PROBING ASTROPHYSICAL ENVIRONMENT WITH EMRIS

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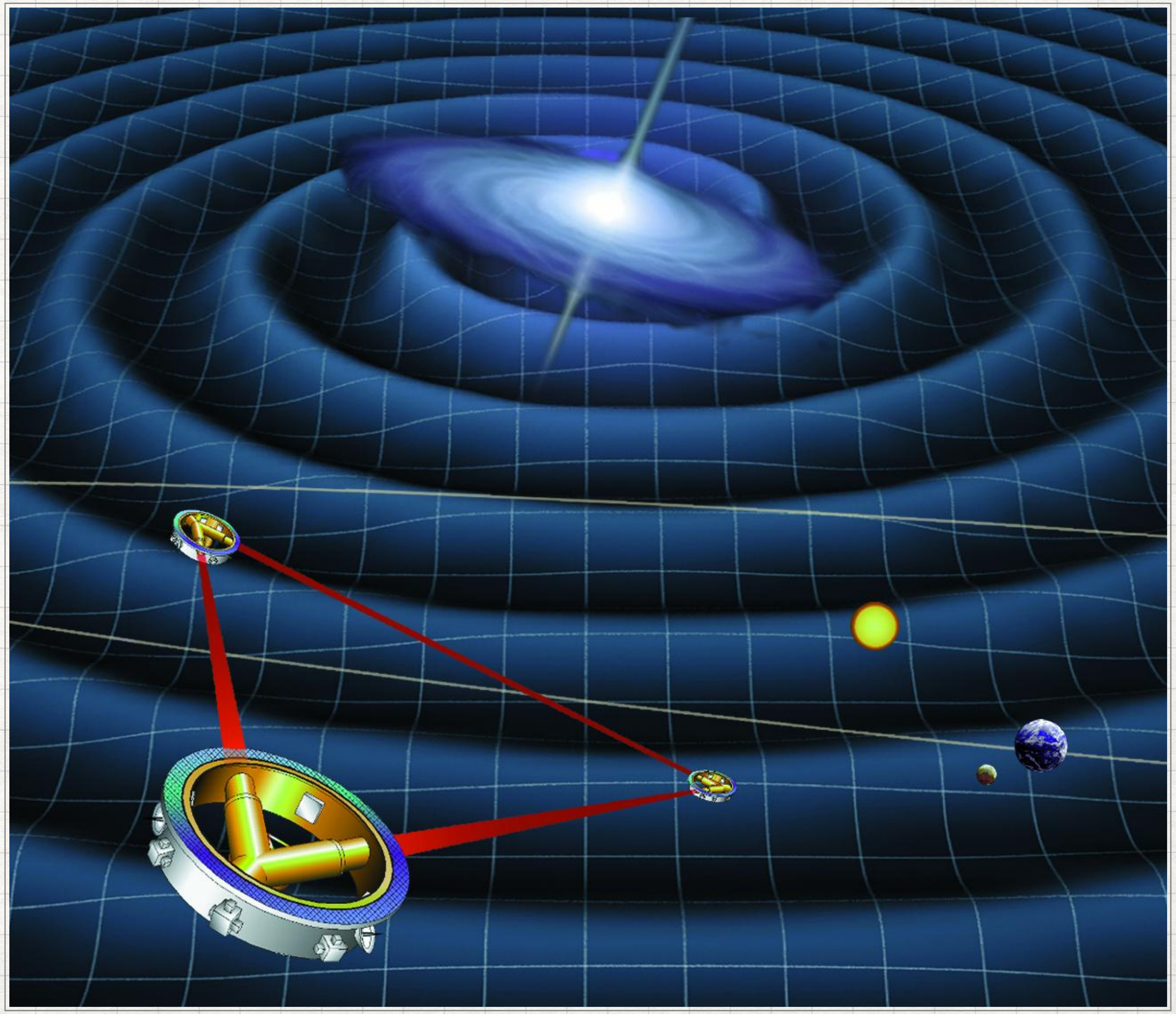

EXTREME MASS RATIO INSPIRAL (EMRI)

EMRI:

- System: Super Massive Black hole (SMBH) : the primary, Stellar Mass compact Object (SMO): the secondary
- Mass Ratio: $q \approx 10^{-4} - 10^{-7}$
- The secondary completes around a million orbits around the primary before plunging
- Thus maps the spacetime around the primary very accurately
- The system emit gravitational waves in millihertz frequencies.

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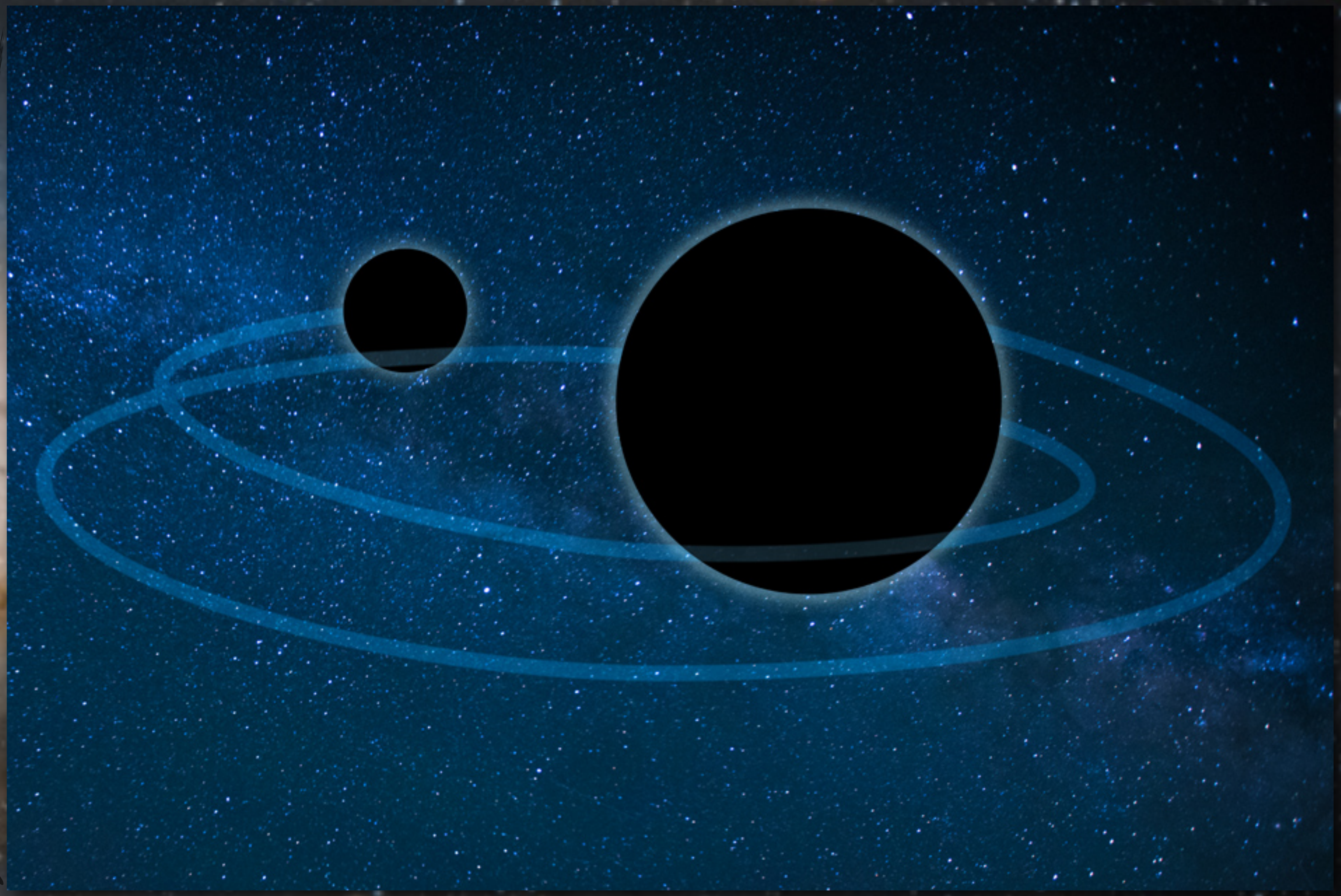

- System: Super Massive Black hole (SMBH) : the primary, Stellar Mass compact Object (SMO): the secondary
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Q: Can we probe the astrophysical environment around the SMBH with EMRI observation?

BLACK HOLE IN DARK MATTER ENVIRONMENT

- **Einstein Cluster Model:** Collisionless particles moving in circular orbits around a static spherically sym. spacetime

[Einstein, Annals Math. 40, 922 (1939)]

$$\langle T_{\mu\nu}\rangle = \frac{n}{m_p}\langle p_\mu p_\nu\rangle \implies T_{\mu\nu} = \mathrm{diag}\{\rho, 0, p_t, p_t\}$$

- $$ds^2 = g^{(0)}_{\mu\nu}dx^\mu dx^\nu = -\left(1-\frac{2M_{\mathrm{BH}}}{r}\right)e^\Gamma dt^2 + \left(1-\frac{2m(r)}{r}\right)^{-1}dr^2 + r^2\left[d\theta^2+\sin^2\theta d\phi^2\right]$$

[Cardoso+, *PRD* 105 (2022) 6,L061501]

$$\Gamma = \sqrt{\frac{M}{\zeta}}\left(-\pi + 2\arctan\left(\frac{r+a_0-M}{\sqrt{M\zeta}}\right)\right), \quad \zeta = 2a_0 - M + 4M_{\mathrm{BH}}$$

- The mass function so chosen that the density follows Hernquist-type density distribution

$$4\pi\rho_{\mathrm{DM}} = \frac{m'(r)}{r^2}; \quad m(r) = M_{\mathrm{BH}} + \frac{Mr^2}{(a_0+r)^2}\left(1-\frac{2M_{\mathrm{BH}}}{r}\right)^2.$$

$$\frac{M}{a_0} \equiv \frac{\text{Mass of the Halo}}{\text{Typical length scale of the Halo}} = \text{Halo Compactness}$$

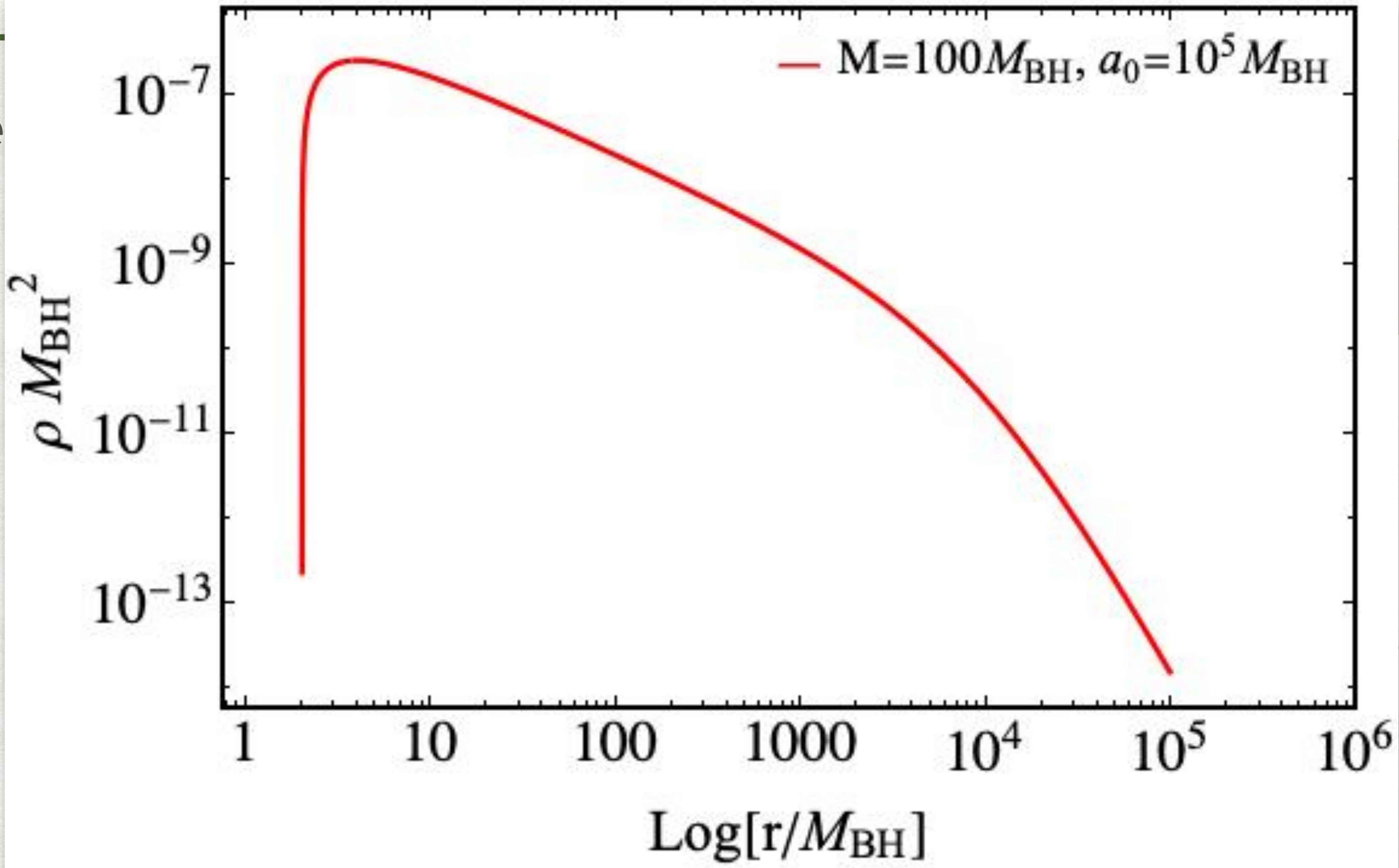
- We examine the possible detectability of dark matter with such a solution acting as a primary source in an EMRI system.

BLACK HOLE IN DARK MATTER ENVIRONMENT

- **Eins** … g in circular orbits around a static spherically sym. space

[Einstein, Annals Math. 40, 922 (1939)]

diag$\{\rho, 0, p_t, p_t\}$



$$\rho = \frac{M\,(a_0 + 2M_{BH})\,(1 - \frac{2M_{BH}}{r})}{2\pi r\,(r + a_0)^3}$$

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- The … Hernquist-type density distribution

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ECCENTRIC ORBITS

- Introduce a particle of mass μ to the spacetime: the secondary, $T_P^{\mu\nu} = \mu \int d\tau \frac{\delta^{(4)}(x^\mu - z_P^\mu(\tau))}{\sqrt{-g}} U_P^\mu U_P^\nu$,

- Consider first the test particle approximation: $\dot{r}^2 = -V_{\text{eff}}(r)$

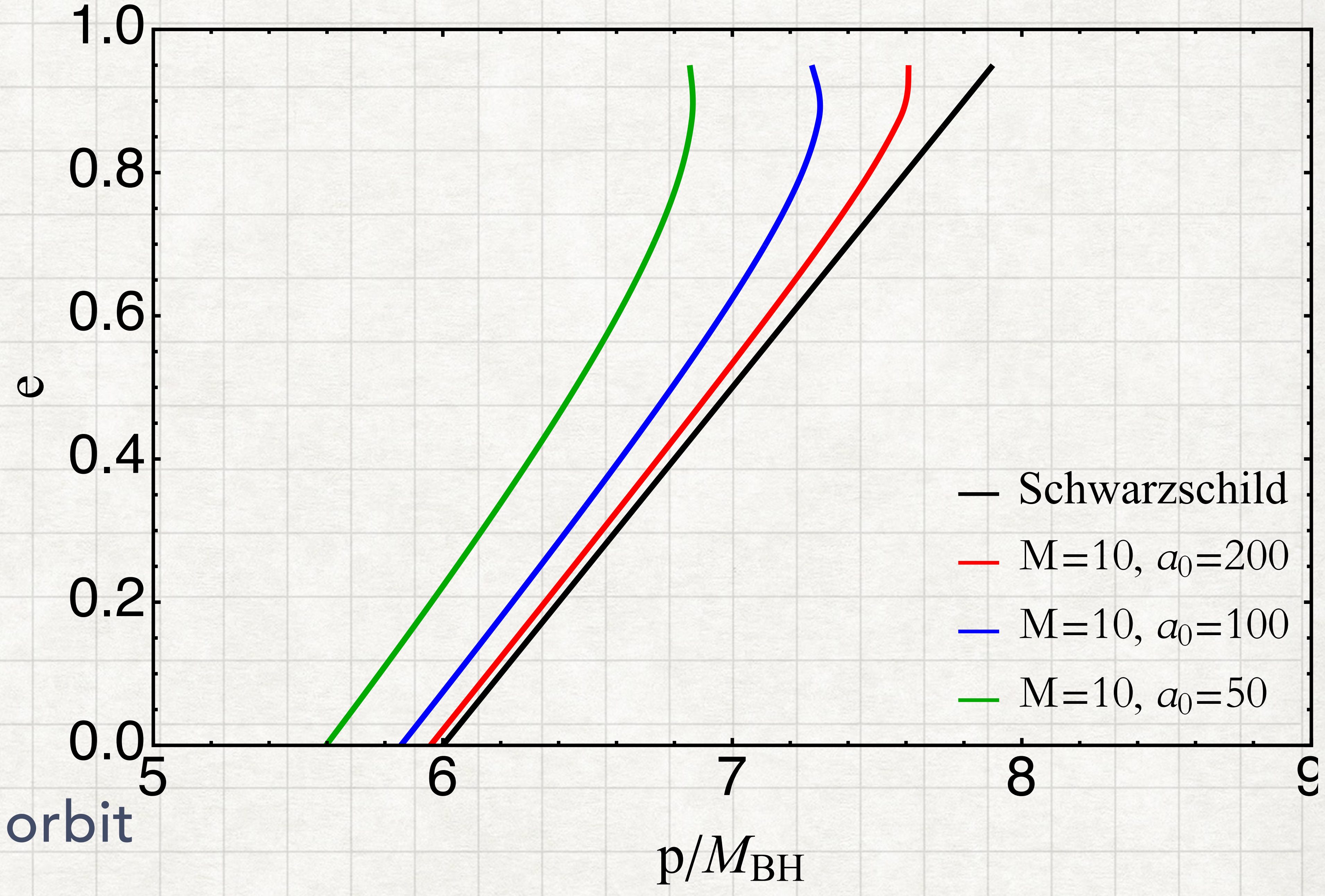
$$V_{\text{eff}}(r_p) = 0\,, \qquad V_{\text{eff}}(r_a) = 0,$$
$$V'_{\text{eff}}(r_p) \leq 0\,, \qquad V'_{\text{eff}}(r_a) > 0\,.$$

$$r_P = r_P(E, J_z)$$
$$r_P = r_P(e, p)$$

- Parameterise: $r = \dfrac{p}{1 + e\cos\chi}$

Equity holds for Unstable circular orbit

$$\Omega_r = -\frac{2\pi}{\int_0^{2\pi} d\chi \frac{E}{g_{tt}\sqrt{-V_{\text{eff}}}}\frac{dr}{d\chi}}\,; \qquad \Omega_\phi \equiv \frac{d\phi}{dt} = -\frac{J_z^2 g_{tt}}{r^2 E}$$



Separatrix

PERTURBATION EQN: REGGE WHEELER ZERILI FORMALISM

- The secondary object perturbs the spacetime: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu}\,; \quad T^{DM}_{\mu\nu} = T^{DM(0)}_{\mu\nu} + T^{DM(1)}_{\mu\nu},$
- Adiabatic Approximation:

$$\left(\frac{dE}{dt}\right)_{orbit} = -\left\langle\frac{dE}{dt}\right\rangle_{\mathrm{GW}}, \qquad \left(\frac{dJ_z}{dt}\right)_{orbit} = -\left\langle\frac{dJ_z}{dt}\right\rangle_{\mathrm{GW}}$$

- In Regge-Wheeler gauge: $g^{(1)}_{\mu\nu} = g^{(1)ax}_{\mu\nu} + g^{(1)pol}_{\mu\nu},\qquad \mathscr{C}^{(1)} = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} \delta\mathscr{C}^{lm}(t,r)Y_{lm}(\theta,\phi)\,, \qquad \mathscr{C} \in \{\rho, p_t, p_r\}$
- The perturbation equation in the axial sector

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V^{ax}\right)\mathscr{R}_{lm\omega} = S^{ax}_{lm\omega}\,, \quad V^{ax} = -\frac{g_{tt}}{r^2}\left(l(l+1) - \frac{6m(r)}{r} + m'(r)\right)\,,$$

- Perturbation equation in the polar sector: $\vec{\psi}_{lm\omega} = (H_1^{lm}, H_0^{lm}, K^{lm}, W^{lm}, \delta\rho^{lm})$

$$(r - r_+)\frac{d\vec{\psi}_{lm\omega}}{dr} = \tilde{\alpha}\vec{\psi}_{lm\omega} + S^{pol}_{lm\omega},$$

- The solution $\vec{\psi}_{lm\omega}(r) = \boldsymbol{\Psi}_{lm\omega}(r)\boldsymbol{\Psi}^{-1}_{lm\omega}(r_B)\vec{\psi}_{lm\omega}(r_B) + \boldsymbol{\Psi}_{lm\omega}(r)\int_{r_B}^{r} dx\ \boldsymbol{\Psi}^{-1}_{lm\omega}(x)\vec{S}^{pol}_{lm\omega}(x)\,, \quad \vec{S}^{pol}_{lm\omega}(x) \propto \delta(r - r_P)$

- Energy and angular momentum flux: $Z^{ax}_{lm\omega} = \mathscr{R}_{lm\omega},\ Z^{pol}_{lm\omega} = \frac{r}{n+1}\left[K^{lm} + \frac{f}{n}\left(H_2^{lm} - r\frac{\partial K^{lm}}{\partial r}\right)\right]$

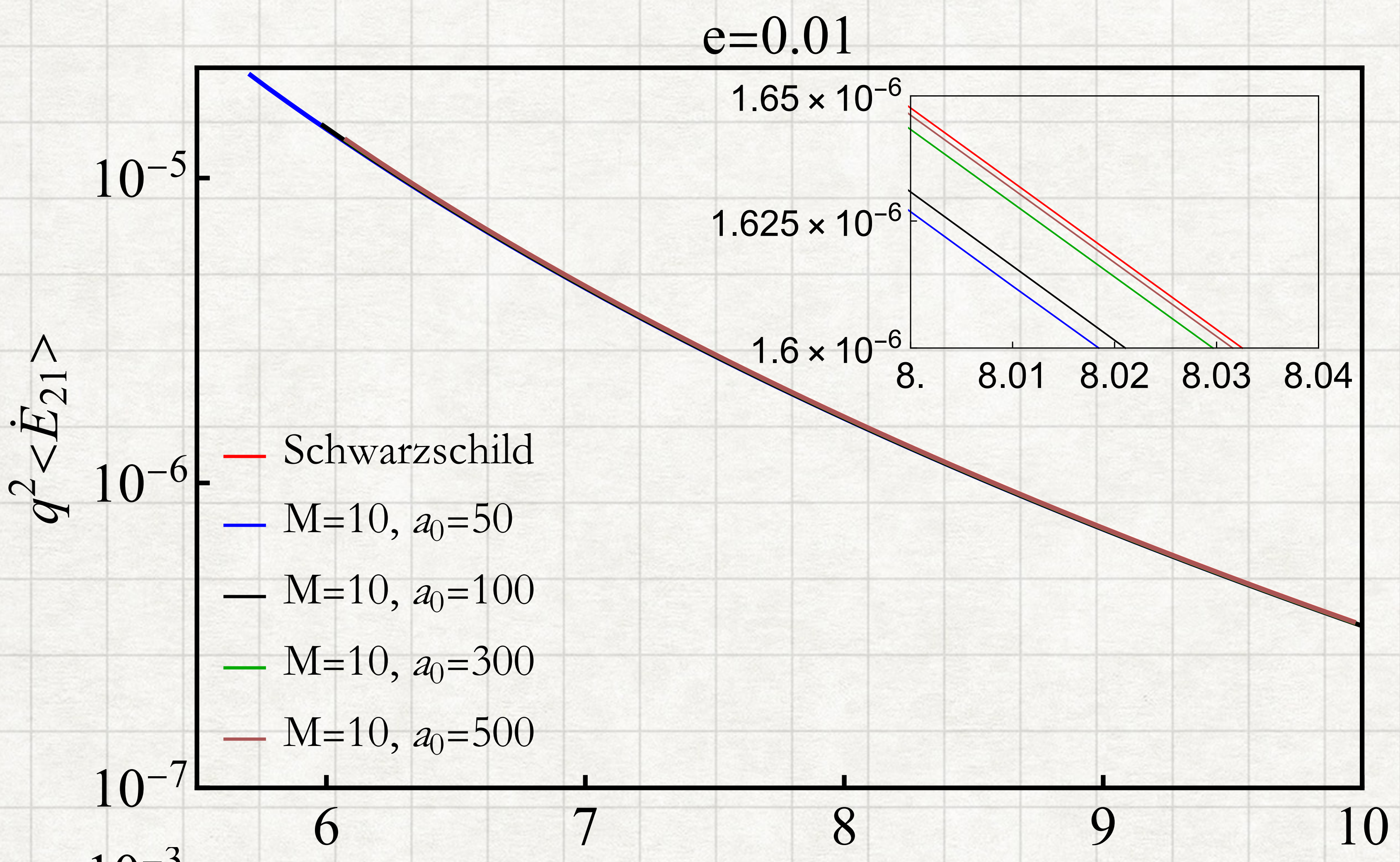
$$\frac{dE}{dt} = \sum_{lm}\frac{dE_{lm}}{dt} = \frac{1}{32\pi}\sum_{lm}\frac{(l+2)!}{(l-2)!}\left[|\dot{Z}^{pol}_{lm\omega}|^2 + 4|Z^{ax}_{lm\omega}|^2\right],$$

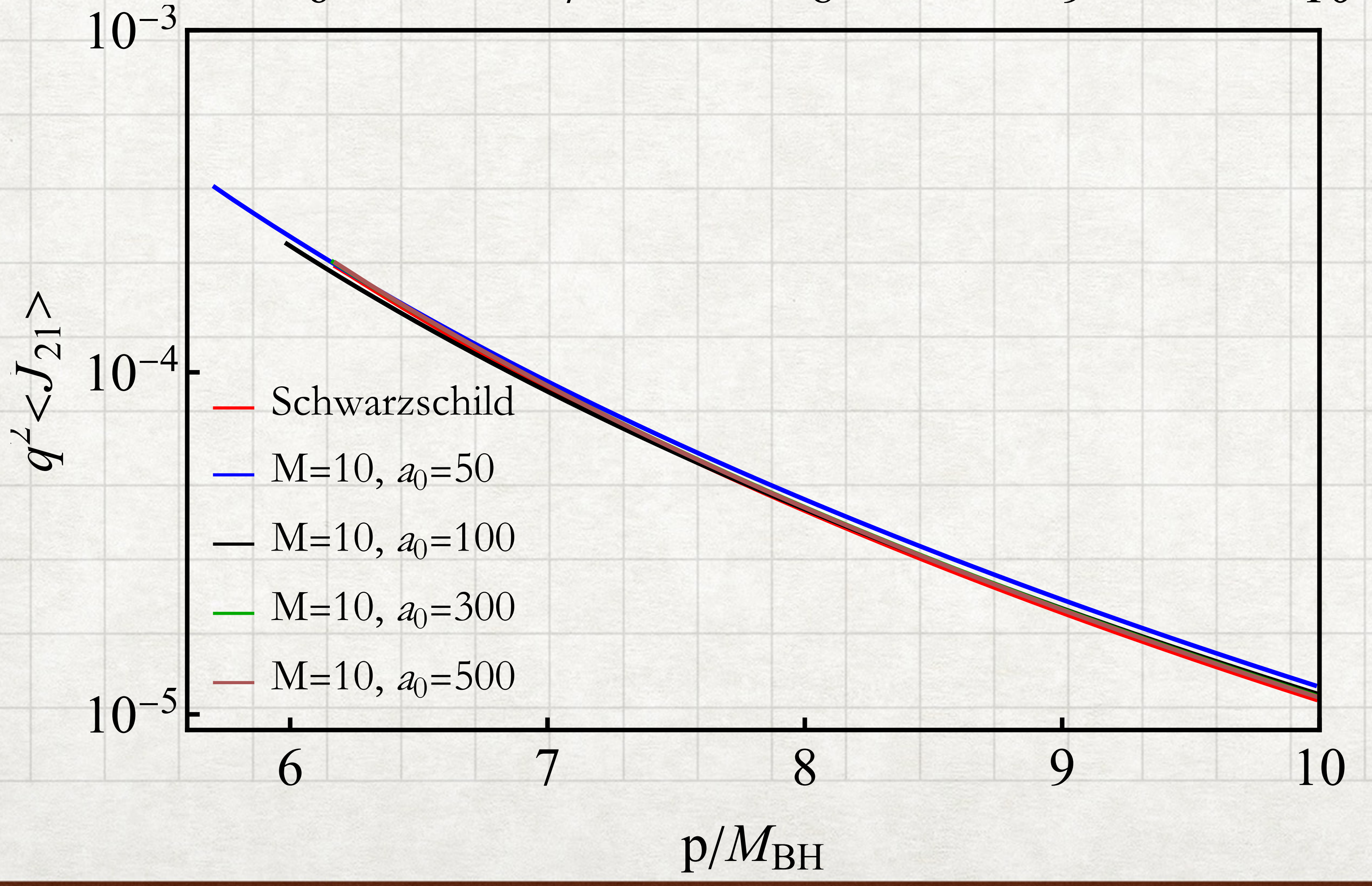
$$\frac{dJ_z}{dt} = \sum_{lm}\frac{dJ_{lm}}{dt} = \frac{1}{32\pi}\sum_{lm} im\frac{(l+2)!}{(l-2)!}\left[\dot{Z}^{pol}_{lm\omega}\tilde{Z}^{pol}_{lm\omega} + 4Z^{ax}_{lm\omega}\int dt\ \tilde{Z}^{ax}_{lm\omega}\right] + \text{c.c}$$

- Orbit averaged flux $\left\langle\frac{dE_{lm}}{dt}\right\rangle_{\text{GW}} = \frac{1}{T_P}\int_0^{2\pi} d\chi\frac{dt}{d\chi}\frac{dE_{lm}}{dt}, \qquad \left\langle\frac{dJ_{lm}}{dt}\right\rangle_{\text{GW}} = \frac{1}{T_P}\int_0^{2\pi} d\chi\frac{dt}{d\chi}\frac{dJ_{lm}}{dt}.$

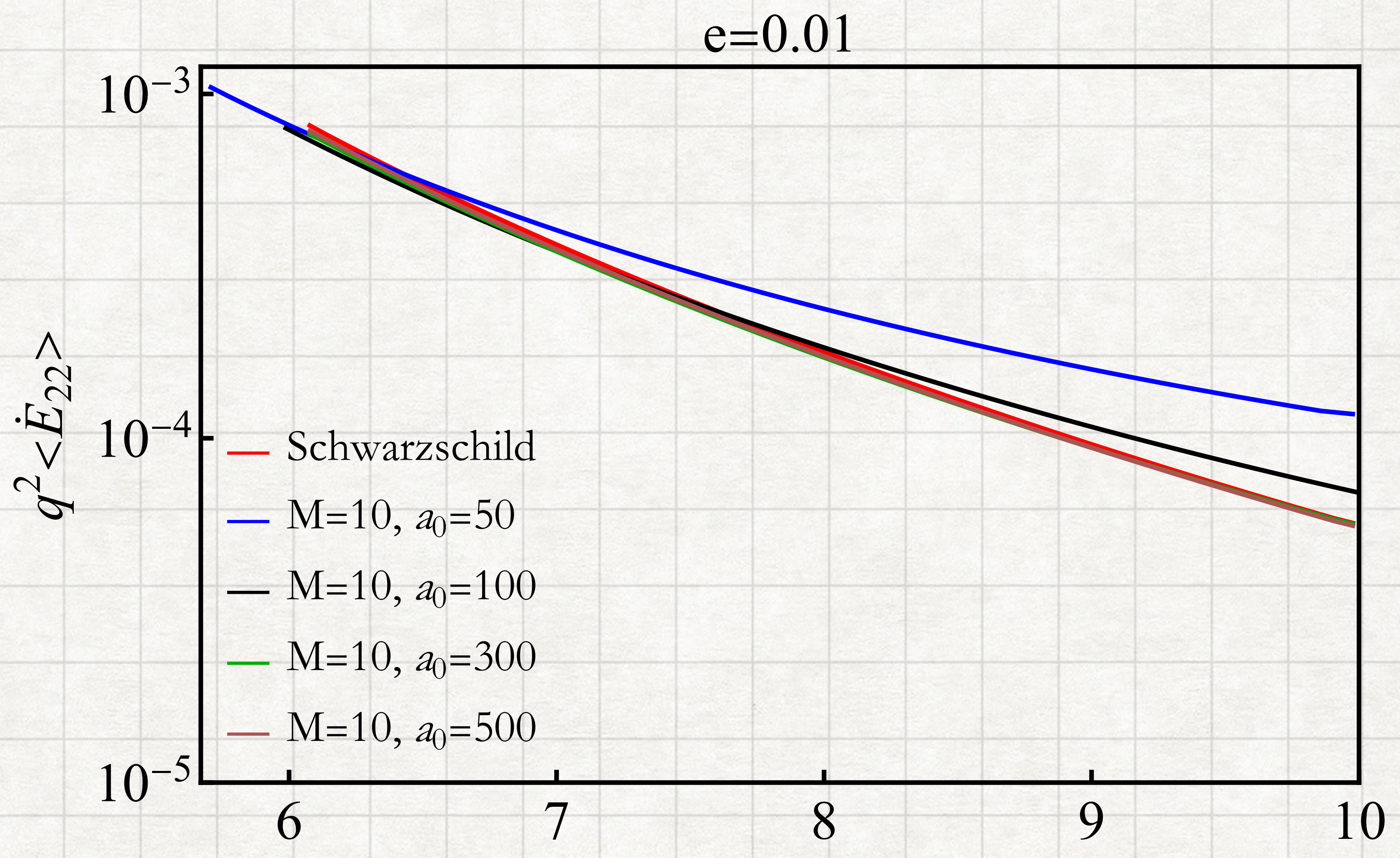
ENERGY AND ANGULAR MOMENTUM FLUX

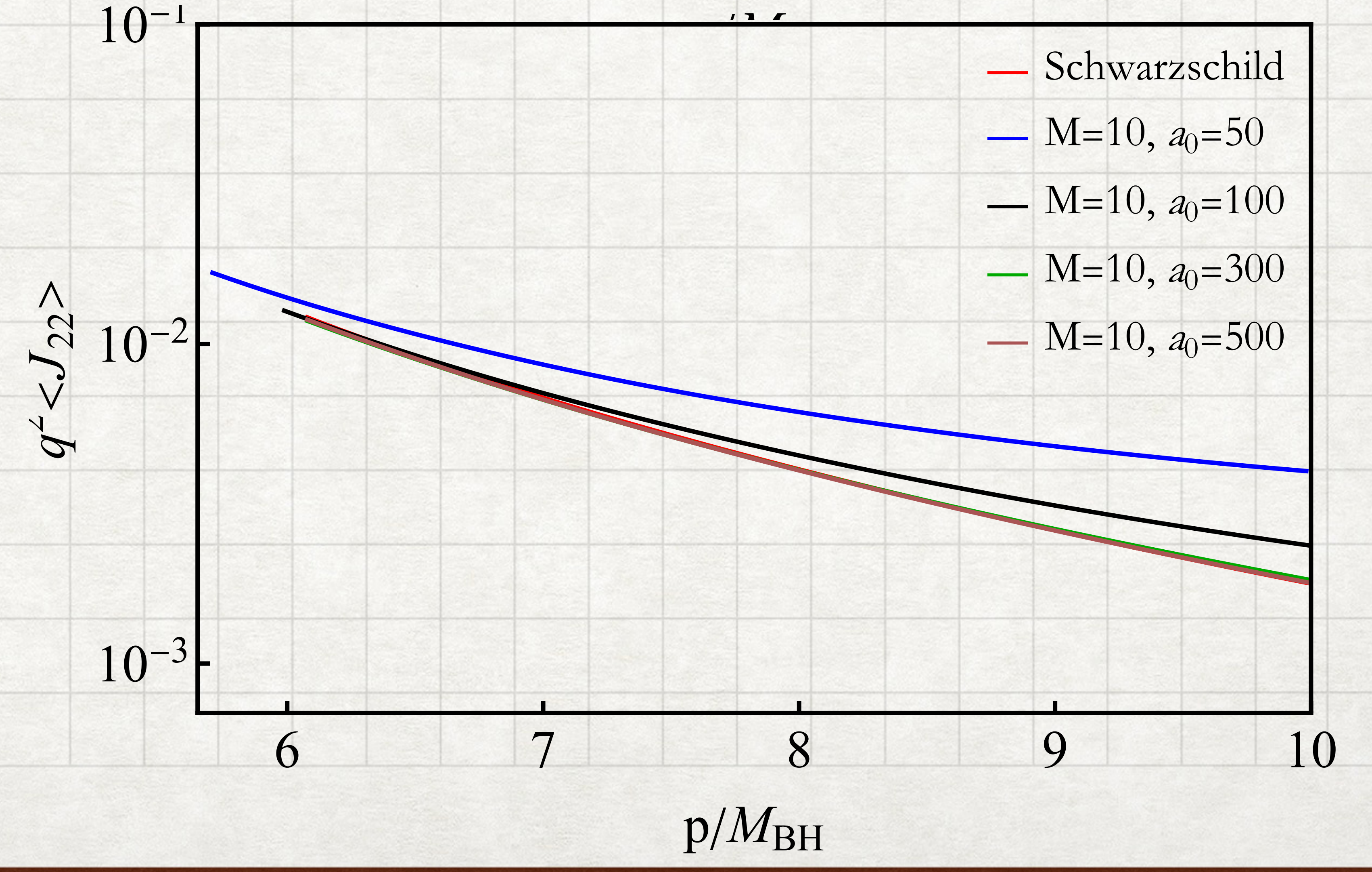
AXIAL SECTOR {2,1} MODE





POLAR SECTOR {2,2} MODE





ENERGY AND ANGULAR MOMENTUM FLUX

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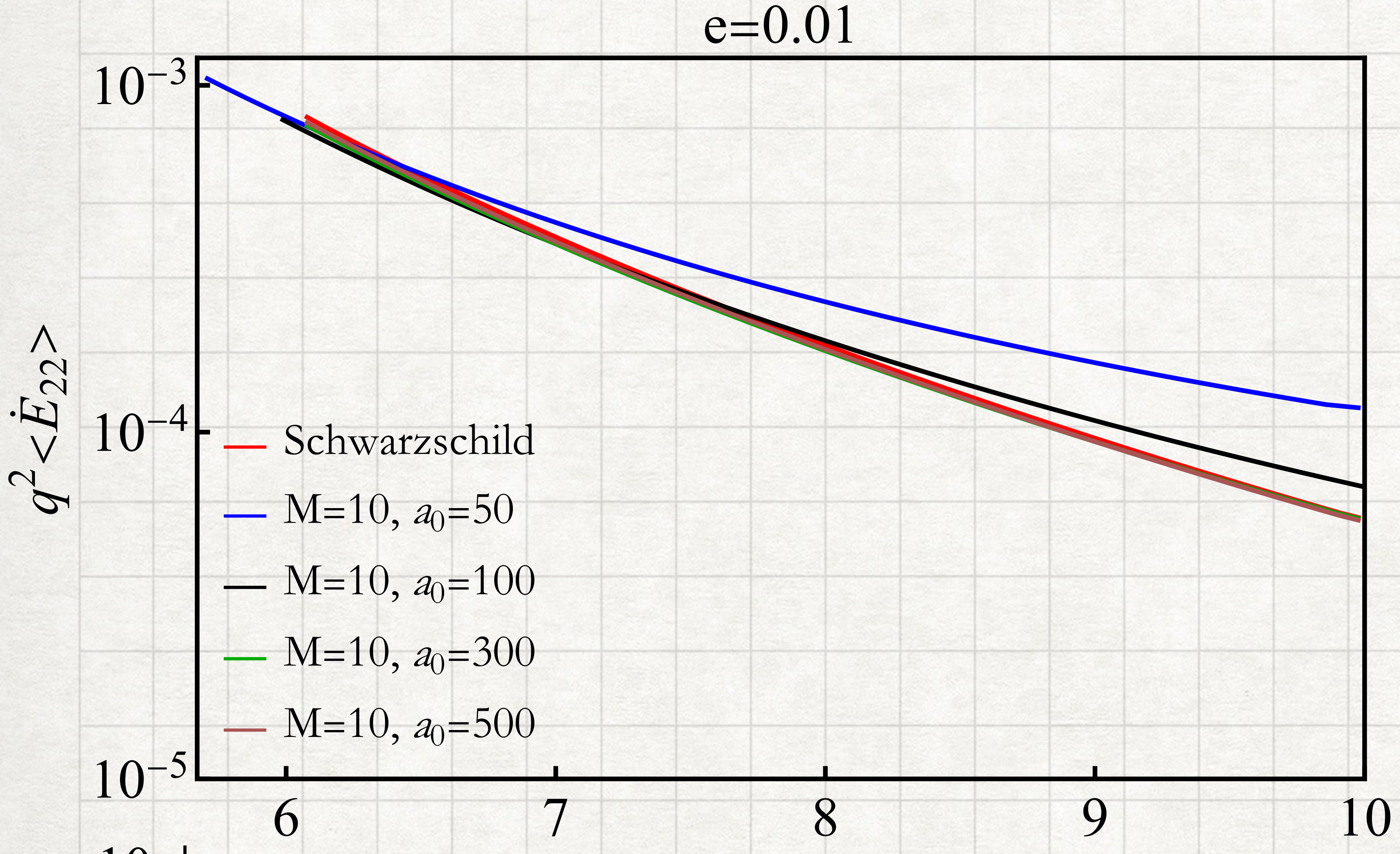
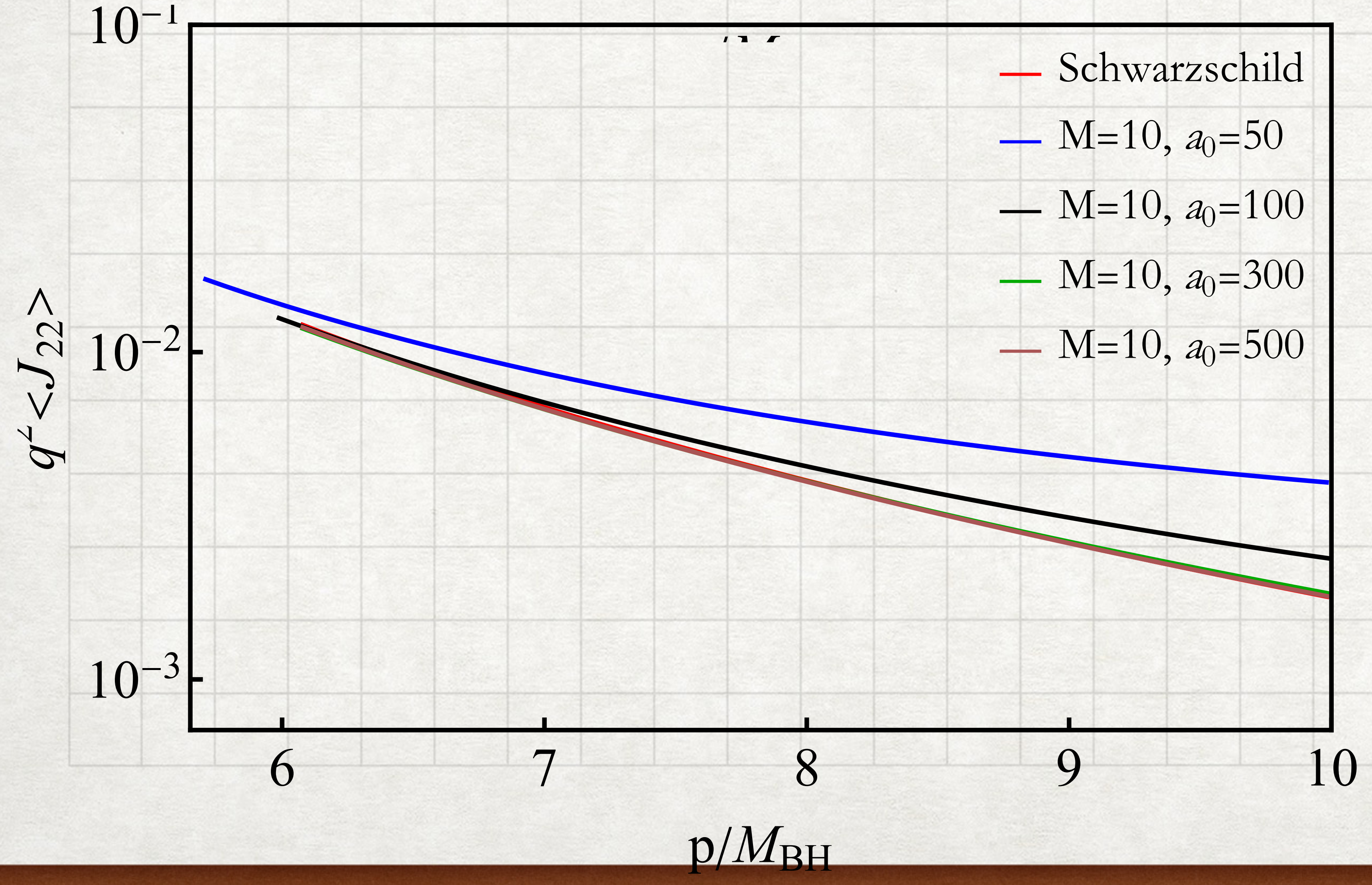
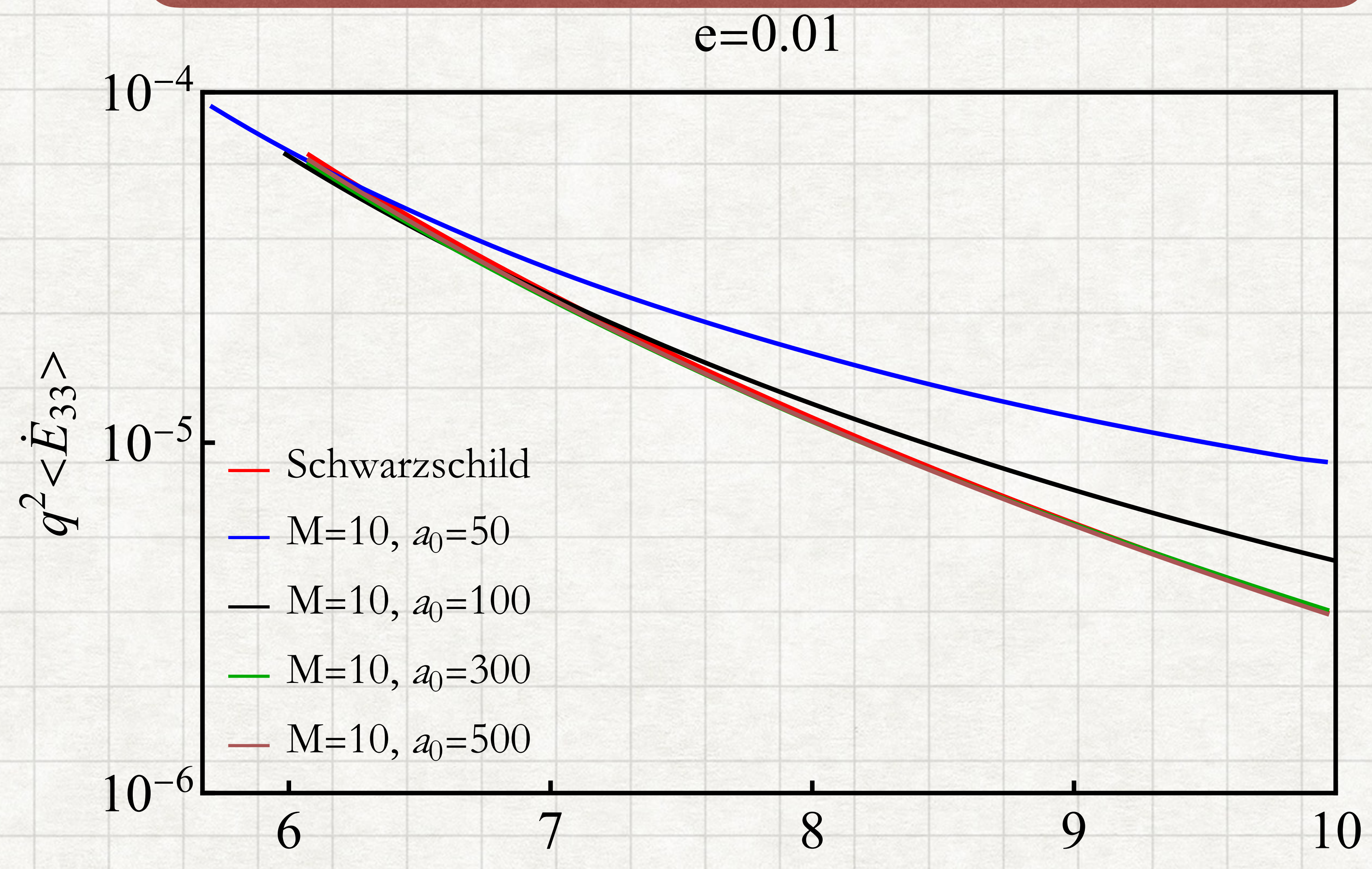
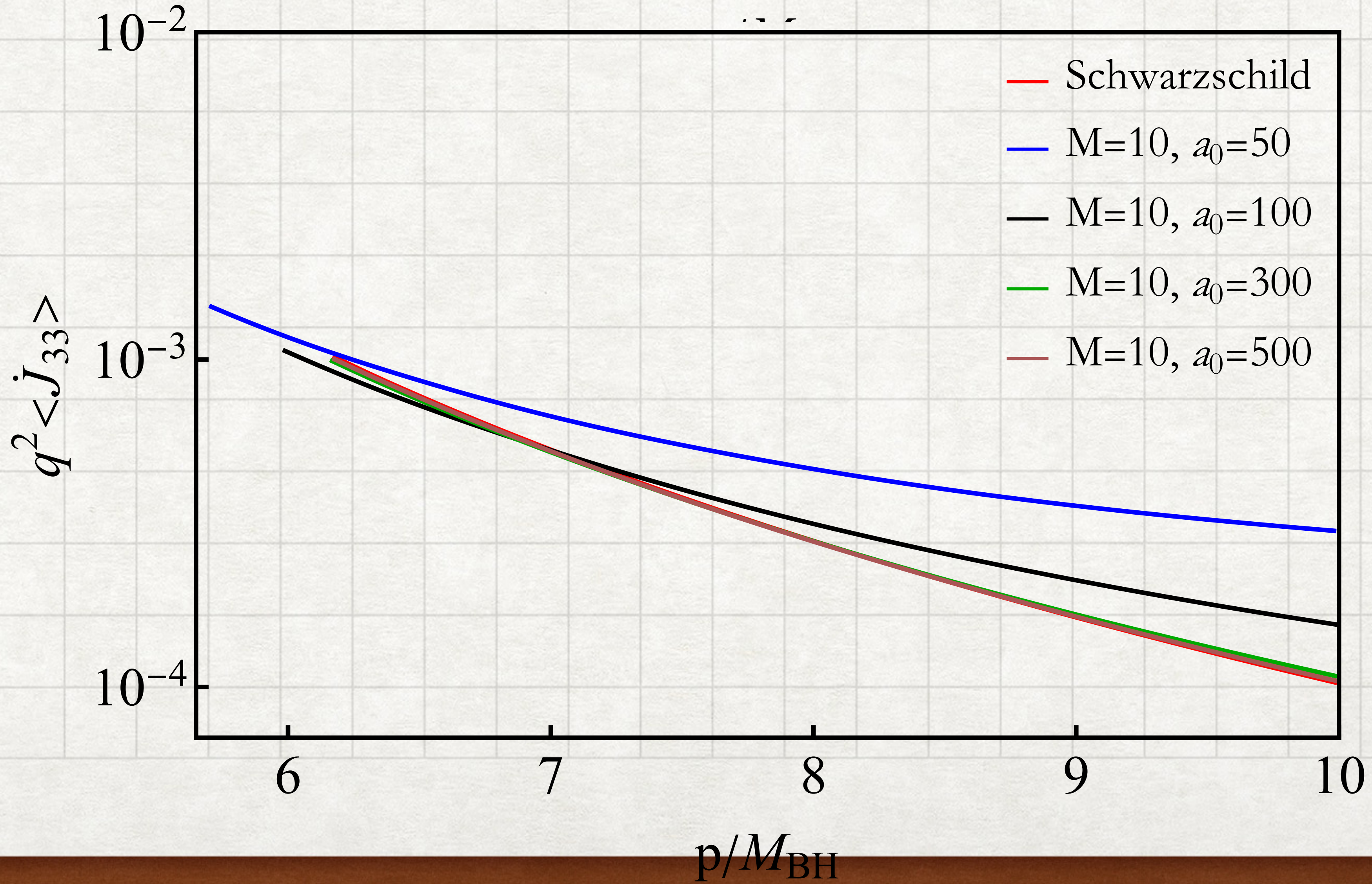




POLAR SECTOR {3,3} MODE



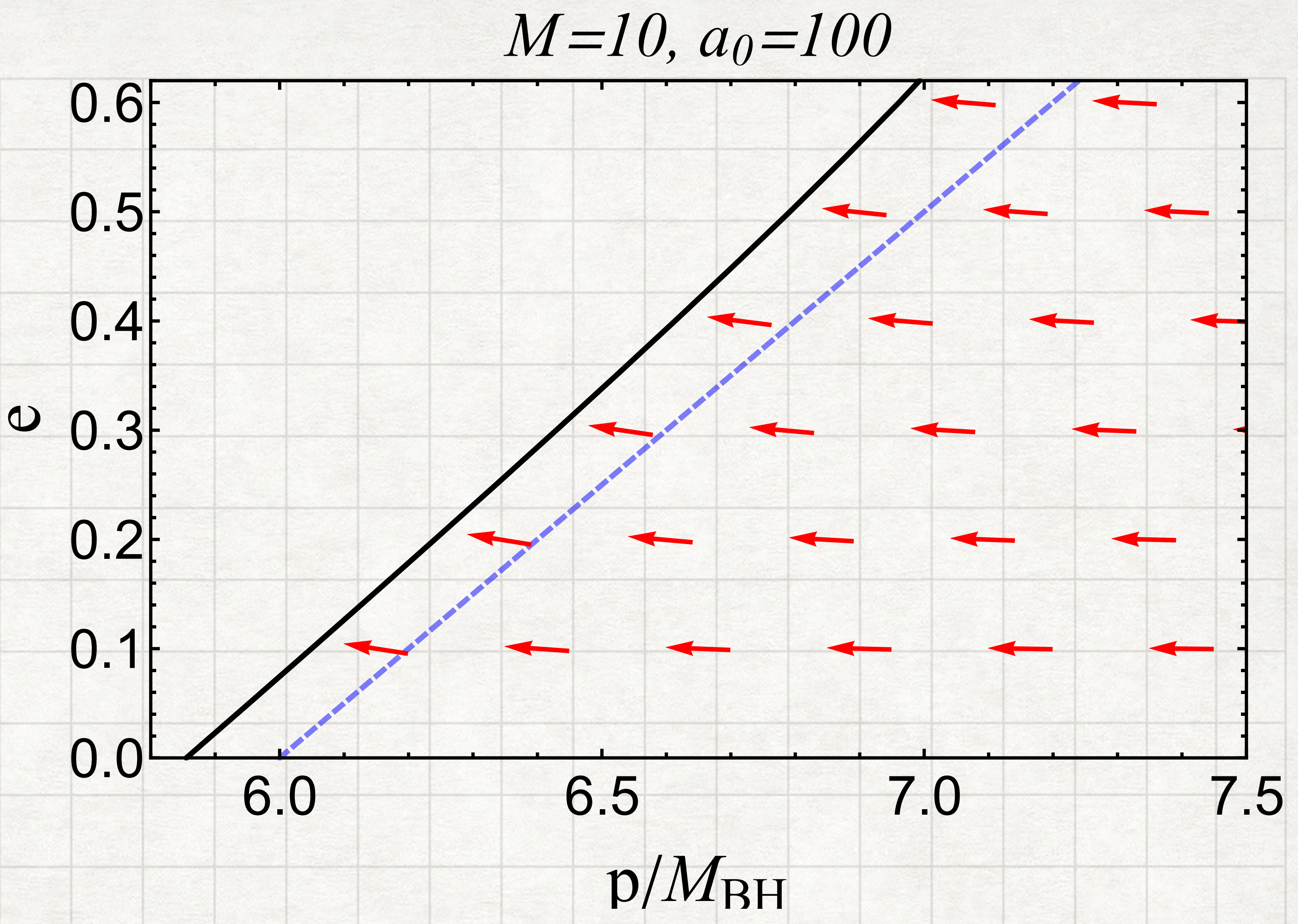


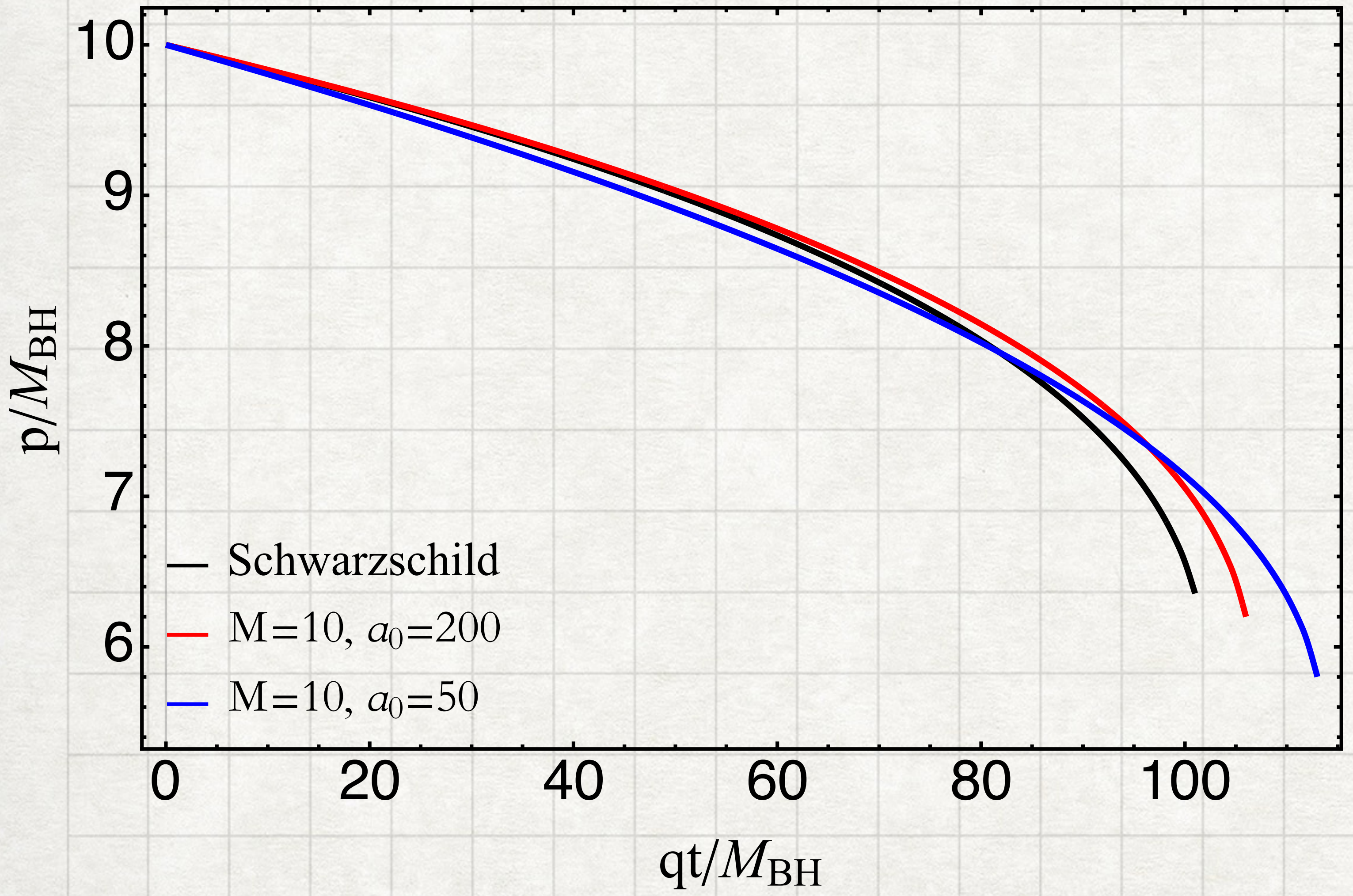
Orbital evolution equation

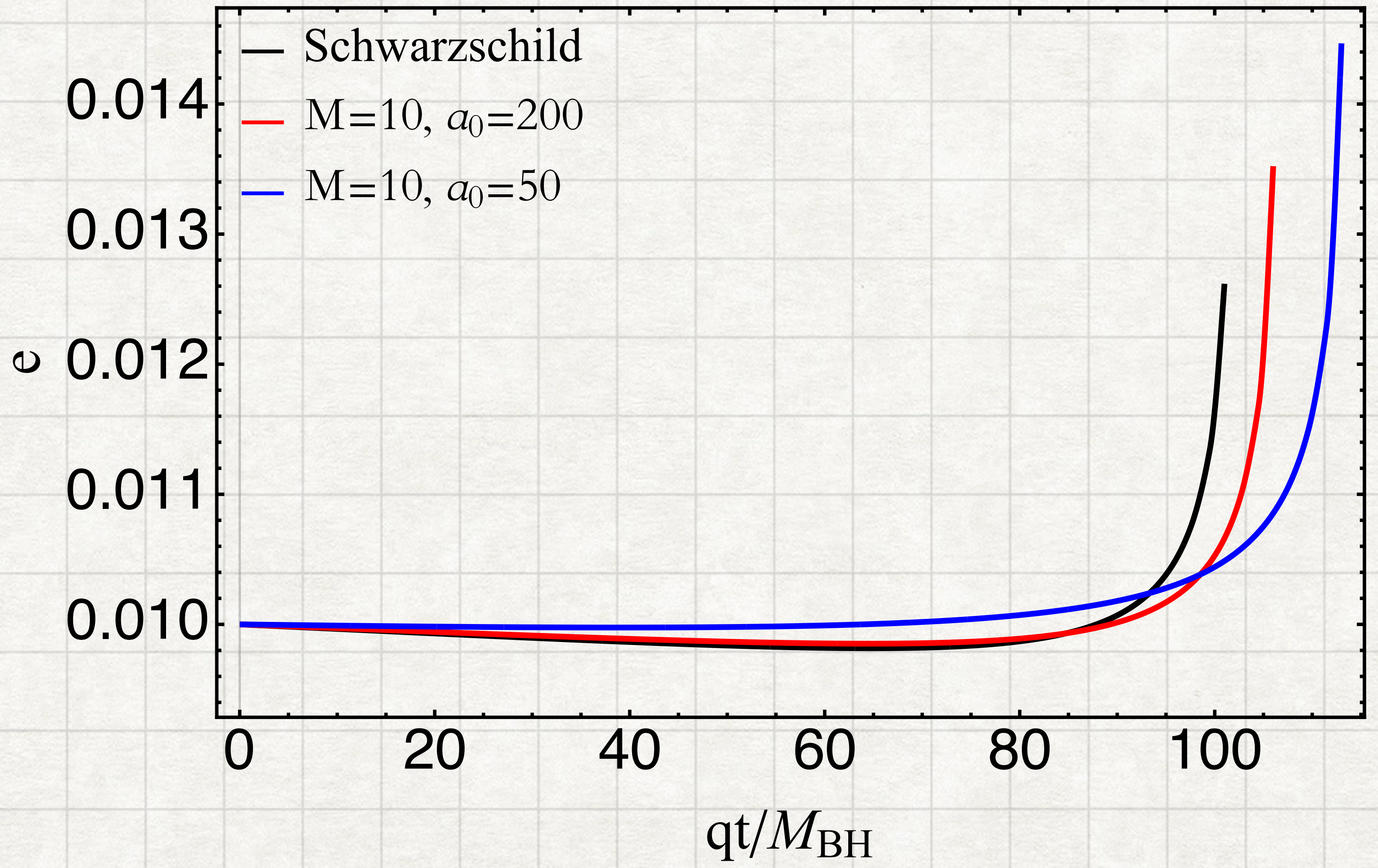
$$\frac{dE}{dt} = \frac{\partial E}{\partial p}\frac{dp}{dt} + \frac{\partial E}{\partial e}\frac{de}{dt}, \quad \frac{dJ_z}{dt} = \frac{\partial J_z}{\partial p}\frac{dp}{dt} + \frac{\partial J_z}{\partial e}\frac{de}{dt}$$

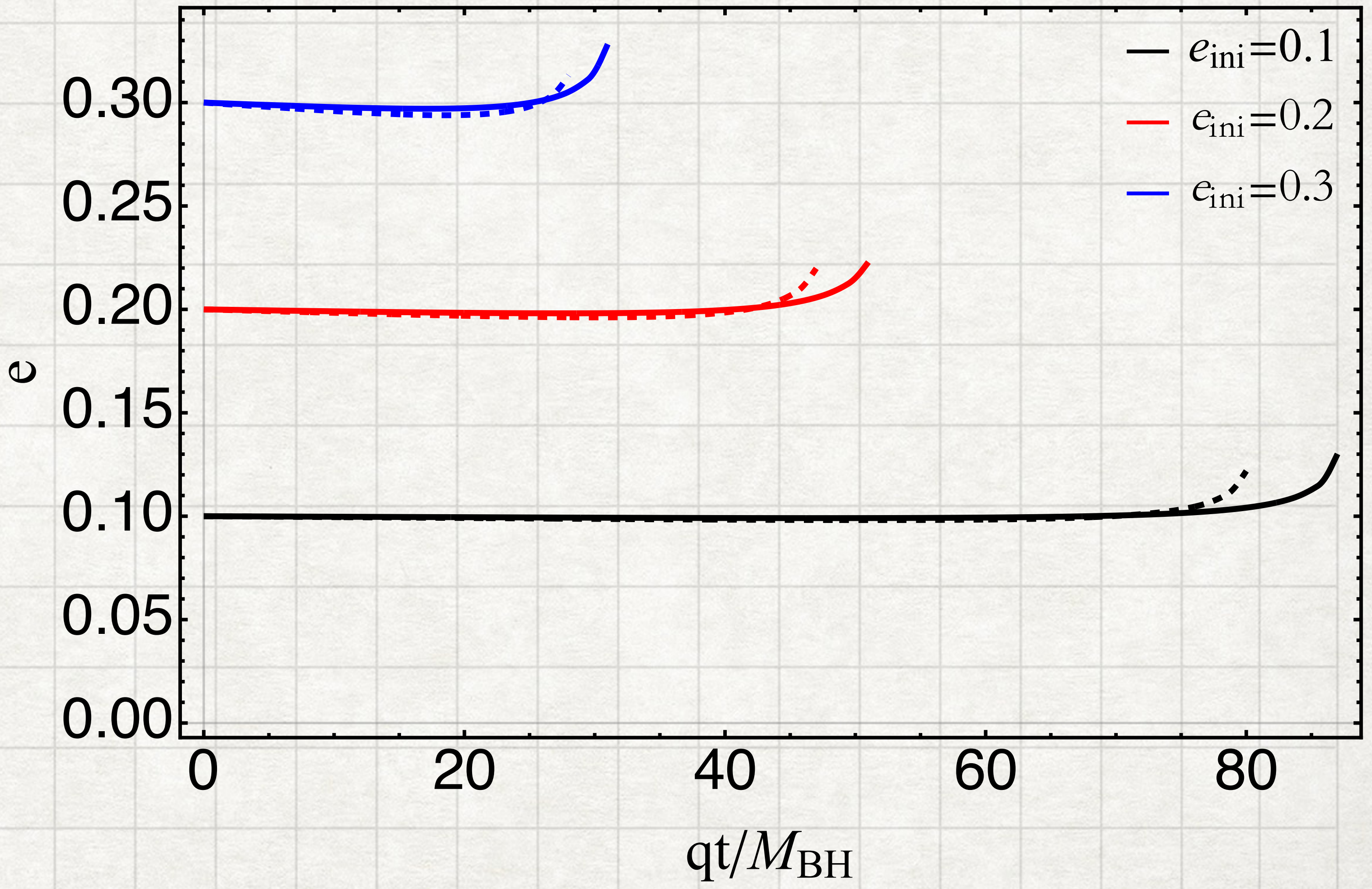
$$\left.\frac{d\ln e}{d\ln p}\right|_{p\to 6+2e, e\gg \varepsilon/4} \sim -\frac{1-e}{e}$$: Schwarzschild Case

[Cutler+, PRD 50, 3816 (1994).]







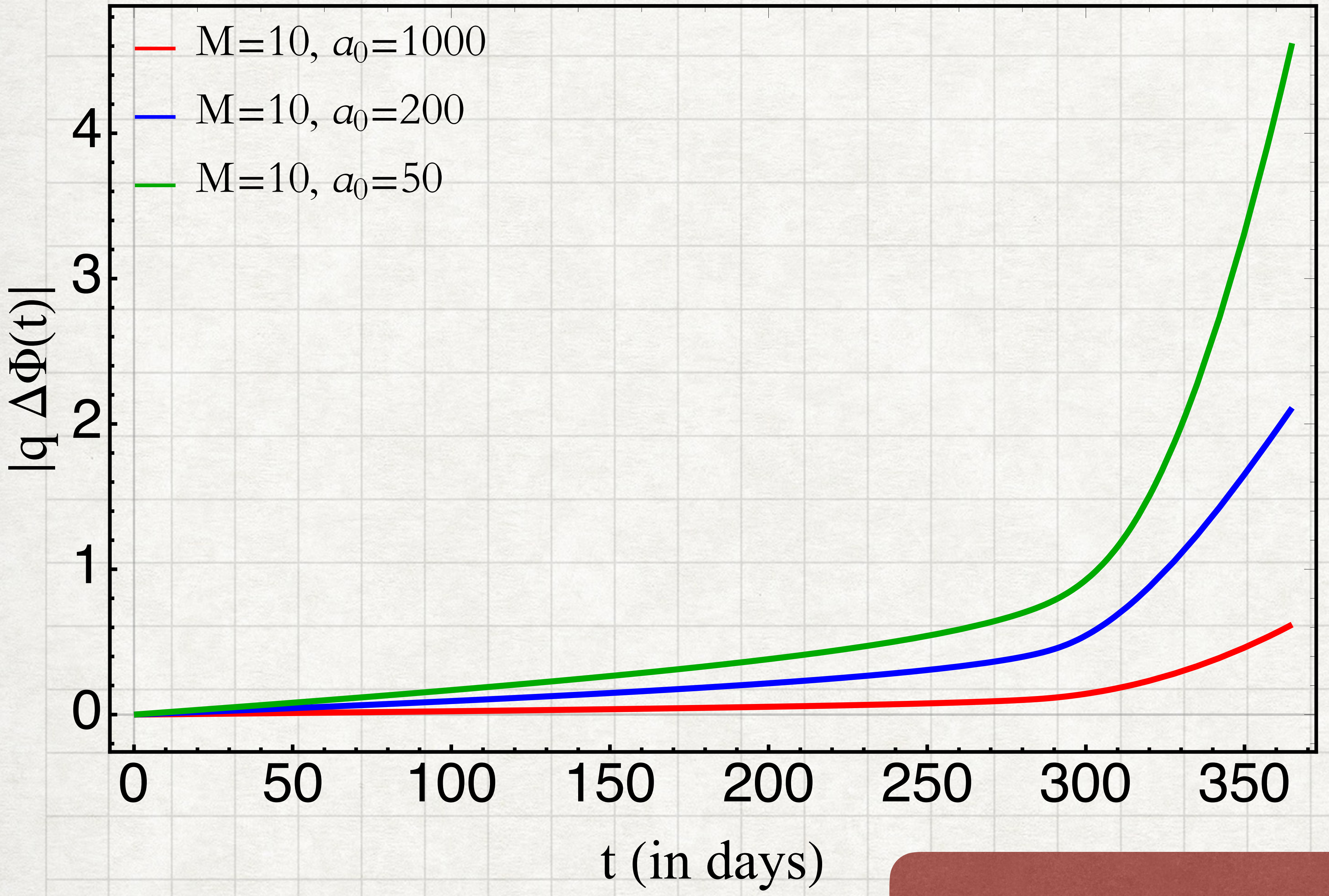


GW PHASE EVOLUTION

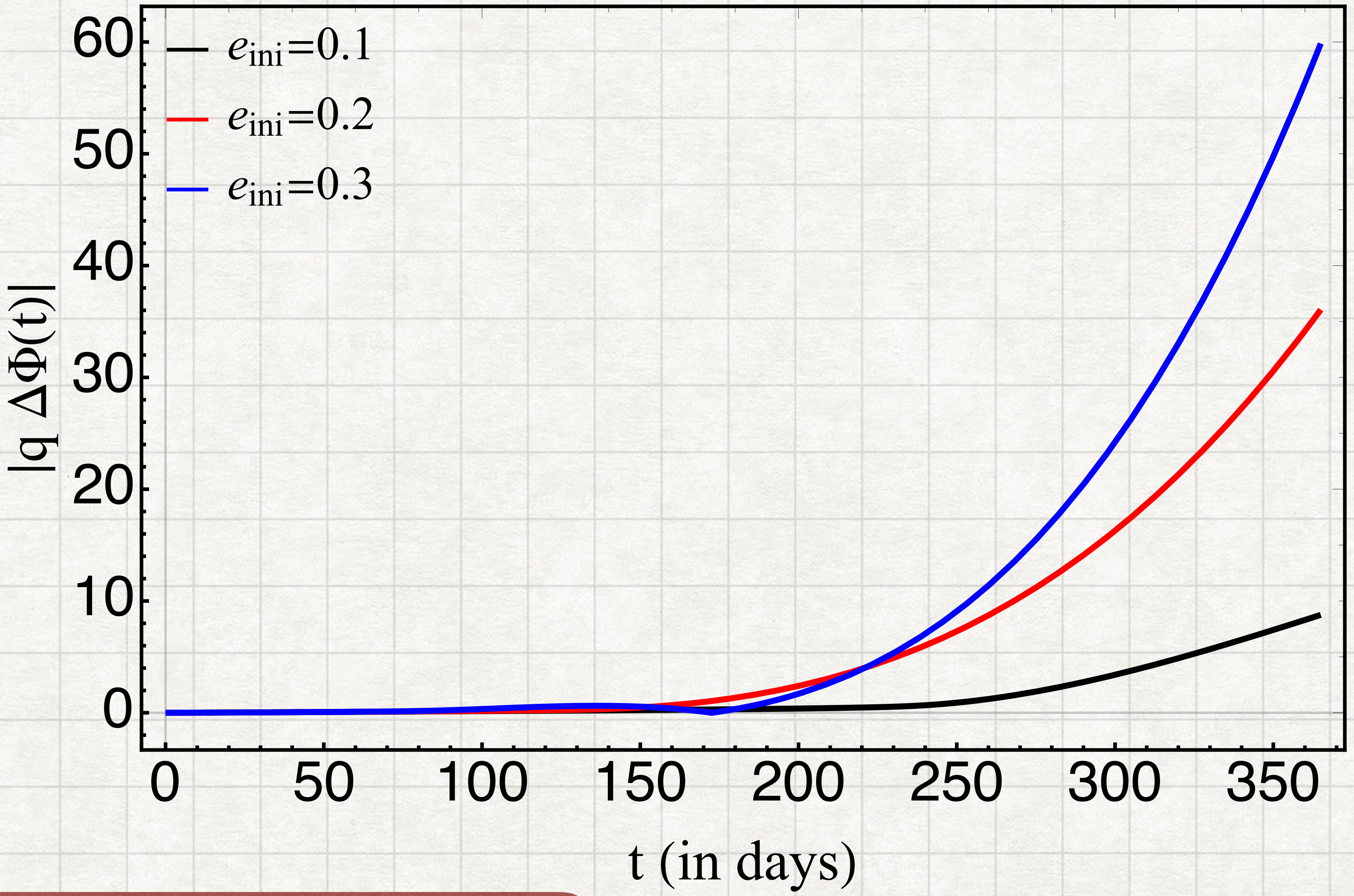
$$\frac{d\varphi_i(t)}{dt} = \left\langle \Omega_i\left(p(t), e(t)\right)\right\rangle = \frac{1}{T_P}\int_0^{2\pi} d\chi \frac{dt}{d\chi}\Omega_i\left(p(t), e(t), \chi\right)\ , \qquad i \in \{\phi, r\}$$

$$\Delta\Phi(t_{\mathrm{obs}}) = \left| \Phi^{\mathrm{DM}}_{\mathrm{GW}}(t_{\mathrm{obs}}) - \Phi^{\mathrm{Schld}}_{\mathrm{GW}}(t_{\mathrm{obs}}) \right|\ .$$

$e_{ini} = 0.01$



$M = 10 M_{BH}\,,\quad a_0 = 100 M_{BH}$



$\Delta\Phi^{th} \geq 0.1$ rad$\,,\quad SNR = 30,$

TAKE AWAY MASSAGE

- The location of the last stable orbit can significantly be influenced by the halo compactness parameter M/a_0
- The energy and momentum flux of $\{2,2\}$ mode is dominant over axial and higher-order polar modes.
- The inspiral time lengthens in the presence of dark matter
- Highly eccentric orbits exhibit shorter inspiral times for a given set of dark matter parameters
- The total accumulated orbital phase during the inspiral rises with higher values of the halo compactness parameter or lower values of the initial eccentricity.
- Dark matter introduces a dephasing of $\Delta\Phi \approx \mathcal{O}(0.5)/q$ rad for $M/a_0 \sim \mathcal{O}(10^{-2})$
- Our results are very encouraging from the point of view of detecting dark matter environments through eccentric EMRI systems.
- If the initial eccentricity is large, the dephasing becomes quite significant, helping us distinguish the effect of the dark matter environment.

Thanks a lot for your attention