LISA (and PTA) and γ-ray telescopes as multi-messenger probes of a first-order cosmological phase transition

YITP Workshop “Gravity and Cosmology”
Yukawa Institute for Theoretical Physics, Kyoto University
Feb. 21, 2024

Alberto Roper Pol
ろぺる ぽる あるべると
University of Geneva

SNSF Ambizione grant: “*Exploring the early universe with gravitational waves and primordial magnetic fields*”

Collaborators: A. Brandenburg (Nordita), T. Boyer (APC), C. Caprini (UniGe/CERN), T. Kahniashvili (CMU), A. Kosowsky (PittU), S. Mandal (CMU), A. Neronov (APC/EPFL), S. Procacci (UniGe), D. Semikoz (APC)

arXiv: 1903.08585, 2009.14174, 2201.05630, 2307.10744, 2308.12943

`https://github.com/AlbertoRoper/cosmoGW` [CosmoGW]

Cosmological GW background

Cosmological GWs have the potential to provide us with *direct information on early universe physics* that is *not accessible via electromagnetic observations, possibly complementary to collider experiments*:

nature of first-order phase transitions (baryogenesis, BSM physics, high-energy physics),
primordial origin of intergalactic magnetic fields.

Probing the early Universe with GWs

Cosmological (pre-recombination) GW background

- Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe.

$$f_* \simeq 1.64 \times 10^{-3} \frac{100}{R_* \mathcal{H}_*} \frac{T_*}{100\,\mathrm{GeV}}\,\mathrm{Hz}$$

- Phase transitions
  - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz
    Peccei-Quinn, B-L, left-right symmetries $\sim 10^7, 10^8$ GeV.
  - Space-based detectors **(LISA)** frequencies are 10^{-5}–10^{-2} Hz
    Electroweak phase transition $\sim$ 100 GeV
  - Pulsar Timing Array (PTA) frequencies are 10^{-9}–10^{-7} Hz
    Quark confinement (QCD) phase transition $\sim$ 100 MeV

- From inflation
  - *B*-modes of CMB anisotropies ($f_c \sim 10^{-18}$ Hz).
  - Can cover all *f* spectrum, depending on end-of-reheating T, and blue-tilted (beyond slow-roll inflation).

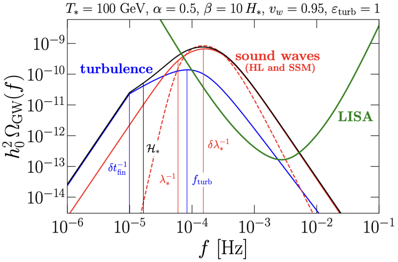
GW sources in the early universe

- Magnetohydrodynamic (MHD) sources of GWs:
    - Sound waves generated from first-order phase transitions.
    - Primordial magnetic fields.
    - (M)HD turbulence from first-order phase transitions.

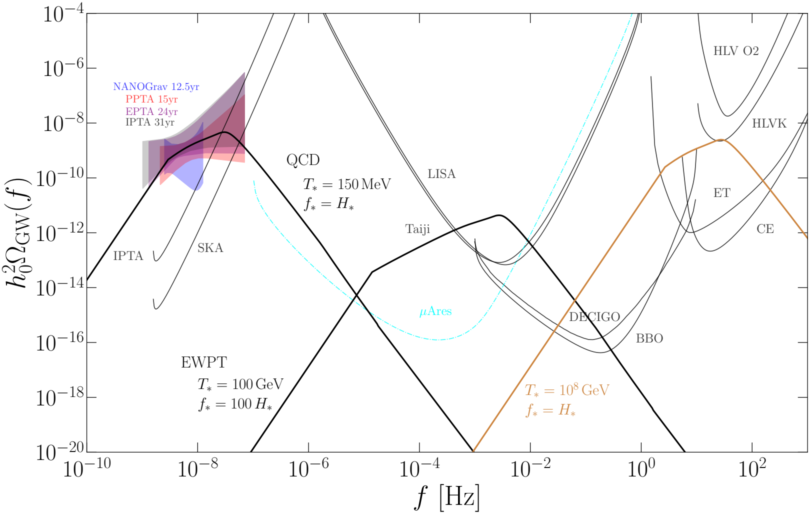
- High-conductivity of the early universe leads to a high-coupling between magnetic and velocity fields.

- Other sources of GWs include
    - Bubble collisions.
    - Cosmic strings.
    - Primordial black holes.
    - Inflation.

ARP *et al.*, 2307.10744, 2308.12943



Gravitational spectrum (turbulence from PTs)[1]

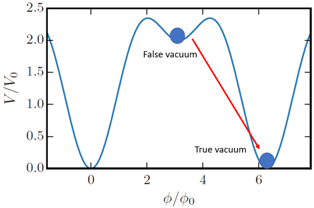


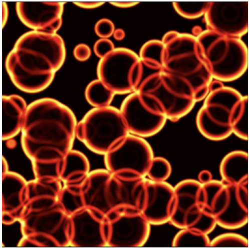
---

[1] ARP, C. Caprini, A. Neronov, D. Semikoz, *PRD* **105**, 123502 (2022)
A. Neronov, ARP, C. Caprini, D. Semikoz, *PRD* **103**, L041302 (2021)
ARP *et al.*, arXiv:2307.10744 (2023).

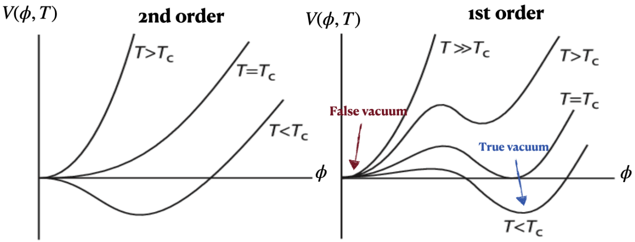
First-order phase transition

$$V(\phi, T) = \frac{1}{2}M^2(T)\phi^2 - \frac{1}{3}\delta(T)\phi^3 + \frac{1}{4}\lambda\phi^4$$



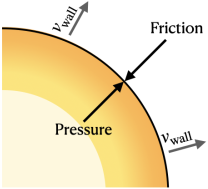


Credits: I. Stomberg



Hydrodynamics of first-order phase transitions[2]

- Broken-phase bubbles are nucleated and expand
- Friction from particles yield a terminal velocity ξ_w of the bubbles
- The bubble can run away when the friction is not enough to stop the bubble's acceleration



$$\nabla_\mu T^{\mu\nu}_{\text{field}} = \frac{\partial V}{\partial \phi}\partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi\,,$$

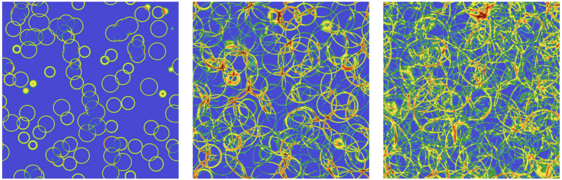
$$\nabla_\mu T^{\mu\nu}_{\text{fluid}} = -\frac{\partial V}{\partial \phi}\partial^\nu \phi - \eta u^\mu \partial_\mu \phi \partial^\nu \phi\,,$$

---

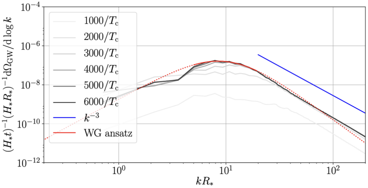
[2] Espinosa, Konstandin, No, Servant, *JCAP* **06** (2010) 028.

GWs from sound waves[3]

- Numerical simulations of the scalar + fluid system can be performed including an effective friction term



- Two scales are found that determine the GW spectrum: R_* and ΔR_* (sound-shell thickness).



(b) Intermediate, $v_{\mathrm{w}} = 0.92$

---

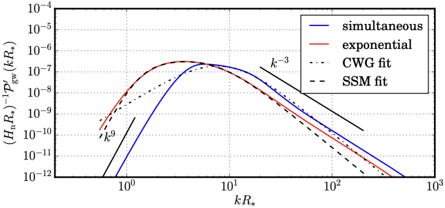
[3] Hindmarsh *et al.*, 2013, 2015, 2017.

GWs from sound waves: Sound Shell Model[4]

- The sound shell model assumes linear superposition of velocity fields from each of the single bubbles and averages over nucleation locations and bubbles lifetimes (semi-analytical model), and the development of sound waves at the time of collisions.

$$\Omega_{\rm GW}(f) \propto S(f)\, K^2\, \tau_{\rm sw}\, R_*$$

- It predicts a steep k^9 spectrum and linear growth with time, according to HH19, and k^{-3} at large frequencies, with an intermediate k between $1/R_*$ and $1/\Delta R_*$.



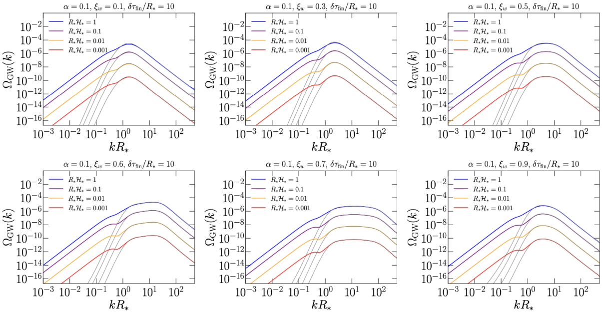
(b) Intermediate, $v_{\rm w} = 0.92$

---

[4] Hindmarsh, 2016; Hindmarsh & Hijazi, 2019.

GWs from sound waves: Sound Shell Model revisited[5]

- Extended Sound Shell model to an expanding Universe and omitted assumptions that were not holding at small k.
- Recovered k^3 at small frequencies and found a $\ln^2(1 + \delta\tau_{\rm sw} H_*)$ time evolution of the causal branch and the "linear-in-time" evolution $\Upsilon = 1 - 1/(1 + \delta\tau_{\rm sw} H_*)$ around the peak, as well as a sharp bump.



---

[5] ARP *et al.*, Phys. Rev. D, arXiv:2308.12943.

Primordial magnetic fields

- Magnetic fields can either be produced at or present during cosmological phase transitions.
- The magnetic fields are strongly coupled to the primordial plasma and inevitably lead to MHD turbulence.[6]
- Present magnetic fields can be amplified by primordial turbulence via dynamo.[7]

---

[6] J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

[7] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. Fluids* **4**, 024608 (2019).

Generation of primordial magnetic fields

- Bubble collisions and velocity fields induced by first-order phase transitions can amplify seed magnetic fields.
- Parity-violating processes during the EWPT are predicted by SM extensions that account for baryogenesis and can produce helical magnetic fields through sphaleron decay or B+L anomalies.[8]

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - i\frac{2\sin\theta_w}{gv^2}\boldsymbol{\nabla}\Phi^\dagger \times \boldsymbol{\nabla}\Phi$$

- Axion fields can amplify and produce magnetic field helicity.[9]

$$\mathcal{L} \supset \frac{\phi}{f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

---

[8] T. Vachaspati, *Phys. Rev. B* **265**, 258 (1991), T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001), J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).

[9] M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

Generation of primordial magnetic fields

- Inhomogeneities in the Higgs field in low-scale electroweak hybrid inflation.[10]
- Magnetic fields from inflation can be present during phase transitions (non-helical[11] and helical[12]).
- Low-scale (QCD and EWPT) magnetogenesis during reheating.[13]
- Chiral magnetic effect.[14]

---

[10] M. Joyce and M. E. Shaposhnikov, *Phys. Rev. Lett.* **79**, 1193 (1997),
J. García-Bellido *et al.*, *Phys. Rev. D* **60**, 123504 (1999).

[11] M. S. Turner and L. M. Widrow, *Phys. Rev. D* **37**, 2743 (1988).

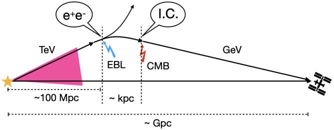
[12] M. Giovannini, *Phys. Rev. D* **58**, 124027 (1998).

[13] R. Sharma, *Phys. Rev. D* **97**, 083503 (2018).

[14] M. Joyce and M. E. Shaposhnikov, *PRL* **79**, 1193 (1997).

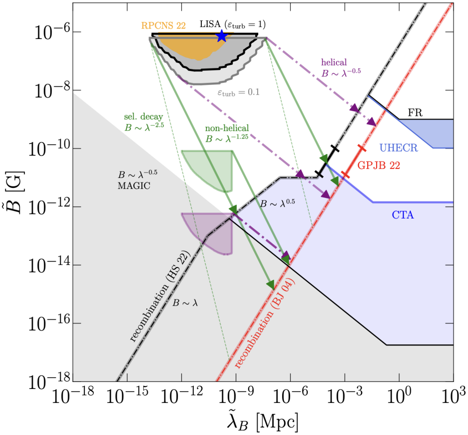
Primordial magnetic fields[3]

- Primordial magnetic fields would evolve through the history of the universe up to the present time and could explain the lower bounds in cosmic voids derived by the Fermi collaboration.[4]



- Maximum amplitude of primordial magnetic fields is constrained by the big bang nucleosynthesis.[5]

- Additional constraints from CMB, Faraday Rotation, ultra-high energy cosmic rays (UHECR).



---

[3] ARP *et al.*, arXiv:2307.10744 (2023).

[4] A. Neronov and I. Vovk, *Science* **328**, 73 (2010).

[5] V. F. Shvartsman, *Pisma Zh. Eksp. Teor. Fiz.* **9**, 315 (1969).

GWs from (M)HD turbulence

- Direct numerical simulations using the PENCIL CODE[15] to solve:
  1. Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken).
  2. Gravitational waves equation.
- In general, large-resolution simulations are necessary to solve the MHD nonlinearities (e.g., unequal-time correlators UETC and non-Gaussianities, which require simplifying assumptions in analytical studies).

---

[15] Pencil Code Collaboration, JOSS **6**, 2807 (2020), `https://github.com/pencil-code/`
ARP *et al.*, *Geophys. Astrophys. Fluid Dyn.* **114**, 130 (2020).

Conservation laws for MHD turbulence

$$T^{\mu\nu}{}_{;\nu} = 0, \quad F^{\mu\nu}{}_{;\nu} = -J^{\mu}, \quad \tilde{F}^{\mu\nu}{}_{;\nu} = 0$$

In the limit of subrelativistic bulk flow:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to[16]

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}\left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho\right) + \frac{1}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right],$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{3}\mathbf{u}\left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho\right) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta J^2\right]$$

$$-\frac{1}{4}\boldsymbol{\nabla} \ln \rho + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho}\boldsymbol{\nabla} \cdot (\rho \nu \boldsymbol{S}),$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J}), \quad \boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B},$$

for a flat expanding universe with comoving and normalized $p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,{\rm phys}}, u_i$, and conformal time t $({\rm d}t = a{\rm d}t_c)$.

---

[16] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996).

GW equation for a flat expanding Universe

- Assumptions: isotropic and homogeneous Universe.
- Friedmann–Lemaître–Robertson–Walker (FLRW) metric $\gamma_{ij} = a^2\delta_{ij}$.
- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = a^2\left(\delta_{ij} + h_{ij}^{\rm phys}\right), \quad |h_{ij}^{\rm phys}| \ll |g_{ij}|$$

- GW equation is[17]

$$\left(\partial_t^2 - \frac{a''}{a} - c^2\nabla^2\right) h_{ij} = \frac{16\pi G}{\mathbf{a} c^2} T_{ij}^{\rm TT}$$

  - h_{ij} are rescaled $h_{ij} = a h_{ij}^{\rm phys}$.
  - Comoving spatial coordinates $\nabla = a\nabla^{\rm phys}$.
  - Conformal time $\mathrm{d}t = a\ \mathrm{d}t_c$.
  - Comoving stress-energy tensor components $T_{ij} = a^4 T_{ij}^{\rm phys}$.
  - Radiation-dominated epoch such that $a'' = 0$.

---

[17] L. P. Grishchuk, *Sov. Phys. JETP* **40**, 409 (1974).

Numerical results for decaying MHD turbulence[18]

Initial conditions

- Initial stochastic magnetic (or velocity) field with fractional helicity $\sigma_{\rm M}$.

$$kB_i(\boldsymbol{k}) = \left(\delta_{ij} - \hat{k}_i \hat{k}_j - i\sigma_{\rm M}\varepsilon_{ijl}\hat{k}_l\right) g_j \sqrt{2\Omega_{\rm M}(k)/k}$$

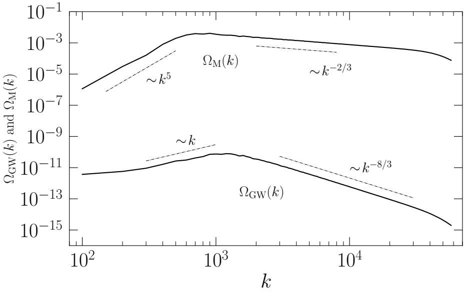
- Batchelor spectrum for magnetic (or vortical velocity) fields, i.e., $\Omega_{\rm M} \propto k^5$ for small $k < k_* \sim \mathcal{O}(\xi_{\rm M}^{-1})$.
- Kolmogorov spectrum in the inertial range, i.e., $\Omega_{\rm M} \propto k^{-2/3}$.

---

[18] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. D* **96**, 123528 (2017).
ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).
ARP *et al.*, *JCAP* **04** (2022), 019.
ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

Numerical results for decaying MHD turbulence[19]

$1152^3, k_* = 2\pi \times 100, \Omega_{\rm M} \sim 10^{-2}, \sigma_{\rm M} = 1$

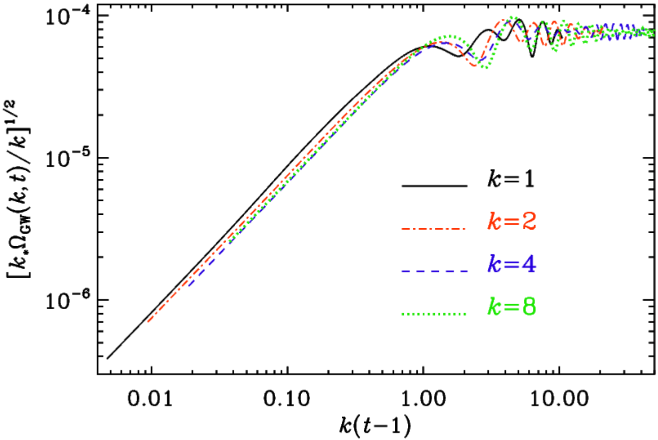


- **Characteristic k scaling in the subinertial range for the GW spectrum.**
- k^2 expected at scales $k < k_*$ and k^3 at $k < H_*$ according to the "top-hat" model (Caprini *et al.*, 2020).

---

[19] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).

Early time evolution of the GW spectrum



Analytical model for GWs from decaying turbulence

- Assumption: magnetic or velocity field evolution $\delta t_{\rm e} \sim 1/(u_* k_*)$ is slow compared to the GW dynamics ($\delta t_{\rm GW} \sim 1/k$) at all $k \gtrsim u_* k_*$.
- We can derive an analytical expression for nonhelical fields of the envelope of the oscillations[20] of $\Omega_{\rm GW}(k)$.

$$\Omega_{\rm GW}(k, t_{\rm fin}) \approx 3 \left(\frac{k}{k_*}\right)^3 {\Omega_{\rm M}^*}^2 \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)}\, p_\Pi\left(\frac{k}{k_*}\right) \\ \times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\, \delta t_{\rm fin} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\, \delta t_{\rm fin} \geq 1. \end{cases}$$

- p_Π is the anisotropic stress spectrum and depends on spectral shape, can be approximated for a von Kárman spectrum as[21]

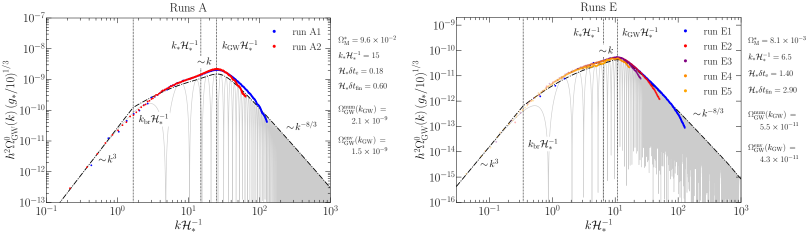
$$p_\Pi(k/k_*) \simeq \left[1 + \left(\frac{k}{2.2 k_*}\right)^{2.15}\right]^{-11/(3\times 2.15)}$$

---

[20] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

[21] ARP *et al.*, arXiv:2307.10744 (2023).

Numerical results for nonhelical decaying MHD turbulence[22]

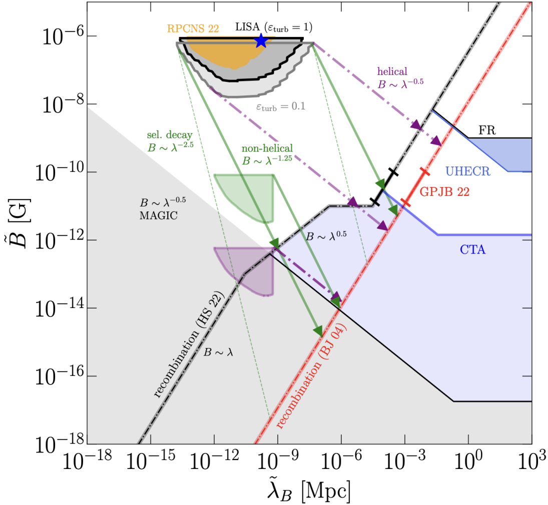


run	$\Omega_{\rm M}^*$	$k_*\mathcal{H}_*^{-1}$	$\mathcal{H}_*\delta t_{\rm e}$	$\mathcal{H}_*\delta t_{\rm fin}$	$\Omega_{\rm GW}^{\rm num}(k_{\rm GW})$	$[\Omega_{\rm GW}^{\rm env}/\Omega_{\rm GW}^{\rm num}](k_{\rm GW})$	n	$\mathcal{H}_*L$	$\mathcal{H}_*t_{\rm end}$	$\mathcal{H}_*\eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	–	–	–	–	–	–	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	–	–	–	–	–	–	512	10π	18	10^{-7}
E3	–	–	–	–	–	–	512	20π	61	10^{-7}
E4	–	–	–	–	–	–	512	30π	114	10^{-7}
E5	–	–	–	–	–	–	512	60π	234	10^{-7}

---

[22] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

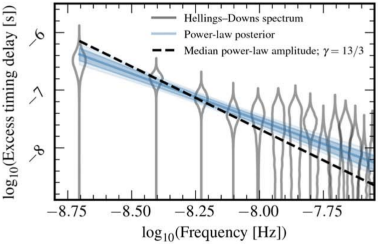
Multi-messenger constraints with LISA and CTA [23]

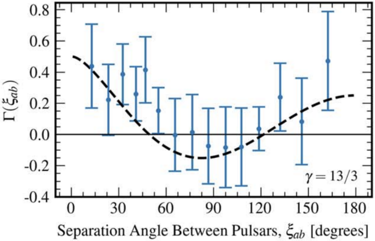


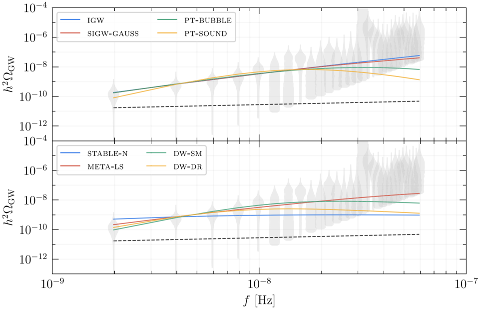
---

[23] ARP *et al.*, arXiv:2307.10744 (2023).

NANOGrav 15 yr data observation[24]



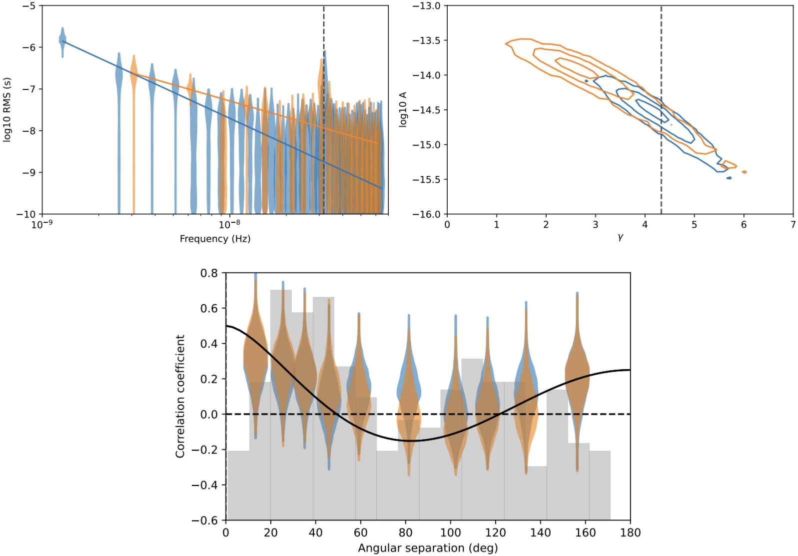




---

[24] [NANOGrav collaboration], *ApJ Lett.* **951**, 8 & 11 (2023).

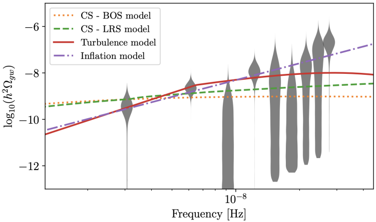
EPTA 24.7 yr data observation (DR 2)[25]

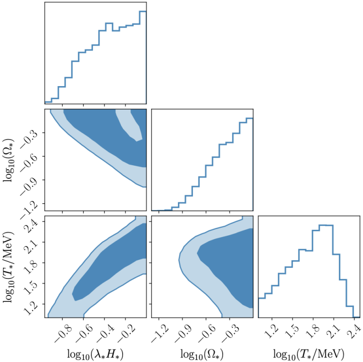


---

[25] [EPTA Collaboration], arXiv:2306.16224.

Primordial magnetic fields constraints with EPTA DR 2[26]





---

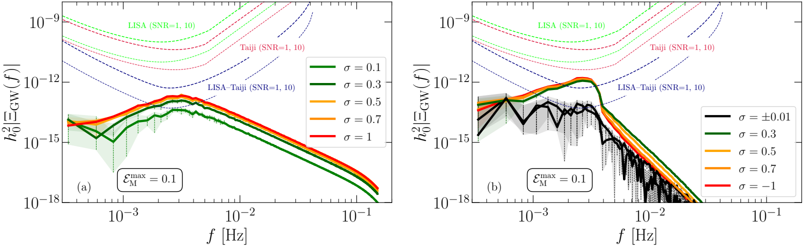
[26] [EPTA collab.] (incl. ARP), arXiv:2306.16227 (2023).

Using LISA and Taiji to detect the GW polarization[29]

- LISA's dipole response function can provide us with a polarized gravitational wave background due to our proper motion.[27]

- Cross-correlation of LISA and an additional space-based GW detector can improve the detectability of a polarized GW background.[28]

$$\mathcal{P}_{\mathrm{GW}}(k)=\frac{\Xi_{\mathrm{GW}}(k)}{\Omega_{\mathrm{GW}}(k)}=\frac{\left\langle \dot{\tilde{h}}_{\times}\dot{\tilde{h}}_{+}^{*}-\dot{\tilde{h}}_{+}\dot{\tilde{h}}_{\times}^{*}\right\rangle}{\left\langle \dot{\tilde{h}}_{+}\dot{\tilde{h}}_{+}^{*}+\dot{\tilde{h}}_{\times}\dot{\tilde{h}}_{\times}^{*}\right\rangle}$$



---

[27] V. Domcke *et al.*, *JCAP* **05** (2020), 028.

[28] G. Orlando, M. Pieroni and A. Ricciardone, *JCAP* **03** (2021), 069.

[29] ARP *et al.*, *JCAP* **04** (2022), 019.

Conclusions

- Velocity and magnetic fields in the early universe can significantly contribute to the stochastic GW background (SGWB) via sound waves and (M)HD turbulence.
- MHD requires, in general, performing high-resolution numerical simulations, which can be done using the PENCIL CODE.
- Since the SGWB is a superposition of different sources, it is extremely important to characterize the different sources, to be able to extract clean information from the early universe physics.
- The interplay between sound waves and the development of turbulence is not well understood. It plays an important role on the relative amplitude of both sources of GWs.
- LISA, PTA, and next-generation ground-based detectors can potentially be used to probe the origin of magnetic fields in the largest scales of our Universe, which is still an open question in cosmology.
- Bubble nuccleation, sound wave production, and magnetogenesis physics can be coupled to our equations for more realistic production analysis (future work).

Thank You!

ありがとう ございます

alberto.roperpol@unige.ch

github.com/AlbertoRoper/cosmoGW
cosmology.unige.ch/users/alberto-roper-pol