Gravitational Lensing of High-Frequency Gravitational Waves by Supermassive Black Holes in the Presence and Absence of the Cosmological Constant

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Outline of the Talk

1. Motivation
2. Spacetime and the Equations of Motion
3. Gravitational Lensing
4. Summary & Outlook on Applications

Motivation

- Why should we investigate gravitational lensing of gravitational waves?
- address questions from fundamental physics:
  - Is gravity really described by general relativity?
  - Is the gravitational interaction described by a massless or a massive particle? Can it be described as a particle at all?
- astrophysics: needed for correct interpretation of observed gravitational waves signals
  - gravitational wave signals from several binary black hole mergers indicate existence of stellar mass black holes with masses higher than anticipated
  - correct identification of lensed gravitational wave signals
  - without better detectors: may provide a better view on gravity or, e.g., neutron star physics from the point of view of the source

Light versus Gravitational Waves

- on first view light and gravitational waves are quite different
- light rays:
  - strongly interacting with their environment
  - wavelength short compared to astrophysical objects
- gravitational waves
  - weakly interacting with their environment
  - wavelength variable but commonly on same or longer length scales as many astrophysical objects
  - consequence: have to be treated as waves in most astrophysical environments
  - exception: as shown by Isaacson (1968) in high-frequency limit gravitational waves move along lightlike geodesics
- in this talk: gravitational lensing of high-frequency gravitational waves (and light) by supermassive black holes

Why Using Analytical Methods?

- one large class of black hole spacetimes in general relativity: Plebanski-Demianski metric (Plebanski and Demianski, 1976)
  - exact solution to Einstein's electrovacuum field equation with cosmological constant
  - includes Kerr-de Sitter metric
  - equations of motion for lightlike geodesics are separable and exactly analytically solvable
- benefits of using analytical methods:
  - arbitrarily precise
  - can reduce calculation time
  - allow very high-resolution calculations
- in this talk: application to calculation of lensing features caused by
  - cosmological constant
  - spin

The Kerr-de Sitter Spacetime

- the line element of the Kerr-de Sitter spacetime reads (Griffiths and Podolský, 2009; $c = G = 1$):

$$g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = \frac{a^2\sin^2\vartheta P(\vartheta) - Q(r)}{\rho(r,\vartheta)}\mathrm{d}t^2 + \frac{2a\sin^2\vartheta\big(Q(r) - (r^2+a^2)P(\vartheta)\big)}{\rho(r,\vartheta)}\mathrm{d}t\mathrm{d}\varphi + \frac{\sin^2\vartheta\big((r^2+a^2)^2P(\vartheta) - a^2\sin^2\vartheta Q(r)\big)}{\rho(r,\vartheta)}\mathrm{d}\varphi^2 + \frac{\rho(r,\vartheta)}{Q(r)}\mathrm{d}r^2 + \frac{\rho(r,\vartheta)}{Q(\vartheta)}\mathrm{d}\vartheta^2, \quad (1)$$

where

$$Q(r) = -\frac{\Lambda}{3}r^4 + \left(1 - \frac{\Lambda}{3}a^2\right)r^2 - 2mr + a^2, \quad (2)$$
$$Q(\vartheta) = 1 + \frac{\Lambda}{3}a^2\cos^2\vartheta, \quad \rho(r,\vartheta) = r^2 + a^2\cos^2\vartheta.$$

  - m: mass parameter
  - a: spin parameter
  - Λ: cosmological constant

Equations of Motion

- the equations of motion of the Kerr-de Sitter metric read, see e.g., Grenzebach *et al.* (2015)

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = (r^2 + a^2)\frac{(r^2+a^2)E - aL_z}{Q(r)} + a\frac{L_z - a\sin^2\vartheta E}{P(\vartheta)},$$

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 = ((r^2 + a^2)E - aL_z)^2 - Q(r)K,$$

$$\left(\frac{\mathrm{d}\vartheta}{\mathrm{d}\lambda}\right)^2 = P(\vartheta)K - \frac{(a\sin^2\vartheta E - L_z)^2}{\sin^2\vartheta},$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\lambda} = a\frac{(r^2+a^2)E - aL_z}{Q(r)} + \frac{L_z - a\sin^2\vartheta E}{\sin^2\vartheta P(\vartheta)},$$

- where the Mino parameter λ (Mino, 2003) is related to the affine parameter s by

$$\frac{\mathrm{d}\lambda}{\mathrm{d}s} = \frac{1}{\rho(r,\vartheta)}$$

Observer Geometry

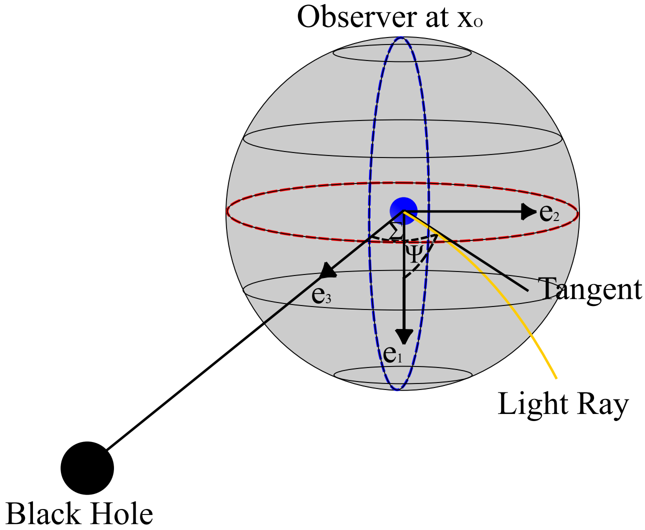


Figure: Illustration of the lens-observer geometry and the tetrad vectors e_1, e_2, and e_3 (from Frost (2022)) as defined in Gernzebach *et al.* (2015).

Carter Observer

- in this work: Carter observer (stationary)
- orthonormal tetrad given by (Grenzebach *et al.*, 2015):

$$e_0 = \left. \frac{(r^2+a^2)\partial_t + a\partial_\varphi}{\sqrt{\rho(r,\vartheta)Q(r)}} \right|_{(x_O)}, \quad e_1 = \left. \sqrt{\frac{P(\vartheta)}{\rho(r,\vartheta)}}\partial_\vartheta \right|_{(x_O)}, \tag{3}$$

$$e_2 = - \left. \frac{\partial_\varphi + a\sin^2\vartheta\partial_t}{\sqrt{\rho(r,\vartheta)P(\vartheta)}\sin\vartheta} \right|_{(x_O)}, \quad e_3 = - \left. \sqrt{\frac{Q(r)}{\rho(r,\vartheta)}}\partial_r \right|_{(x_O)}$$

- relate constants of motion E, L_z, and K to angles on the celestial sphere of the observer
- advantage: can also be used for observers inside the ergoregion

Integration Approach

- reparameterise equations of motion using angles on the celestial sphere
- derive radius coordinates of photon orbits in terms of celestial longitude
- use celestial latitude and longitude to distinguish different types of motion
- solve equations of motion using elementary and Jacobi's elliptic functions and Legendre's elliptic integrals
- use analytic solutions to:
  1. calculate Mino parameter $\lambda_L < \lambda_O = 0$ from fixed $r_O < r_L$
  2. determine number of turning points of the ϑ motion
  3. calculate $\vartheta_L(\Sigma, \Psi)$ and $\varphi_L(\Sigma, \Psi)$ (for $\varphi_O = 0$) and use them to define lens map
  $$(\Sigma, \Psi) \to (\vartheta_L(\Sigma, \Psi), \varphi_L(\Sigma, \Psi)) \tag{4}$$
  4. calculate redshift from r_O, ϑ_O, r_L, and $\vartheta_L(\Sigma, \Psi)$
  5. calculate travel time $T(\Sigma, \Psi)$ (for $t_O = 0$)

Lens Map: Sphere of Light Sources

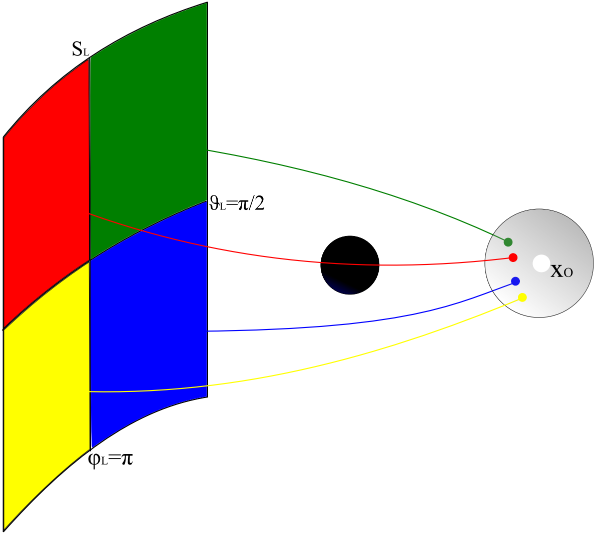
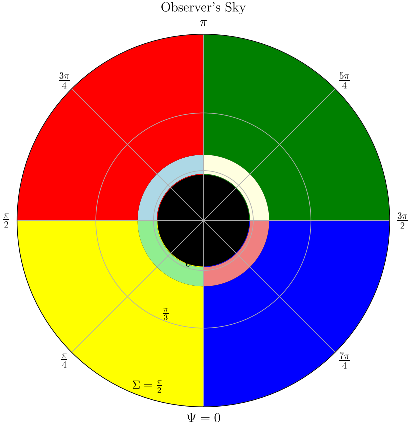


Figure: Illustration of the lens map (adapted from Frost (2022)) following the colour convention in Bohn *et al.* (2015).

Lens Map: Influence of the Cosmological Constant

Schwarzschild Metric



Schwarzschild-de Sitter Metric

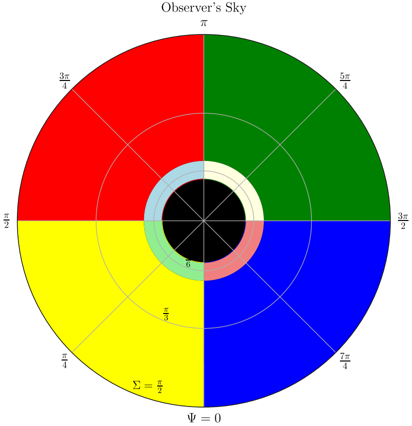
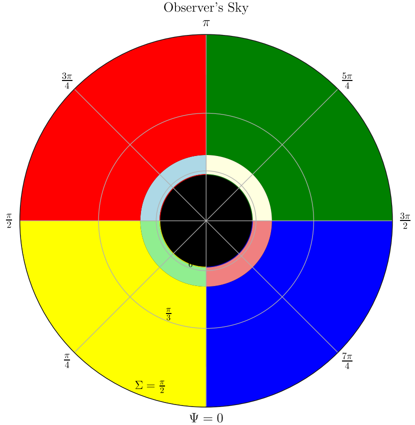


Figure: Lens maps for $\Lambda = 1/(200m^2)$, $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Lens Map: Effects of the Spin

Schwarzschild Metric



Kerr Metric

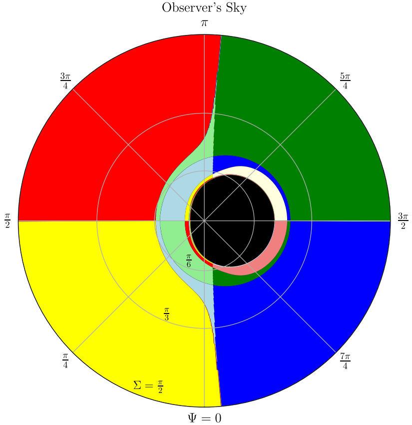


Figure: Lens maps for $a = 95m/100$, $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Redshift: Definition

- energy measured at position of the light source and the observer:

$$\text{observer:} \quad E_O = -p_\mu \dot{x}^\mu_O \quad \text{source:} \quad E_L = -p_\mu \dot{x}^\mu_L \tag{5}$$

- p_μ: four-momentum of the light ray $\dot{x}^\mu$: four-velocities of observer and light source
- $p_t = -E$ and $p_\varphi = L_z$
- using the definitions of the energies the redshift now reads:

$$z = \frac{E_L}{E_O} - 1 \tag{6}$$

- Schwarzschild metric: $z = -0.057$
- Schwarzschild-de Sitter metric: $z = 0.648$
- in general: depends directly and indirectly on celestial coordinates

Redshift: Effects of the Spin - Static Light Sources

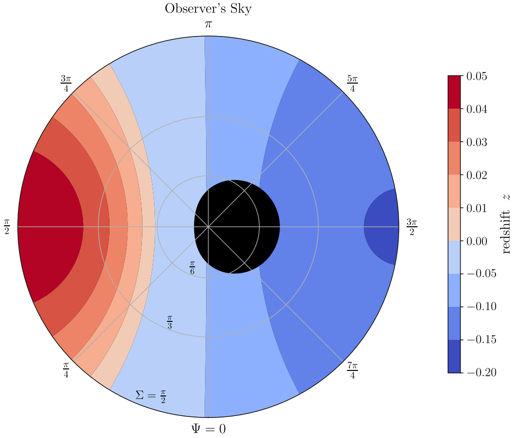


Figure: Redshift map for $a = 95m/100$, $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

- Schwarzschild metric: $z = -0.057$

Redshift: Effects of the Spin - Carter Light Sources

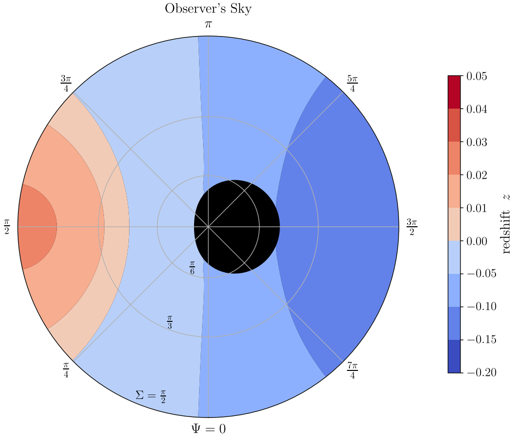


Figure: Redshift map for $a = 95m/100$, $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

- Schwarzschild metric: $z = -0.057$

Travel Time: Effects of the Cosmological Constant

Schwarzschild Metric | Schwarzschild-de Sitter Metric

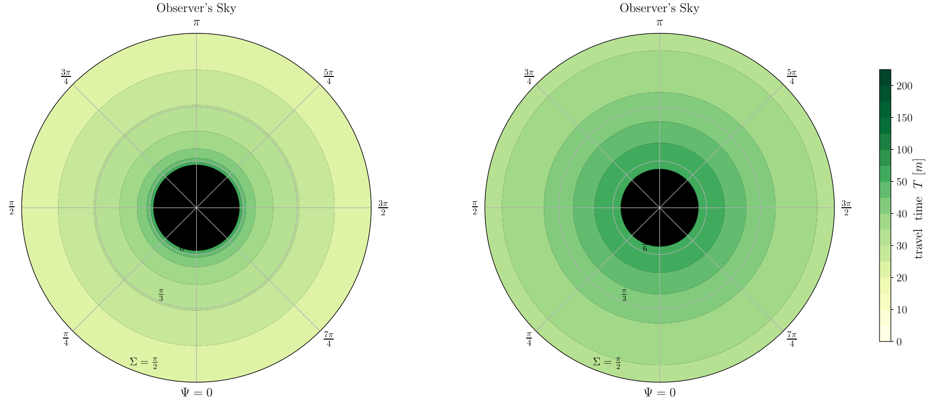
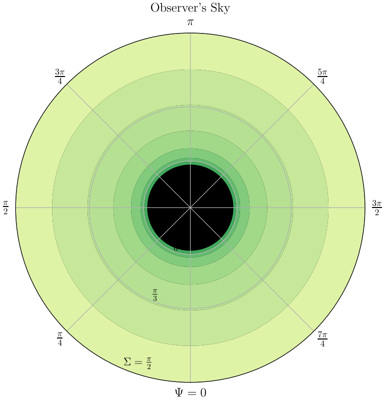


Figure: Travel time maps for $\Lambda = 1/(200m^2)$, $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Travel Time: Effects of the Spin

Schwarzschild Metric



Kerr Metric

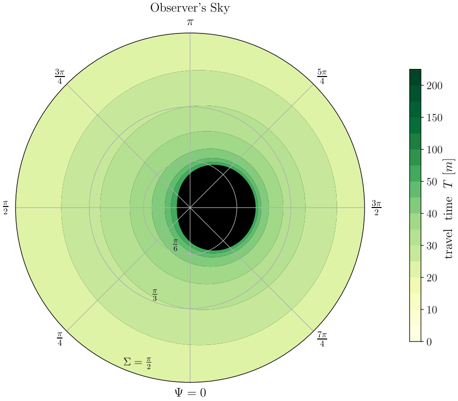
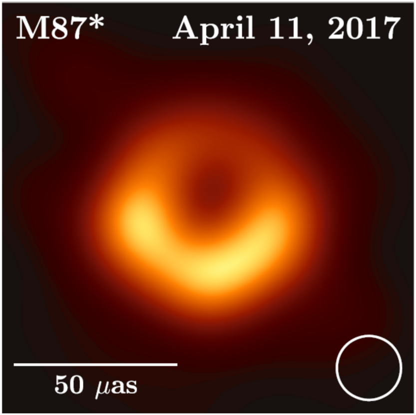


Figure: Travel time maps for $a = 95m/100$, $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Event Horizon Telescope Images

M87



Milky Way

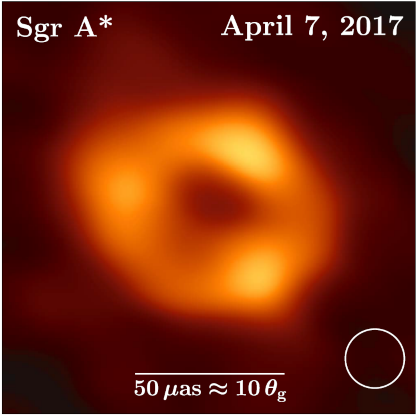


Figure: Event Horizon Telescope images of the shadows of the supermassive black holes in the centres of the galaxy M87 (left) and the Milky Way (right). Taken from Fig. 3 in EHT *et al.* (2019) and Fig. 3 in EHT *et al.* (2022).

Summary & Outlook on Applications

- cosmological constant:
  - shadow shrinks and images shift to lower celestial latitudes
  - redshift and travel time increase (in our case significantly)
- spin:
  - noncircular shadow
  - images up to fourth order visible
  - redshift is function on celestial sphere
  - asymmetry in travel time maps
- potential applications to gravitational wave astrophysics:
  - lensing of stellar mass binary black hole mergers close to supermassive black holes
  - reconstructing the shadow when combined with detection of gravitational waves for which wave optics becomes important?
- next step: extend approach to wave optics

Longterm Goal: Einstein-Maxwell-Pauli Observatory

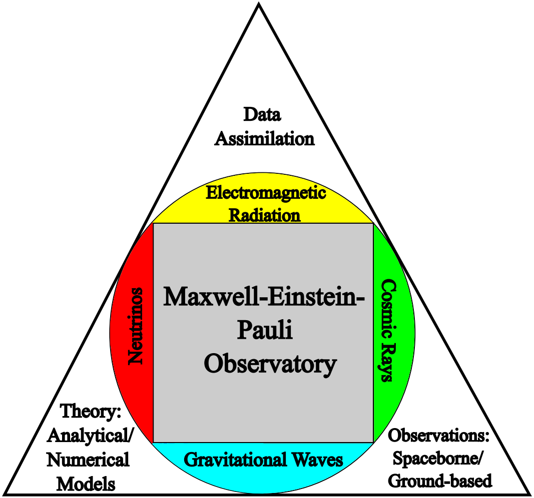


Figure: Schematic of the Maxwell-Einstein-Pauli Observatory.

Announcement

- On March 13th, 2024, we start a new seminar series on Gravity and Cosmology. Feel welcome to join! Use the QR code or ask us for the subscription link.

References Part I

- [1] R. A. Isaacson. Gravitational Radiation in the Limit of High Frequency. I. The Linear Approximation and Geometrical Optics. Phys. Rev. 166(5), 1263-1271, 1968. doi: 10.1103/PhysRev.166.1263.
- [2] J. F. Plebanski and M. Demianski. Rotating, charged, and uniformly accelerating mass in general relativity. Annals of Physics (N. Y.), 98(1):98–127, 1976. doi: 10.1016/0003-4916(76)90240-2.
- [3] J. B. Griffiths and J. Podolský: Exact Space-Times in Einstein's General Relativity. Cambridge University Press, Cambridge, 2009. ISBN: 978-0-521-88927-8.
- [4] A. Grenzebach, V. Perlick and C. Lämmerzahl: Photon regions and shadows of accelerated black holes. Int. J. Mod. Phys. D, 24(9), 1542024, 2015. doi: 10.1142/S0218271815420249.
- [5] Y. Mino: Perturbative approach to an orbital evolution around a supermassive black hole. Phys. Rev. D, 67(8), 084027, 2003. doi: 10.1103/PhysRevD.67.084027.

References Part II

- [6] T. C. Frost. Gravitational lensing in the charged NUT–de Sitter spacetimePhys. Rev. D, 105(6), 064064, 2022. doi: 10.1103/PhysRevD.105.064064.
- [7] A. Bohn, W. Throwe, F. Hebert, K. Henriksson, D. Bunandar, M. A. Scheel and N. W. Taylor: What does a binary black hole merger look like? Class. Quantum Grav., 32(6), 065002, 2015. doi: 10.1088/0264-9381/32/6/065002.
- [8]The Event Horizon Telescope Collaboration et al. First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole. Astrophys. J. Lett., 875(1), L1, 2019. doi: 10.3847/2041-8213/ab0ec7.
- [9] The Event Horizon Telescope Collaboration et al. First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way. Astrophys. J. Lett., 930(2), L12, 2022. doi: 10.3847/2041-8213/ac6674.