Multipole tidal effects in the post-Newtonian gravitational-wave phase of compact binary coalescences

Λ Σ $\Lambda^{(3)}$

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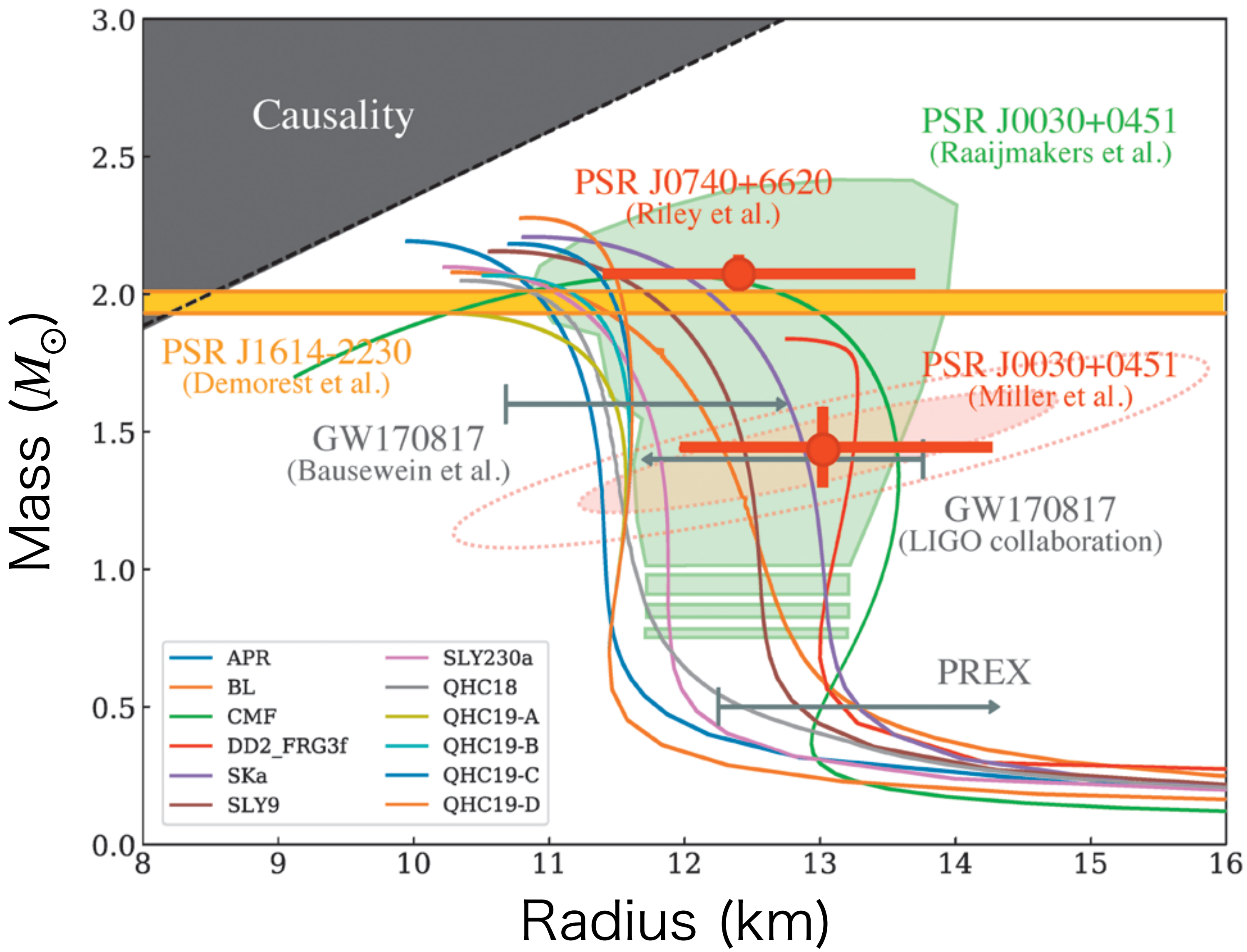
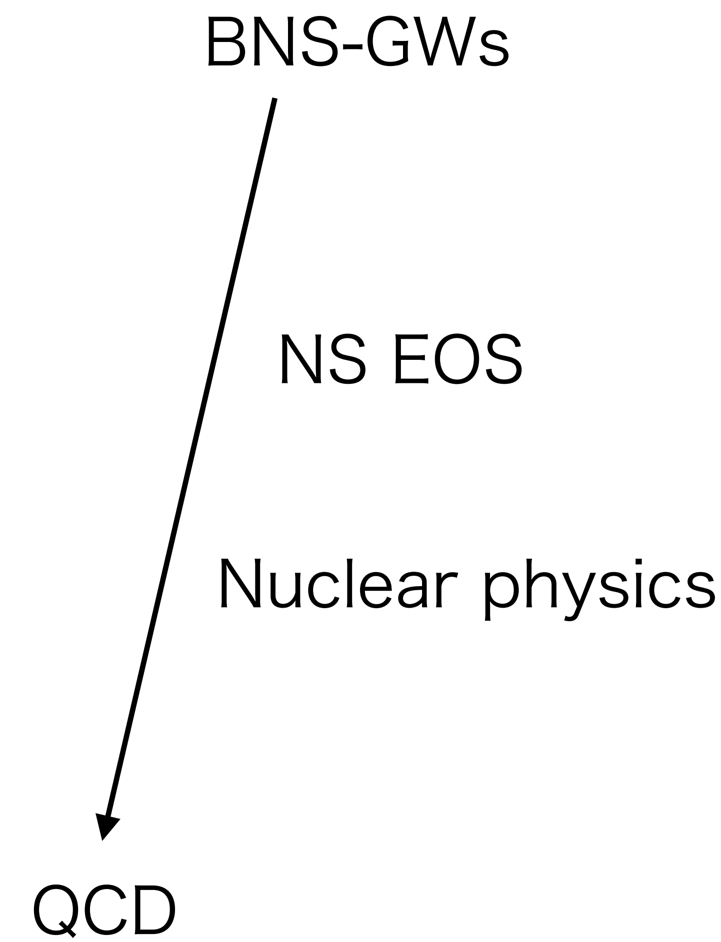
Science targets of data analyzing BNS-GWs

BNS coalescences are valuable laboratories for nuclear astrophysics

Mass-Radius relation for neutron star (NS)





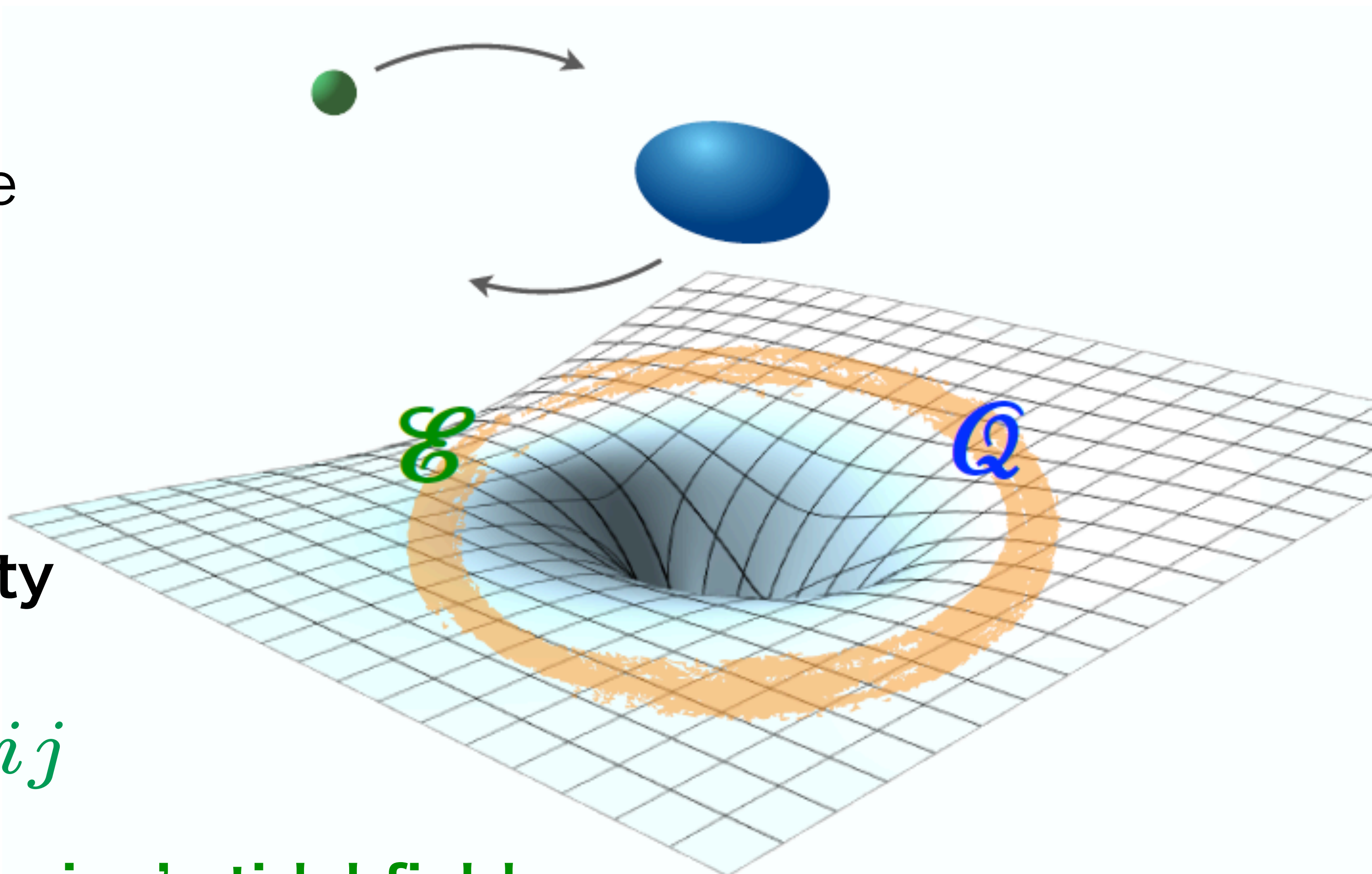
[Enoto, Yasutake, JPSJ'21]

BNS-GWs can provide complementary information on the macroscopic properties of neutron stars and the dense matter.

Review [Lattimer&Prakash2016; Baiotti2019; Dietrich, Hinderer, Samajdar 2021; Chatziioannou2020]

Tidal deformability

When binary orbital separations are small, each neutron star is tidally distorted by its companion.



[Dietrich, Hinderer, Samajdar, '20]

"Mass"-type quadrupole

Tidal deformability

$$\mathcal{Q}_{ij} = -\lambda \mathcal{E}_{ij}$$

(Tidal-induced) Quadrupole moment

Companion's tidal field

> Tidal deformability
> 1) characterizes NS EOS , 2) affects GW phase

Binary tidal deformability

$$\tilde{\Lambda} = \frac{16}{13}\left[(1+11X_B)X_A^4\Lambda_A + (A \leftrightarrow B)\right]$$

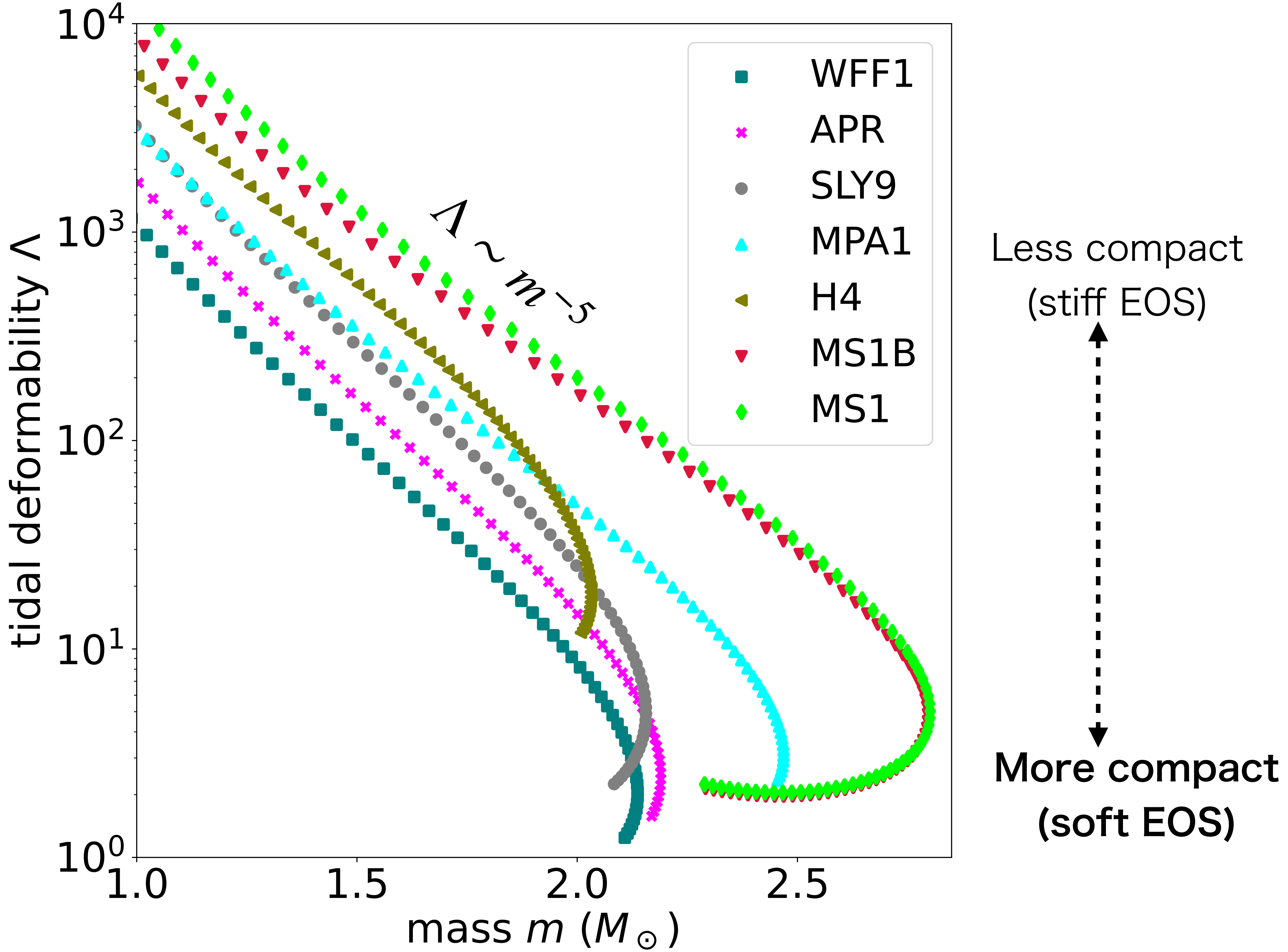
$\Lambda_{A,B} = \lambda_{A,B}/m_{A,B}^5$: individual ones

$X_{A,B} = m_{A,B}/(m_A + m_B)$: mass ratio

Λ

[Flanagan, Hinderer, '07;
Hinderer '08;
Vines, Flanagan, Hinderer '11]

Tidal deformability characterizes NS EOS



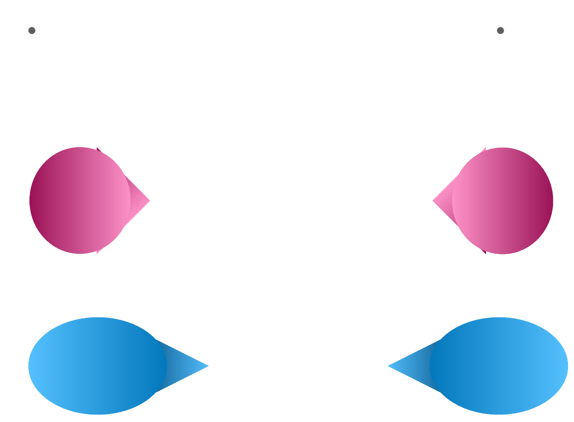
Tidal effects on waveform

Binary evolution depends on neutron star (NS) EOSs.

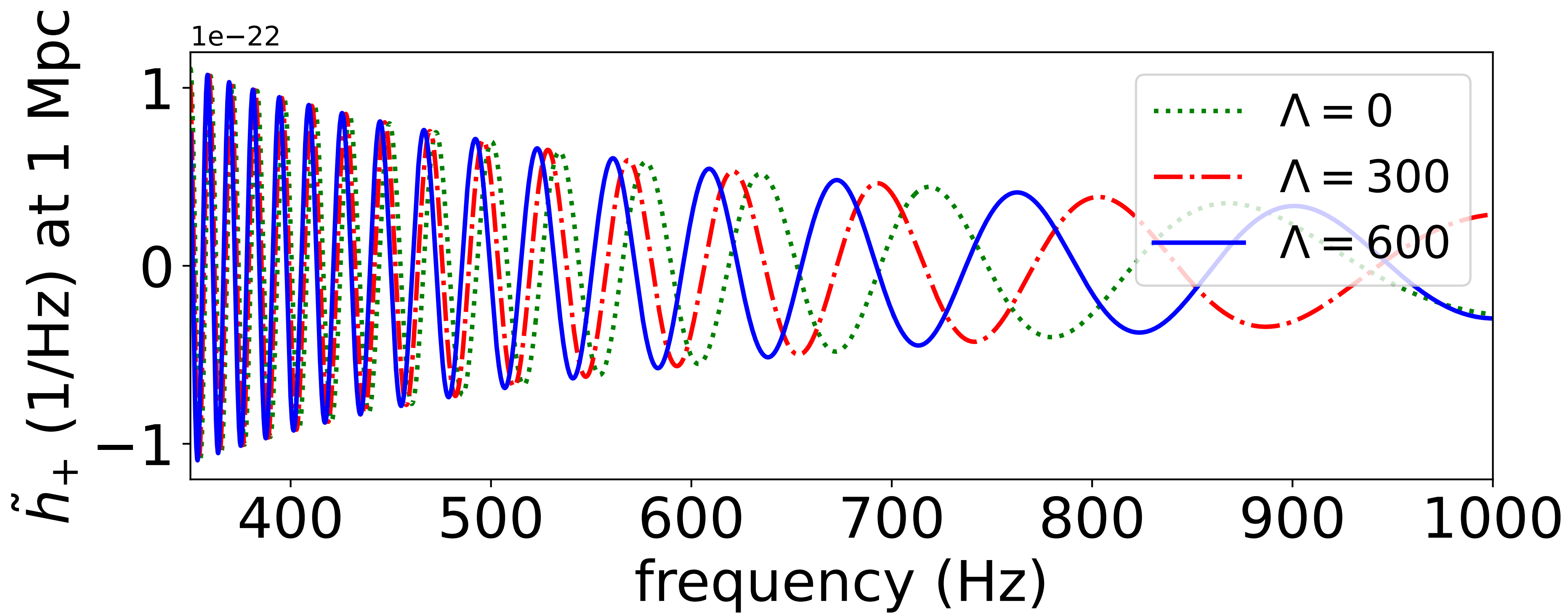
Point-particle (Binary BH) (Λ=0) -> slowest evolution

More compact NSs (small Λ) -> slow evolution

Less compact NSs (large Λ) -> fast evolution



Frequency-domain BNS waveforms (TaylorF2_PNTidal)



Tidal effects appear above 300 Hz.

Post-Newtonian GW phase

[Luc Blanchet's talk in symposium]

PN phase can efficiently describe the GW emission in the inspiral regime.

Newton gravity + GR correction: $\mathcal{O}((v/c)^0) + \mathcal{O}((v/c)^2) + \mathcal{O}((v/c)^3) + \cdots$.

Energy balance

Energy loss vs GW energy rad. rate

$$\dot{E}_{\rm binary} = -\dot{E}_{\rm GW} \longrightarrow$$

Phase

$$\dot{\phi}(t) = \frac{v^3}{M} \qquad \dot{v}(t) = -\frac{\dot{E}_{\rm GW}(v)}{dE_{\rm binary}/dv}$$

0-3.5PN (4.5PN) 5-7.5PN

$$\Psi_{\rm BNS}(f) = \Psi_{\rm BBH}(f) + \Psi_{\rm Tidal}(f)$$

η: symmetric mass ratio

$\chi_{\rm eff}$: spin

$$\sim \mathcal{M}^{-5/3} f^{-5/3} \left[1 + a_{1\rm PN} x + a_{1.5\rm PN} x^{3/2}\right.$$

$\mathcal{M}$:chirp mass

$$\left. + a_{2\rm PN} x^2 + \ldots + a_{5\rm PN} x^5 + \ldots \right]$$

κ: spin-induced quad. Λ: tidal deformability

v: orbital velocity

$$x = (\pi M f)^{2/3} = v^2$$

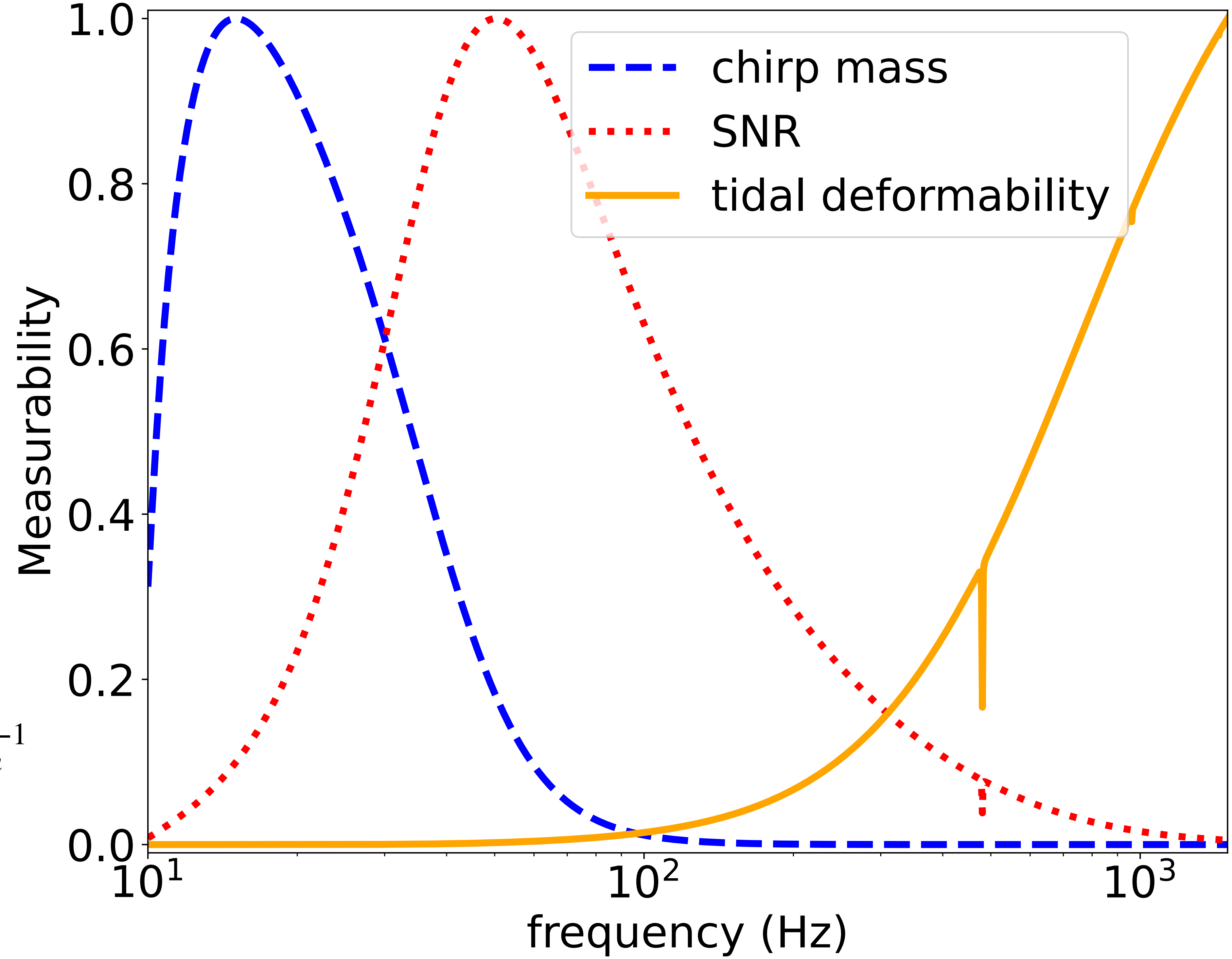
PN is theoretically rigid and base of Numerical Relativity calibrated models.

Measurability of binary parameters with GWs

used in [Damour+ 2012]

BNS signal, Adv LIGO design sensitivity

SNR: γ(f)f, middle frequency band



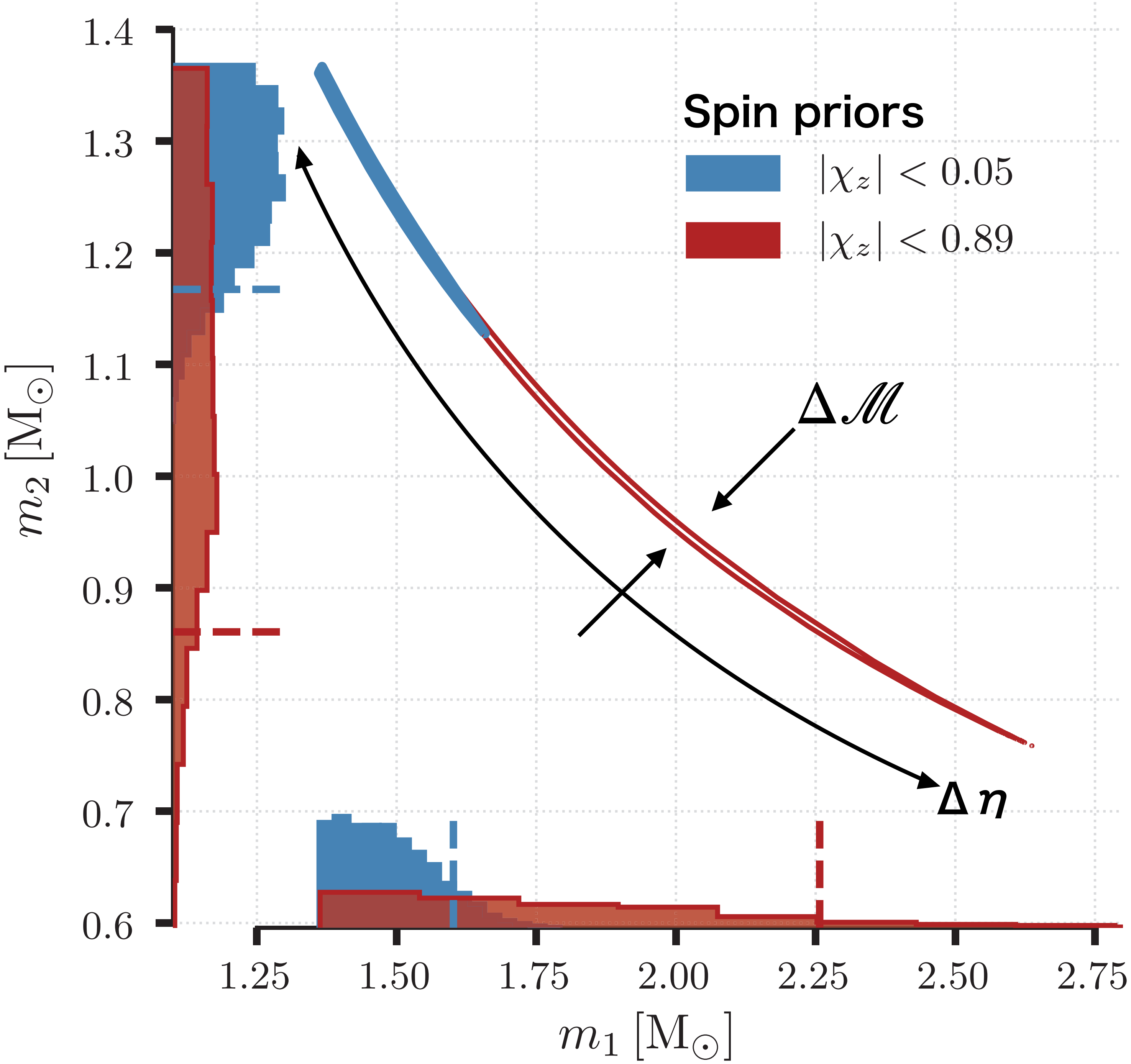
$\gamma(f) \propto f^{-7/3} S_n^{-1}$

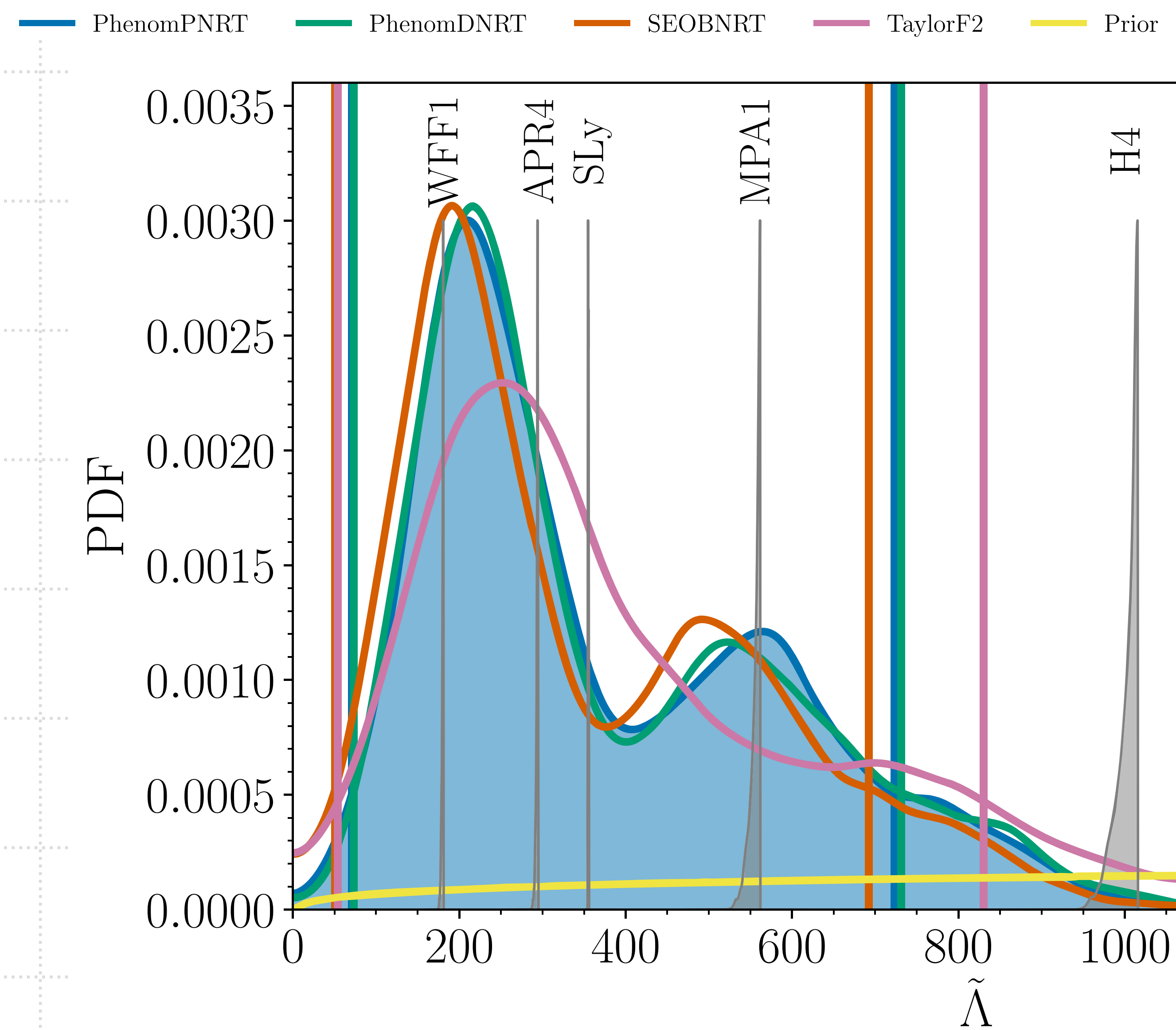
Chirp mass: γ(f)f/x(f)^5
low frequency band <100 Hz

Tidal effects: γ(f)fx(f)^5, high frequency band

GW inference on GW170817

[LVC 2017, 2018a]





$\tilde{\Lambda}$: measured, less compact EOS models are disfavored.

[De+'18; Dai+'18; LVC'17'18a'18b'19; Landry+'18'19; Capano+'19; Narikawa+'19; Chatziioannou'20, ⋯]

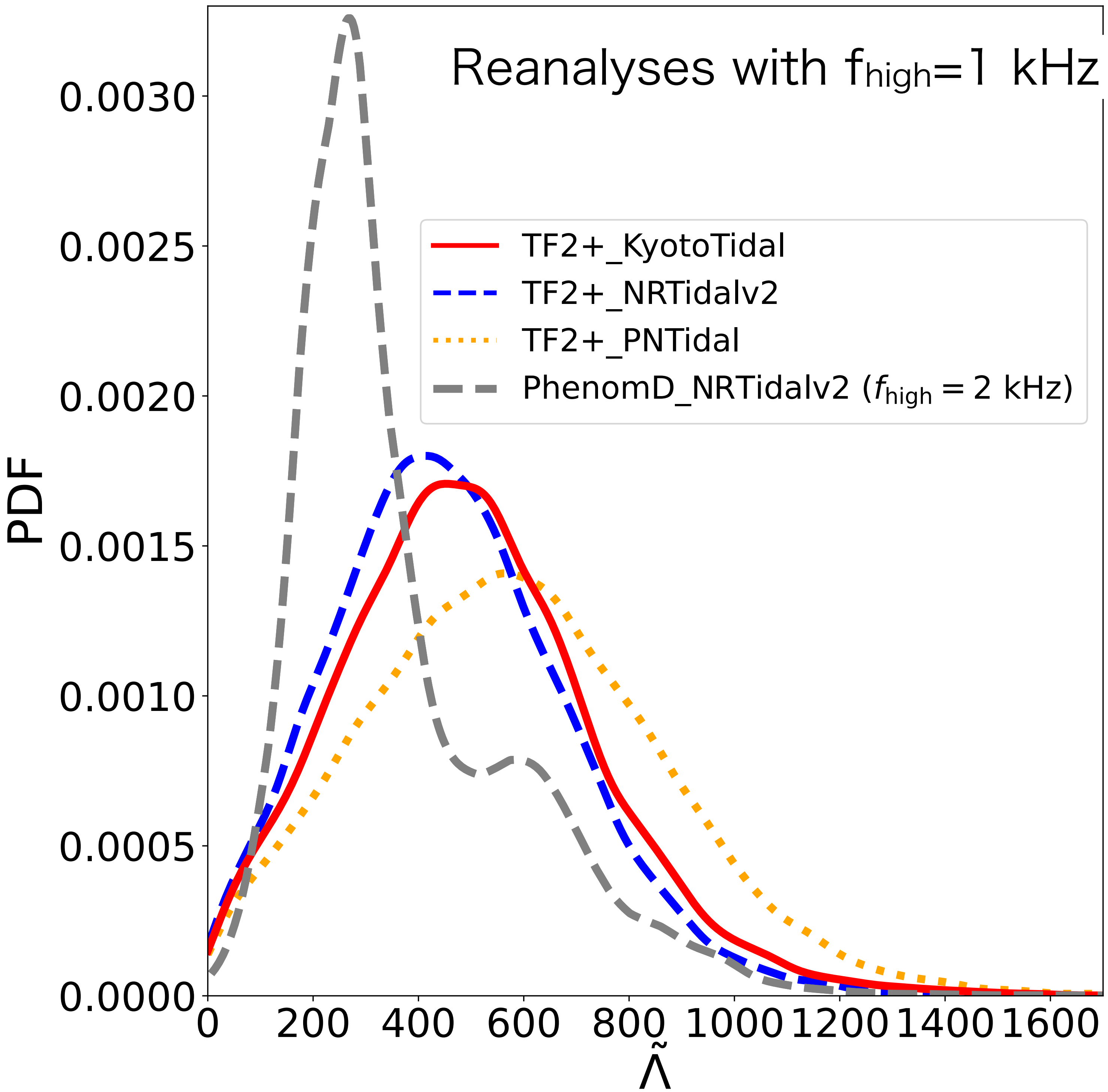
(f_{high}=2 kHz [LVC '18])

$\mathscr{M}$: measured well.

η: not measured well.

η - χ_{eff} correlation

GW170817: Waveform systematics, high-freq noise



Reanalyses with f_{high}=1 kHz focusing on inspiral regime

Inspiral-only analyses, with f_{high}=1 kHz, give less waveform biases and larger $\tilde{\Lambda}$ than the IMR analyses, with f_{high}=2 kHz.

[Dai+'18; Narikawa+'19; Gamba+'21]

(f_{high}=2 kHz [LVC '18])

High-freq noise issues in Livingston data [Narikawa+ '18]

KyotoTidal and **NRTidalv2** give smaller estimates of $\tilde{\Lambda}$ for GW170817 than **PNTidal** (and TEOBResumS) (within statistical uncertainties).

[LVC'19; Narikawa+'20; Gamba+'21; Ashton&Dietrich'21; ⋯]

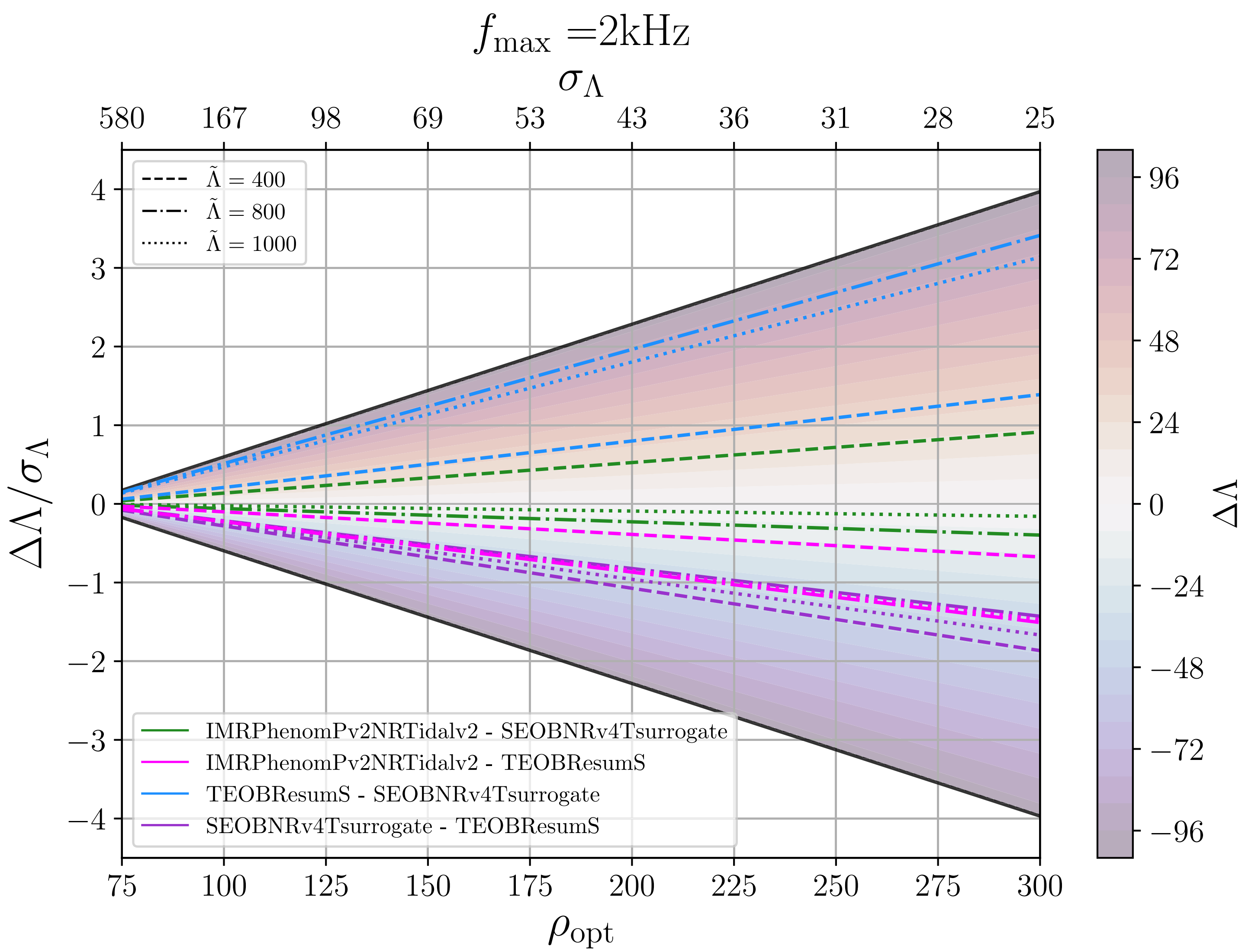
Current estimated BNS merger rate $320^{+490}_{-240}\ \mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$ [LVC 2021]

Projected EOS constraints from expected BNS coalescences

For GW170817, $\sigma_{\tilde{\Lambda}} \sim 650$ at SNR 32.
For GW170817-like BNS, for A+, $\sigma_{\tilde{\Lambda}} \sim 100$ and $\Delta R \sim 500\ \mathrm{m}$ at SNR~200,
for 3G, $\sigma_{\tilde{\Lambda}} \sim 20$ and $\Delta R \sim 100\ \mathrm{m}$ at SNR~1000. [Landry+'20; Chatziioannou'22;⋯]

Systematic bias $\Delta\tilde{\Lambda}$ vs statistical uncertainty $\sigma_{\tilde{\Lambda}}$ with 3G detectors

[Gamba+'21]



$\Delta\tilde{\Lambda}/\sigma_{\tilde{\Lambda}} \sim 1$ at SNR~175-200 for all $\tilde{\Lambda}$ values.

Therefore, in 3G detectors era, systematic bias will be larger than statistical uncertainty for BNS GW waveform models.

→ **Further improve waveform model to avoid bias.**

Tidal polarizabilities

(Tidal-induced) multipole moment

Companion's tidal field

dimensionless tidal polarizabilities

"Mass"-type quadrupole

$$\mathcal{Q}_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\Lambda \equiv \frac{\lambda}{m^5}$$

"Current"-type quadrupole

$$\mathcal{S}_{ij} = -\sigma \mathcal{H}_{ij}$$

$$\Sigma \equiv \frac{\sigma}{m^5}$$

"Mass"-type octupole

$$\mathcal{Q}_{ijk} = -\lambda^{(3)} \mathcal{E}_{ijk}$$

$$\Lambda^{(3)} \equiv \frac{\lambda^{(3)}}{m^7}$$

[cf. Luc Blanchet's talk in symposium]

Complete and correct PN tidal waveform, 7.5PN

Update for the "mass" quad. (Λ), "current" quad. (Σ), and "mass" oct. ($\Lambda^{(3)}$)
by Henry, Faye, Blanchet, '20 (HFB-form). within MPM-PN formalism

Tidal polarizabilities Mass quad. Λ Current quad. Σ Mass oct. $\Lambda^{(3)}$

φ_{tidal}	Mass Quadrupole	Current Quadrupole	Mass Octupole
5PN (L)	[FH08, F14, DNV12, VF13, VHF11] ✔	✗	✗
6PN (NL)	[DNV12, VF13, AGP18] ✔	[VF13, AGP18] ✓	✗
6.5PN (tail)	[DNV12, AGP18] ✓	✗	✗
7PN (NNL)	✓	✓	[AGP18, L18] ✓
7.5PN (tail)	✓	✓	✓

[FH08] Flanagan, Hinderer '08; [F14] Favata '14; [DNV12] Damour, Nagar, Villain '12; [VF13] Vines, Flanagan '13; [VHF11] Vines, Hinderer, Flanagan '11; [AGP18] Abdelsalhin, Gualtieri, Pani '18; [BV20] Banihashemi, Vines '20; [L18] Landry '18

The contributions obtained by Henry, Faye, Blanchet, '20 are indicated as a check mark ✔.

Here, uncalculated coefficients at 7PN order are completed and coefficients at 7.5PN order for mass quad. are corrected.

Rewrite HFB-form to "more familiar" form for Λ

convenient form for data analysis [Narikawa, Uchikata, T. Tanaka '21]

Λ Mass quadrupole $G\mu_A^{(2)} \equiv \left(\frac{Gm_A}{c^2}\right)^5 \Lambda_A = \frac{2}{3} k_A^{(2)} R_A^5,$ **5-7.5PN**

$$\Psi_{\rm Tidal}^{\rm Mass-Quad}(f) \sim -\tilde{\Lambda} x^{5/2} \left[1 + a_{\rm 6PN}^{\rm Mass-Quad} x + a_{\rm 6.5PN}^{\rm Mass-Quad} x^{3/2} + a_{\rm 7PN}^{\rm Mass-Quad} x^2 + a_{\rm 7.5PN}^{\rm Mass-Quad} x^{5/2}\right]$$

complete 7PN **corrected 7.5PN**

$$x = (\pi M f)^{2/3}$$

The component form is used for PN tidal base of NRTidalv3 (latest NR calibrated model).

[Abac+ '23 and private communication for corrections with them.]

Rewrite HFB-form to "more familiar" form for Σ and Λ(3)

convenient form for data analysis [Narikawa '23]

Σ Current quadrupole $G\sigma_A^{(2)} \equiv \left(\frac{Gm_A}{c^2}\right)^5 \Sigma_A = \frac{1}{48} j_A^{(2)} R_A^5,$ **6-7.5PN**

$$\Psi_{\rm Tidal}^{\rm Current-Quad}(f)$$
$$\sim -\tilde{\Sigma} x^{5/2}\left[x + a_{\rm 7PN}^{\rm Current-Quad} x^2 + a_{\rm 7.5PN}^{\rm Current-Quad} x^{5/2}\right]$$

$\Lambda^{(3)}$ Mass octupole $G\mu_A^{(3)} \equiv \left(\frac{Gm_A}{c^2}\right)^7 \Lambda_A^{(3)} = \frac{2}{15} k_A^{(3)} R_A^7,$ **7PN**

$$\Psi_{\rm Tidal}^{\rm Math-Oct}(f) \sim -\tilde{\Lambda}^{(3)} x^{5/2}\left[x^2\right]$$

$$x = (\pi M f)^{2/3}$$

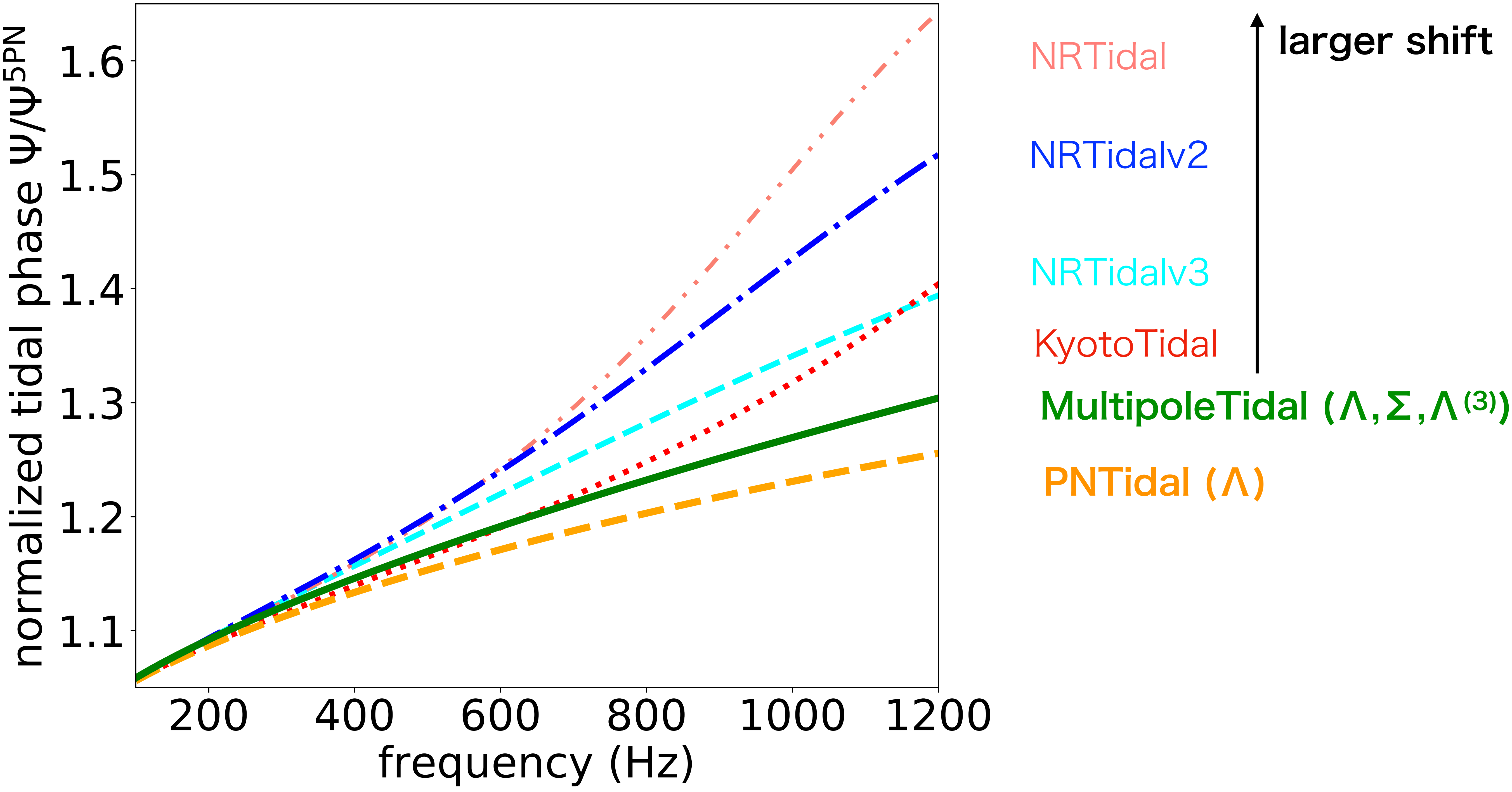
Here, coefficients up to 7.5PN order are completed.

Phase evolution

Compare waveform models

With quasiuniversal (Multipole Love) relations [Yagi '13]

Equal mass $m_A=m_B=1.35\,M_\odot$, $\Lambda_A=\Lambda_B=300$, $\Sigma_A=\Sigma_B=3.1$, $\Lambda^{(3)}_A=\Lambda^{(3)}_B=483$



MultipoleTidal ($\Lambda,\Sigma,\Lambda^{(3)}$) gives a larger phase shift than **PNTidal (Λ)**, and is closer to the NRTidalv2.

Application to GW170817

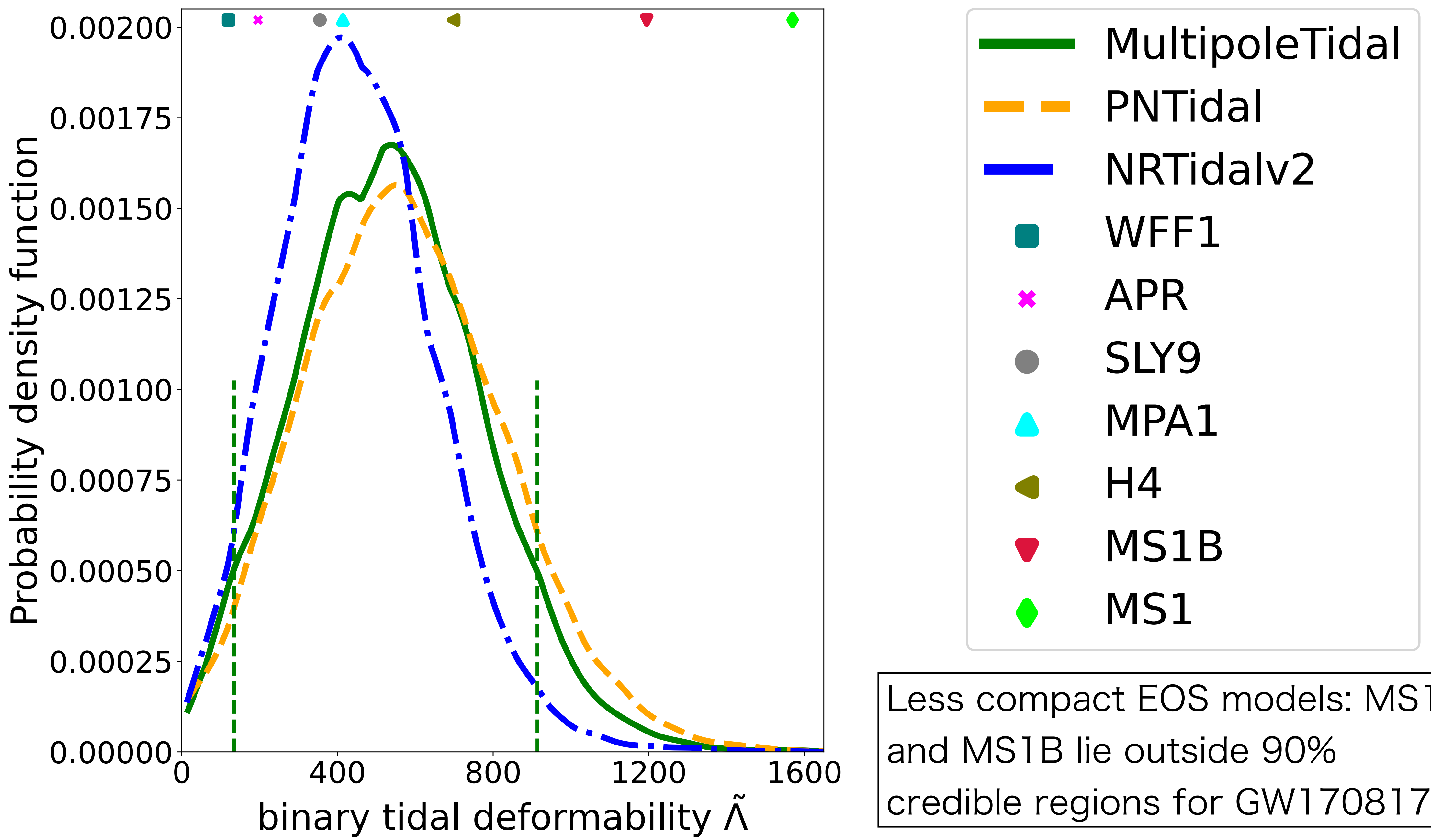
Compare waveform models

With quasiuniversal (Multipole Love) relations [Yagi '13]

Posteriors of $\tilde{\Lambda}$

GW170817

low-spin prior $|\chi|<0.05$, $f_{high}=1000$ Hz



Less compact EOS models: MS1 and MS1B lie outside 90% credible regions for GW170817.

MultipoleTidal (Λ,Σ,$\Lambda^{(3)}$) gives a smaller inferred $\tilde{\Lambda}$ than **PNTidal (Λ)**, and is closer to the NRTidalv2, which is consistent with the phase shift.

MultipoleTidal (Λ,Σ,$\Lambda^{(3)}$) is not significant impact on the estimates of $\tilde{\Lambda}$ for GW170817 (consistent with the results in [Pradhan+ '22]).

Summary

Rewrite the updated PN tidal phase for Λ, Σ, and $\Lambda^{(3)}$ to the convenient form for the data analysis.
It is useful for waveform modeling.

Conclusion

MultipoleTidal (Λ,Σ,$\Lambda^{(3)}$) gives a smaller inferred $\tilde{\Lambda}$ than **PNTidal (Λ)**, and is closer to the **NRTidalv2**, which is consistent with the phase shift.

MultipoleTidal (Λ,Σ,$\Lambda^{(3)}$) is not significant impact on the estimates of $\tilde{\Lambda}$ for GW170817.

Thank you very much for the attention.