香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Gravitational Waves from More Attractive Dark Binaries

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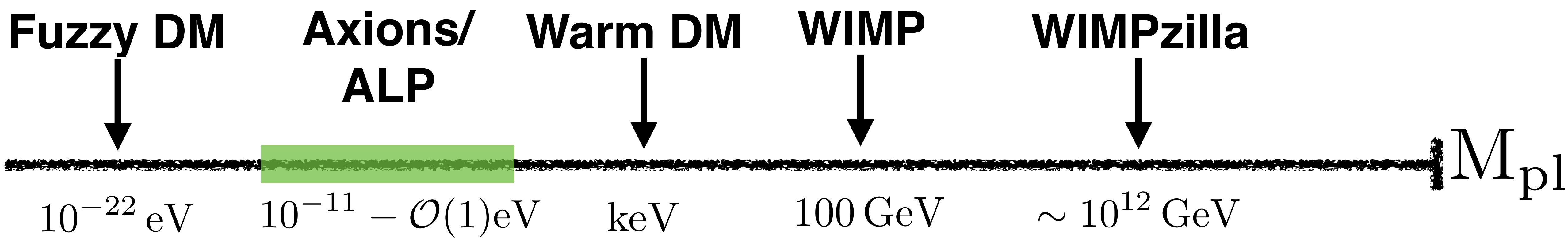
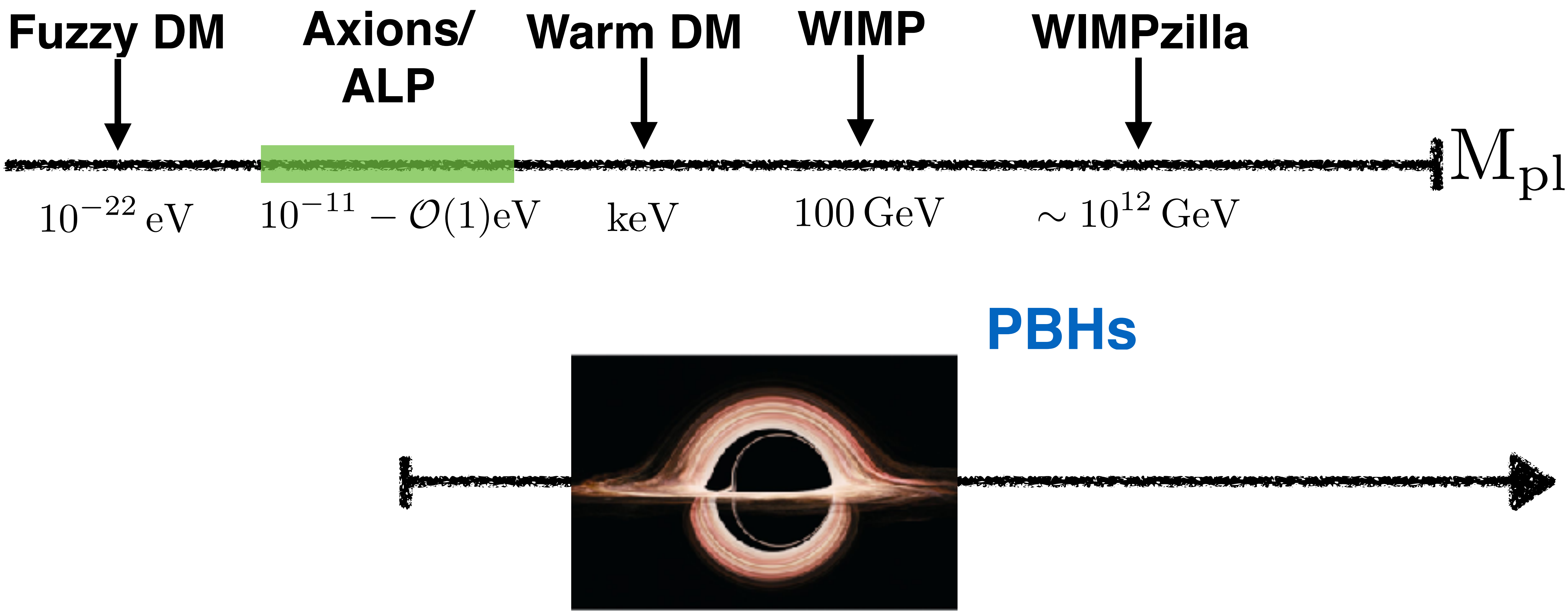
based on 2312.13378, w/ Yang Bai and Nicholas Orlofsky

DM Zoo


Fuzzy DM
Axions/ ALP
Warm DM
WIMP
WIMPzilla
$M_{\rm pl}$
$10^{-22}\,\mathrm{eV}$
$10^{-11} - \mathcal{O}(1)\mathrm{eV}$
keV
$100\,\mathrm{GeV}$
$\sim 10^{12}\,\mathrm{GeV}$

DM Zoo



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PBHs

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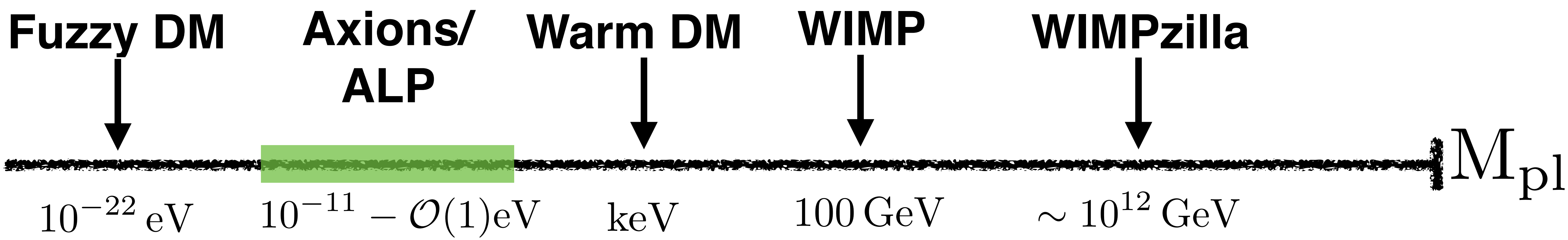
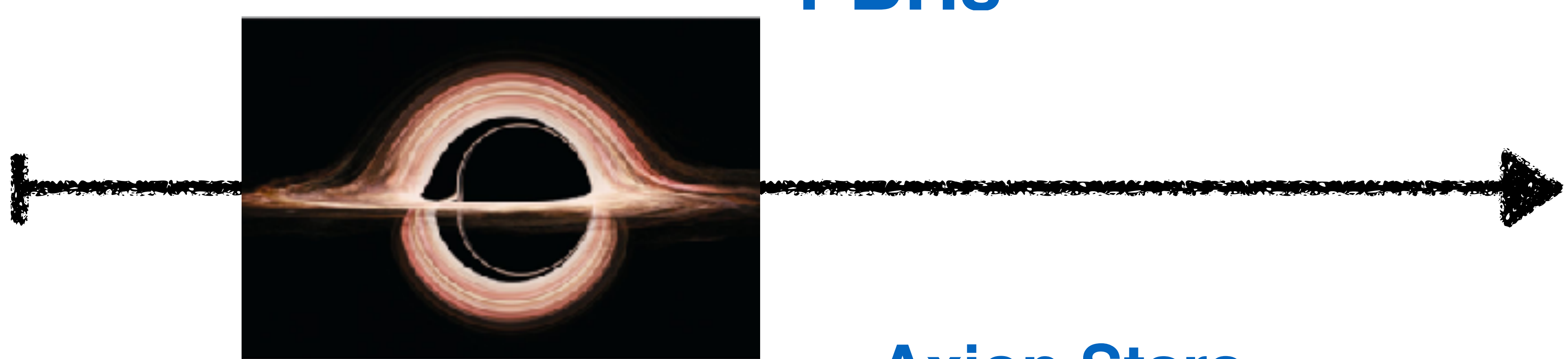


PBHs



+ **Axion Stars**
+ **Dark Quark Nuggets**
+ **Q-balls**
+ **…**

MDMs from Phase Transitions

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

MDMs from Phase Transitions

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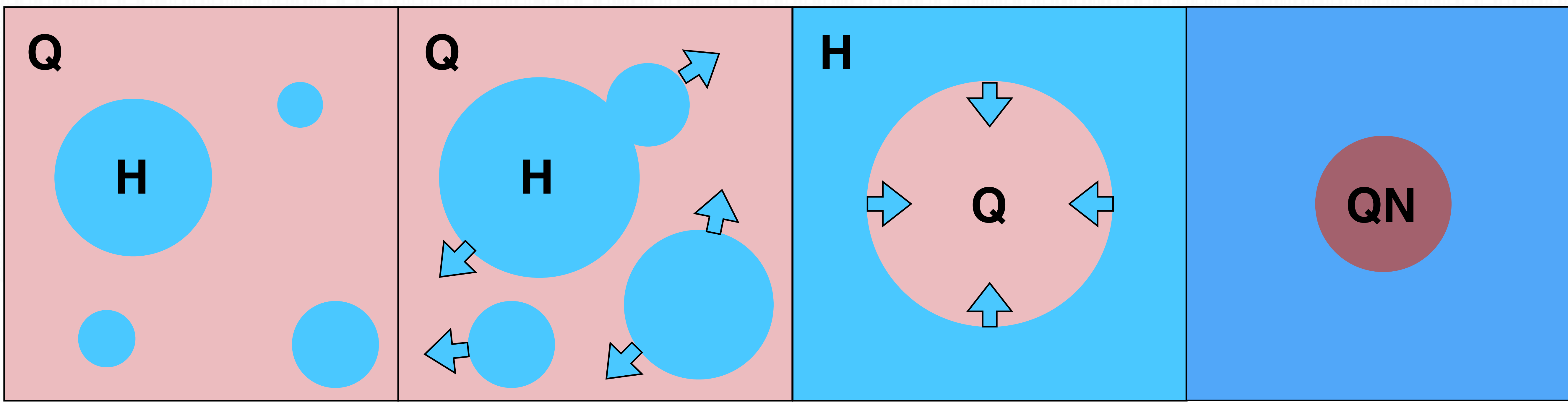
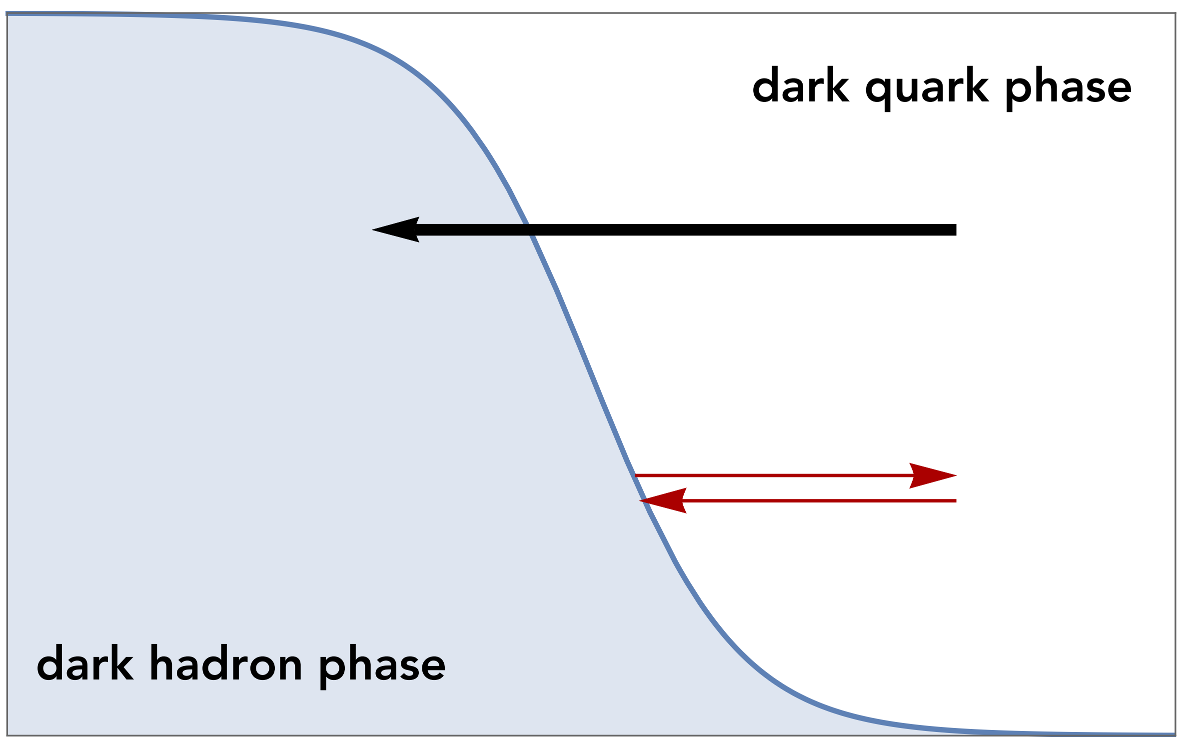
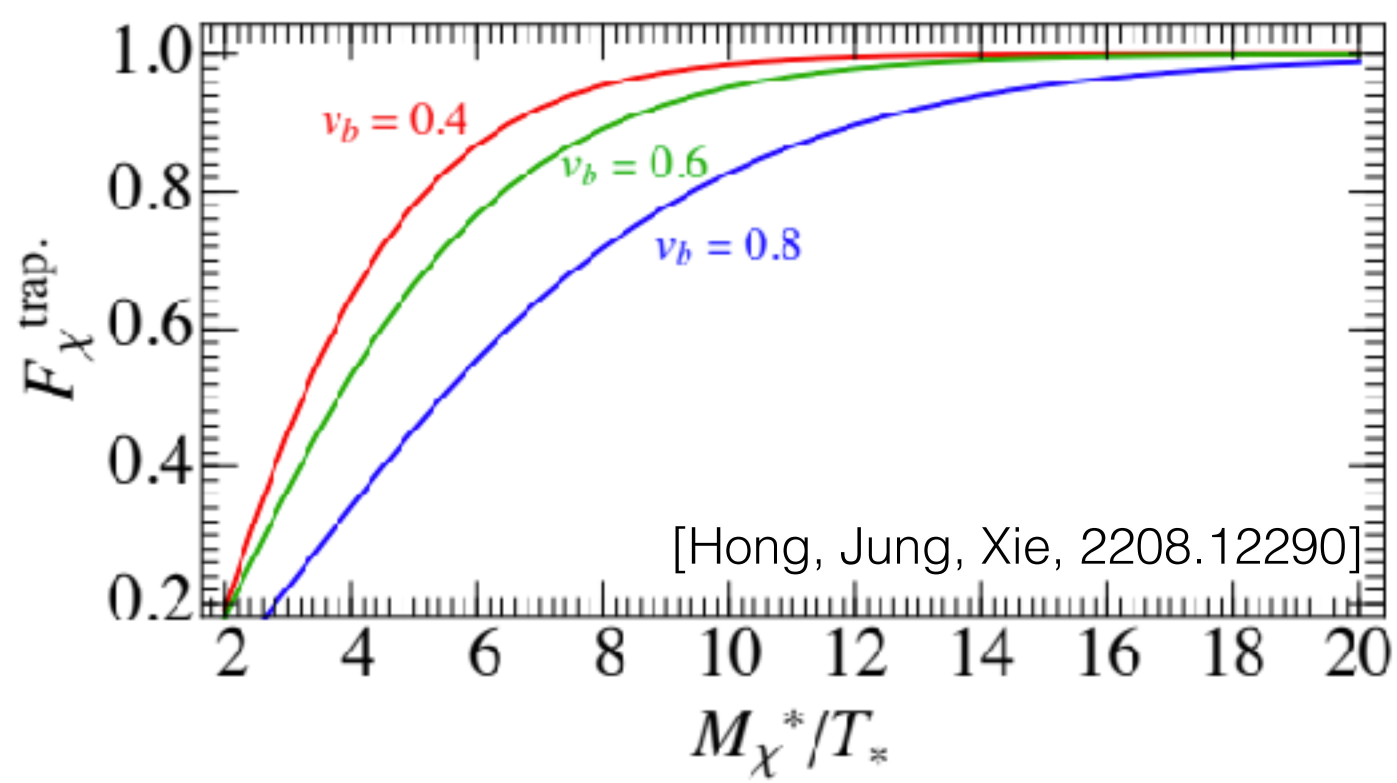


Q: quarks H: hadrons QN: quark nuggets

Trapped in the False Vacuum

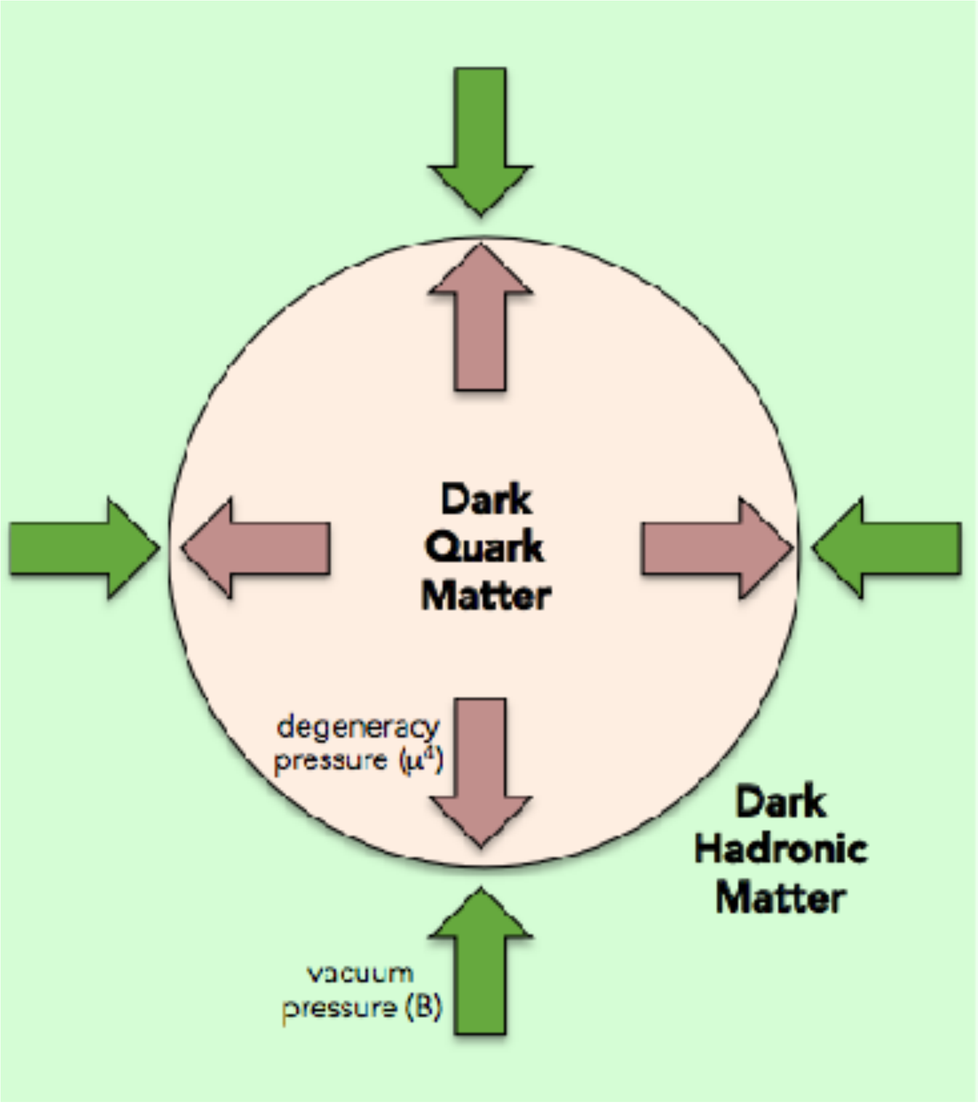
- **Some particles in the unbroken phase may not be energetic enough to tunnel through**
  - ➡ Bubble wall velocity
  - ➡ Mass of the corresponding state in the broken phase



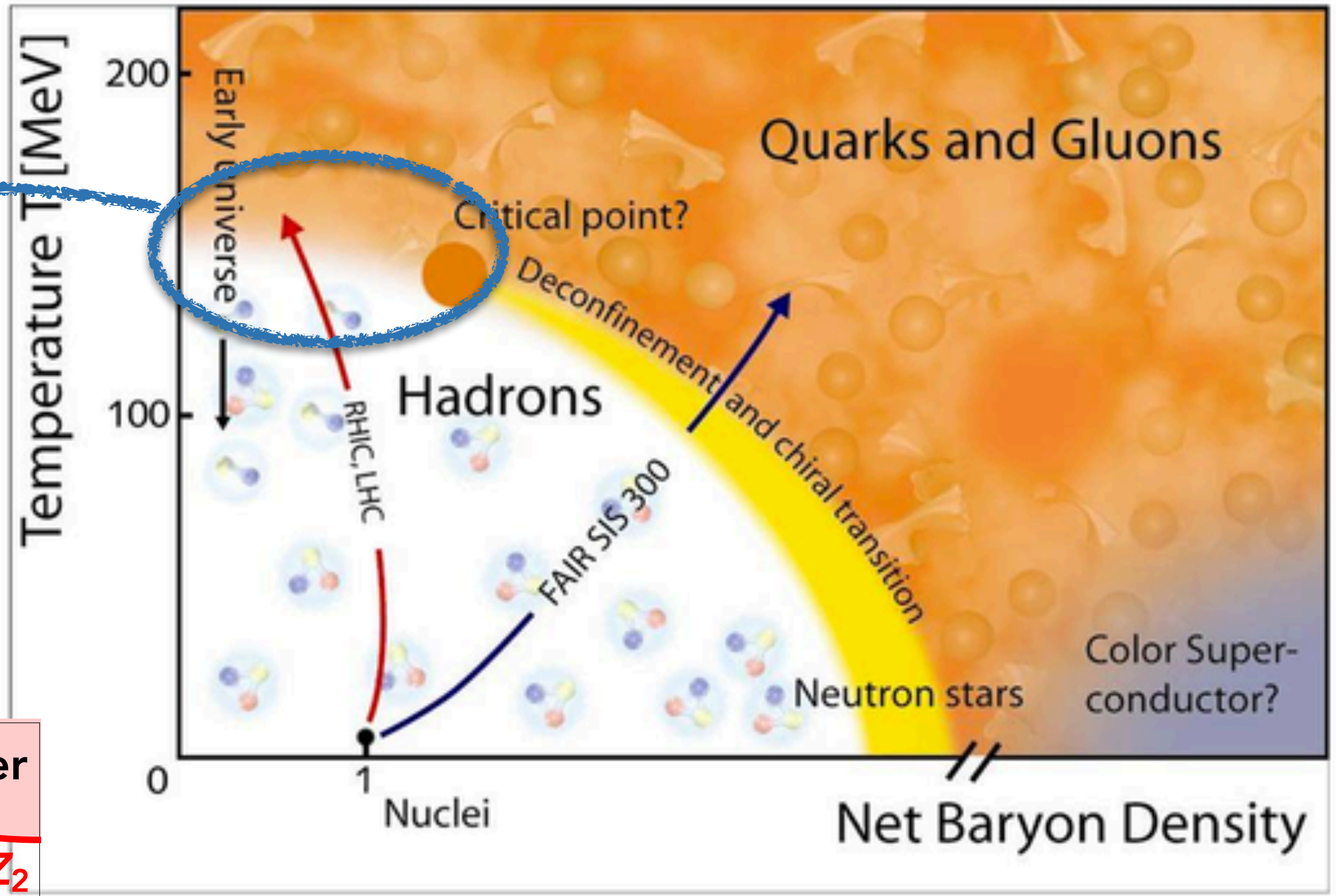


Balancing the Vacuum Pressure

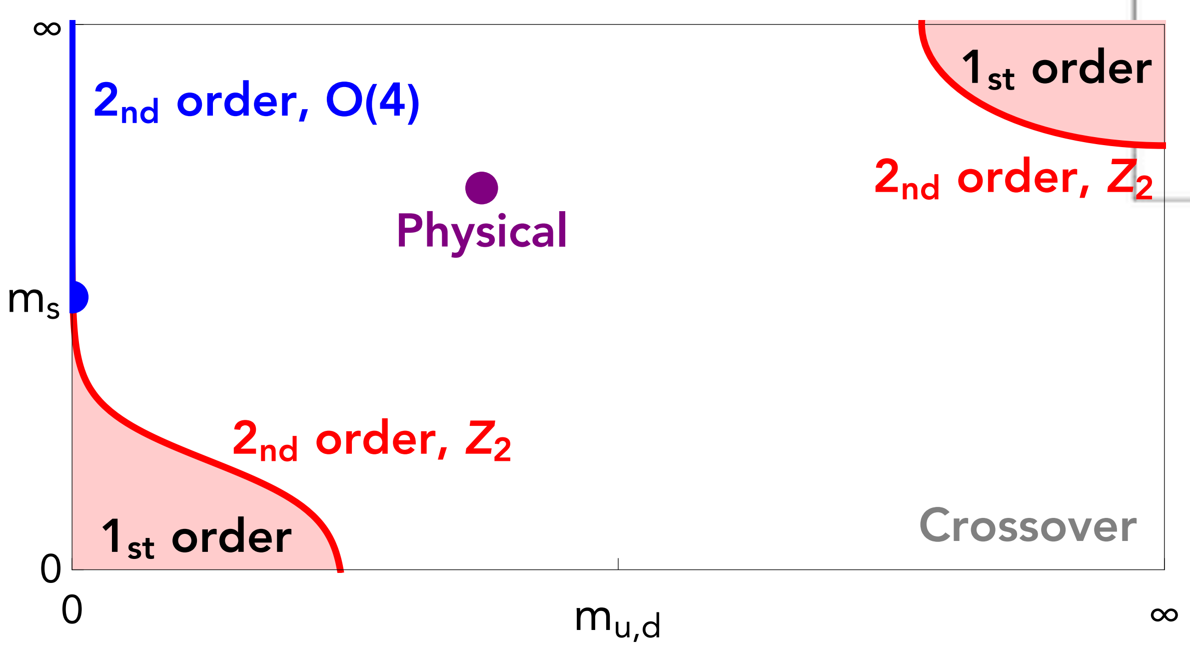
- Pressure from the true vacuum is countered by the Fermi degeneracy pressure



QCD is Upsetting…



Continuous Crossover



…While the Dark Sector is Still Fine

- **A direct FOPT from a scalar potential**
- **Or being lazy and just copy the QCD into the dark sector (but have light dark quarks s.t. FOPT can happen)**

$$\mathcal{L}_{\mathrm{dQCD}} = \sum_{i=1}^{N_f} \left[\bar{\psi}_i i\gamma^\mu D_\mu \psi_i - m_{\psi_i}\,\bar{\psi}_i\psi_i\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}$$

➡ The "dark quark nuggets"

[Bai, Long, *SL*, 1810.04360]

MDMs Being More Attractive

- **A direct FOPT from a scalar potential**
- **Or being lazy and just copy the QCD into the dark sector (but have light dark quarks s.t. FOPT can happen)**

$$\mathcal{L}_{\rm dQCD} = \sum_{i=1}^{N_f} \left[\bar{\psi}_i i\gamma^\mu D_\mu \psi_i - m_{\psi_i}\, \bar{\psi}_i \psi_i\right] - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a}$$

➡ The "dark quark nuggets"

[Bai, Long, *SL*, 1810.04360]

- **Adding an additional attraction between the MDMs**

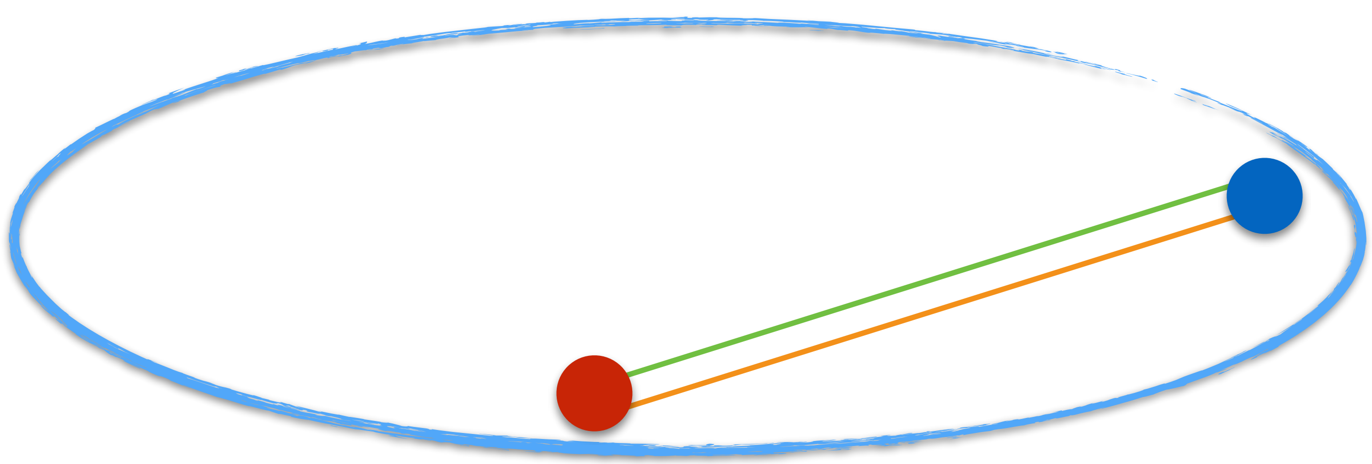
$$\mathcal{L}_{\rm Yukawa} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \sum_i (m_{\psi_i} + y_i\,\phi)\,\bar{\psi}_i\,\psi_i - V_0(\phi)\,, \quad V_0(\phi) = \frac{1}{2} m_{\rm med}^2 \phi^2$$

MDMs Being More Attractive

- **MDM binaries with a dark force**

$$F = -\frac{Gm_1 m_2}{r^2}\left(1 - \alpha e^{-m_{\rm med} r}(1 + m_{\rm med} r)\right)$$

$$\alpha = y^2 q_1 q_2/(4\pi G m_1 m_2)$$



MDMs Being More Attractive

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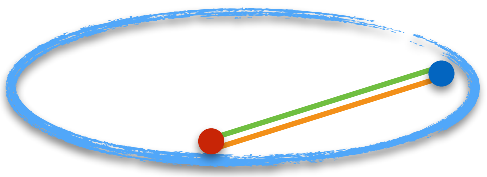


- ➡ Massless mediator limit

$$F = -G'm_1m_2/r^2, \quad G' = (1-\alpha)G \equiv \beta G$$

$$\omega^2 = \frac{G'm}{a^3}, \quad E = -\frac{G'm^2\eta}{2a} \quad e^2 = 1 + \frac{2EL^2}{G'^2m^5\eta^3}$$

a: semi-major axis
e: eccentricity

- **Just a "rescaled" gravity?**

Energy Spectrum from the Binary

- **Energy emission from the binary is different now**
  - Energy emission through GW portal

$$\langle \dot{E}_{\rm GW} \rangle = \frac{32 G G'^3 \eta^2 m^5}{5 a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

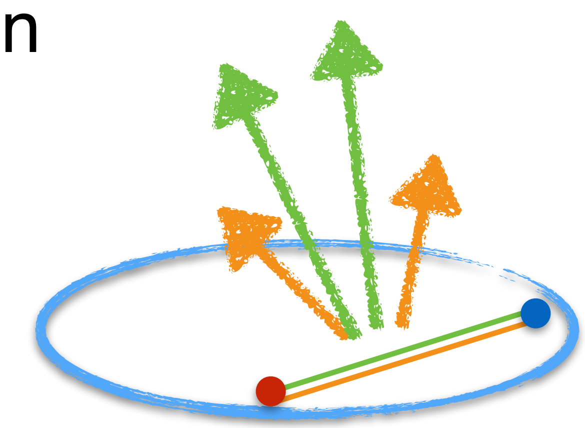
  - Energy emission through the dark force portal via dipole

$$\langle \dot{E}_{\rm DF} \rangle = \frac{G G'^2}{12\pi} \eta^2 m^4 \left(\frac{g q_1}{\sqrt{G} m_1} - \frac{g q_2}{\sqrt{G} m_2} \right)^2 \frac{1}{a^4} \frac{2+e^2}{(1-e^2)^{5/2}}$$

    - charge separation from a parity violating bubble

[Kharzeev, Zhitnitsky, 0706.1026]

    - (possibly large) fermion flavor fluctuation at formation



$$m = m_1 + m_2 \,, \quad \eta = \frac{m_1 m_2}{m^2}$$

Energy Spectrum from the Binary

- **The GW energy spectrum of the binary**
  - ➡ GW frequency is related to orbital frequency: $f_{\mathrm{GW},s} = \omega/\pi$

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW},s}} = \frac{\dot{E}_{\mathrm{GW}}}{\dot{f}_{\mathrm{GW,s}}} = \frac{\pi \dot{E}_{\mathrm{GW}}}{\dot{\omega}} = \frac{\pi \dot{E}_{\mathrm{GW}}}{-\frac{3\sqrt{2}}{G' m^{5/2} \eta^{3/2}} \sqrt{-E}\dot{E}}$$

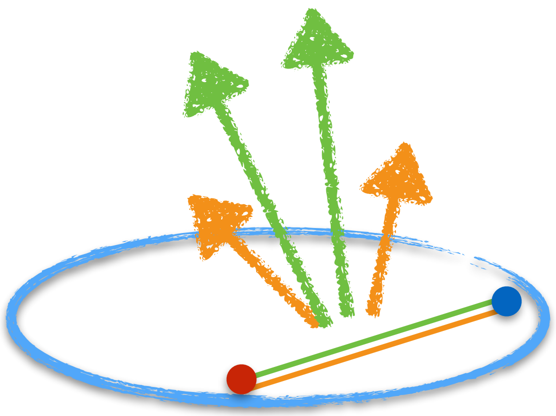
$$\dot{E} = \dot{E}_{\mathrm{GW}} + \dot{E}_{\mathrm{DF}}$$

same mass, opposite charge

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW},s}} = \frac{\pi\sqrt{a}\left(37e^4 + 292e^2 + 96\right) G'^{3/2} M_{\mathrm{obj}}^{5/2}}{3\sqrt{2}\left(\underbrace{10\,a(1-e^2)(2+e^2)(\beta-1)}_{\text{DF portal}} + \underbrace{\left(37e^4 + 292e^2 + 96\right) G' M_{\mathrm{obj}}}_{\text{GW portal}}\right)}$$

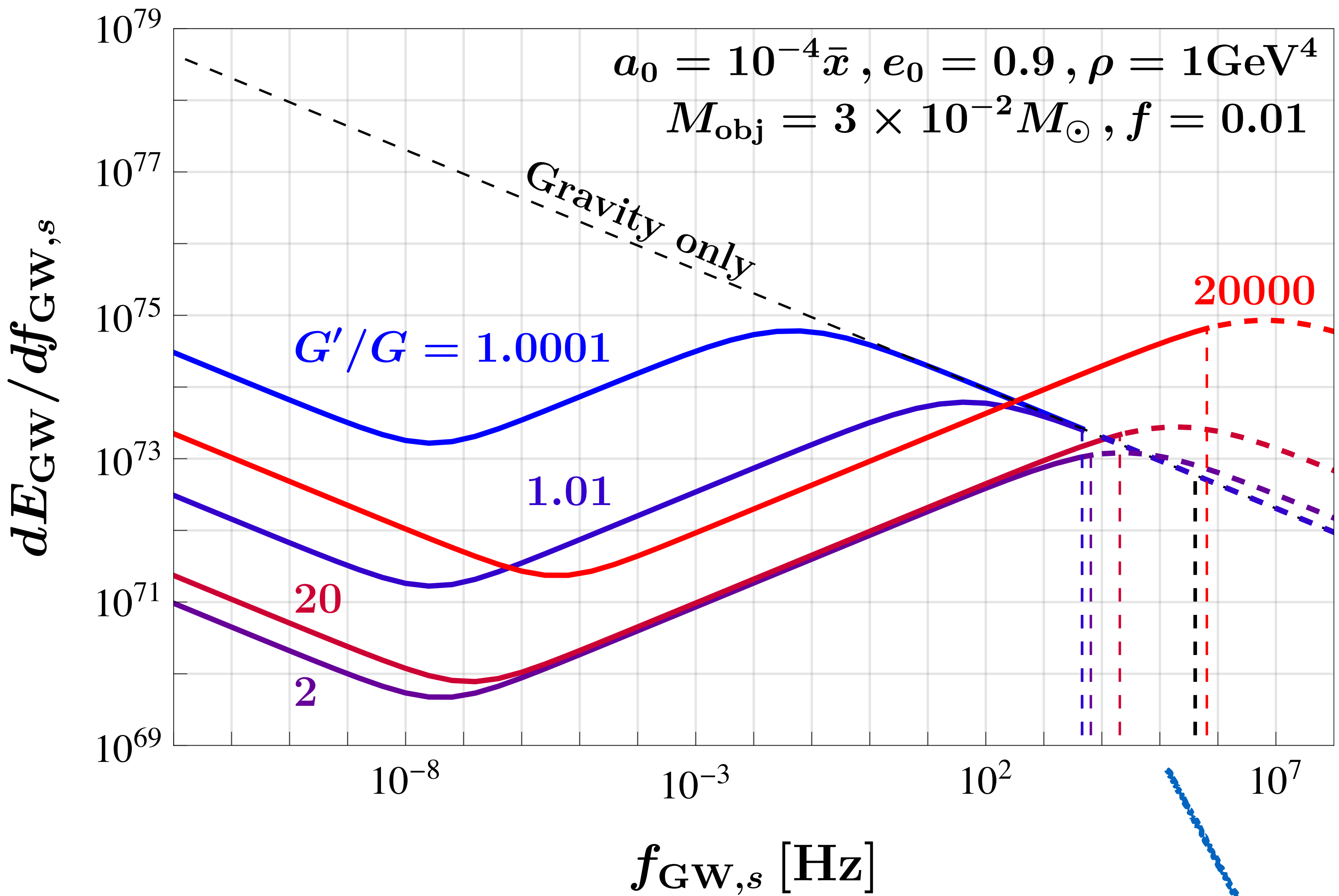
  - ➡ Without the dark force

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW,s}}} \propto \sqrt{a} \propto f_{\mathrm{GW,s}}^{-1/3}$$



Energy Spectrum from the Binary

- The GW energy spectrum of the binary



assuming the inspiral ends when the binary touches each other

SGWB from Dark Binaries

- **Convolution over cosmic history**
  - For primordial black holes (gravity only)

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_0^{z_{\rm sup}} dz \frac{R(z)}{(1+z)H(z)}\frac{dE_{\rm GW}}{df_{{\rm GW},s}}((1+z)f_{\rm GW})$$

  - With additional interactions, orbital geometry becomes extremely important

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_{\sqrt{1-c_2^2}}^{e_{0,\rm max}} de_0 \int d\tau \frac{n_{\rm obj}}{4}\mathcal{P}(\tau, e_0)\frac{dE_{\rm GW}}{df_{{\rm GW},s}}\left[(1+z(t))f_{\rm GW}\right]$$

orbital eccentricity **merger lifetime, related to semi-major axis**

The Merger Rate

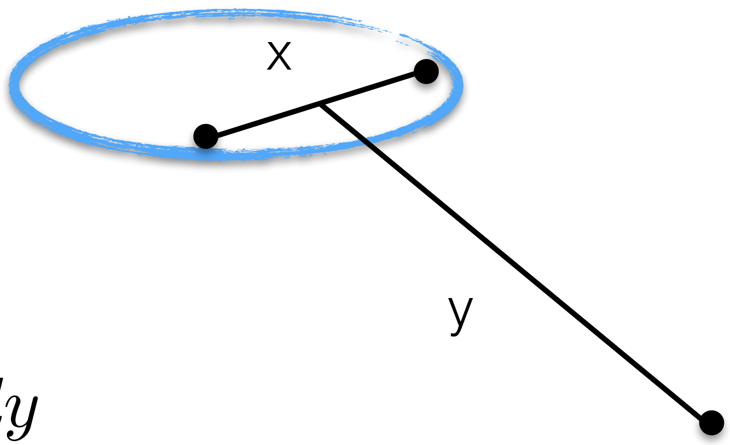
- The merger rate depends on the geometry of the binary and its nearest neighbor

$$R(x,y)=\frac{1}{2}\frac{n_{\rm obj}}{2}P=\frac{1}{2}\frac{3H_0^2}{8\pi G}\frac{f\,\Omega_{\rm DM}}{2M_{\rm obj}}P(x,y)$$

x: comoving distance between the binary

y: comoving distance to the nearest neighbor

f: fraction of the binaries among all DM



➡ Assuming random formation

$$P(x,y)\,dxdy=\frac{9x^2y^2}{\bar{x}^6}e^{-(y/\bar{x})^3}dxdy$$

➡ In terms of the orbital parameters

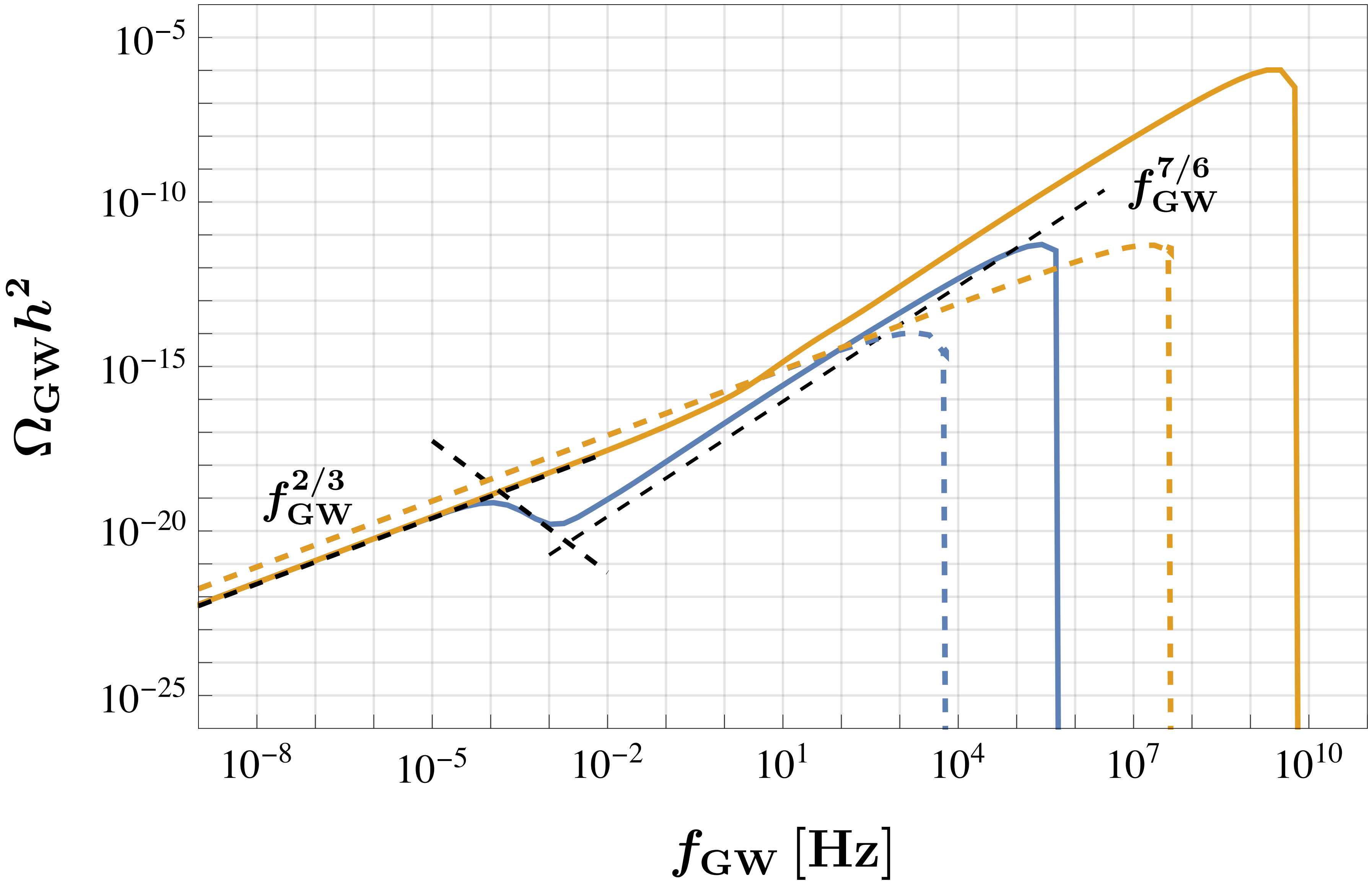
$$a_0=\frac{c_1}{\beta}\frac{1}{f}\frac{x^4}{\bar{x}^3}\,,b_0=c_2\left(\frac{x}{y}\right)^3a_0\,,e_0=\sqrt{1-\left(\frac{b_0}{a_0}\right)^2}\,.$$

$$\bar{x}=\frac{1}{1+z_{\rm eq}}\left(\frac{8\pi GM_{\rm obj}}{3H_0^2\,f\,\Omega_{\rm DM}}\right)^{1/3}$$

[Ioka, Chiba, Tanaka, Nakamura, astro-ph/9807018]

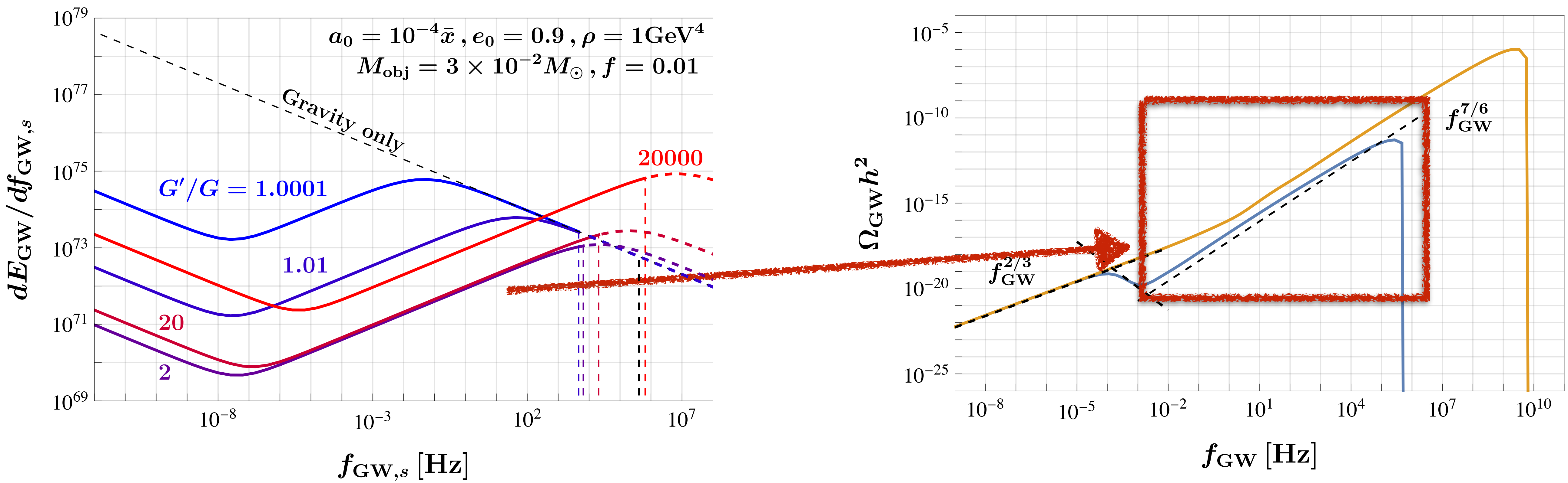
Spectral Shape

- A two- or three-stage power-law spectrum



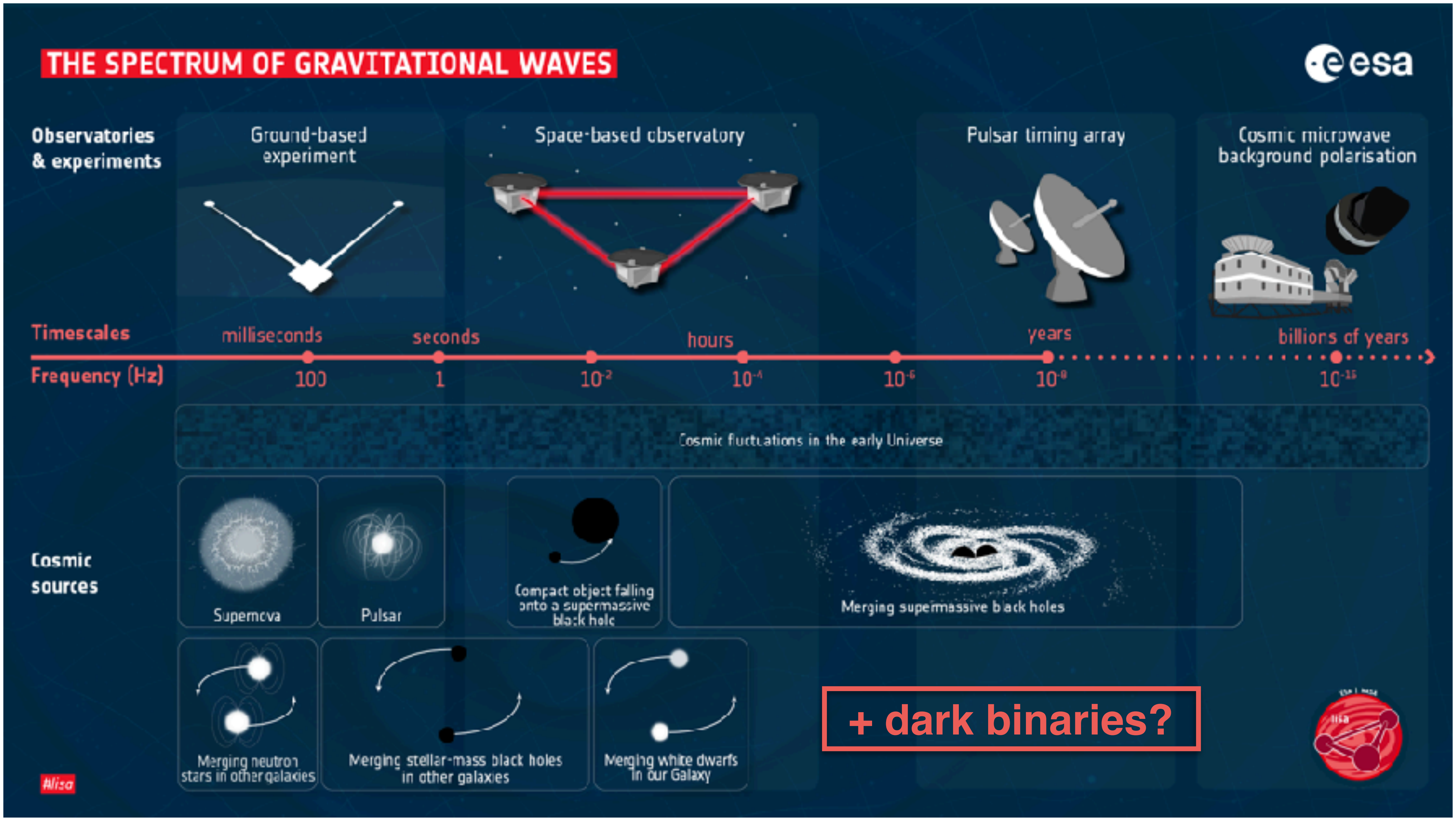
Spectral Shape

- **The high-frequency region**
  - Mainly from dark force dominated mergers at MD



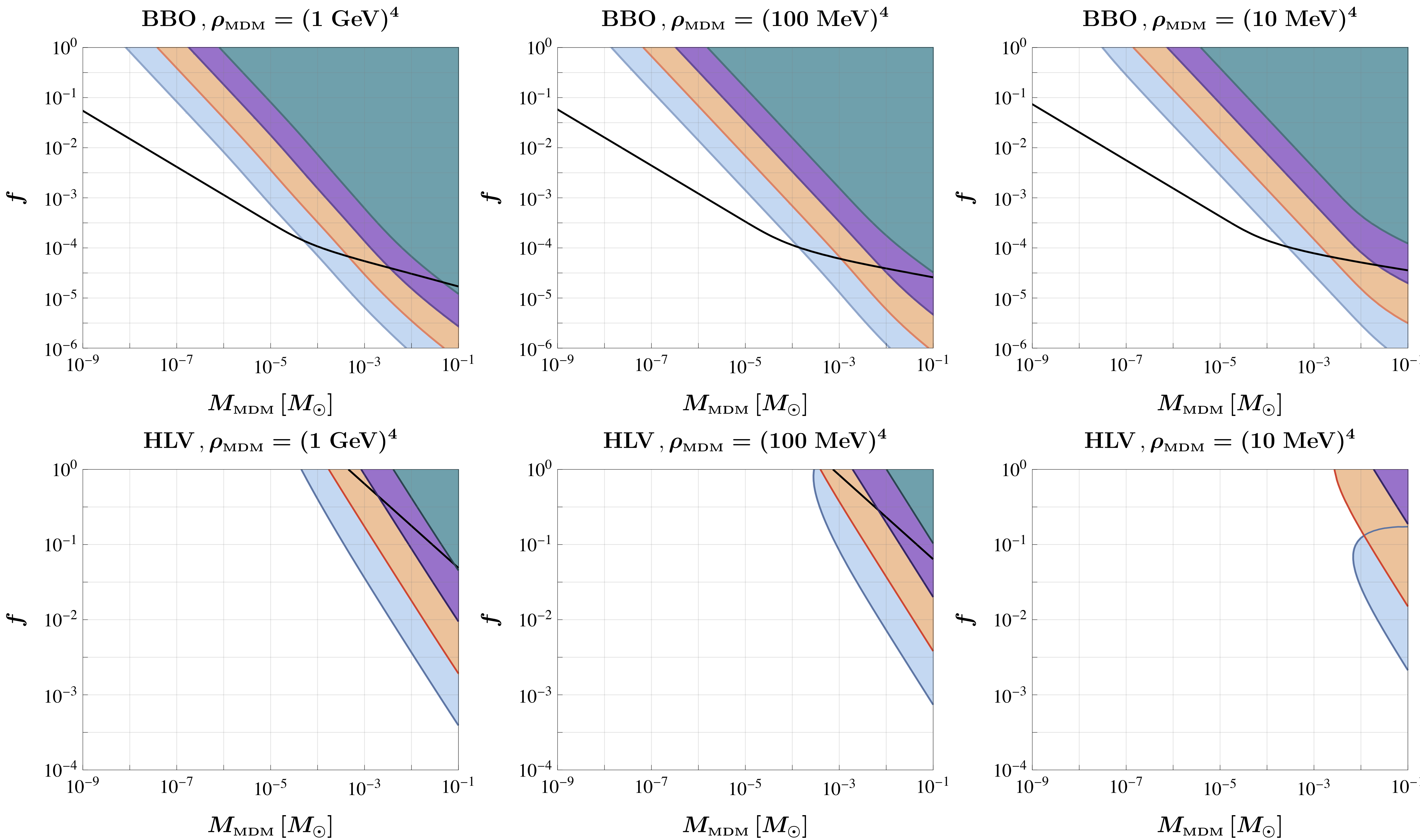
$$\Omega_{\rm GW} \approx \frac{4 \times 2^{\frac{5}{6}} \times 13^{\frac{1}{18}} \pi^{\frac{59}{36}} f^{\frac{13}{9}} G^{\frac{55}{36}} \beta^{\frac{67}{36}} (1+z_{\rm eq})^{\frac{1}{3}} \rho_{\rm obj}^{\frac{1}{12}} H_0^{\frac{1}{9}} M_{\rm obj}^{\frac{13}{9}} \Omega_{\rm DM}^{\frac{10}{9}}}{45 \times 3^{7/36} e^{13/9} c_1^{1/3} c_2^{5/9} (\beta-1)^{8/9} \Omega_{\rm M}^{1/18}} f_{\rm GW}^{7/6}$$

Sensitivity at Experiments



➡ Consider SKA, LISA, BBO and LIGO-Virgo (HLV)

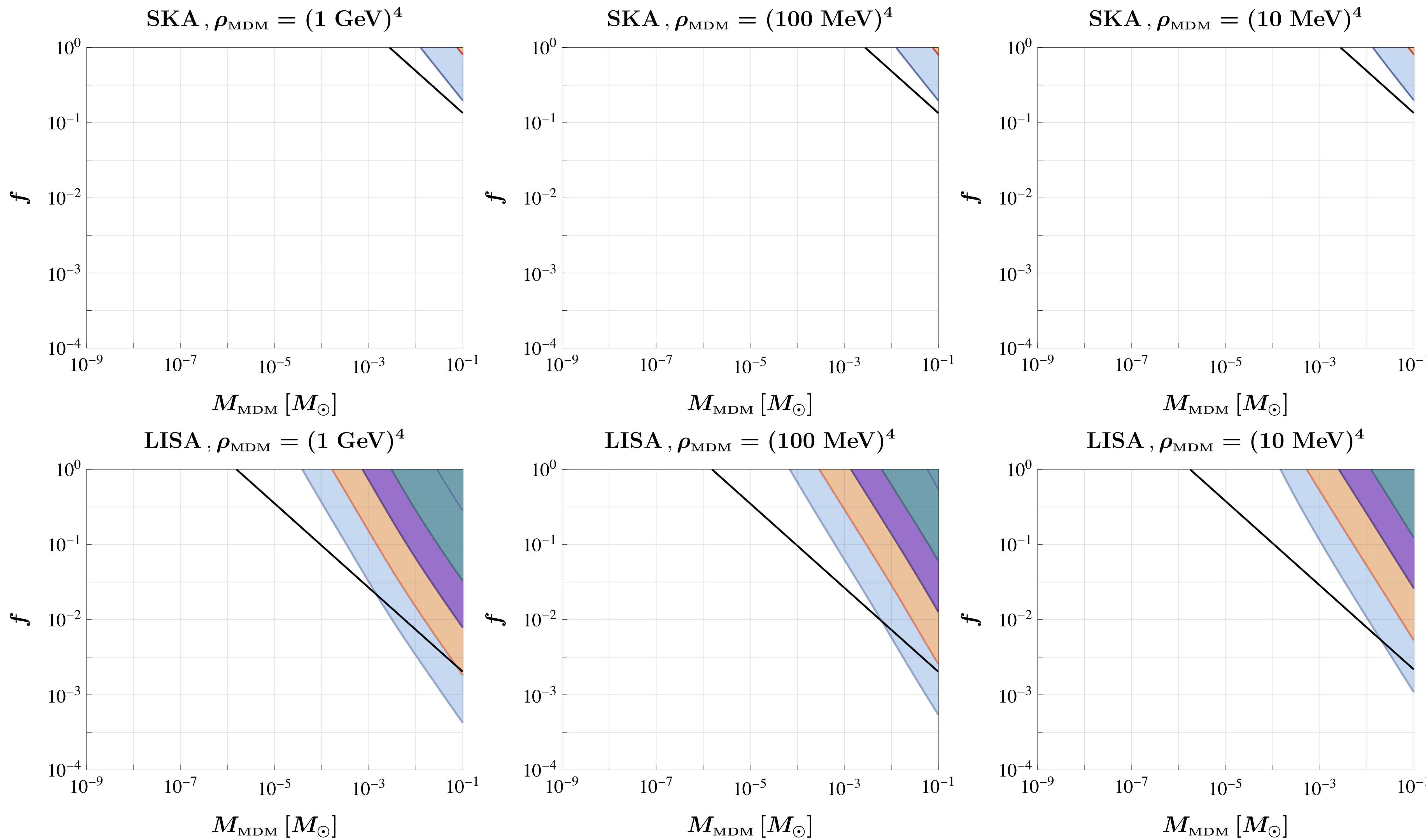
Sensitivity at Experiments



$\varrho_{\rm th} = 1\,,\ T_{\rm obs} = 20$ yr (SKA)/1 yr (others)

$: \ \beta \equiv G'/G = 10^4, 10^3, 10^2, 10$

Sensitivity at Experiments

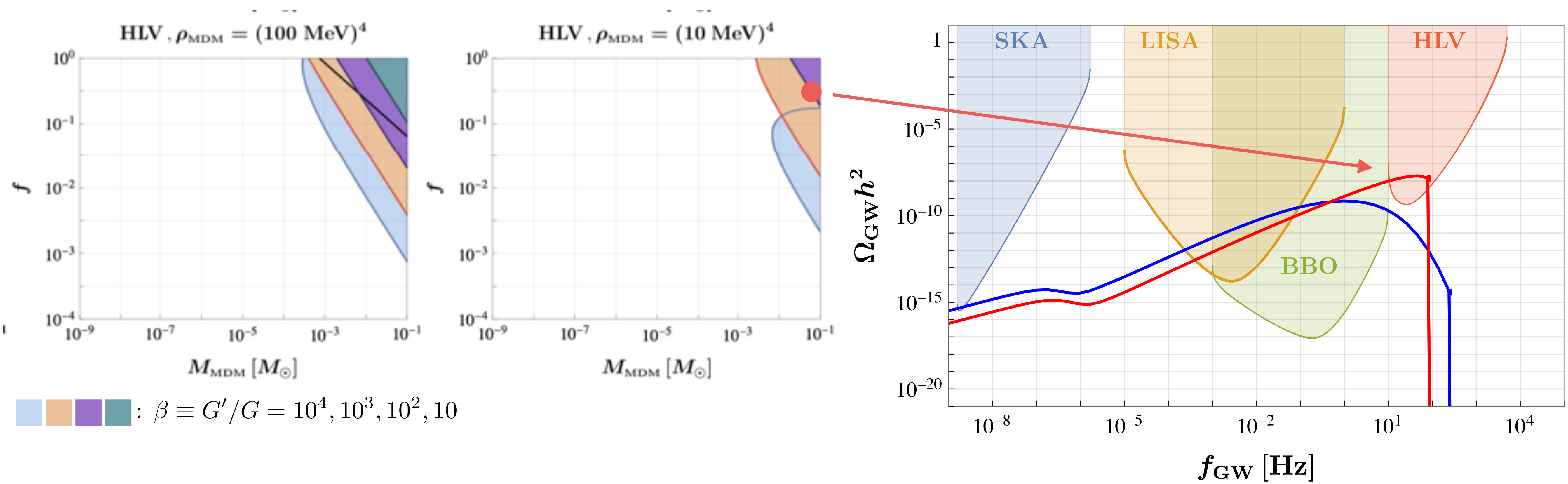


$\varrho_{\rm th} = 1\,,\, T_{\rm obs} = 20$ yr (SKA)/1 yr (others)

: $\beta \equiv G'/G = 10^4, 10^3, 10^2, 10$

Sensitivity at Experiments

- **A larger interaction doesn't always come with a larger signal**



  - ➡ Too large an interaction makes the merger happens too early and thus doesn't contribute to the corresponding frequency

Complications from the Model

- **The Yukawa sector**

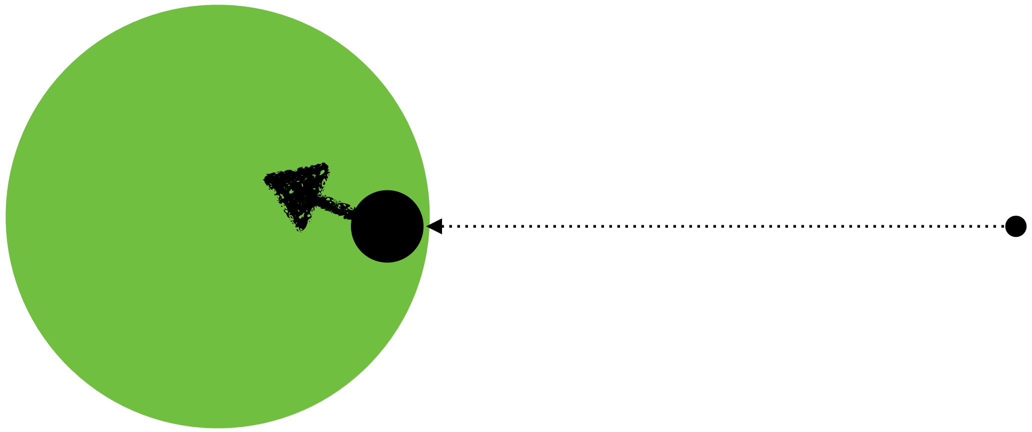
$$\mathcal{L}_{\rm dQCD} = \sum_{i=1}^{N_f} \left[\bar{\psi}_i i\gamma^\mu D_\mu \psi_i - m_{\psi_i}\, \bar{\psi}_i\psi_i\right] - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a}$$

$$\mathcal{L}_{\rm Yukawa} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \sum_i (m_{\psi_i} + y_i\,\phi)\,\bar{\psi}_i\,\psi_i - V_0(\phi)\,, \quad V_0(\phi) = \frac{1}{2} m_{\rm med}^2 \phi^2$$

- **Finite density effect**

$$V_1 = g\,\frac{1}{2\pi^2}\int_0^{k_F} dk\, k^2 \sqrt{k^2 + (m_\psi + y\,\phi)^2} \approx \frac{g}{8\pi^2}\, \underline{\mu^2}\, \left[\mu^2 - (m_\psi + \underline{y\,\phi})^2\right]$$

➡ The mediator becomes heavy inside: a screening effect!



Complications from the Model

- **Requiring the dark force range to cover the MDM**

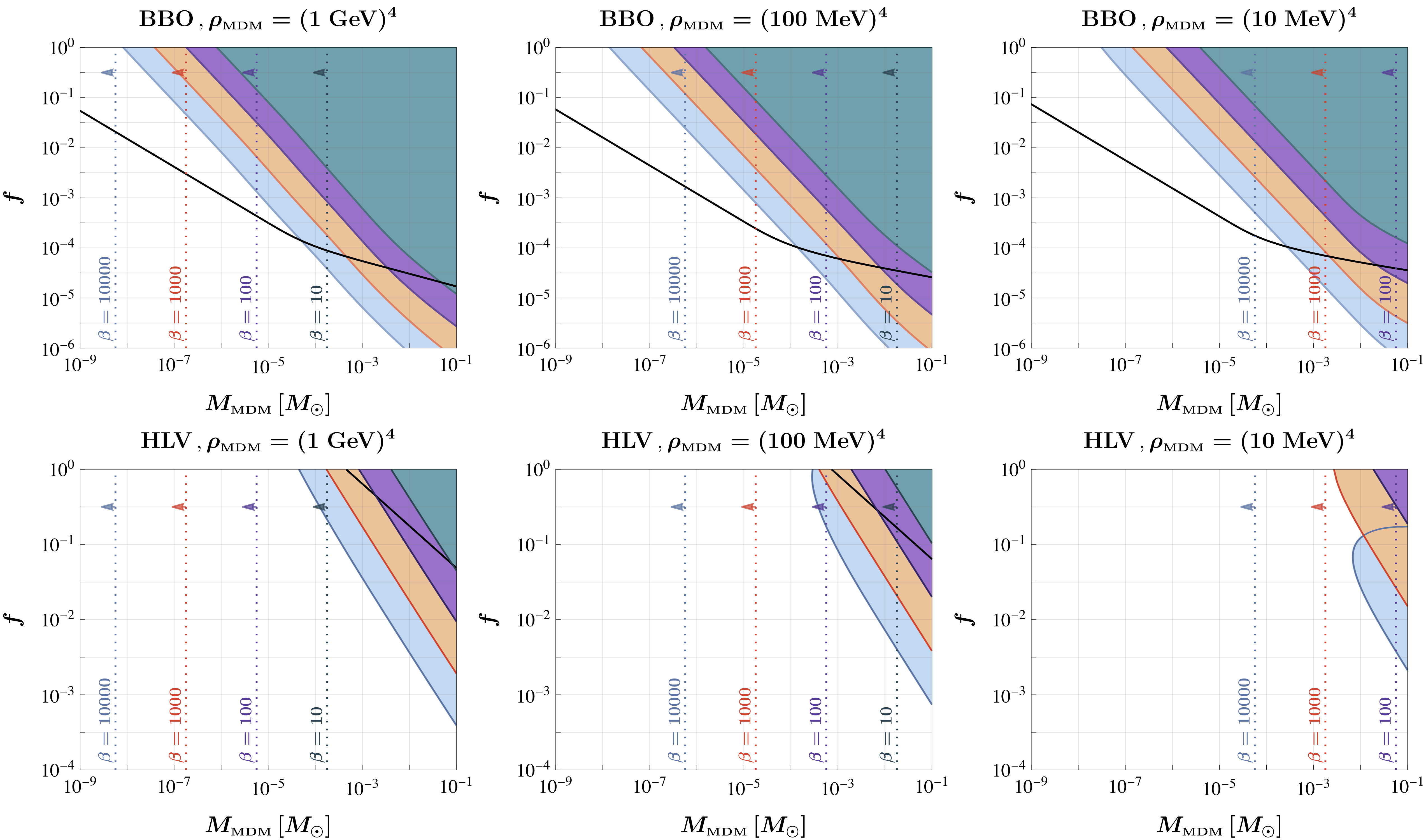
$$|m_{\rm in}|R < 1$$

$$\Rightarrow y < 2^{19/12} 3^{-7/12} \pi^{5/6} g^{-1/4}\, \Lambda_d^{1/3}\, M^{-1/3} = (2 \times 10^{-19}) \left(\frac{\Lambda_d}{1\,\mathrm{GeV}}\right)^{1/3} \left(\frac{M_\odot}{M}\right)^{1/3}$$

$$\alpha = \frac{3\, g\, y^2\, m_\psi^2}{128\pi^3\, G\, \Lambda_d^4} \lesssim (0.02) \left(\frac{m_\psi/\Lambda_d}{0.5}\right)^2 \left(\frac{1\,\mathrm{GeV}}{\Lambda_d}\right)^{4/3} \left(\frac{M_\odot}{M}\right)^{2/3}$$

- ➡ DF may not be arbitrarily strong for MDM
- ➡ Model dependent

Complications from the Model



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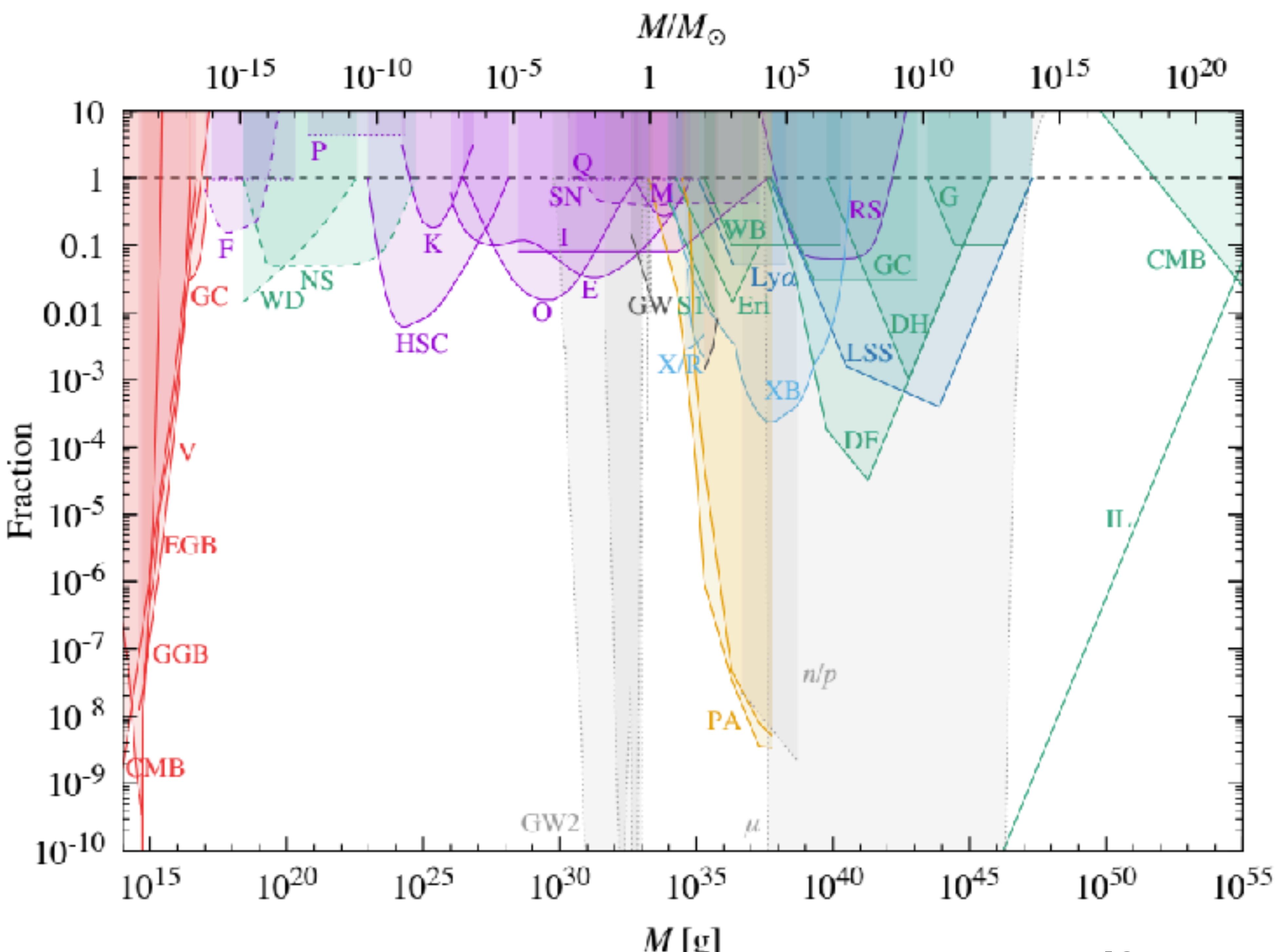
: $\beta \equiv G'/G = 10^4, 10^3, 10^2, 10$

Lensing



[Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

Conclusion

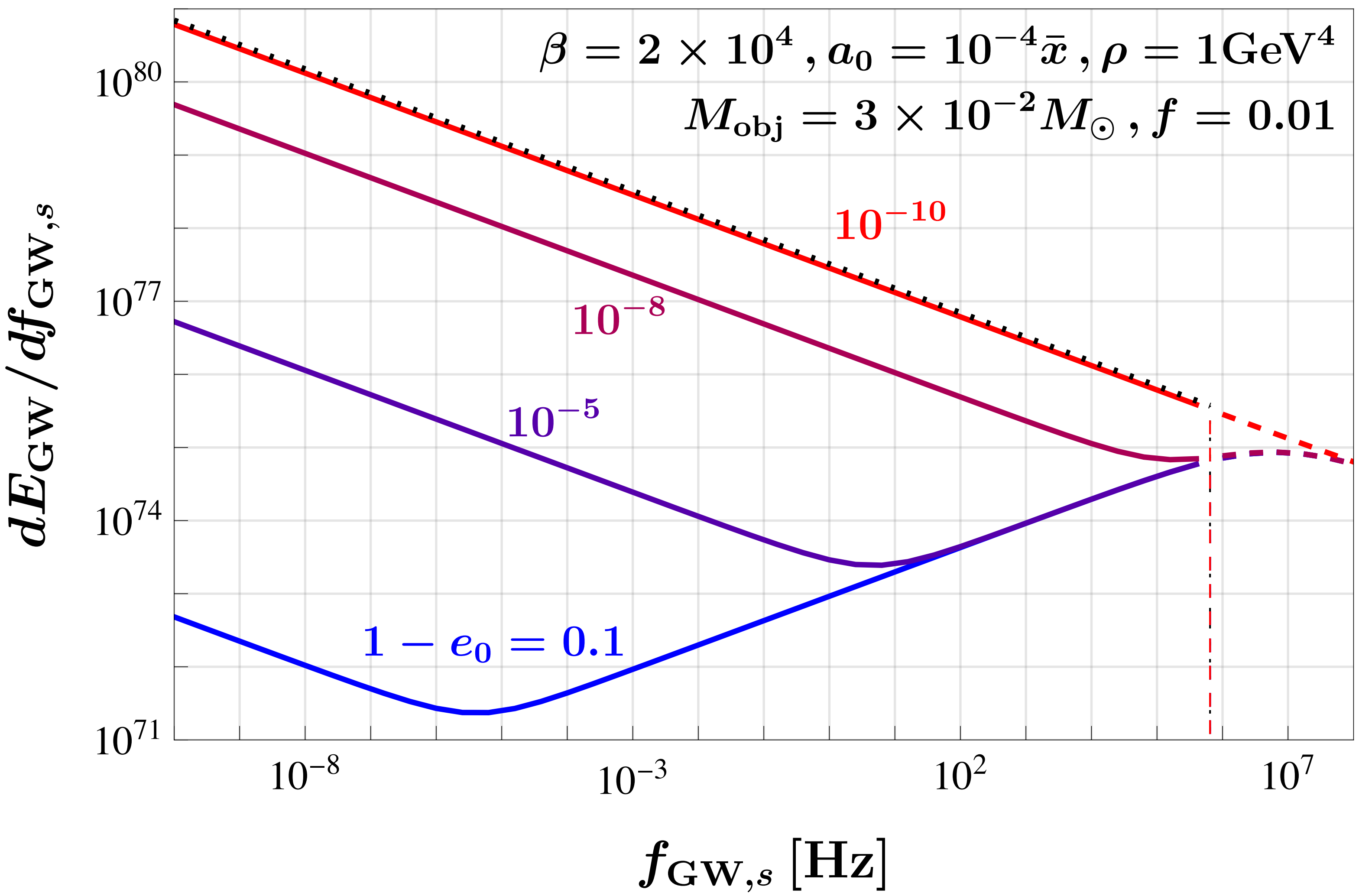
- Macroscopic DM may be formed from a cosmic phase transition, and later organize themselves into binaries
- Mergers of these binaries can be a new source of SGWB
- With an additional attractive interaction, the SGWB from the dark binaries can distinctively different
- From the aspects of both signal strength and model building, the additional interaction may not be arbitrarily strong

Thank you!

Backup

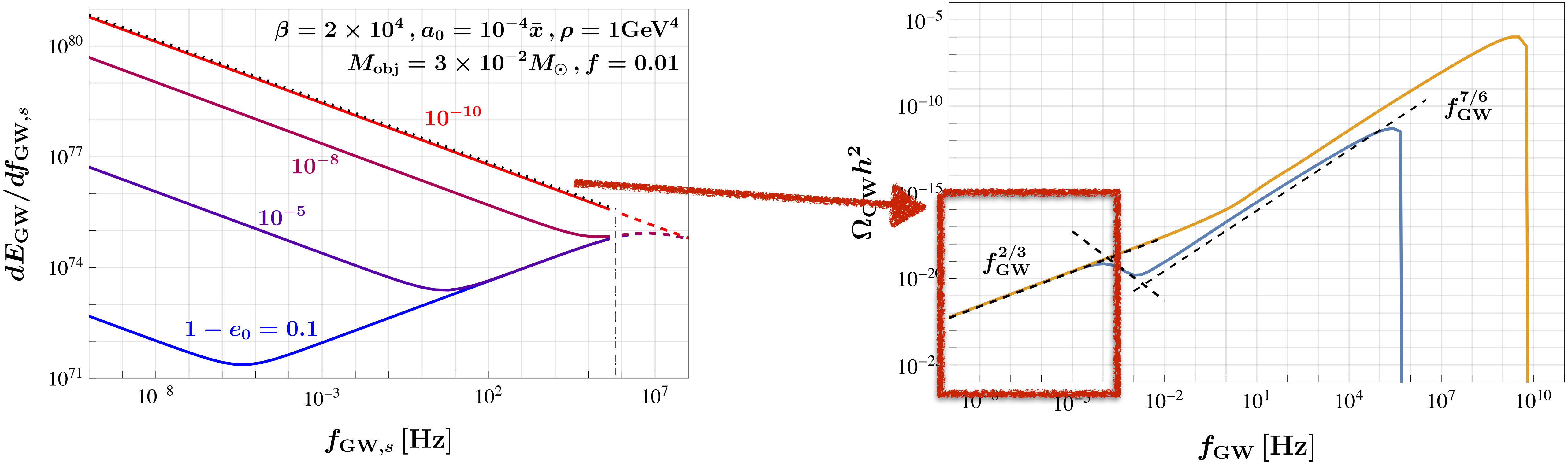
Energy Spectrum from the Binary

- **The GW energy spectrum of the binary**
  - Small a_0 or e_0



Spectral Shape

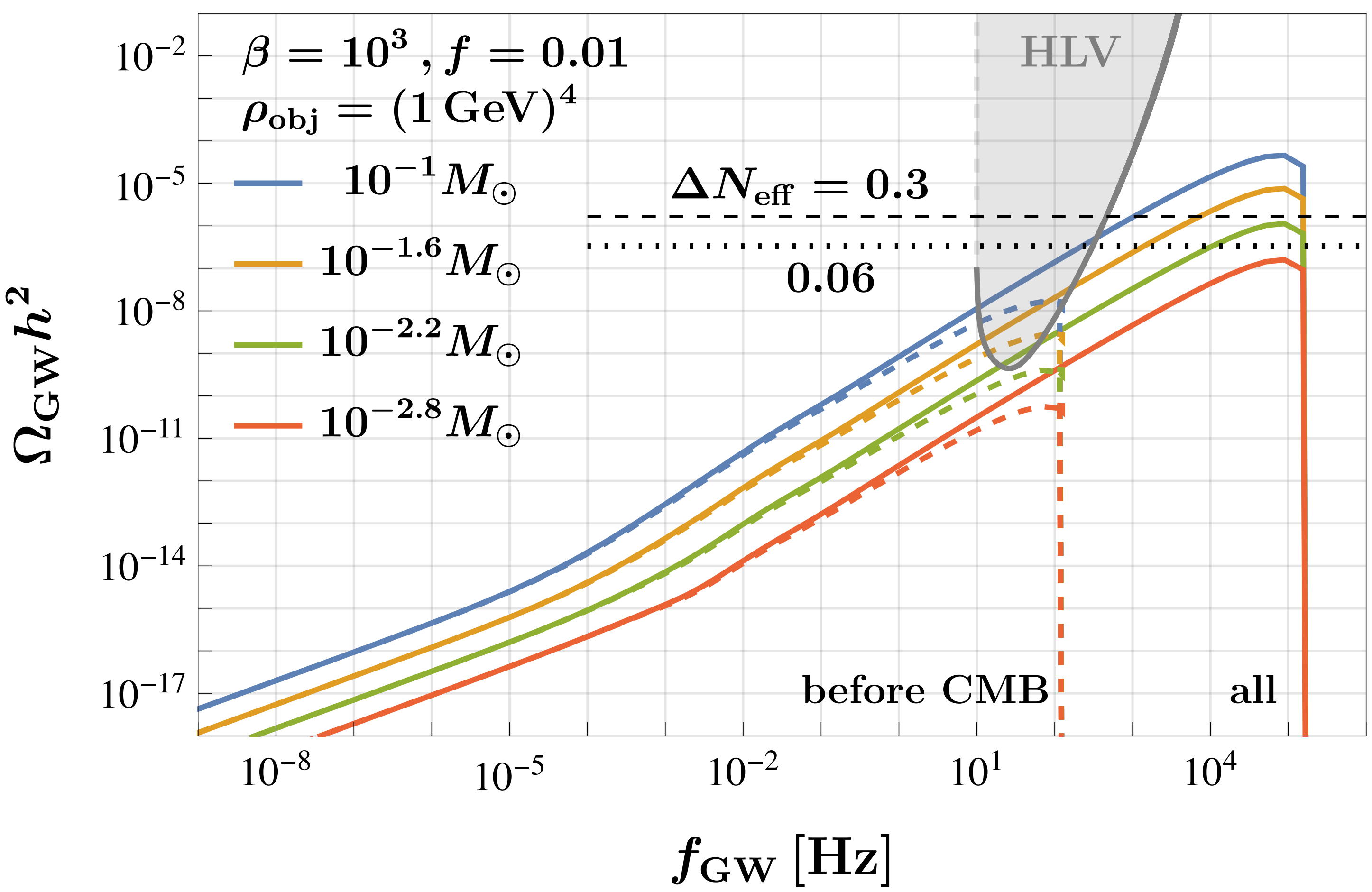
- **The low-frequency region**
  - ➡ A relic from the very early, GW-dominated mergers



$$\Omega_{\rm GW} \approx \frac{85^{\frac{2}{5}}\pi^{\frac{8}{15}}\Gamma^{\frac{3}{5}}\left(\frac{7}{3}\right)f^{\frac{6}{5}}\left(G^{14}\beta^{17}H_0^4M_{\rm obj}^{14}(1+z_{\rm eq})\Omega_{\rm DM}^{17}\right)^{\frac{1}{15}}}{2^{\frac{77}{15}}3^{\frac{4}{15}}c_1^{\frac{4}{5}}c_2^{\frac{4}{5}}(\beta-1)^{\frac{2}{5}}}f_{\rm GW}^{2/3}$$

Constraints from N_{eff}

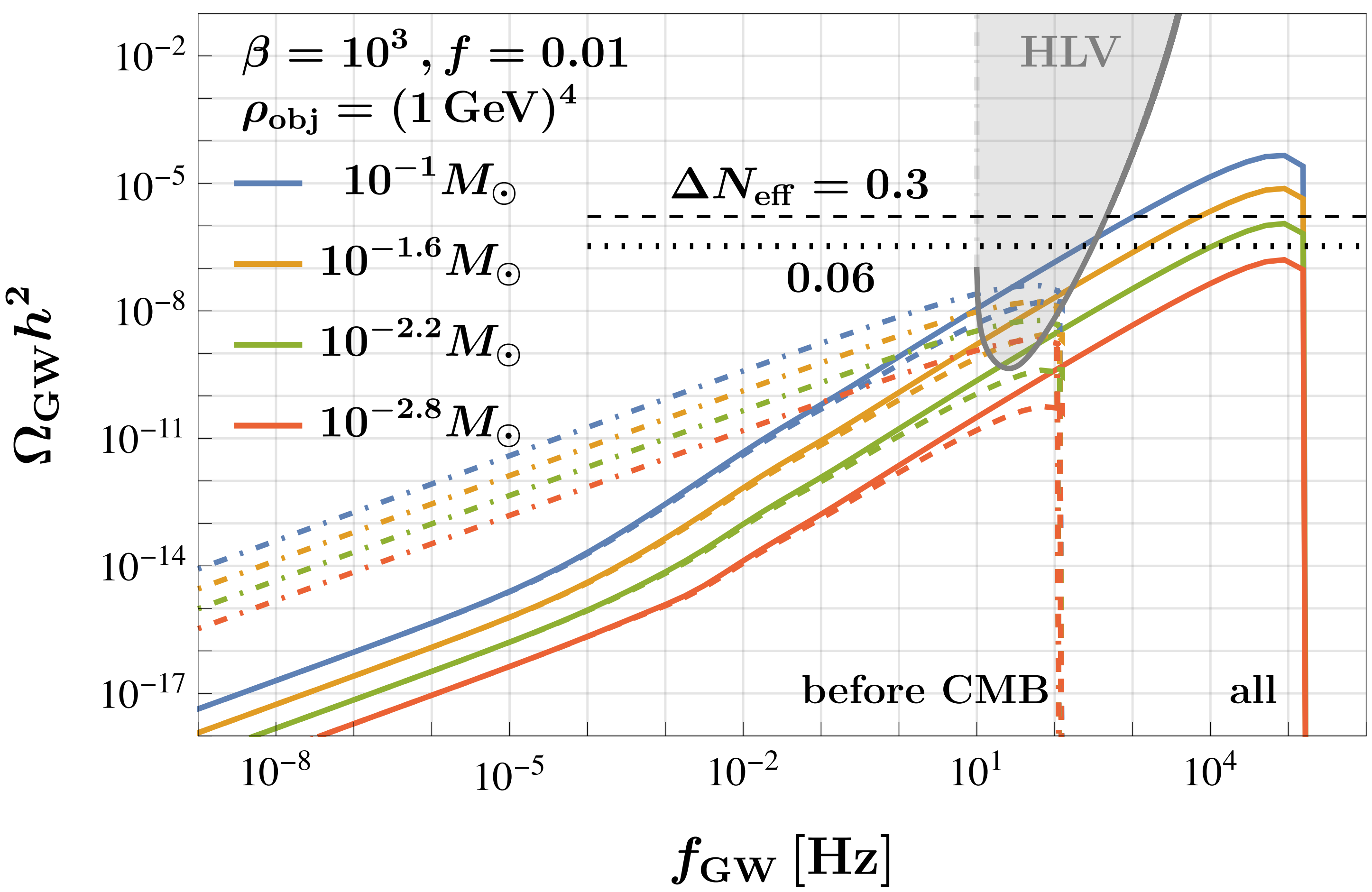
- Counting contributions from only before the CMB



➡ Amplitude of that part is ~$10^{3.5}$ smaller compared with the full spectrum for the GW

Constraints from N_{eff}

- **Counting contributions from only before the CMB**



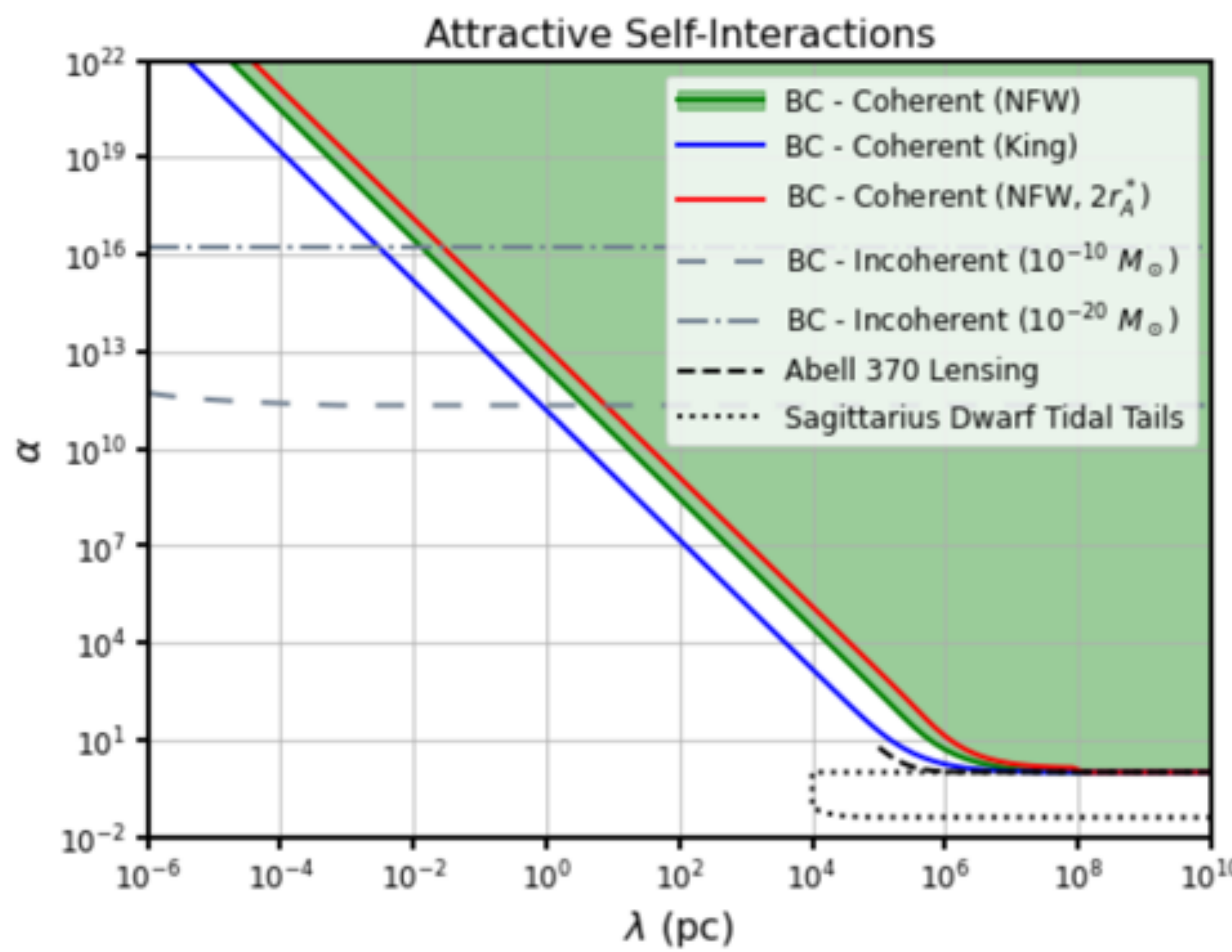
➡ DF emission is not a huge issue either

DF Mediator Mass

- **Typical distance**

$$\bar{x} = \frac{1}{1+z_{\rm eq}}\left(\frac{8\pi G M_{\rm obj}}{3H_0^2\, f\,\Omega_{\rm DM}}\right)^{1/3} \approx 0.1\,{\rm pc}\left(\frac{M_{\rm obj}}{M_\odot}\right)^{1/3}\left(\frac{1}{f}\right)^{1/3} \sim (6\times 10^{-23}\,{\rm eV})^{-1}\left(\frac{M_{\rm obj}}{M_\odot}\right)^{1/3}\left(\frac{1}{f}\right)^{1/3}$$

- **Cosmological constraints from bullet clusters**



[Bogorad, Graham, Ramani, 2311.07648]

Formation of the Dark Binaries

- **The dark binaries are considered formed when they decouple from the Hubble flow**
  - ➡ Compare the centripetal force with the dragging from the Hubble flow

[Ioka, Chiba, Tanaka, Nakamura, astro-ph/9807018]

  - ➡ Additional G'/G to compensate for the dark force

$$G'\bar{\rho}_{\rm obj} \equiv G' \cdot f \frac{\rho_{\rm eq}}{2} \frac{\bar{x}^3 R_{\rm eq}^3}{x^3 R^3} = G\rho_r$$

  - ➡ Should decouple before matter-radiation equality

More on the Screening

- **The scalar field with the effective potential**

$$\partial_r^2\phi + \frac{2}{r}\partial_r\phi = \frac{\partial V_{\rm eff}(\phi)}{\partial\phi}$$

$$V_{\rm eff}(\phi) = -\,a\,\phi\,\Theta(R-r) + \frac{1}{2}\,\left[m_{\rm in}^2\,\Theta(R-r) + m_{\rm med}^2\,\Theta(r-R)\right]\,\phi^2$$

- **The solution**

$$\phi_{\rm out} = c_1 e^{-m_{\rm med}\,r}\,, \qquad \phi_{\rm in} = a/m_{\rm in}^2 + c_2(e^{-m_{\rm in}\,r} - e^{m_{\rm in}\,r})/r$$

$$\frac{q_{\rm eff}\,y}{4\pi} = c_1 = a\,\frac{e^{m_{\rm med}\,R}[m_{\rm in}\,R\,\cosh\,(m_{\rm in}\,R) - \sinh\,(m_{\rm in}\,R)]}{m_{\rm in}^2[m_{\rm med}\,\sinh\,(m_{\rm in}\,R) + m_{\rm in}\,\cosh\,(m_{\rm in}\,R)]}$$

$$m_{\rm med}R \ll 1\,,\; m_{\rm in}R \ll 1 \Longrightarrow c_1 = a\,R^3/3$$

$$m_{\rm med}R \ll 1\,,\; m_{\rm in}R \gg 1 \Longrightarrow c_1 = a\,\frac{R^3}{(m_{\rm in}\,R)^2}$$

Scalar and Vector Emission

- **Emission through vector or scalar takes the same form in the massless mediator limit**

$$\langle \dot{E}_S \rangle = \frac{1}{3}\eta^2 m^2 \omega^4 r^2 g_S(m_S, e)(\tilde{\mathrm{q}}_1 - \tilde{\mathrm{q}}_2)^2,$$

$$\langle \dot{E}_V \rangle = \frac{2}{3}\eta^2 m^2 \omega^4 r^2 g_V(m_V, e)(\tilde{\mathrm{q}}_1 - \tilde{\mathrm{q}}_2)^2,$$

$$g_S(m_S, e) = \sum_{n=1}^{\infty} 2n^2 \left[\mathcal{J}_n'^2(ne) + \left(\frac{1-e^2}{e^2}\right)\mathcal{J}_n^2(ne)\right] \times \left[1 - \left(\frac{m_S}{n\omega}\right)^2\right]^{3/2},$$

$$g_V(m_V, e) = \sum_{n=1}^{\infty} 2n^2 \left[\mathcal{J}_n'^2(ne) + \left(\frac{1-e^2}{e^2}\right)\mathcal{J}_n^2(ne)\right] \times \left[1 - \left(\frac{m_V}{n\omega}\right)^2\right]^{1/2} \left[1 + \frac{1}{2}\left(\frac{m_V}{n\omega}\right)^2\right],$$

[Krause, Kloor, Fischbach, 1994]

[Alexander, McDonough, Sims, Yunes, 1808.05286]

SGWB from Dark Binaries

- **With these discussed**

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_{\sqrt{1-c_2^2}}^{e_{0,\max}} de_0 \int d\tau \frac{n_{\rm obj}}{4}\mathcal{P}(\tau, e_0)\frac{dE_{\rm GW}}{df_{{\rm GW},s}}\left[(1+z(t))f_{\rm GW}\right]$$

$$t = t_{\rm dec} + \tau$$

- **Boundaries of the τ-integrations**
  - Formation of the macroscopic DM

$$t_{\rm dec} > t_{\rm form}\ ,\ t < t_0$$

we take $t_{\rm form} \sim \rho_{\rm obj}^{1/4}$

  - Decouple before matter-radiation equality

$$\tau < c_1^3 f\,\beta h(e_0)\bar{\tau}$$

  - For a given GW frequency to be produced at the source

$$\frac{G'm}{a_0^3} < \pi^2(1+z(t))^2 f_{\rm GW}^2 < \frac{G'm}{(2R_{\rm obj})^3}\ .$$

$$f_{\rm GW,max} = \sqrt{\frac{G'\rho_{\rm obj}}{3\pi}}$$

SGWB from Dark Binaries

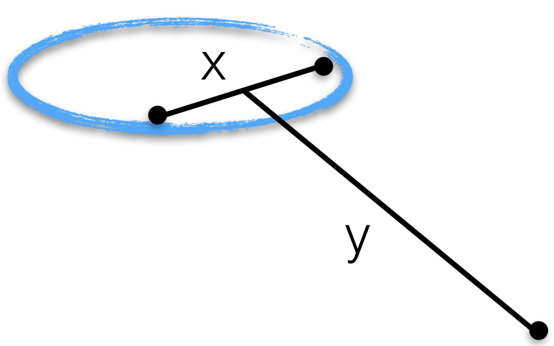
- **With these discussed**

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c} \int_{\sqrt{1-c_2^2}}^{e_{0,\rm max}} de_0 \int d\tau \frac{n_{\rm obj}}{4} \mathcal{P}(\tau, e_0) \frac{dE_{\rm GW}}{df_{{\rm GW},s}} \left[(1+z(t))f_{\rm GW}\right]$$

$$t = t_{\rm dec} + \tau$$

- **Boundaries of the e0-integrations**
    - Lower boundary comes from the requirement of y>x

$$y > x \Rightarrow e_0^2 > 1 - c_2^2$$



    - Upper boundary comes indirectly from the phase space of the τ-integration

Higher Harmonics

- **With an eccentric orbit, the binary should emit GW at all harmonics of the orbital frequency**
  - ➡ We are effectively assuming all energy are emitted through the n=2 channel
  - ➡ To account for the other modes

$$\frac{dE_{\rm GW}}{dt} = \frac{32GG'^3\eta^2 m^5}{5a^5}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} = \frac{32GG'^3\eta^2 m^5}{5a^5}\sum_n g(n,e)\,,$$

$$g(n,e) = \frac{n^4}{32}\Bigg\{\left[J_{n-2}(ne)-2eJ_{n-1}(ne)+\frac{2}{n}J_n(ne)+2eJ_{n+1}(ne)-J_{n+2}(ne)\right]^2 + (1-e^2)\left[J_{n-2}(ne)-2eJ_n(ne)+J_{n+2}(ne)\right]^2+\frac{4}{3n^2}J_n^2(ne)\Bigg\}\,,$$

[Peters and Mathews, Phys. Rev. 131 (1963) 435-439]

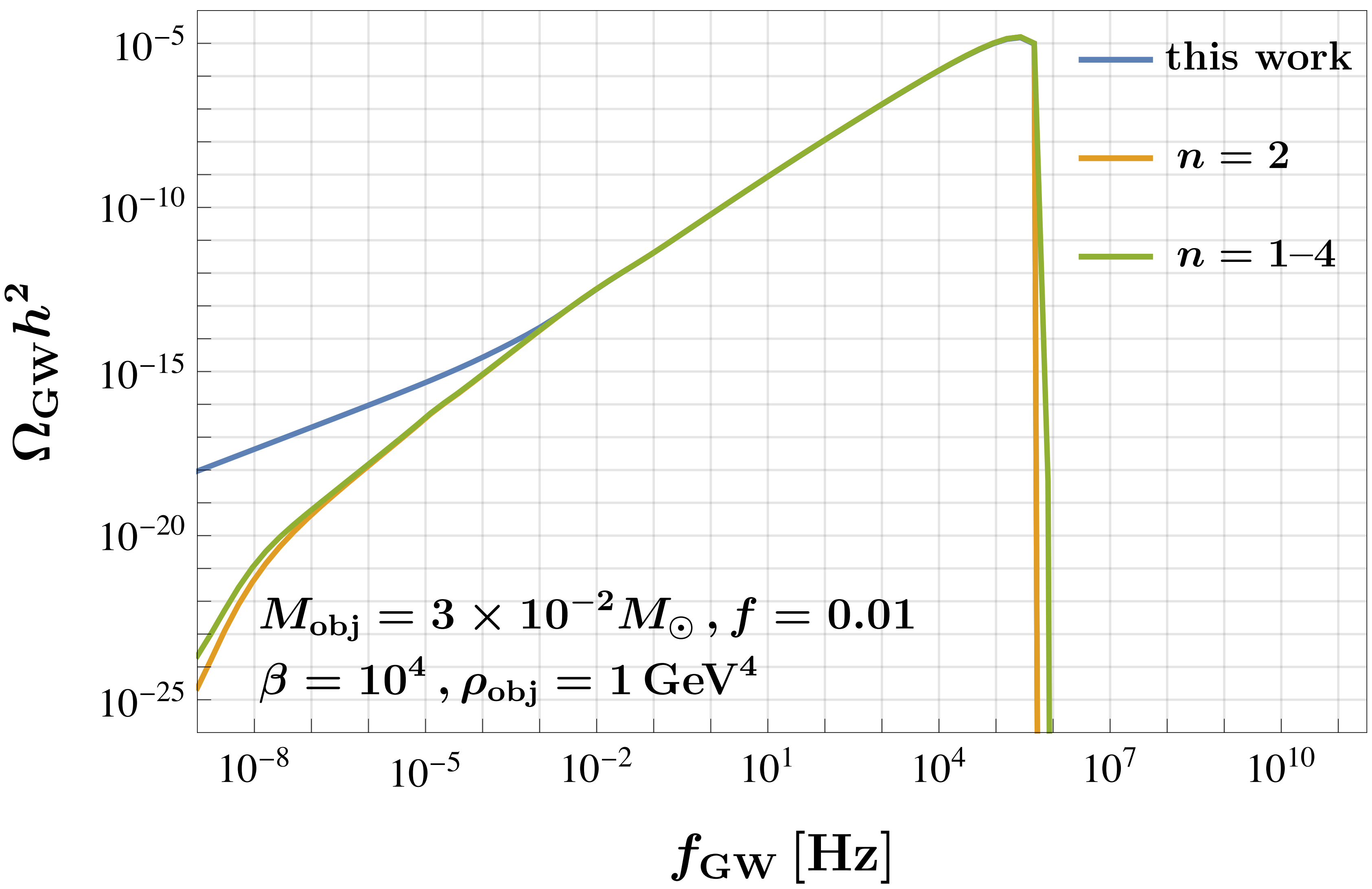
[Enoki, Nagashima, astro-ph/0609377]

Higher Harmonics

- **The following calculation is straight-forward**

$$
\begin{aligned}
\frac{d^2 E_{\rm GW}}{dt\, df_{{\rm GW},s}} &= \frac{32 G G'^3 \eta^2 m^5}{5a^5} \sum_n g(n,e)\delta(f_{{\rm GW},s} - n f_{\rm orb})\,, \\
\frac{dE_{\rm GW}}{df_{{\rm GW},s}} &= \sum_n \int dt \frac{32 G G'^3 \eta^2 m^5}{5a^5} g(n,e)\delta(f_{{\rm GW},s} - n f_{\rm orb}) \\
&= \sum_n \int \frac{de}{de/dt} \frac{32 G G'^3 \eta^2 m^5}{5a^5} g(n,e)\delta(f_{{\rm GW},s} - n f_{\rm orb}) \\
&= \sum_n \int de \frac{32 G G'^3 \eta^2 m^5}{5a^5} \frac{a^3(1-e^2)^{3/2}}{4eG^2(\mathcal{G}-1)\mathcal{G}M_{\rm obj}^2} \frac{g(n,e)}{n} \delta(f_{\rm orb} - f_{{\rm GW},s}/n) \\
&= \sum_n \frac{32 G G'^3 \eta^2 m^5}{5a^5} \frac{a^3(1-e^2)^{3/2}}{4eG^2(\mathcal{G}-1)\mathcal{G}M_{\rm obj}^2} \frac{g(n,e)}{n} \frac{1}{\left|\frac{df_{\rm orb}}{de}\right|_{e=e_n}} \\
&\equiv \sum_n \left(\frac{E_{\rm GW}}{df_{{\rm GW},s}}\right)_n \,,
\end{aligned}
$$

Higher Harmonics



Sensitivity at Experiments

- **Signal-to-noise ratio for multiple detectors where cross-correlation can be performed**

$$\varrho^2 = n_{\mathrm{det}} T_{\mathrm{obs}} \int df_{\mathrm{GW}} \left(\frac{\Omega_{\mathrm{GW}}}{\Omega_{\mathrm{noise}}} \right)^2$$

[Schmitz, 2002.04615]

  - ➡ n_det=2 for cross-correlation, and 1 for auto-correlation (if applicable)

  - ➡ Nontrivial noise subtraction is required for auto-correlation
    - TDI interfereometry?

[Smith and Caldwell, 1908.00546]