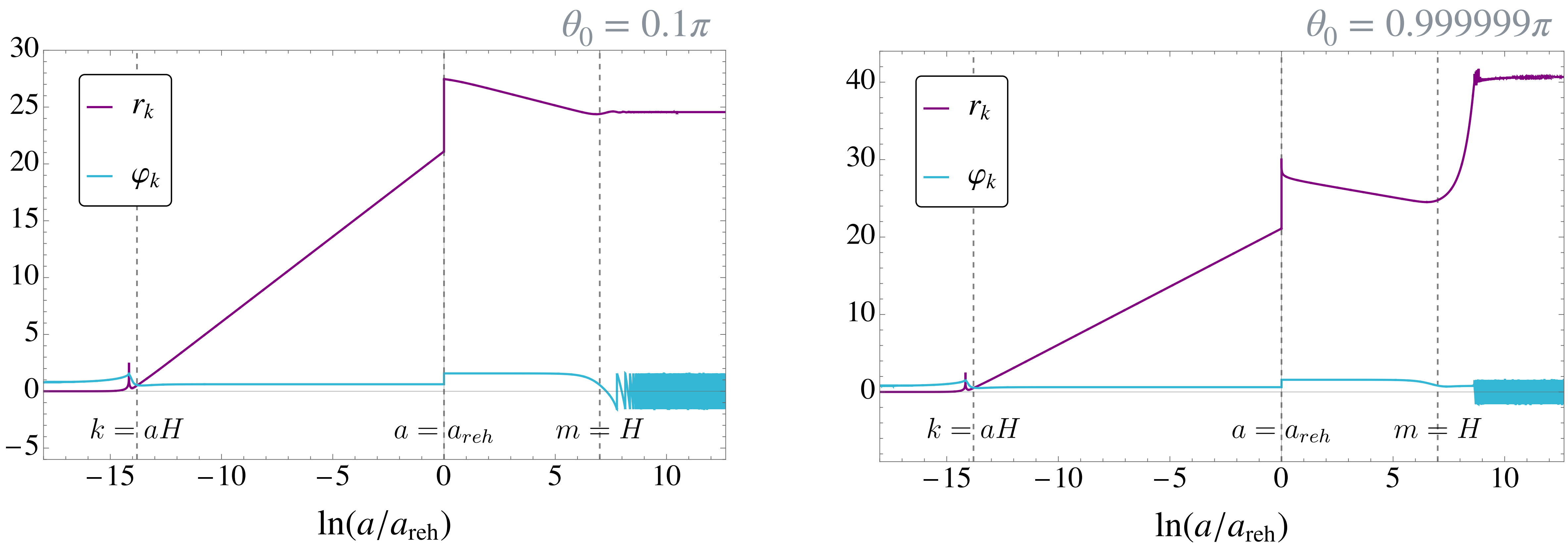
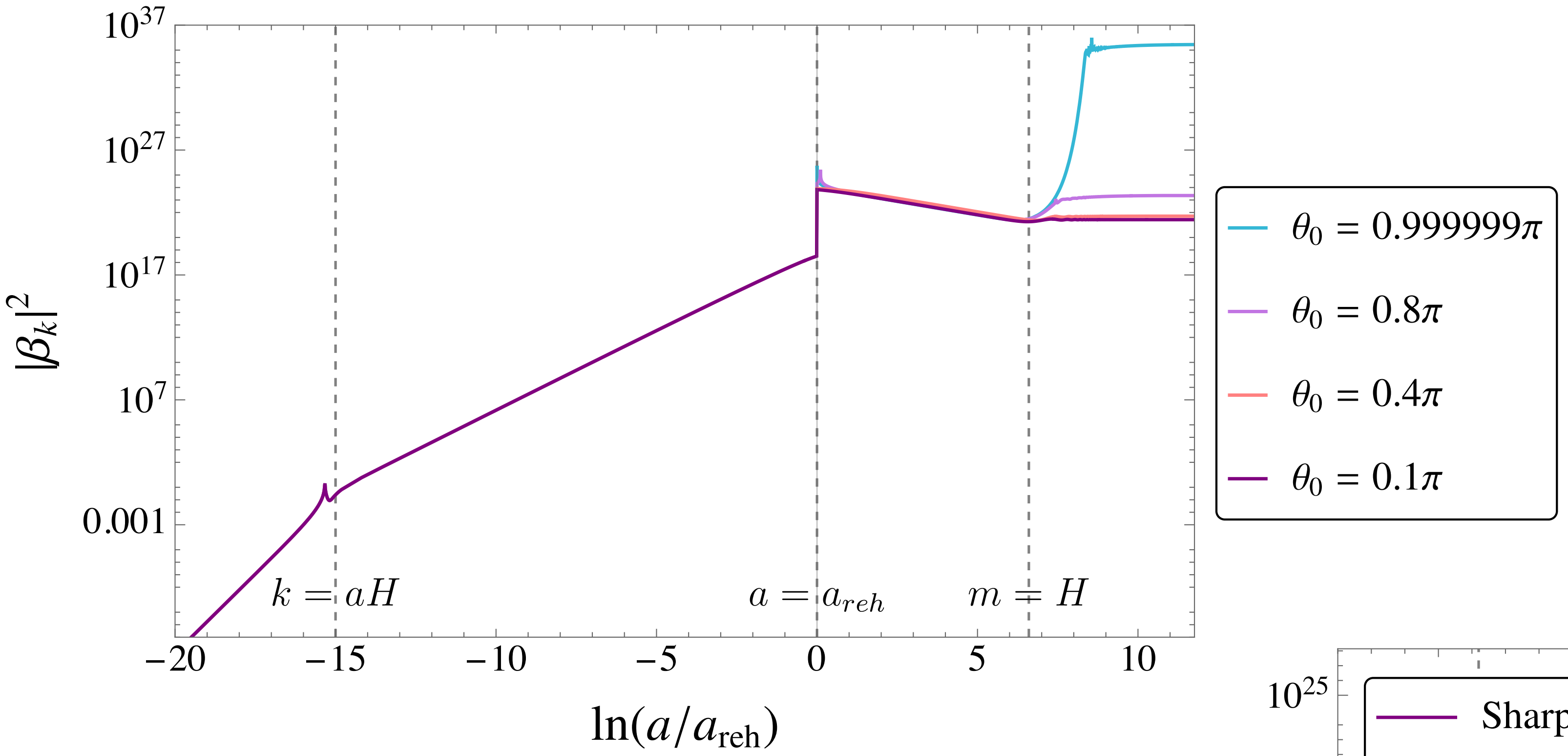


✓ Approaching the hilltop of the potential, r_k increases, as $|\beta_k|^2$ did

✓ Anharmonic effects give extra squeezing at the onset of the oscillations

✓ φ_k , linked with the rotation of the ellipse, oscillate among $-\pi$ and π after the onset of the oscillations

Our results in the late time limit are not affected:

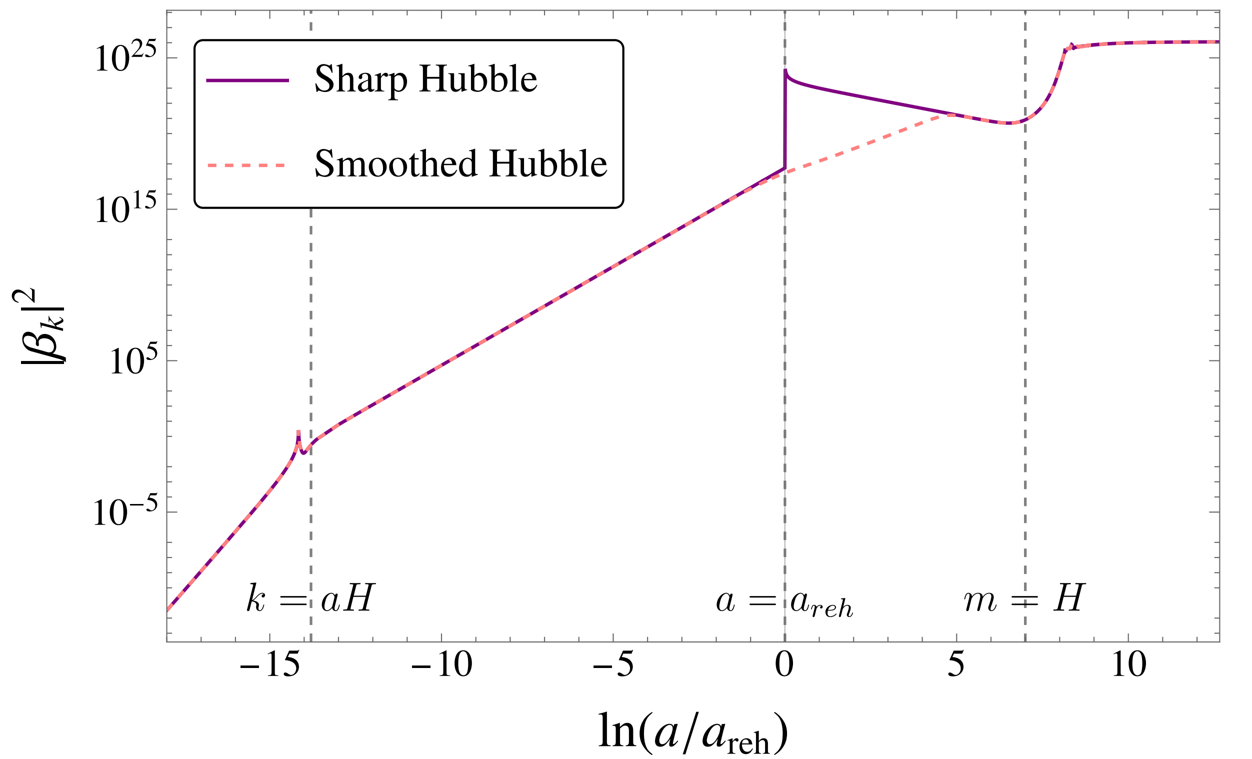


Quasi - de Sitter Inflation

$$H(\eta) = \begin{cases} m_\varphi \sqrt{\dfrac{1}{3} - \dfrac{2}{3}\eta} & \textit{inflation} \\ \dfrac{m_\varphi}{\sqrt{3}} e^{-2\eta} & \textit{radiation} \end{cases}$$

Smoothed Reheating

$$H(\eta) = H_{inf} \frac{e^{-2\eta}}{e^{-2\eta} + 1}$$



Conclusions and Future Directions

Conclusions

✓ Anharmonic effects produce an enhancement in the number of particles created due to the expansion

✓ Anharmonic effects increase also the amount of squeezing of the perturbations

✓ The ellipse in phase space keeps rotating

Future Directions

✓ Compute the bispectrum of the axion perturbations

✓ A cosmological Bell experiment could be designed

THANKS FOR THE ATTENTION