2024. 2. 23 GC2024 @ YITP, Kyoto

Wave Nature of GW Lensing and its Applications

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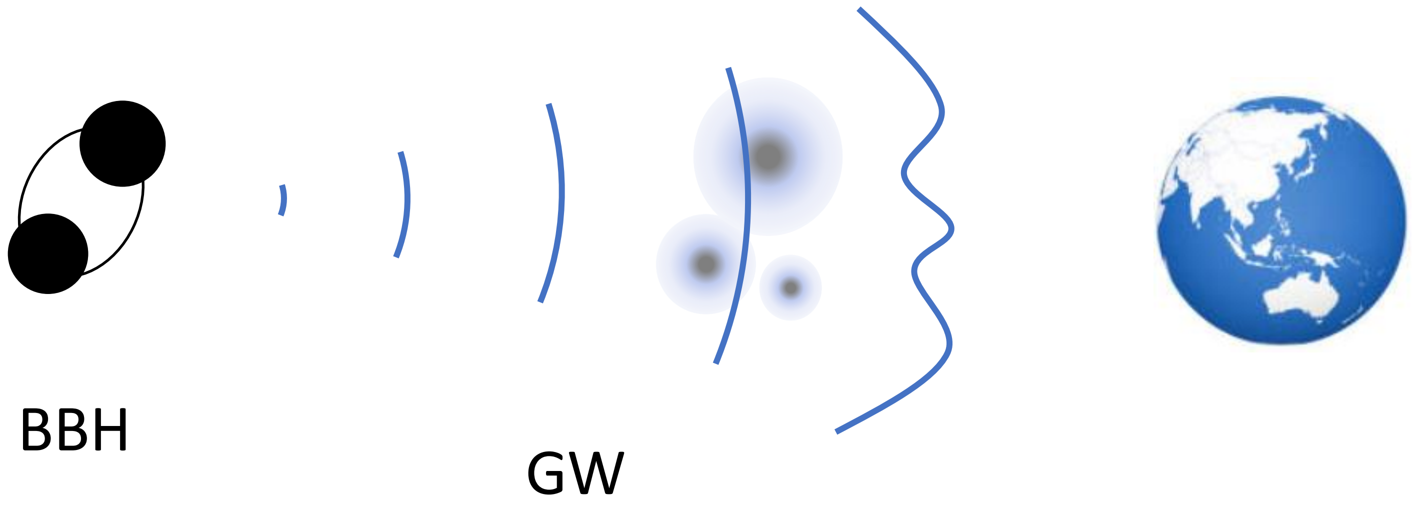
Based on

- "Small-scale shear: peeling off diffuse subhalos with gravitational waves"
  HGC, Chanung Park and Sunghoon Jung, Phys. Rev. D **104**, 063001 (2021)
- "Co-Existence test of Primordial black holes and Particle Dark Matter"
  HGC, Sunghoon Jung, Philip Lu, and Volodymyr Takhistov, arXiv:2311.17829

GW Lensing

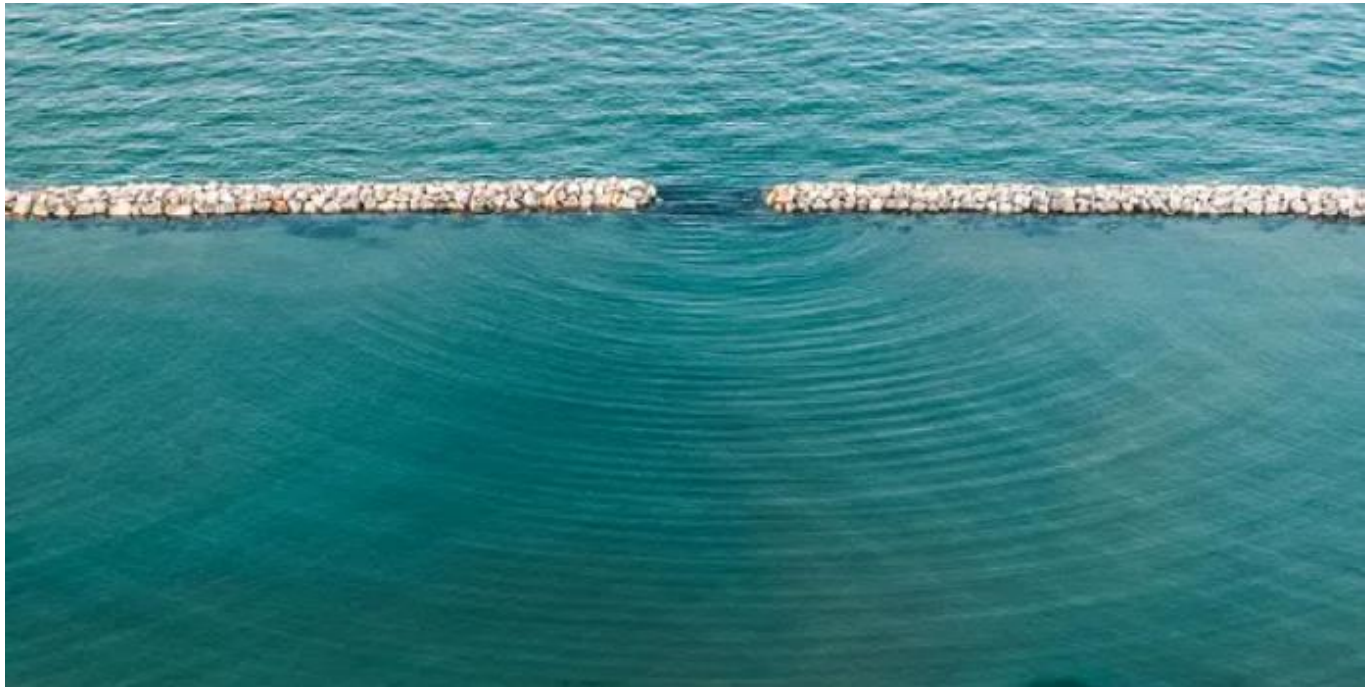
- Gravitational Lensing of Gravitational Wave “GW Lensing”
- GW Lensing as a probe of **dark matter** properties
  - Lensing probability, GW spectrum distortions
- **Wave nature** matters!



- LIGO, ET, CE
- LISA, DECIGO
- Etc..

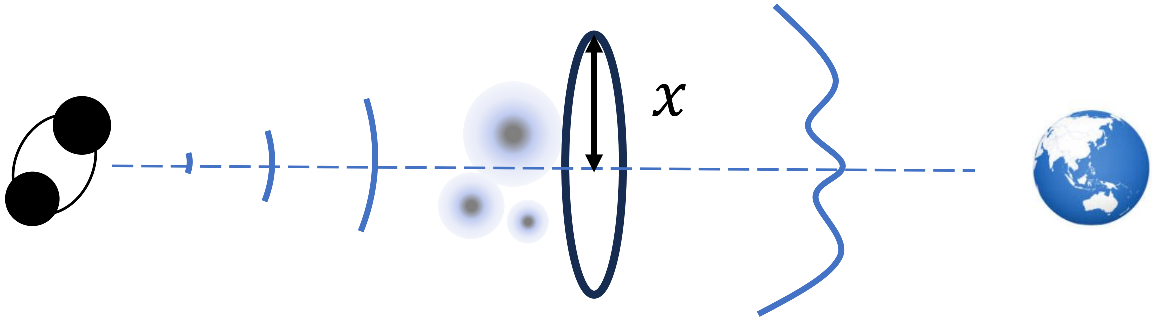
Wave Optics

- $h_{\mu\nu}(x) \sim \phi(x) e_{\mu\nu} \Rightarrow \Box_g \phi = 0$
- $\Rightarrow (\nabla^2 + w^2)\phi(x) = 4w^2 U(x)\,\phi(x)$
- Wave Optics provides a solution : $\phi(x) \propto \oint dx' e^{i\,\Psi(x';x)}$ cf) Huygens' principle



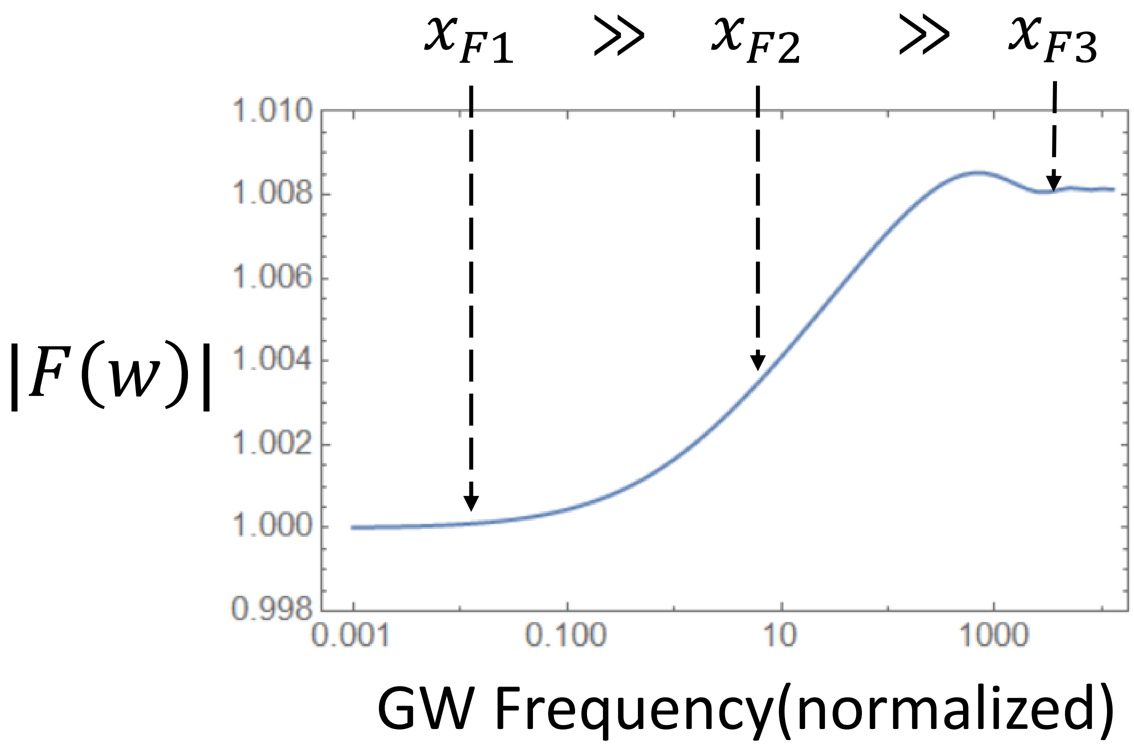
Lensing Amplification Factor

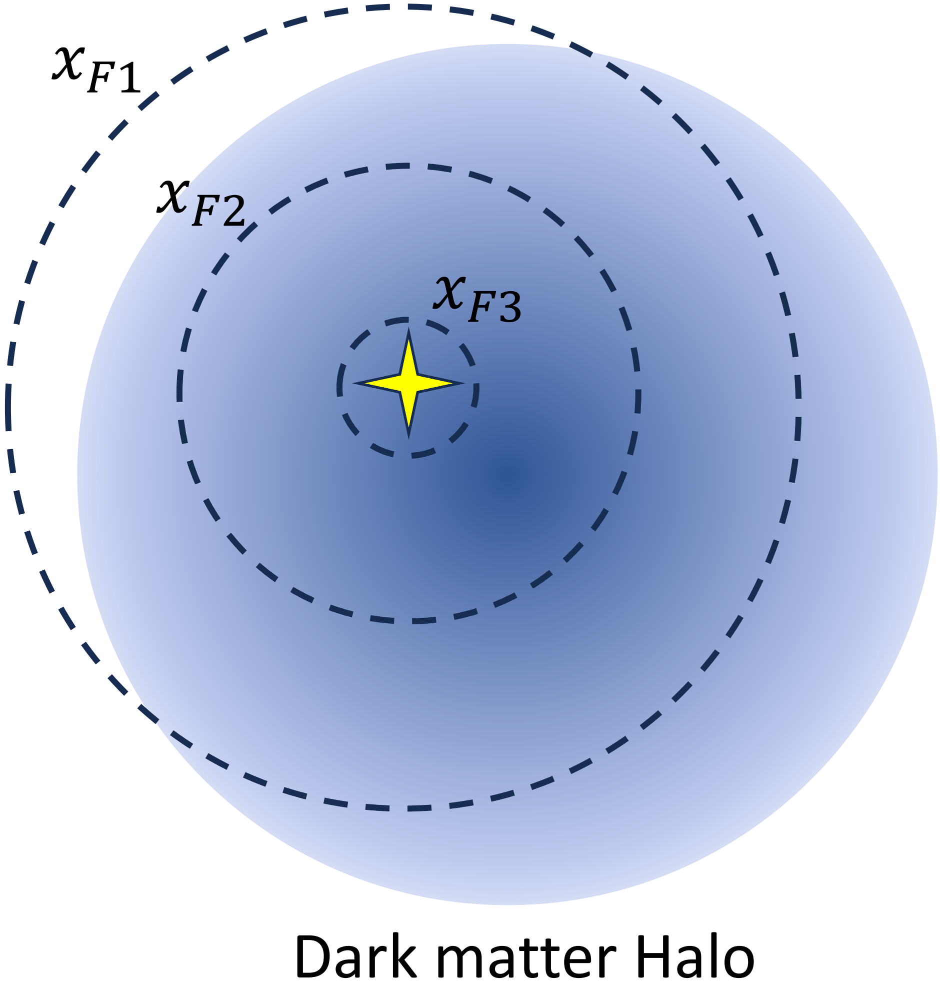
- Lensing amplification factor : $F(w) \equiv \phi(w)/\phi_o(w)$ $\phi_o(w) = A\, e^{iwr}/r$
- $F(w) = 1$ without lens object
- $F(w) \simeq 1 + \bar{\kappa}(x_F\sqrt{i})$ for weakly lensed signals **HGC**, Park, Jung 2021
- $\bar{\kappa}(x) \propto$ Mean line-of-sight mass density within a radius x
- $x_F \equiv \sqrt{\frac{c}{w}\frac{d_l d_{ls}}{d_s}}$: Fresnel length, $O(1)\ pc \sim \sqrt{1\ Gpc/1Hz * c}$



Lensing Amplification Factor

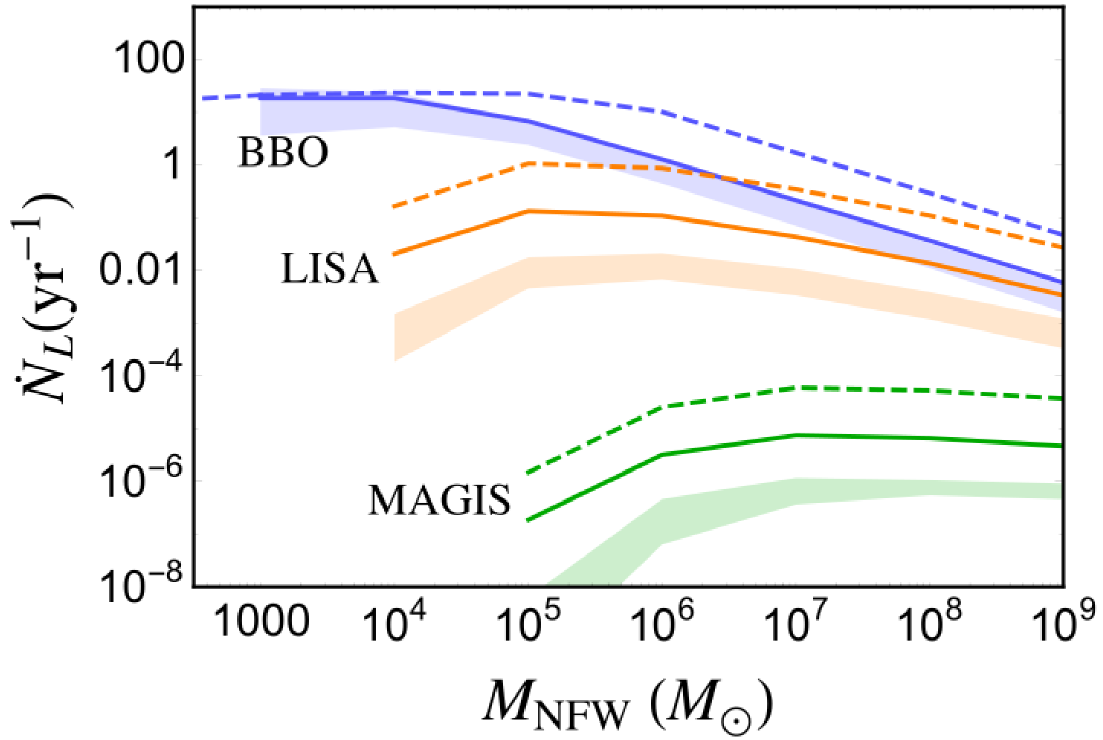
- Lensing amplification factor of dark matter halo (NFW)





DM Halo Detection

- DM Halo with $\sim 1\ pc$ size induces the non-trivial spectrum, phase distortion within $0.01\ Hz \sim 1\ Hz$ GWs
- Assuming standard cold dark matter halo(NFW profile) and Super-Massive BBH sources, several(BBO) or marginally 1(LISA) light DM halo can be detected

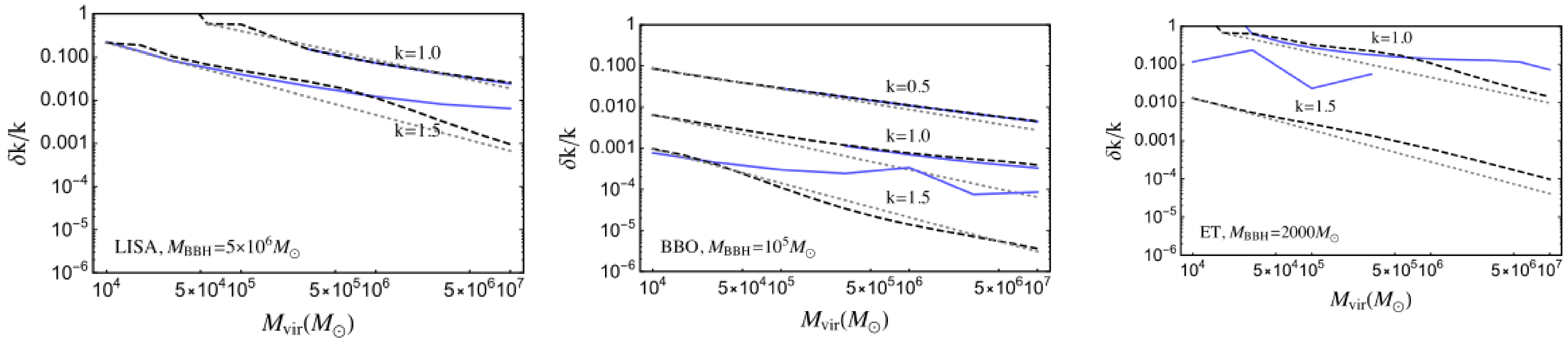


BBH Merger rate
Solid : 0.01 $Gpc^{-3}yr^{-1}$
Shaded : astrophysical (Bonetti 2018)

Dashed : No halo mass distribution

Lens profile measurement

- Using $F(w) \simeq 1 + \bar{\kappa}(x_F\sqrt{i})$, lens profile measurement is possible
- We tested the power-law profile cases $\bar{\kappa}(x) \propto x^{k-2}$, $k < 2$
- Power-law index 'k' can be measured with good accuracy



Identification of Dressed BH

- Can we distinguish lensing of BH w/ and w/o DM halo?

Dressed BH

- If a BH is the Primordial BH(PBH), it likely has a huge DM halo surrounding it (= Dressed PBH) Mack 2006

$M_{halo} \sim 100\ M_{PBH}$

$R_{halo} \sim 0.6\ pc\ \left(M_{halo}/M_{\odot}\right)^{1/3}$

$\rho(r) \propto r^{-9/4}$

- The DM halo can be a discriminating feature of PBH from the stellar-origin BH

- Dressed BH can be detected by microlensing searches
    - FRB lensing (Oguri 2022), lensing survey (Cai 2022), etc

Dressed BH

- At high frequency limit, μ_r and Δt are the only lensing observables

$$F(w) \propto 1 + \sqrt{\mu_r} e^{i\, w\, \Delta t - i\, \pi/2}$$

- But this combination always can be interpreted as point mass lensing

$$y_s = \sqrt{\mu_r^{1/2} + \mu_r^{-1/2} - 2}$$

Impact parameter of point mass lens

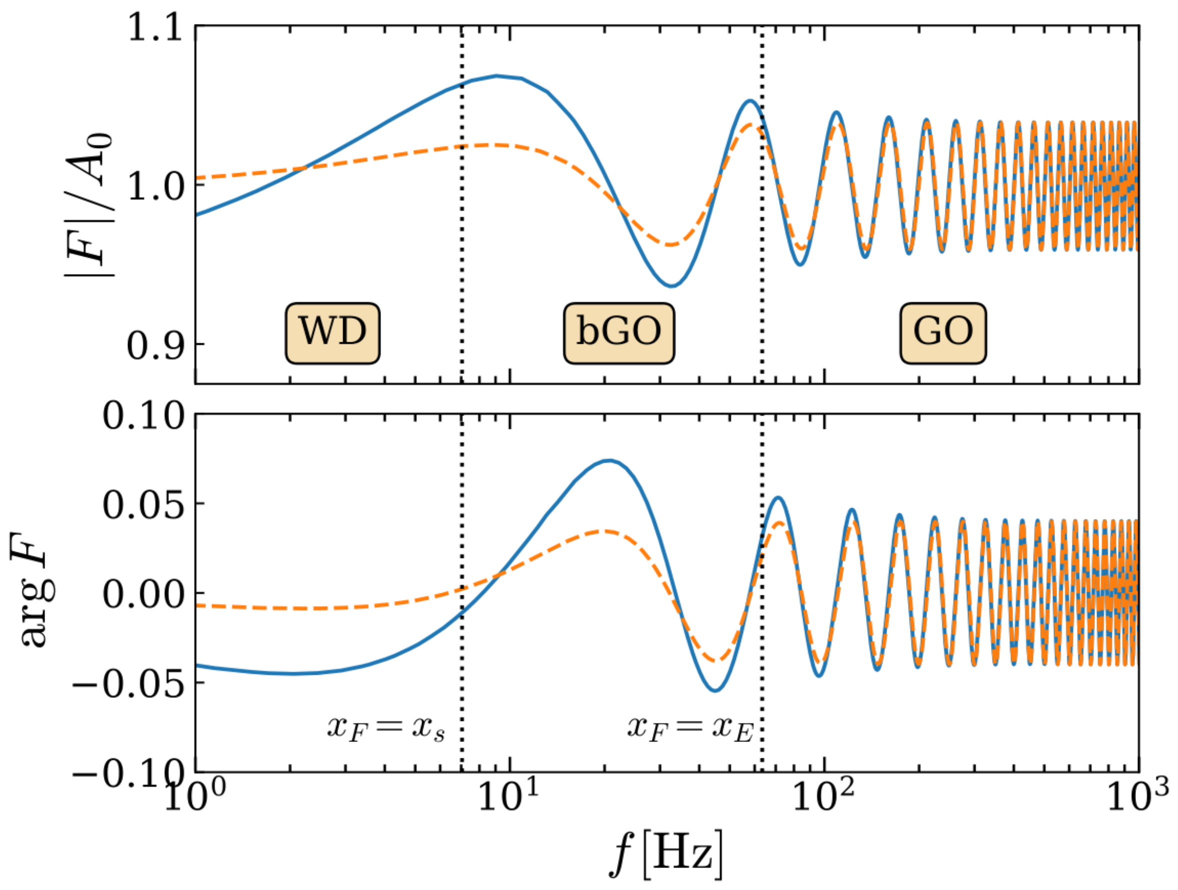
$$M_l = \frac{\Delta t}{2\left(\sqrt{\mu_r + \mu_r^{-1} - 2} - \ln \mu_r\right)}$$

Redshifted mass of point mass lens

- **GW lensing with Diffraction** can break the degeneracy **HGC,** Jung, Lu, Takhistov 2023

GW lensing of Dressed BH

- The DM halo induces distinguished diffraction patterns

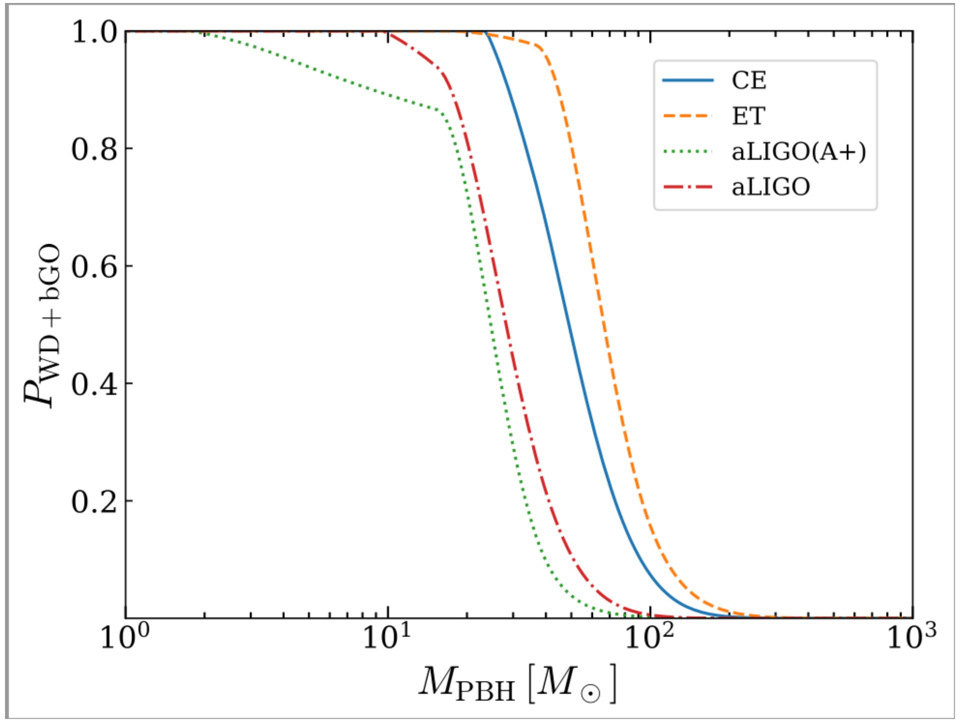


Solid : Dressed BH
Dashed : Bare BH

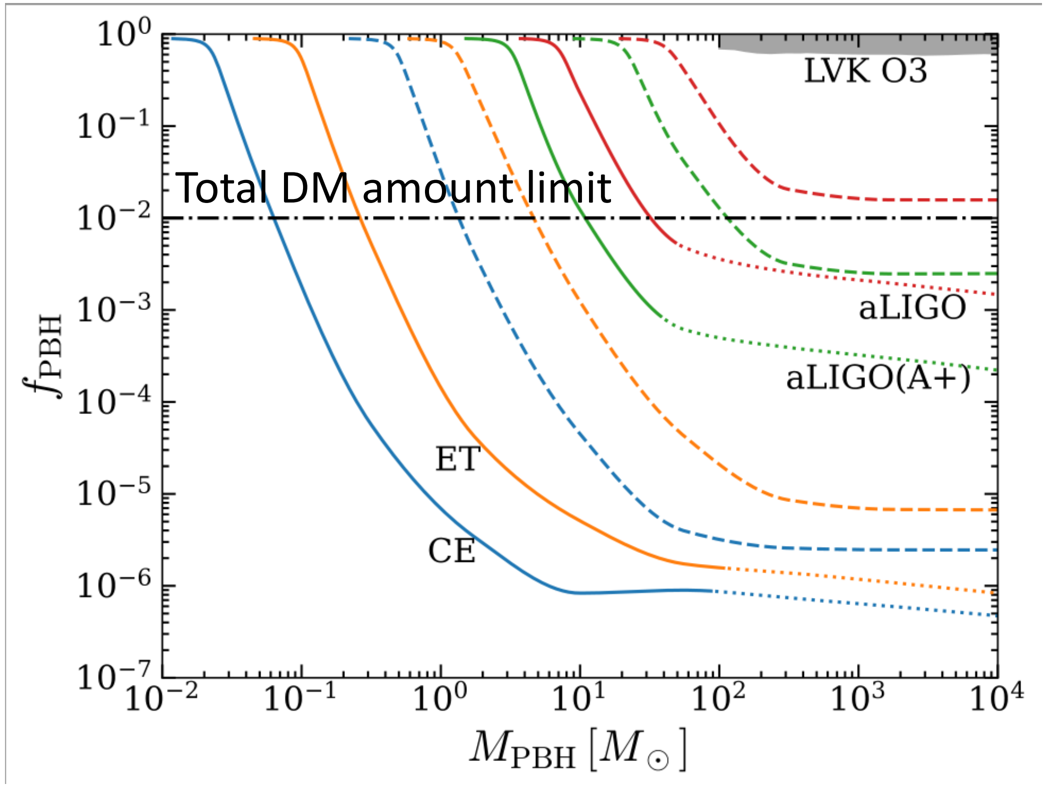
WD : Weak Diffraction
bGO : beyond Geometric Optics

Detection of Dressed BH

- Not every lensing event contains diffraction. It needs more precise alignment.
  - P = (Diffraction cross-section) / (Detection cross-section)
- Dressed (P)BH lighter than 100 solar mass can be identified.



(5 years)



Solid : Dressed BH
Dashed : Bare BH
Dotted : Indistinguishable

Summary

1. Wave nature of GW lensing links lensing amplification to lens profile.

2. GW lensing can be used to probe light DM halos.

3. Dressed BH and Bare BH can be distinguished by GW lensing w/ diffraction.

4. Wave nature of GW lensing will be powerful tool for studying a dark matter properties.