Magnetically Confined Mountains on Neutron Stars in General Relativity

Pedro H. B. Rossetto

OzGrav - The ARC Centre of Excellence for Gravitational Wave Discovery
School of Physics
University of Melbourne

Outline

1. Background and Motivation

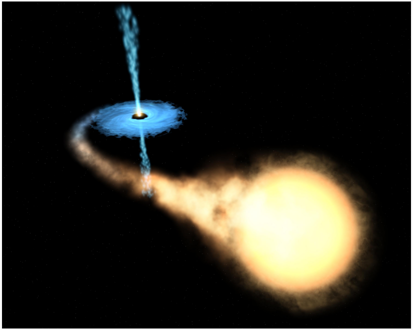
2. Magnetically Confined Mountains on Neutron Stars

3. Hydromagnetic Structure

4. Continuous Gravitational Waves

Background and Motivation

Spin-up of Neutron Stars

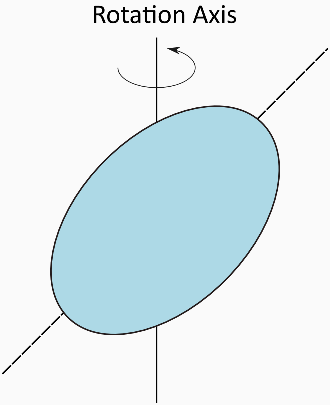


ESA, NASA, and Felix Mirabel

- Neutron stars (NSs) are sometimes found in binary systems such as Low Mass X-Ray Binaries (LMXBs);
- The infalling matter should spin-up NS close to their breaking frequency. But that is not experimentally verified[1];
- NSs in accreting systems also show reduced magnetic fields.

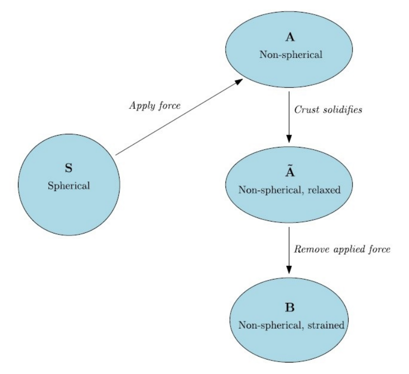
[1] Chakrabarty et al., 2003

Spin-down Mechanisms



- We need mechanisms for the NS to lose angular momentum;
- One possibility is for the star to spin down due to GW emission;
- Therefore, we need a time-varying quadrupole;
- For a rotating star, this can be non-axisymmetric deformations, the so-called 'mountains'.

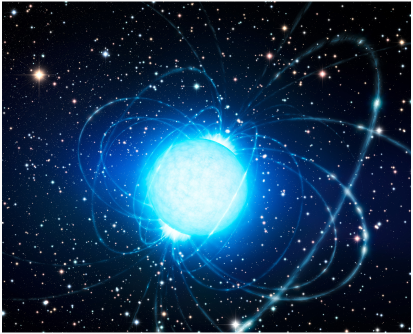
Elastic of Mountains



Gittins, Andersson and Jones, 2021

- The two main categories of mountains are: elastic (see image) and magnetic;
- The elastic mountains come from elastic deformation of the crust;
- Mountain size is determined by the maximum stress the crust can sustain.

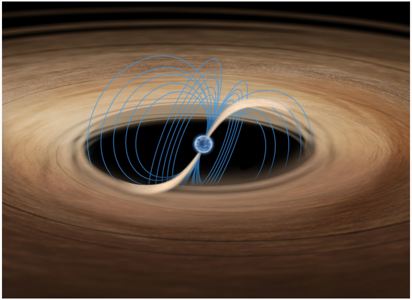
Magnetic Mountains



- Magnetic mountains are present in isolated and accreting systems;
- In isolated systems, the presence of magnetic field itself deforms the NS away from spherical symmetry;
- This is more relevant for magnetars and depends on the internal magnetic field.

Magnetically Confined Mountains on Neutron Stars

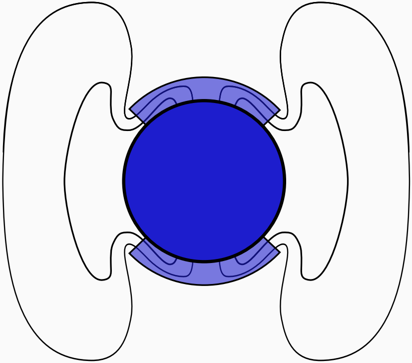
Magnetically Driven Accretion



Source: NASA

- In the inner regions of the accretion disk, the fluid motion is dominated by the magnetic field (magnetosphere);
- Matter is accreted on the magnetic poles;
- The accreted plasma distorts the magnetic field of the star.

Magnetically Confined Mountain



- Mountains start to form in the polar regions;
- Gravity tries to smooth out the mountain;
- The mountain deforms the magnetic field;
- Magnetic field resists and holds the mountains in place.

Newtonian Analysis

- The Newtonian analysis of this mechanism is already very explored in the literature [2];
- The overall result is a decrease of the magnetic dipole moment of the star and an increase of the mass-quadrupole moment;
- These results partially help understand the observed properties of LMXBs.

[2] See, for example, Uchida, 1981; Payne and Melatos, 2004; Vigelius and Melatos, 2008; Wette et al., 2010; Fujisawa et al., 2022.

Formulation of the Problem in General Relativity

- The goal of my research was to approach this problem in General Relativity;
- The first principle equations are:

Rest mass conservation

$$\nabla_a(\rho u^a) = 0$$

Conservation of Energy Momentum

$$\nabla_a T^{ab} = \nabla_a(T^{ab}_{fluid} + T^{ab}_{EM}) = 0$$

Formulation of the Problem in General Relativity

Einstein's Equations

$$G_{ab} = 8\pi T_{ab}$$

Maxwell's Equations

$$\mathrm{d}\mathbf{F} = 0$$

$$\mathrm{d} \star \mathbf{F} = \star \mathbf{J}$$

Relevant Approximations

- In the small accretion limit, we fix a background geometry of spacetime as Schwarzschild;

$$\mathrm{d}s^2 = -e^{2\Phi}\,\mathrm{d}t^2 + e^{-2\Phi}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2$$

where

$$\Phi(r) = \frac{1}{2}\ln\left(1 - \frac{2M_*}{r}\right)$$

- The fluid is a perfect conductor ($\sigma \to \infty$)

$$E_b = u^a F_{ab} = 0$$

Relevant Approximations

- The system evolves through quasi-static steps of magnetostatic equilibrium;

$$u^a = e^{-\Phi} t^a$$

- The system is axisymmetric ($\mathcal{L}_\lambda F = 0$), where $\lambda_a = (\mathrm{d}\phi)_a$ is the axisymmetric Killing vector;
- The magnetic field is poloidal;

$$F = \frac{2}{|\lambda|} \mathrm{d}\phi \wedge \mathrm{d}\psi$$

- Using these approximations in the first principle equations, we obtain:

GR Grad-Shafranov Equation

$$\frac{1}{\lambda}\left(\nabla^a\nabla_a\psi - \frac{1}{\lambda}\nabla^a\lambda\nabla_a\psi\right) = -\rho h F'(\psi)$$

In spherical coordinates:

$$\frac{1}{r^2\sin^2\theta}\left[\frac{\partial^2\psi}{\partial r^2} - \frac{\partial}{\partial r}\left(\frac{2M}{r}\frac{\partial\psi}{\partial r}\right) + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta}\right)\right] = \\ = -\rho h F'(\psi)$$

The Grad-Shafranov Equation

- The function $F(\psi)$ is defined self-consistently by the magnetic flux freezing condition and imposing a rest mass-flux ratio.

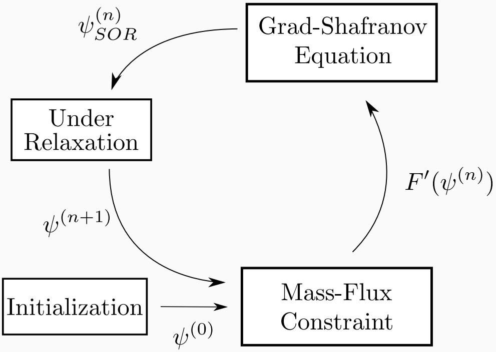
GR self-consistant F

$$F(\psi) = \left(\frac{\mathrm{d}M}{\mathrm{d}\psi}\right)^{1+c_s^2} \left(\frac{2\pi}{c_s^2} \int_C r \sin\theta |\nabla\psi|^{-1} e^{-(\Phi-\Phi_0)/c_s^2} \, \mathrm{d}s\right)^{-(1+c_s^2)}$$

- The general form of the problem is maintained in GR.

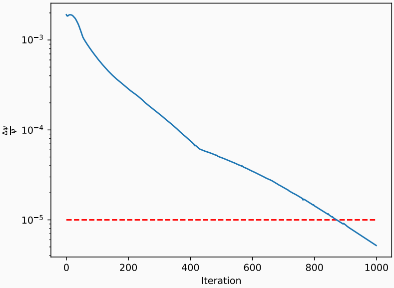
Numerical Approach

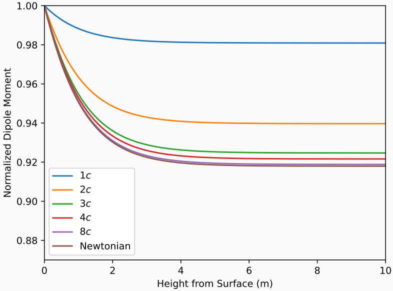
- The GR Grad-Shafranov equation is solved with Successive Over Relaxation (SOR);
- The mass-flux constraint is maintained through an iterative process.



Numerical Convergence

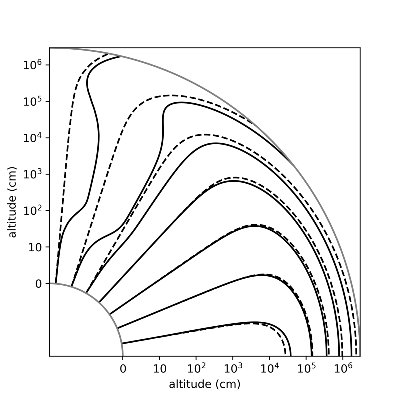
- The system does reach a final equilibrium.
- The relativistic results reproduce the Newtonian one in the appropriate limit ($c \to \infty$).



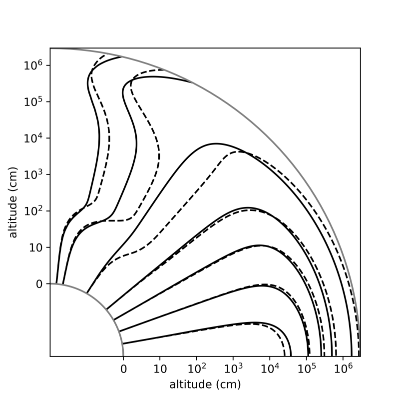


Hydromagnetic Structure

- The distortion of the magnetic field lines is attenuated in GR.

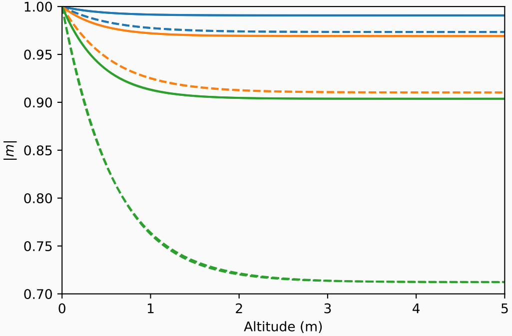


(a) GR vs. Dipole



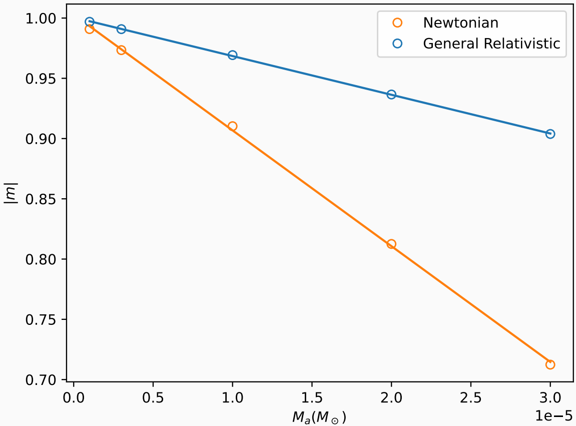
(b) GR vs. Newtonian

- The attenuation of the magnetic field is also found in the magnetic dipole moment:



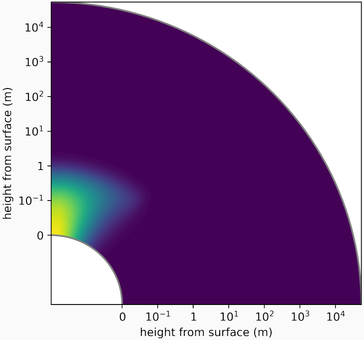
Reduction of the magnetic dipole for different masses: $10^{-6} M_\odot$ (blue), $10^{-5} M_\odot$ (orange) and $3 \times 10^{-5} M_\odot$ (orange). General relativistic results in solid lines, and Newtonian in dashed lines.

- And in the total magnetic dipole moment per accreted mass:

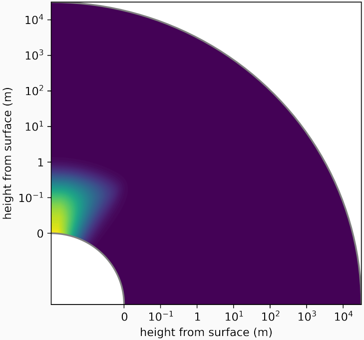


- In the plot above the GR curve is 3 times less steep than the Newtonian one!

- Because of the different deformation of the magnetic field, the density distribution also changes.



(a) Newtonian



(b) GR

Paper Published

- These results were summarized in a paper published at MNRAS.

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Magnetically confined mountains on accreting neutron stars in general relativity

Pedro H B Rossetto, Jörg Frauendiener, Ryan Brunet, Andrew Melatos

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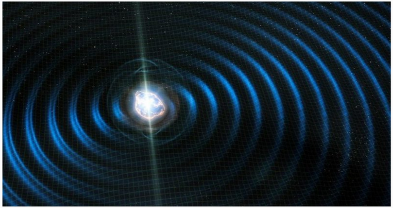
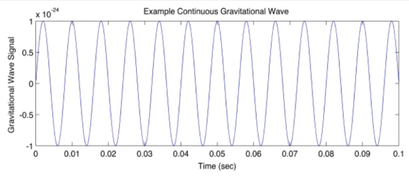
Continuous Gravitational Waves

Continuous Gravitational Waves

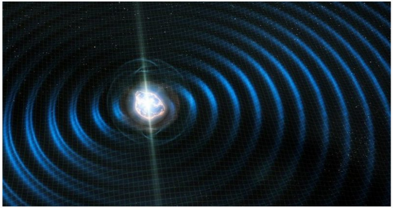


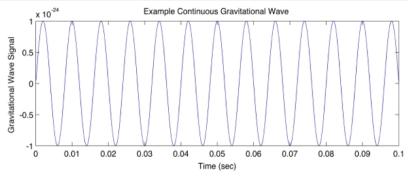


scienceblog.com/496929/ with alterations.

- Continuous gravitational waves (CGWs) are GW with constant frequency;
- The best source candidates for these waves are massive rotating non-axisymmetric objects;

Continuous Gravitational Waves

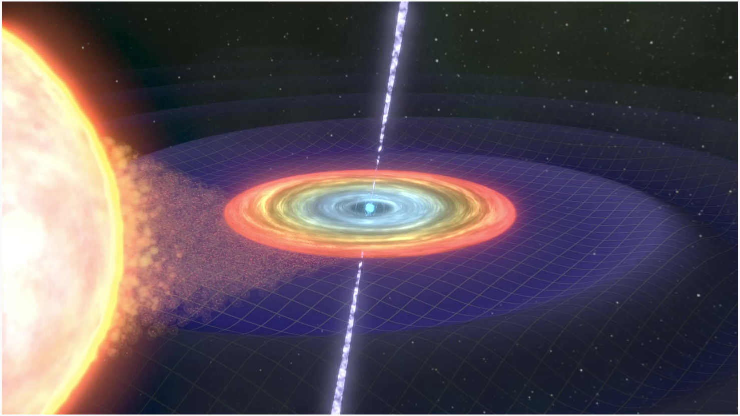




scienceblog.com/496929/ with alterations.

- Continuous gravitational waves (CGWs) are GW with constant frequency;
- The best source candidates for these waves are massive rotating non-axisymmetric objects;
- That is: neutron stars with mountains!

- In accreting systems, the continuous aspect is due to the spin torque balance;



Source: OzGrav ARC Centre of Excellence

Ellipticity

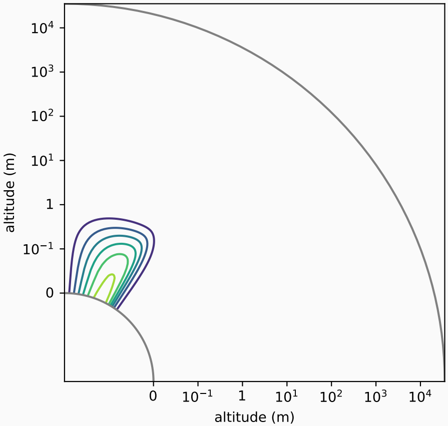
- Given the hydromagnetic structure of the solutions, we can calculate the mass ellipticity of the star:

$$\epsilon = \frac{\pi}{I_0} \int_{V'} \left(e + \frac{B^2}{2} \right) r'^4 \sin\theta (3\cos^2\theta - 1) \, \mathrm{d}r' \, \mathrm{d}\theta$$

- Which is related to the amplitude of continuous gravitational waves via:

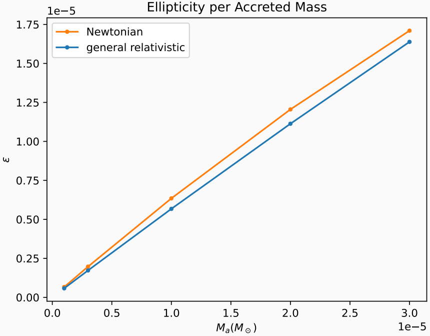
$$h_0 = \frac{4\pi^2 I_0 f_{GW}^2}{r} \epsilon$$

Magnetic Contribution



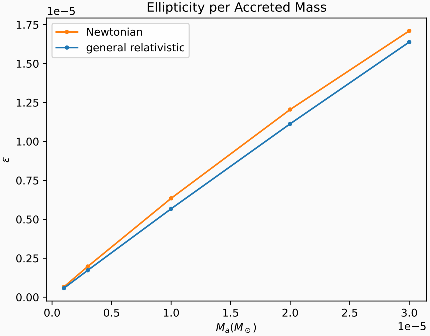
- An interesting theoretical contribution is due to the magnetic field $B^2/2$;
- In our case the ellipticities are small, in the order of $\epsilon \sim 10^{11}$;
- This contribution is relevant for magnetars, but their rotation frequency is small which hinders gravitational wave emission.

Ellipticity



- We observe a reduction of the ellipticity of the star in GR;
- For some parameters of the model, the difference between relativistic and Newtonian can get to 10%;
- The ellipticity values are too high due to the simplifying assumptions of the model (isothermal equation of state and rigid stellar surface).

Ellipticity



- We observe a reduction of the ellipticity of the star in GR;
- For some parameters of the model, the difference between relativistic and Newtonian can get to 10%;
- The ellipticity values are too high due to the simplifying assumptions of the model (isothermal equation of state and rigid stellar surface).

Conclusion

- GR has an important role to play in the model of magnetically confined mountains on neutron stars;
  - The relativistic effects on the magnetic properties of the star are large, yielding three times less magnetic screening;
  - Relativistic effects reduce the ellipticity of the star, reducing the amplitude of gravitational waves;
- The general relativistic model needs to be expanded.

Future Work

- Several improvements can be done as future work in order to have more complete and astrophysically relevant model. They include:
- – Generalising the equation of state;
- – Consider time-dependent effects, mountain sinking and Ohmic diffusion;
- – Analyse the stability using GRMHD solvers, such as *GRHydro*;
- – Consider the effects of the neutron star rotation in the curvature of spacetime.

Acknowledgements

I want to thank:

- The organizing committee for the invitation to give this presentation;
- My supervisors, Prof. Jörg Frauendiener and Prof. Andrew Melatos;
- And all of you have listened to the talk.

Thank you!

Any questions?