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Post-Newtonian approximations in higher-order scalar-tensor theories

Jin Saito (Rikkyo University)

Based on work with [arXiv:2402.10459]

JS, Zhibang Yao (Rikkyo U), Tsutomu Kobayashi (Rikkyo U)

Introduction

- There are many modified gravity theories.

  e.g.) Horndeski theory, DHOST theory, Spatially covariant theory, etc

- We would like to obtain a gravitational theory which is consistent with our universe.
  - We need to constrain a gravitational theory with experiments.
  - Convenient parametrizations to see the consistency with experiments:

PPN parameters

$$\gamma^{\mathrm{PPN}}, \beta^{\mathrm{PPN}}, \alpha^{\mathrm{PPN}}_{1,2,3}, \zeta^{\mathrm{PPN}}_{1,2,3,4}, \xi^{\mathrm{PPN}}$$

⟷

EFT of DE parameters

$$M, \alpha_T, \alpha_H, \alpha_L, \beta_1, \beta_2, \beta_3$$

Our aims are to obtain all the PPN paramters

and to get the relation between these parameters.

U-DHOST theory (A. De Felice, et al arXiv: 1803.06241)

- Action

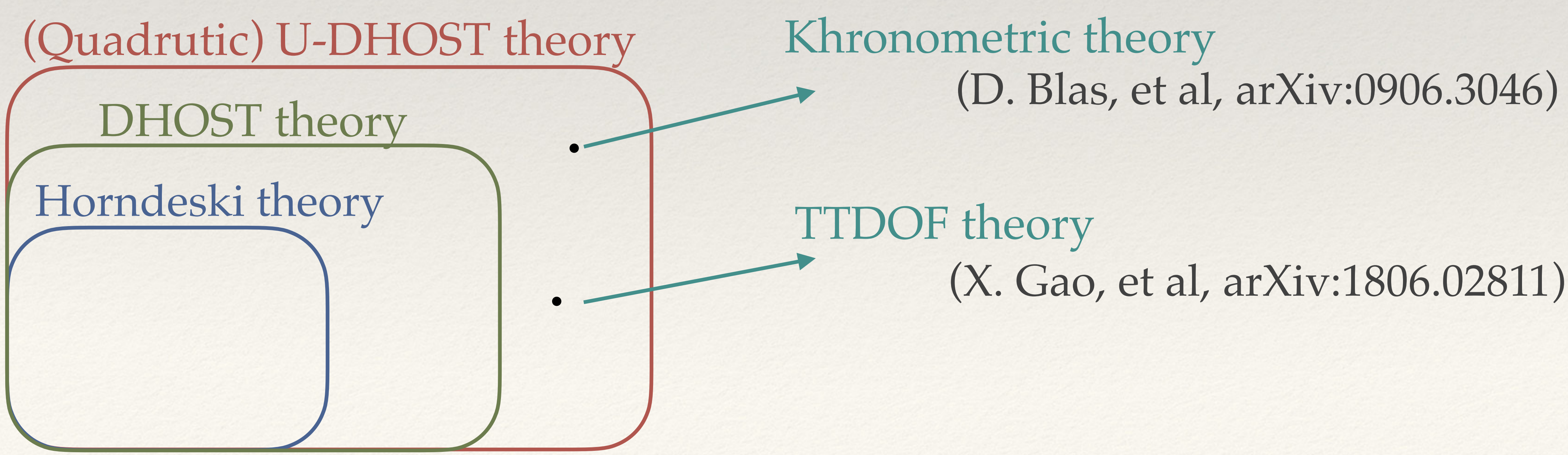
$$\phi_\mu := \nabla_\mu \phi, \quad \phi_{\mu\nu} := \nabla_\mu \nabla_\nu \phi, \quad X := -\phi^\mu \phi_\mu / 2, \quad ' := \partial / \partial X$$

$$S_{\rm grav} = \int \mathrm{d}^4 x \sqrt{-g} \left[P(\phi, X) + f(\phi, X)\mathcal{R} + A_1(\phi, X)\phi_{\mu\nu}\phi^{\mu\nu} + A_2(\phi, X)(\Box\phi)^2 + A_3(\phi, X)\phi^\mu\phi_{\mu\nu}\phi^\nu\Box\phi \right.$$
$$\left. + A_4(\phi, X)\phi^\mu\phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho + A_5(\phi, X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2 \right]$$

$$A_1 = a_1 - \frac{f}{2X}, \quad A_2 = a_2 + \frac{f}{2X}, \quad A_3 = \frac{f}{2X^2} - \frac{f'}{X} + 2a_1 a_3 + 2\left(3a_3 + \frac{1}{2X}\right) a_2,$$

$$A_4 = a_4 + \frac{f'}{2X} - \frac{f}{2X^2} + \frac{a_1}{X}, \quad A_5 = \frac{a_4}{2X} - \frac{f'}{4X^2} + a_1\left(\frac{1}{4X^2} + 3a_3^2 + \frac{a_3}{X}\right) + a_2\left(3a_3 + \frac{1}{2X}\right)^2,$$

  - It satisfies degeneracy conditions under the unitary gauge.



EFT of DE parameters + extra

(E. Bellini, et al, arXiv:1404.3713) (D. Langlois, et al, arXiv:1703.03797)

- EFT of DE parameters ($M, \alpha_T, \alpha_H, \alpha_L, \beta_1, \beta_2, \beta_3$):
  - It is introduced in the effective field theory (EFT) of dark energy (DE).
  - Convenient for classifying and characterizing the scalr-tensor theory.
  - Physical significance of each parameter:

    e.g.) $c_{\rm GW}^2 = 1 + \alpha_T, \quad M^{-2} = 8\pi G_{\rm GW} c_{\rm GW}^2$

- additional parameters (δ_1, δ_2):

$$M^2 \delta_1 := 2X(f - 2Xf')', \quad M^2 \delta_2 := -8X \left[X(f' + A_1 - XA_4) \right]'$$

About α_L

$\alpha_L = 0$

Horndeski theory
Ia DHOST theory

The Vainshtein mechanism works.

$\alpha_L \neq 0$

Khronometric theory
TTDOF theory

The Vainshtein mechanism does not work.

✓ The PPN formalism

- We consider only the case of $\alpha_L \neq 0$.
- Our analysis does not include Horndeski and Ia DHOST theory.

Energy-momentum tensor

- Assuming the perfect fluids:

four-velocity $u^{\mu} := (u^0, u^0\vec{v})$

$$T^{\mu\nu} = [\rho + \rho\Pi + p]\, u^{\mu}u^{\nu} + pg^{\mu\nu}$$

rest mass energy (ρ), internal energy ($\rho\Pi$), isotropic pressure (p)

- Energy conservation law: $\nabla_{\mu}T^{\mu\nu} = 0$
- Baryon number density conservation:

conserved mass density $\rho^* := \rho\sqrt{-g}u^0$

$$\nabla_{\mu}(\rho u^{\mu}) = 0 \Rightarrow \partial_t\rho^* + \partial_i(\rho^* u^i) = 0$$

- Post-Newtonian bookkeeping:

Bookkeeping parameter: $\epsilon \ll 1$

$$U \sim v^2 \sim p/\rho \sim \Pi \sim \mathcal{O}(\epsilon^2), \quad \partial_t \sim \mathcal{O}(\epsilon)\partial_i$$

Newtonian gravitational potential $\Delta U = -4\pi G_{\rm N}\rho^* \Leftrightarrow U = \int \mathrm{d}^3x' \frac{\rho^*(t,\vec{x}')}{|\vec{x}-\vec{x}'|}$

PPN formalism

(C. M. Will, Cambridge University Press, 2011)

- Convenient for verifying the consistency with solar-system tests
- It is characterized by
  - The PPN parameters: $\gamma^{\rm PPN}, \beta^{\rm PPN}, \alpha^{\rm PPN}_{1,2,3}, \zeta^{\rm PPN}_{1,2,3,4}, \xi^{\rm PPN}$
  - The gravitational constant: $G_{\rm N}$
- PN expansions (standard PPN gauge):

$$g_{00} = -1 + 2U + 2\left(\psi - \beta^{\rm PPN}U^2\right)\epsilon^4 + \mathcal{O}(\epsilon^6),$$

$$g_{0i} = B_i\epsilon^3 + \mathcal{O}(\epsilon^5),$$

$$g_{ij} = \left(1 + 2\gamma^{\rm PPN}U\epsilon^2 + 2C\epsilon^4\right)\delta_{ij} + \mathcal{O}(\epsilon^6),$$

$$\phi = q(t + \gamma^{\rm PPN}_{\mathcal{X}}\dot{\mathcal{X}}\epsilon^3) + \mathcal{O}(\epsilon^5)$$

Leading correction $\sim \mathcal{O}(\epsilon^3)$

unitary gauge: $\phi = qt$

$t \to t + \lambda_1\dot{\mathcal{X}}$ $\sim \mathcal{O}(\epsilon^3)$

standard PPN gauge

● :Linear conbination of the PPN potentials $(U, \mathcal{X}, V_i, \Phi_{1,2,3,4,6,W})$

e.g.) $\Delta\Phi_1 = -4\pi G_{\rm N}\rho^* v^2, \quad \Delta\Phi_2 = -4\pi G_{\rm N}\rho^* U, \ldots$

Calculations of the PPN parameters

- Outline for determining the PPN parameters:

  1. Expand the field equations $\mathcal{E}_{\mu\nu} = T_{\mu\nu}$:

$$\mathcal{E}_{\mu\nu} = \mathcal{E}^{(0)}_{\mu\nu} + \mathcal{E}^{(2)}_{\mu\nu}\epsilon^2 + \mathcal{E}^{(3)}_{\mu\nu}\epsilon^3 + \mathcal{E}^{(4)}_{\mu\nu}\epsilon^4 + \mathcal{O}(\epsilon^5), \quad T_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{(2)}_{\mu\nu}\epsilon^2 + T^{(3)}_{\mu\nu}\epsilon^3 + T^{(4)}_{\mu\nu}\epsilon^4 + \mathcal{O}(\epsilon^5)$$

  2. Compare the coefficients of each potential in LHS with that in RHS

     e.g.) $\mathcal{E}_{00} = \mathscr{A}_1 \Delta\Phi_1 + \mathscr{A}_2 \Delta\Phi_2, \quad T_{00} = \mathscr{B}_1 \Phi_1 \Rightarrow \begin{cases} \mathscr{A}_1 = \mathscr{B}_1 \\ \mathscr{A}_2 = 0 \end{cases}$

  We investigate the equations at each order:

  - $\mathcal{O}(\epsilon^2) \to \ \mathcal{E}^{(2)}_{00} = T^{(2)}_{00}, \quad \delta^{ij}\mathcal{E}^{(2)}_{ij} = \delta^{ij} T^{(2)}_{ij}$
  - $\mathcal{O}(\epsilon^3) \to \ \mathcal{E}^{(3)}_{0i} = T^{(3)}_{0i}$
  - $\mathcal{O}(\epsilon^4) \to \ \mathcal{E}^{(4)}_{00} = T^{(4)}_{00}, \quad \delta^{ij}\mathcal{E}^{(4)}_{ij} = \delta^{ij} T^{(4)}_{ij}$

Results

$$8\pi G_{\rm N} = \left[\frac{2(1+\alpha_T)}{2(1+\alpha_H)^2 - (1+\alpha_T)\beta_3}\right]\frac{1}{M^2}, \qquad \gamma^{\rm PPN} = \frac{1+\alpha_H}{1+\alpha_T},$$

$$\alpha_1^{\rm PPN} = 4\left[\frac{2(1+\alpha_H)^2}{1+\alpha_T} - \frac{1+\alpha_H}{1+\alpha_T} - 1 - \beta_3\right],$$

$$\beta^{\rm PPN} = \frac{4\gamma^{\rm PPN}\left[c_{\rm GW}^2\gamma^{\rm PPN}\left(1+\gamma^{\rm PPN}\right) + 2\delta_1\right] - \beta_3\left(3+\gamma^{\rm PPN}\right) - 2\delta_2}{4\left[2c_{\rm GW}^2(\gamma^{\rm PPN})^2 - \beta_3\right]},$$

$$\alpha_2^{\rm PPN} = \frac{3\left[2\left(\gamma^{\rm PPN}+\beta_1\right) - 2c_{\rm GW}^2(\gamma^{\rm PPN})^2 - \beta_3\right]^2}{2\left[2c_{\rm GW}^2(\gamma^{\rm PPN})^2 - \beta_3\right]}\left(\frac{1}{\alpha_L}+1\right) - 1 + c_{\rm GW}^2(\gamma^{\rm PPN})^2 + 6\beta_1 - \frac{\beta_3}{2} + \frac{\beta_2 - 6\beta_1^2 - 12\beta}{2c_{\rm GW}^2(\gamma^{\rm PPN})^2}$$

$$\alpha_3^{\rm PPN} = \zeta_1^{\rm PPN} = \zeta_2^{\rm PPN} = \zeta_3^{\rm PPN} = \zeta_4^{\rm PPN} = \xi^{\rm PPN} = 0,$$

- $\mathcal{O}(\epsilon^2)$
- $\mathcal{O}(\epsilon^3)$
- $\mathcal{O}(\epsilon^4)$

:Terms which deviate from that of GR

The values of the PPN parameters in GR:

$$\gamma^{\rm PPN} = \beta^{\rm PPN} = 1, \quad \alpha_{1,2,3}^{\rm PPN} = \zeta_{1,2,3,4}^{\rm PPN} = \xi^{\rm PPN} = 0$$

Reproduction of khronometric case

Unit time-like vector: $u_\mu := -\phi_\mu/\sqrt{2X}$

- Action of khronometric theory: (D. Blas, et al, arXiv:0906.3046)

$$S = \frac{M_*^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left[\mathcal{R} + c_1\nabla_\mu u_\nu \nabla^\mu u^\nu + c_2(\nabla_\mu u^\mu)^2 + c_3\nabla_\mu u_\nu\nabla^\nu u^\mu + c_4 u^\mu u^\nu \nabla_\mu u_\lambda \nabla_\nu u^\lambda\right]$$

$$c_{1,2,3,4} = const$$

- The PPN parameters : (D. Blas, et al, arXiv: 1007.3503)

$$8\pi G_\mathrm{N} = \frac{1}{(1-c_4/2)M_*^2}, \quad \gamma^{\mathrm{PPN}} = 1, \quad \beta = 1, \quad \alpha_1 = -\frac{4(2c_3+c_4)}{1+c_3},$$

$$\alpha_2^{\mathrm{PPN}} = -2 + \frac{4}{1+c_3} - \frac{2c_2}{c_2+c_3} - \frac{3(2-3c_2-c_3)}{3c_2+c_3-c_4} + \frac{(1-2c_2-c_3)c_4}{(1+c_3)(c_2+c_3)},$$

$$\alpha_3^{\mathrm{PPN}} = \zeta_1^{\mathrm{PPN}} = \zeta_2^{\mathrm{PPN}} = \zeta_3^{\mathrm{PPN}} = \zeta_4^{\mathrm{PPN}} = \xi^{\mathrm{PPN}} = 0$$

Our results reproduce the PPN parameters in khronometric theory by selecting the EFT of DE parameters as

$$M^2 = (1+c_3)M_*^2, \quad \alpha_T = \alpha_H = -\frac{c_3}{1+c_3}, \quad \alpha_L = -\frac{3}{2}\frac{c_2+c_3}{1+c_3},$$

$$\beta_1 = \beta_2 = 0, \quad \beta_3 = \frac{c_4}{1+c_3}, \quad \delta_1 = \delta_2 = 0.$$

The case of TTDOF theory

- TTDOF theory has just two tensorial DOF as same as GR.

(X. Gao, et al, arXiv:1806.02811)

- The EFT of DE parameters in TTDOF theory:

$$\bar{b}_{0,1,2}, \bar{a}_{2,4} = const$$

$$M^2 = \frac{\bar{b}_0}{1+\bar{b}_2 q}, \quad M^2(1+\alpha_T) = \bar{a}_2 + \bar{a}_4 q, \quad M^2(1+\alpha_H) = \bar{a}_2,$$

$$\alpha_L = \frac{\left(\bar{b}_2 - \bar{b}_1\right) q}{1+\bar{b}_1 q}, \quad \beta_1 = \beta_2 = \beta_3 = \delta_1 = \delta_2 = 0$$

- The PPN parameters:

$$8\pi G_{\rm N} = \frac{\bar{a}_2 + \bar{a}_4 q}{\bar{a}_2^2}, \quad c_{\rm GW}^2 = \frac{(\bar{a}_2 + \bar{a}_4 q)\left(1+\bar{b}_2 q\right)}{\bar{b}_0}, \quad \gamma^{\rm PPN} = \frac{\bar{a}_2}{\bar{a}_2 + \bar{a}_4 q}, \quad \beta^{\rm PPN} = \frac{1+\gamma^{\rm PPN}}{2},$$

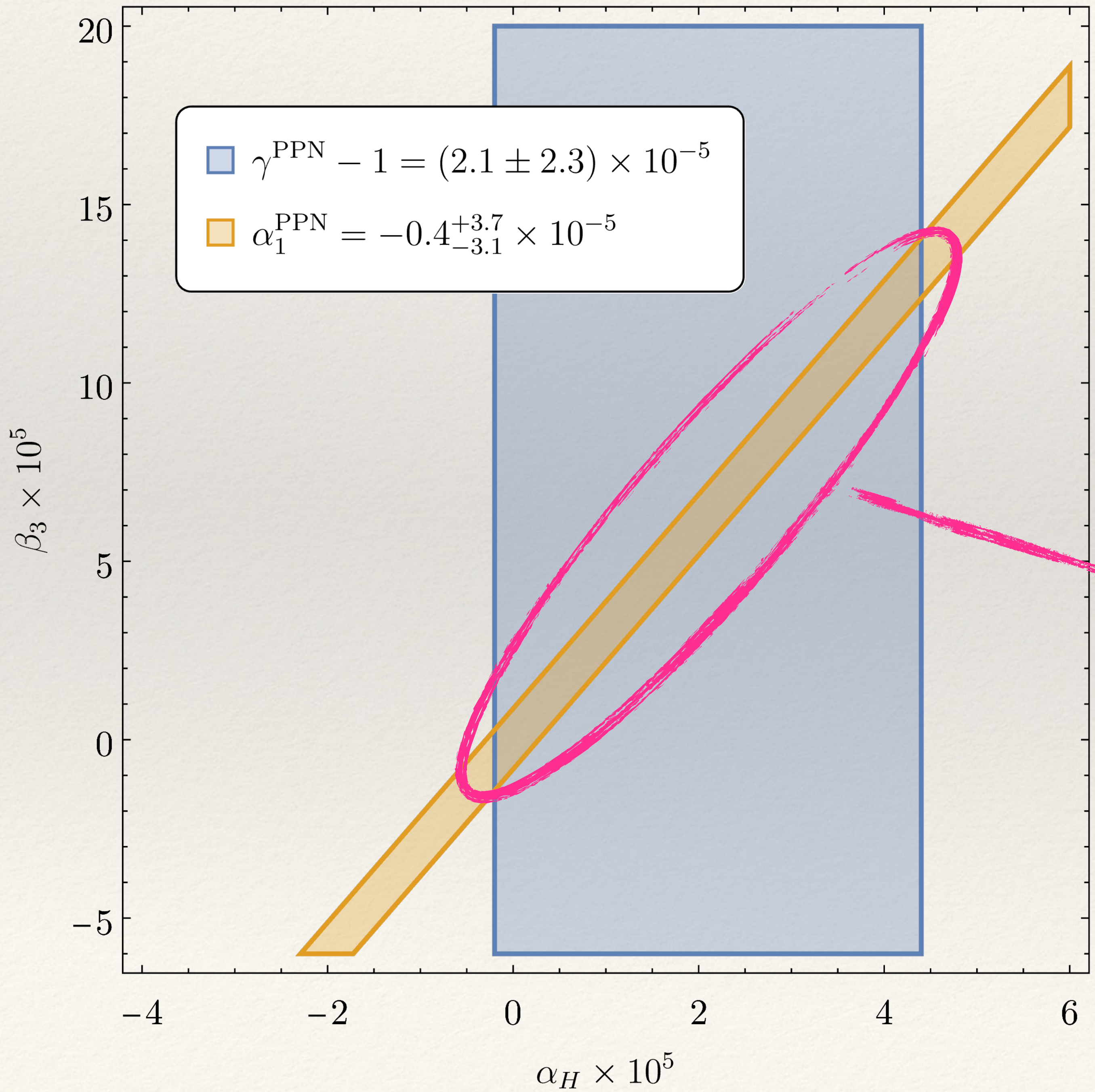
$$\alpha_1^{\rm PPN} = 4\left[2c_{\rm GW}^2(\gamma^{\rm PPN})^2 - \gamma^{\rm PPN} - 1\right], \quad \alpha_2^{\rm PPN} = \frac{3\left(1 - c_{\rm GW}^2\gamma^{\rm PPN}\right)^2}{c_{\rm GW}^2}\left(\frac{1}{\alpha_L} + 1\right) - 1 + c_{\rm GW}^2(\gamma^{\rm PPN})^2$$

$$\gamma^{\rm PPN} = c_{\rm GW} = 1 \ \left(\bar{a}_2 = \frac{\bar{b}_0}{1+\bar{b}_2 q}, \quad \bar{a}_4 = 0\right) \quad \Leftrightarrow \quad G_{\rm N} = G_{\rm GW}, \quad \beta^{\rm PPN} = 1, \quad \alpha_1^{\rm PPN} = \alpha_2^{\rm PPN} = 0$$

→ We cannot distinguish TTDOF theory from GR.

Constraints on the EFT parameters

Here, we assume $c_{\rm GW} = 1 \Leftrightarrow \alpha_T = 0$



In this case, $\gamma^{\rm PPN}$ and $\alpha_1^{\rm PPN}$ are given by

$$\begin{cases} \gamma^{\rm PPN} = 1 + \alpha_H, \\ \alpha_1^{\rm PPN} = 4\left[2(\gamma^{\rm PPN})^2 - \gamma^{\rm PPN} - 1 - \beta_3\right] \end{cases}$$

The area allowed by the constraints on $\gamma^{\rm PPN}$ and $\alpha_1^{\rm PPN}$

Conclusions

- The PPN formalism is convenient for discussing the consistency of a gravitational theory with observations.
- We obtain all the PPN parameters in higher-order scalar-tensor theories (only in the case of $\alpha_L \neq 0$) in terms of the EFT parameters.
- Our results reproduce the PPN parameters in khronometric theory.
- In future work, we investigate the gravitational radiation in U-DHOST theory.

Thank you for attention!!