@ **Gravity and Cosmology 2024** (Feb. 26th, 2024)

Relation of observables and the internal physics of neutron stars under F(R) gravity

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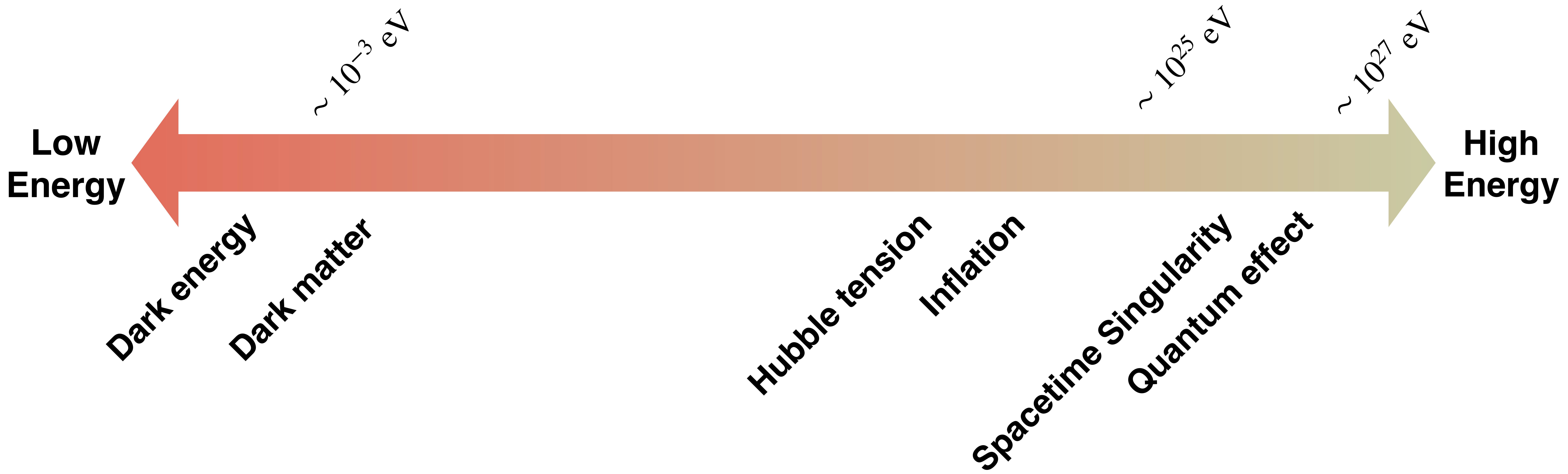
Based on PRD **107** 104019 (2023), and on-going works.

Problems of GR and Modified Gravity



Present universe model with GR & SM matters has several problems in wide range of energy-scale.

⇒ GR can be effective theory. **More accurate gravity theory** can be exist.

⇒ Modified Gravity

$F(R)$ Gravity theory

Metric Description

Action: $S_F = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R)$

EOM:
$$F_R(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} - \left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right]F_R(R) = \kappa^2 T_{\mu\nu}$$

Auxiliary equation:
$$F_R(R)R - 2F(R) + 3\Box F_R(R) = \kappa^2 T$$

ST Description

Action: $S_F = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[\Phi R - V(\Phi)]$

EOM:
$$G_{\mu\nu} = \frac{\kappa^2}{\Phi}T_{\mu\nu} + \frac{1}{\Phi}\left[\nabla_\mu\nabla_\nu\Phi - g_{\mu\nu}\left(\Box\Phi + \frac{1}{2}V\right)\right]$$
$$\Box\Phi = \frac{1}{3}\left[\Phi V'(\Phi) - 2V(\Phi) + \kappa^2 T\right]$$

$$\Phi \equiv F_R(R)(>0)$$
$$V(\Phi) \equiv R(\Phi)\Phi - F(\Phi)$$

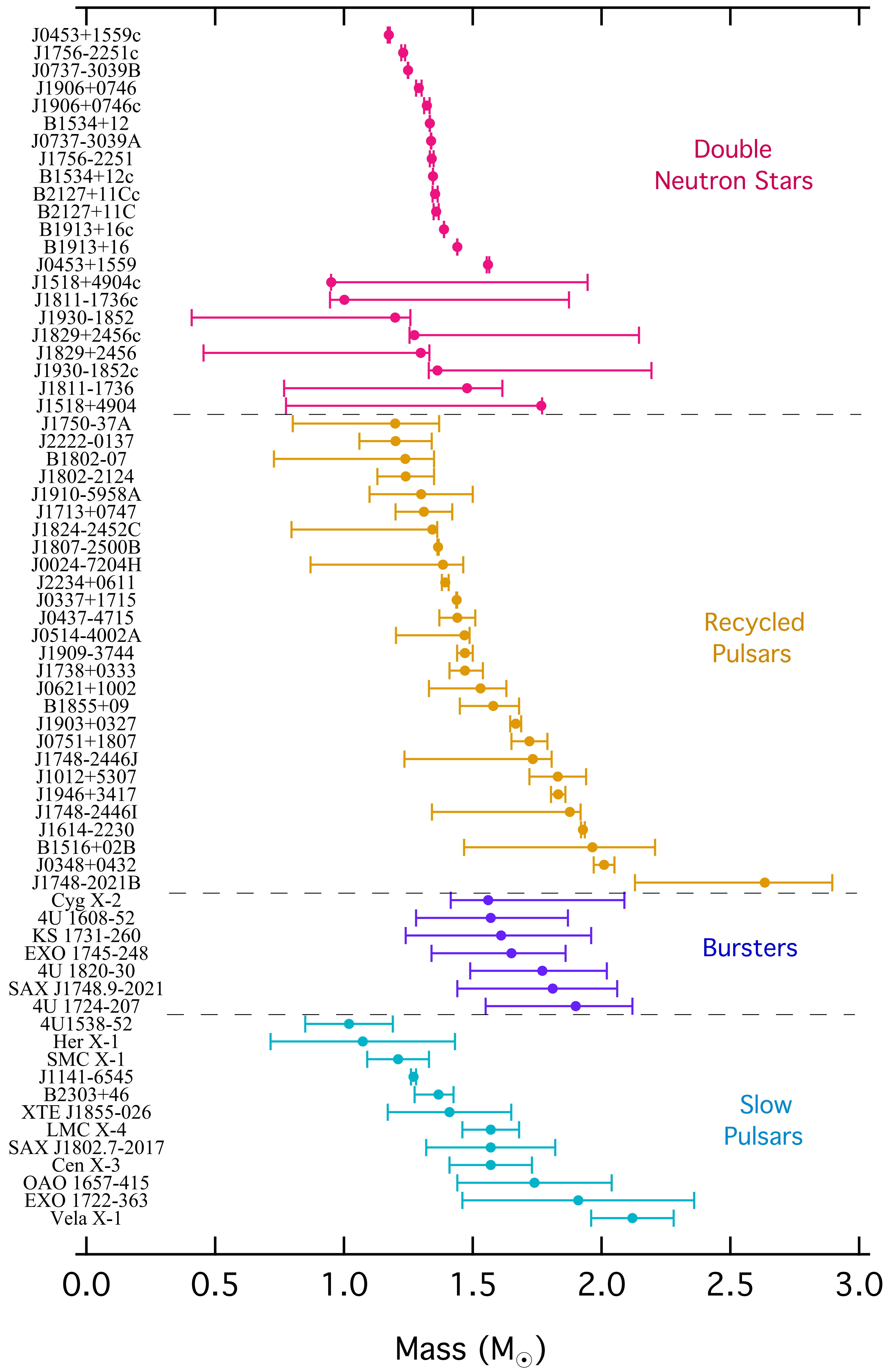
DOF: 2+1 →
- **Dark energy** [S. Capozziello, 2002], [S. M. Carroll+, 2004], etc…
- **Dark matter** [S. Nojiri+, 2008], [T. Katsuragawa+, 2017], etc…

Probe: Neutron Stars (NSs)

From observational point of view

- Compact → Strong gravity
- Accessible through various obs. ways (GWs, X-ray, etc…)

→ **Deviation from GR can be found in NSs' physical quantities.**



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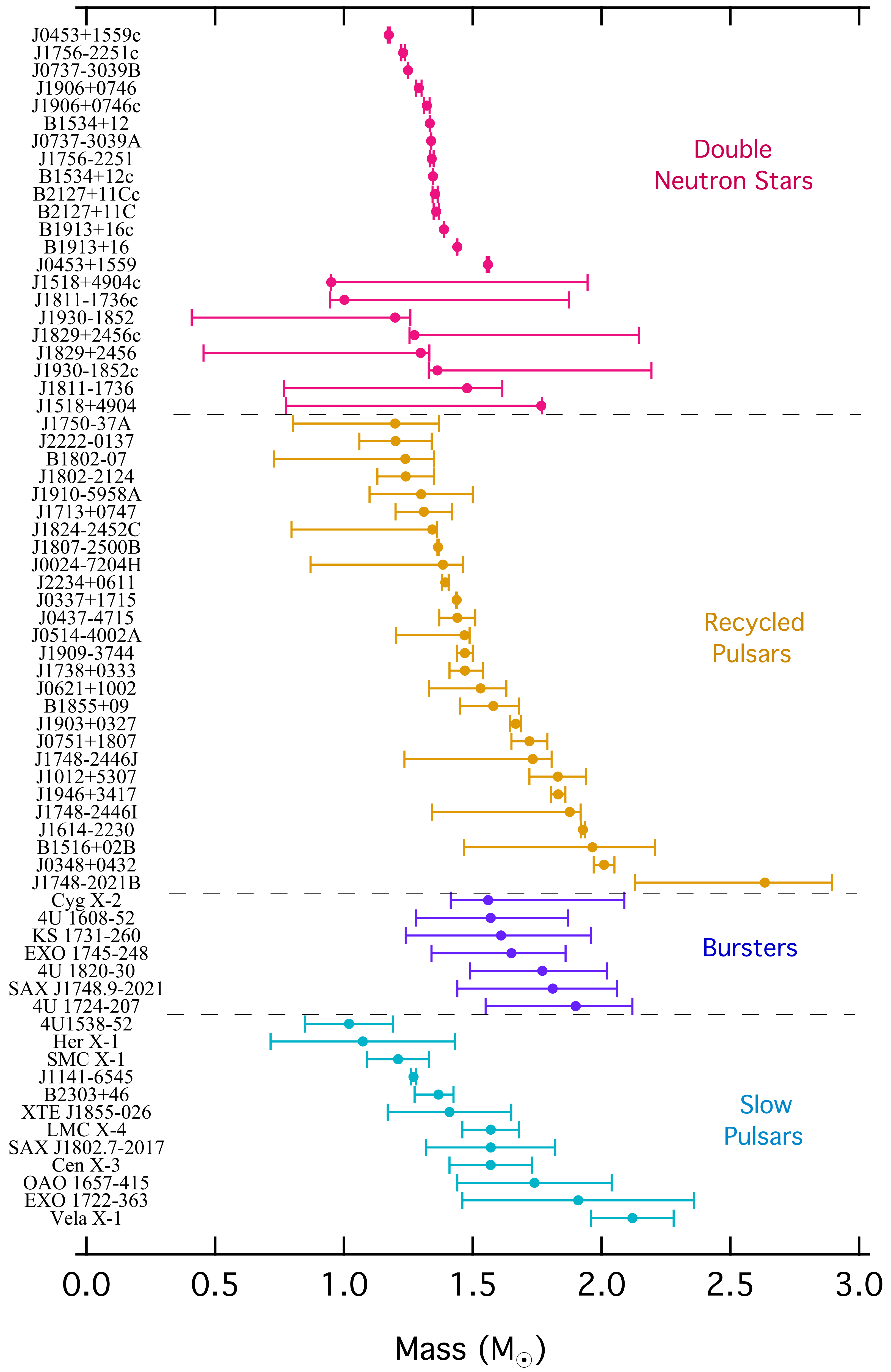
From theoretical point of view

- **Background**: TOV problem ↔ M-R relation
- (Static) perturbation: Tidal deformation ↔ Love number
- Time evolution: Thermal evolution ↔ Surface temp.

How does additional DOF affect NS physics & observables?
(internal behavior? scalar-hair?)



R^2 Model as Massive Scalar Case

[A. A. Starobinsky, 1980] etc..

Model

$$F(R) = R + \alpha R^2$$

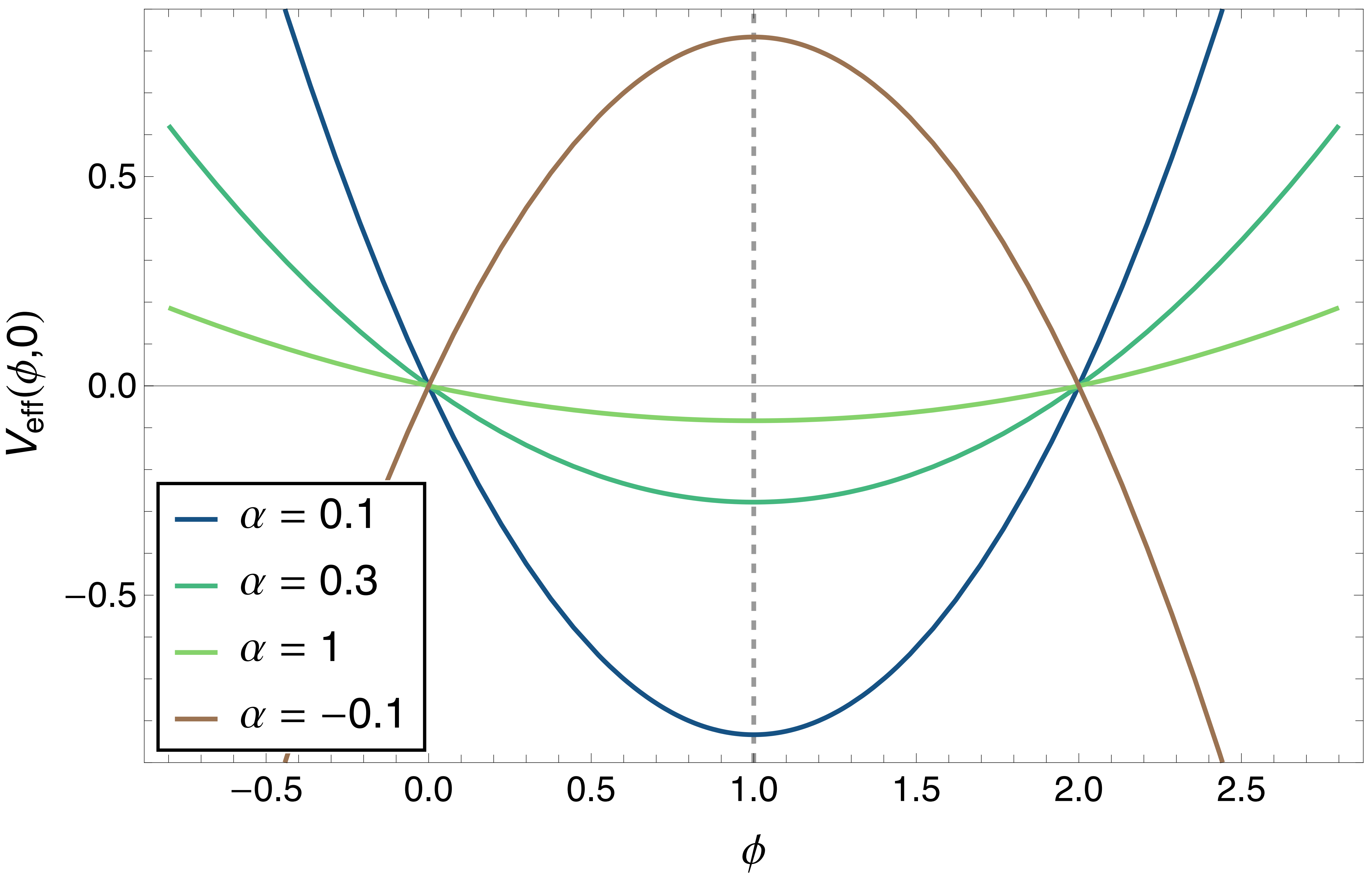
Minimal modification for high curvature regime

Chameleon potential (effective potential of scalar):

$$V_{\rm eff}(\Phi, T) = \frac{1}{12\alpha}(\Phi - \Phi_{\rm min})^2 - \frac{\Phi_{\rm min}^2}{12\alpha} \quad \Phi_{\rm min} = 1 - 2\kappa^2 \alpha T$$

Chameleon mass (effective mass): $m_\Phi = \dfrac{1}{\sqrt{6\alpha}},$

⇒ **Scalar behaves as a massive particle everywhere.**



Problems in Previous Works

Several problems still remain in TOV problem under R^2 model:

- **"Asymptotic solution" outside the star**
  "Asymptotically flat" was often assumed.
  → How do solutions become "flat"?

Exp. decay? [A. V. Astashenok+, 2018] etc…
Exact Schwarzscild? [A. Ganguly+, 2014] etc…
Damping oscillation? [M. A. Resco+, 2016] etc…

- **Scalar's behavior inside the star**
  Its (internal) behavior and influences had not been well investigated.
  → Tidal deformation / NS cooling etc…

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Our aim

Revisit TOV problem in R^2 gravity, paying attention on

- **Asymptotic behaviors of solutions**
- **Scalar = curvature profile inside the star**

Proper Boundary Conditions in R^2 Model

Assumption Static, spherical symmetric, and asymptotically flat

$$ds^2 = -\mathrm{e}^{2\nu}dt^2 + \mathrm{e}^{2\lambda}dr^2 + r^2\Omega_{AB}dx^A dx^B$$

Uniquely reduces to Schwarzschild sol in GR (∵ Birkoff-Jebsen's theorem)

EOM for scalar (deviation from GR solution):

$$\Box\Phi = m_\Phi^2(\Phi - \Phi_{\min}), \quad \left(m_\Phi = \frac{1}{\sqrt{6\alpha}}, \ \Phi_{\min} = 1 - 2\kappa^2\alpha T\right)$$

⇒ For distant vacuum region ($r \gg \alpha^{-1/2}$) $\Phi \sim 1 + \frac{C}{r}\mathrm{e}^{-m_\Phi r}$

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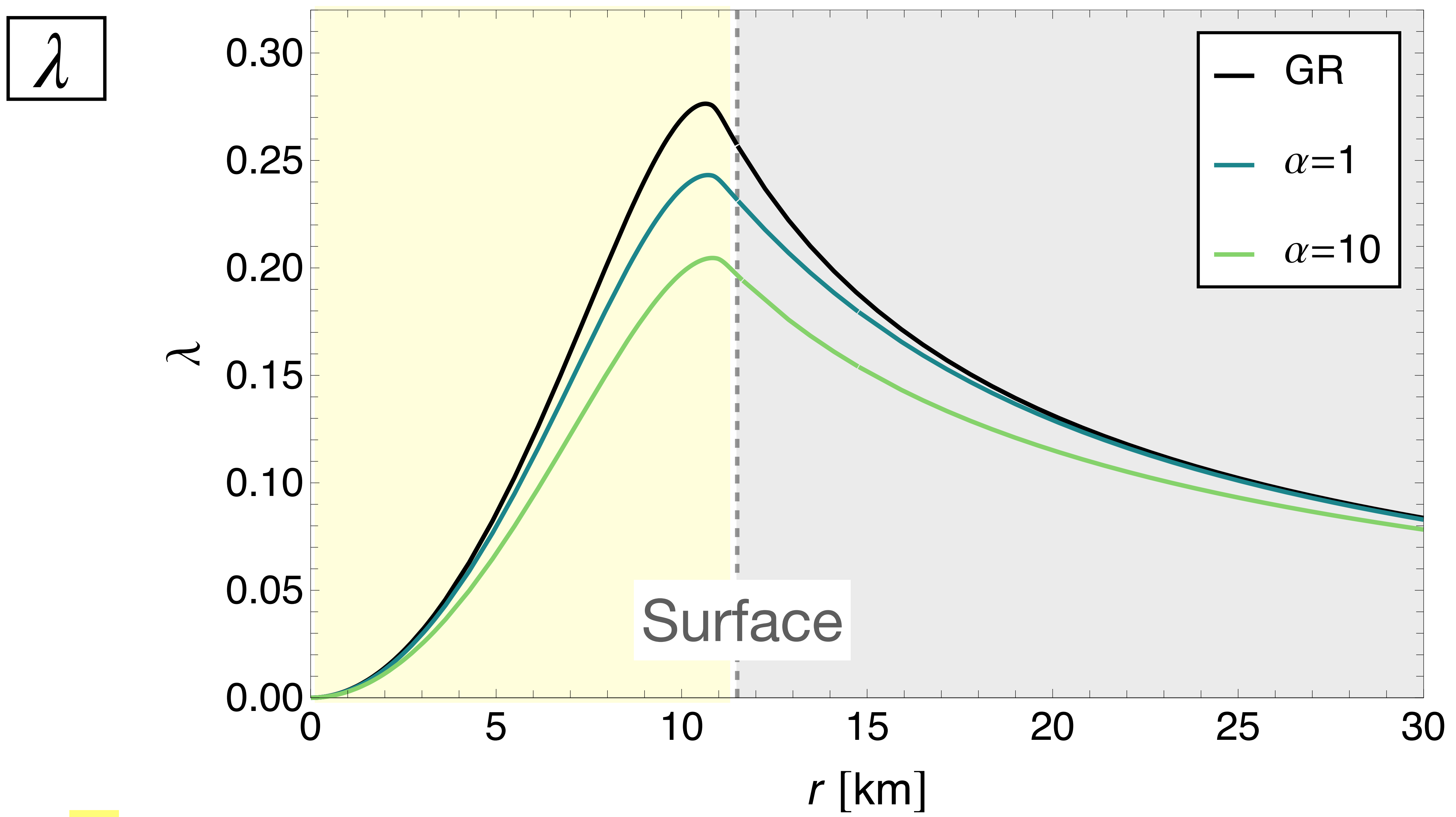
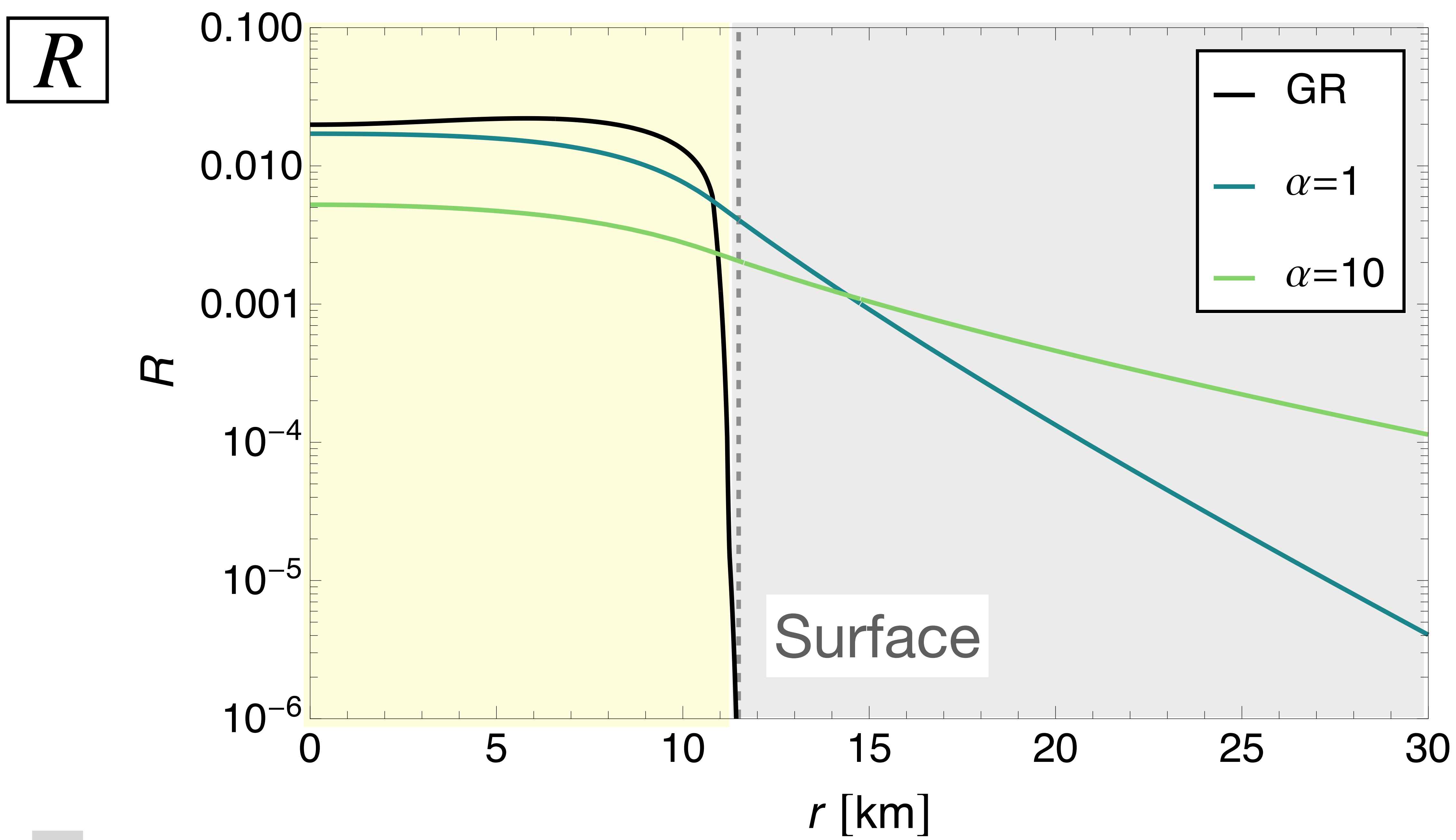
GR Modification effect: exponentially decreasing

→ **Asymptotic Schwarzschild**

Results of R^2 case: Geometry

[A. V. Astashenok+, 2018]
[K. Numajiri+, 2022]





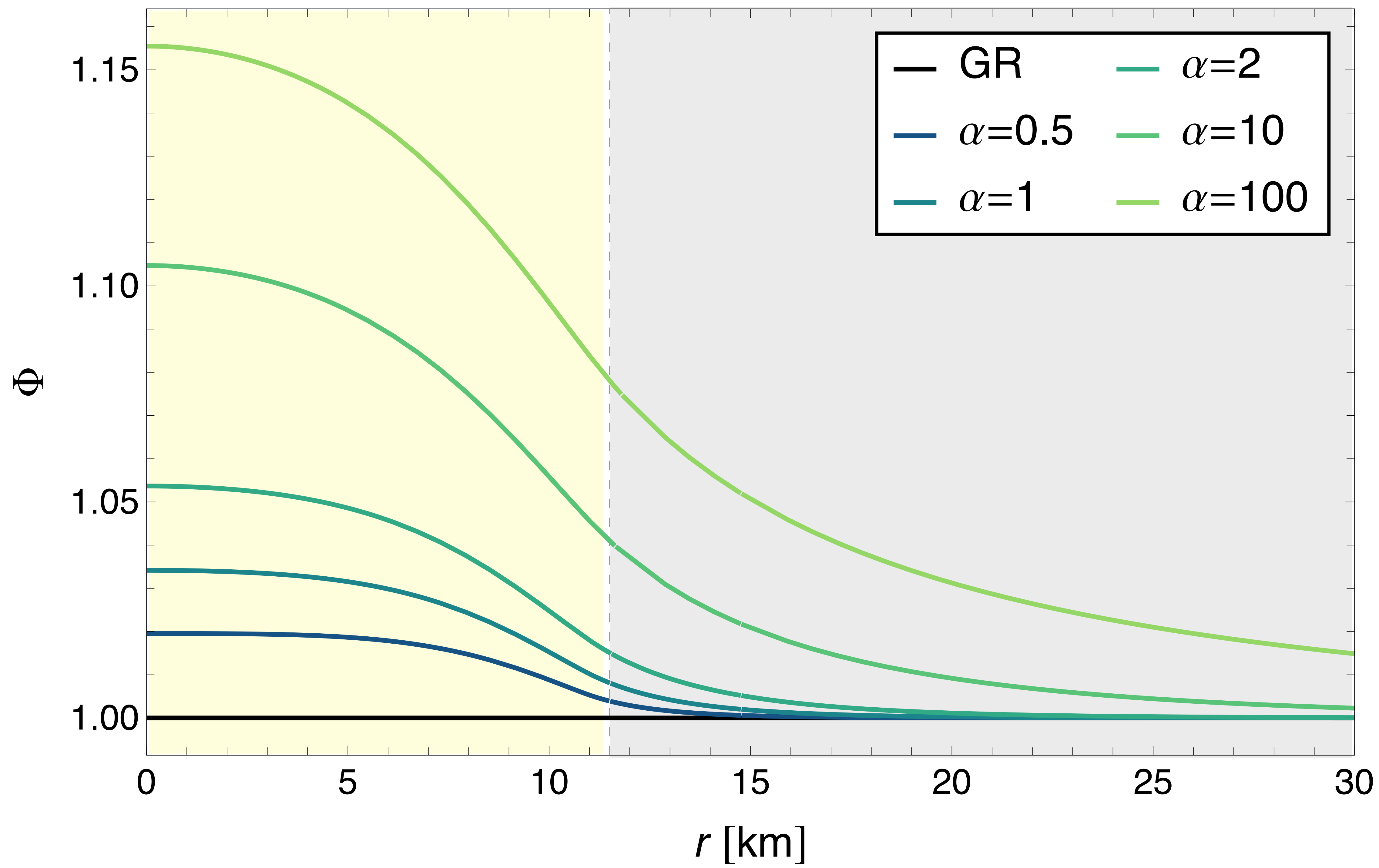
Interior:

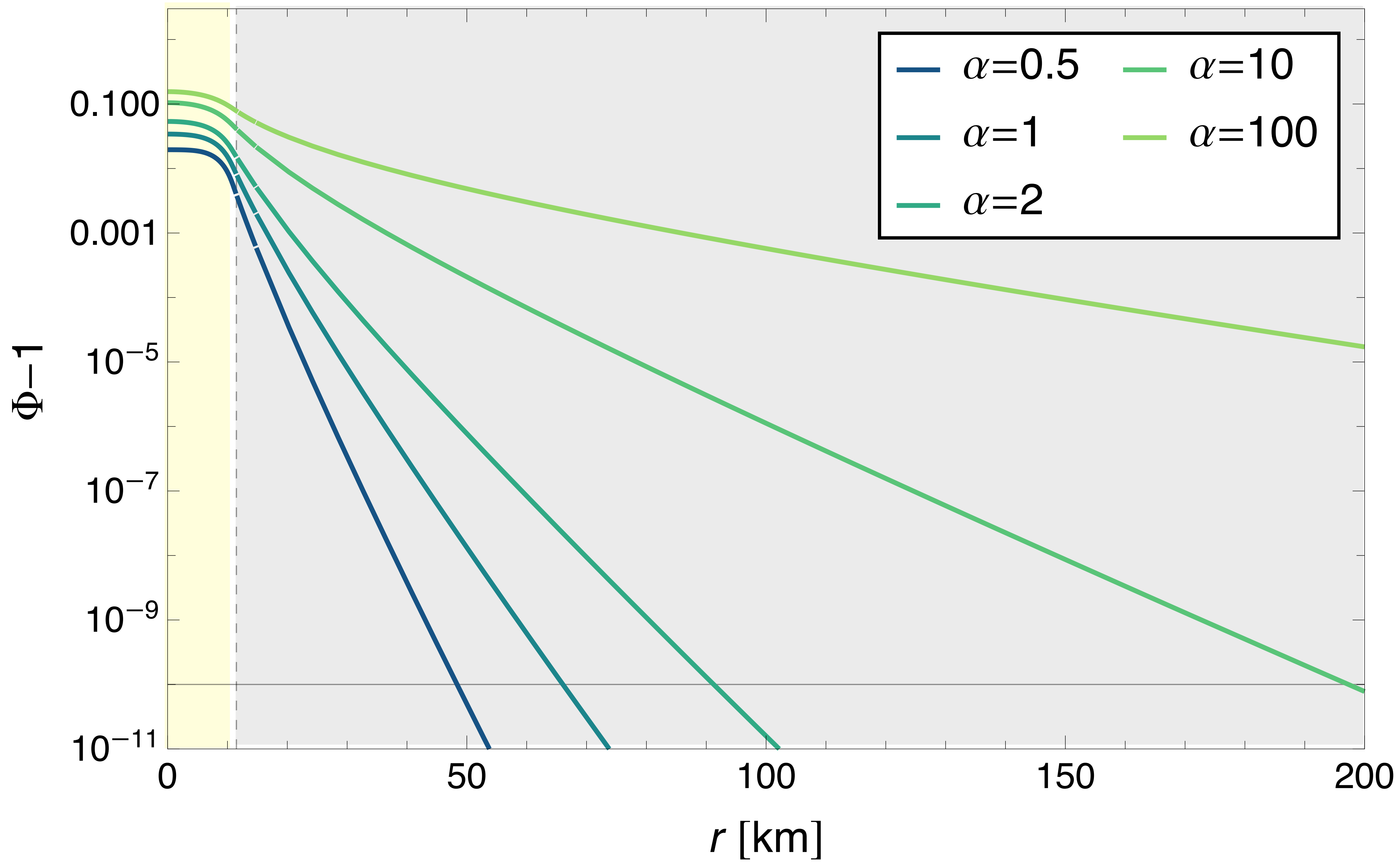
- Metric:
  Largely deviates around the surface
- Curvature
  Largely deviate for large α

Exterior:

- Metric:
  Coincides with Sch. as distant from surface
- Curvature
  Exponentially decreasing (**scalar-hair**)

Results of R^2 case: Scalar Distribution



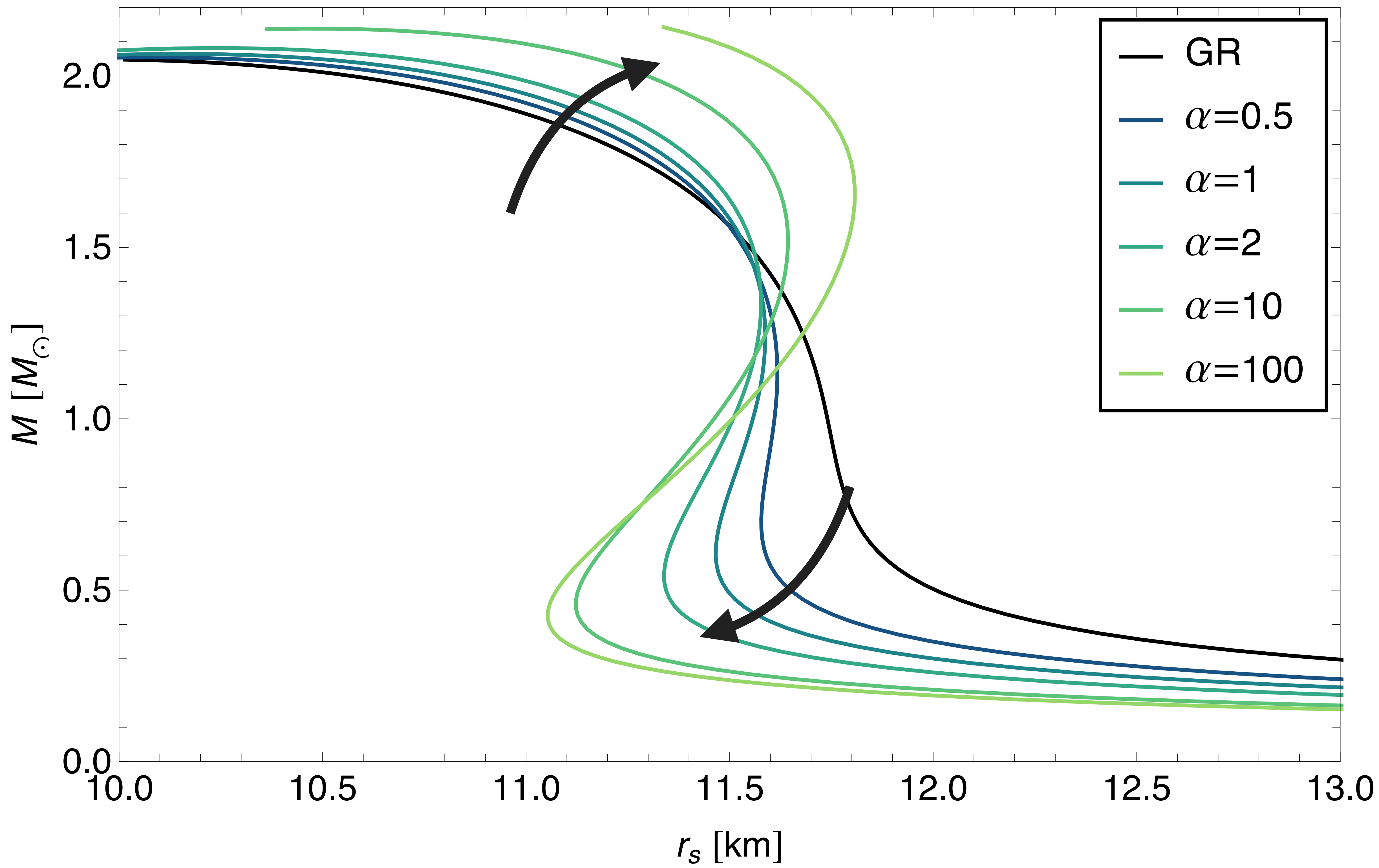


- Takes non-trivial value
  (particularly around **the center**)
- **Not necessarily decrease monotonically**
  (depending on central energy density)

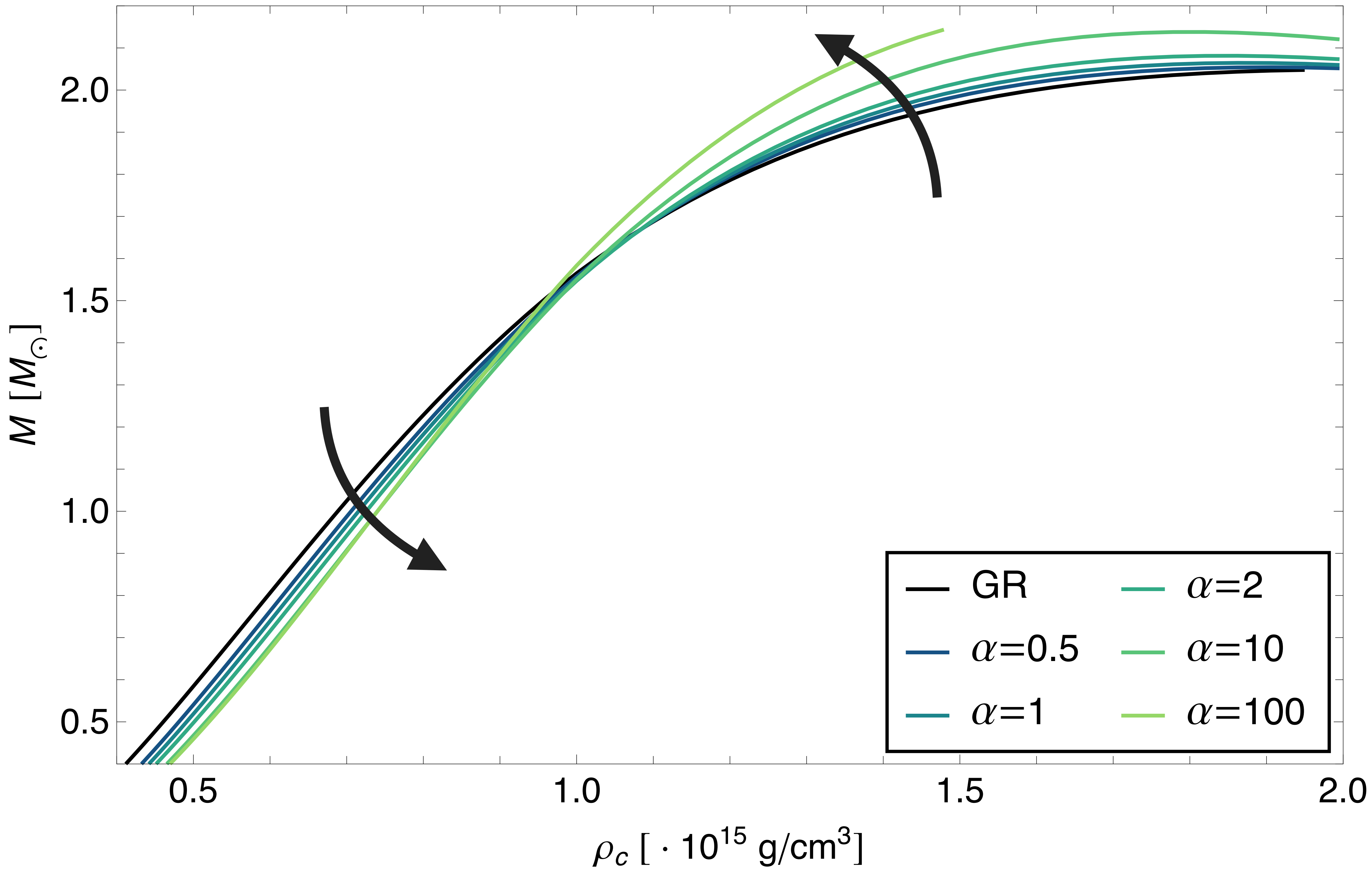
- Exponentially decreasing scalar-hair
  Hair radius ∝ Compton length of scalar

Results of R^2 case: M-R Relation

Mass vs Radius



Mass vs Central energy density



The mass curve "rotates" as α increases.

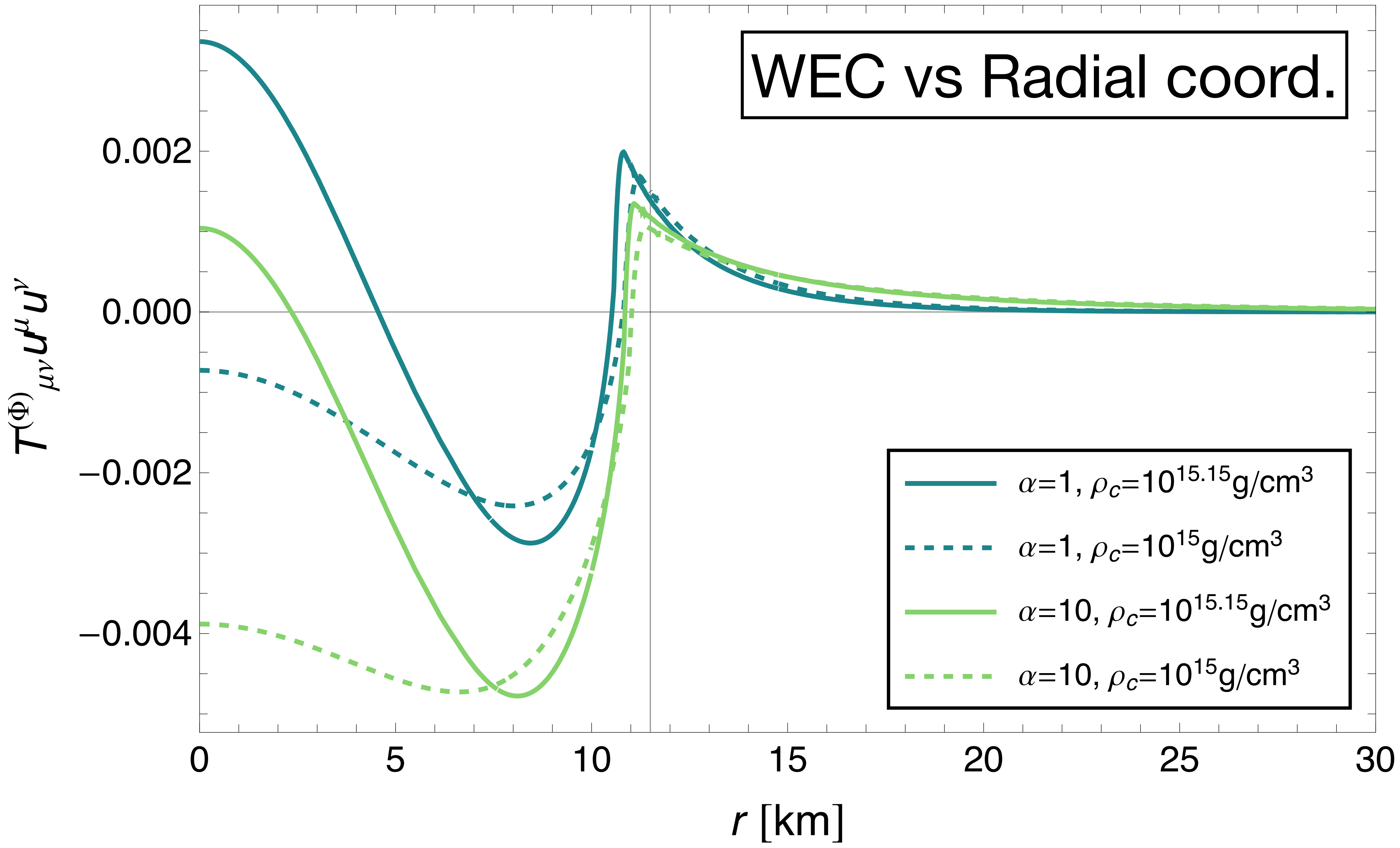
Consistent with [A. V. Astashenok+, 2018] etc…

Why They "Rotate"?

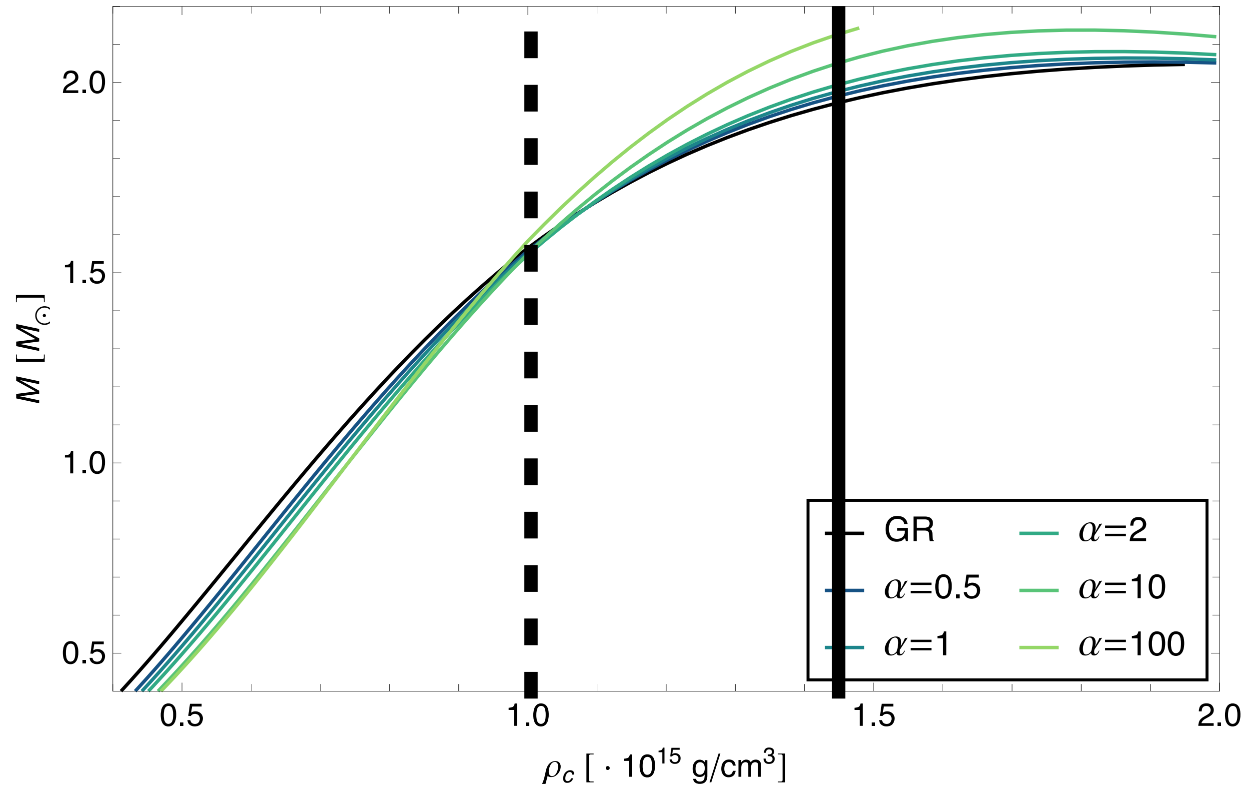
Effective energy-momentum of scalar:

$$T_{\mu\nu}^{(\Phi)} = \frac{1}{\kappa^2\Phi}\left[\nabla_\mu\nabla_\nu\Phi - g_{\mu\nu}\left(\Box\Phi + \frac{1}{2}V(\Phi)\right)\right]$$

Weak energy condition (~effective energy density)



[K. Numajiri+, 2022]



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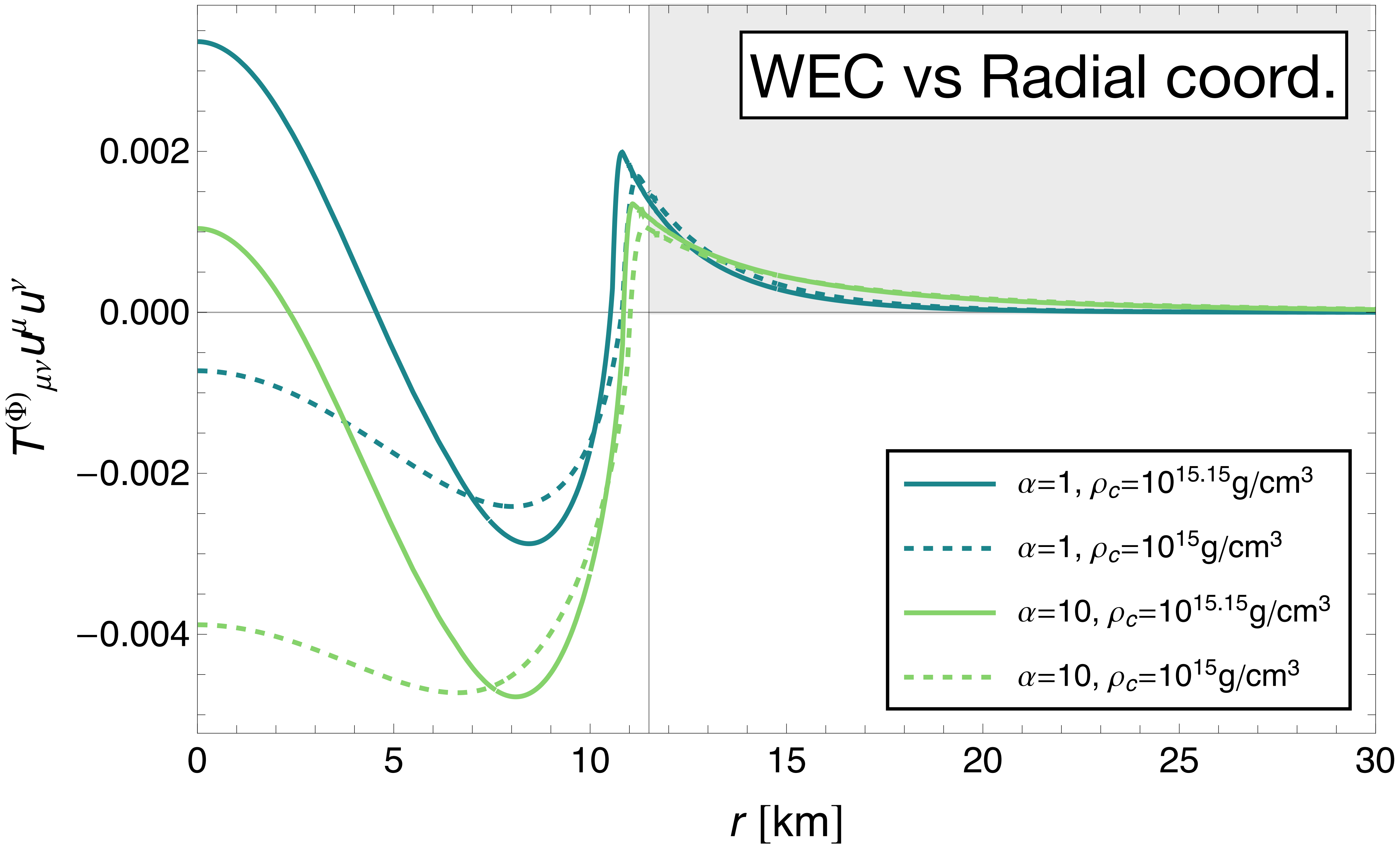
Outside: Positive → Scalar hair **weighs the mass.**



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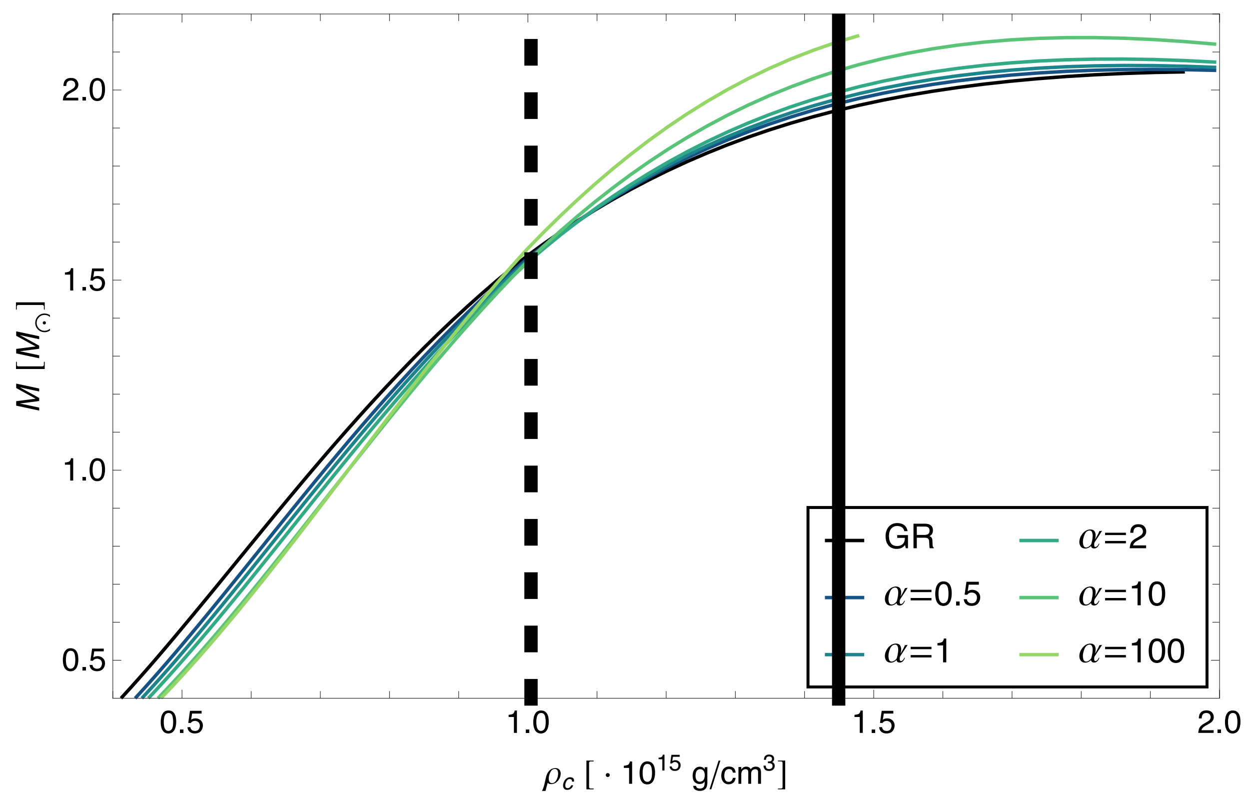

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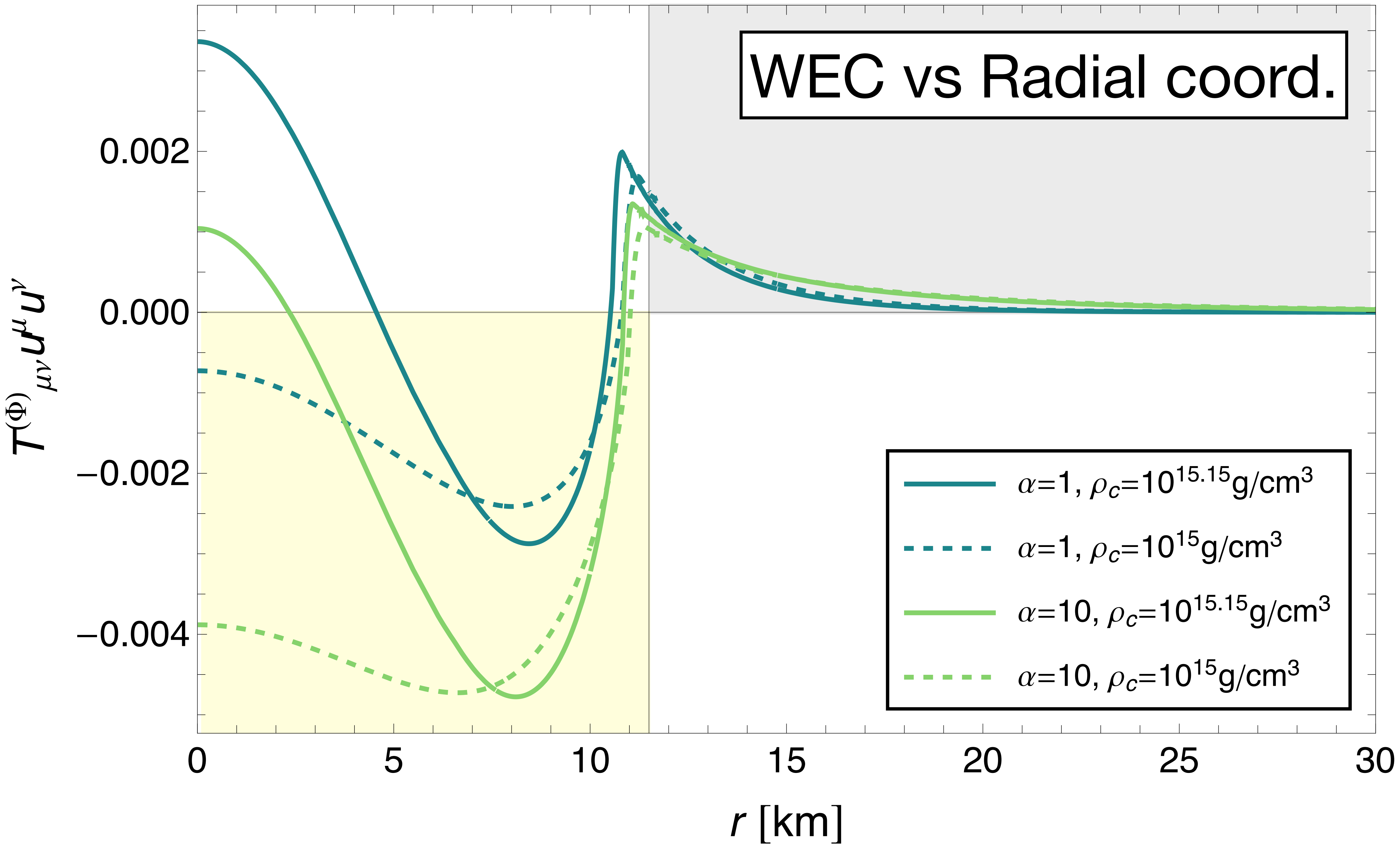
Outside: Positive → Scalar hair **weighs the mass.**

Inside: Negative in almost everywhere (for low ρ_c).
Negative partially (for high ρ_c).

→ **Reduces the mass effectively.**

→
- **Total mass**
- **Sch mass at the surface (→ Metric deviation)**



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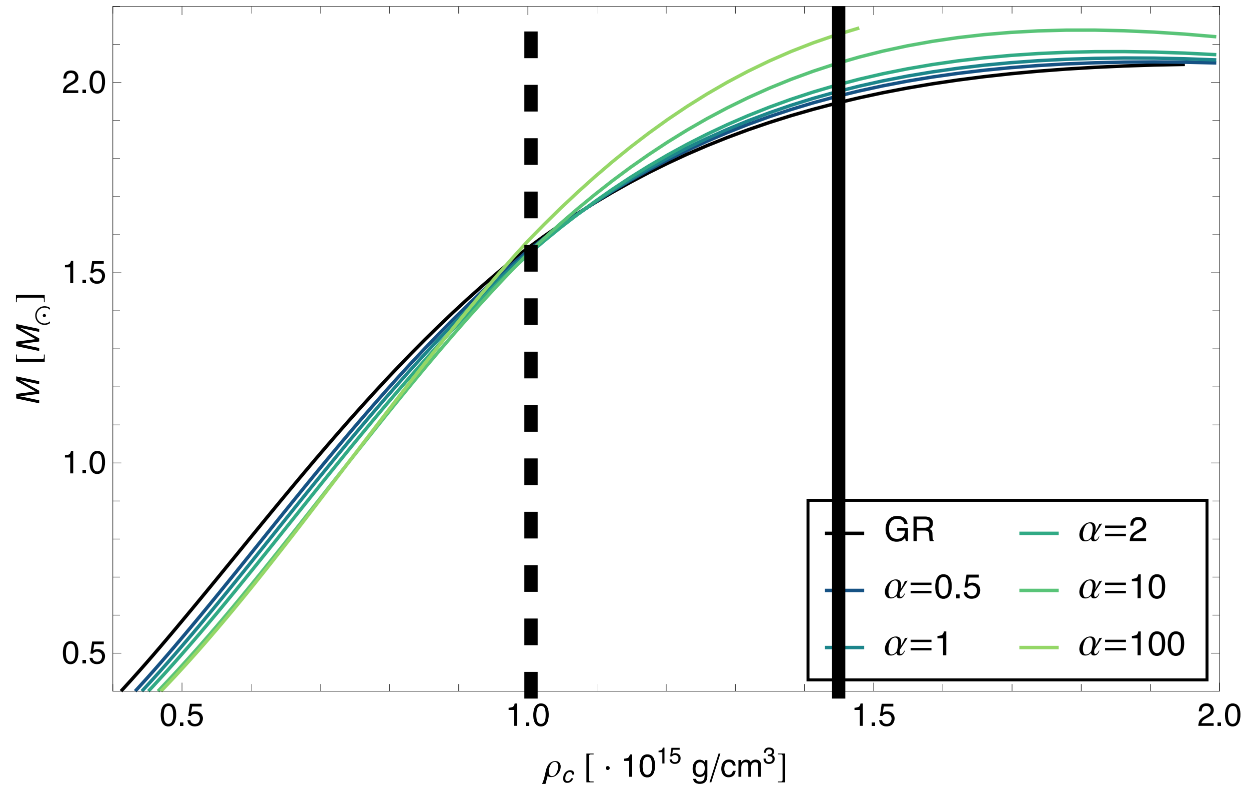
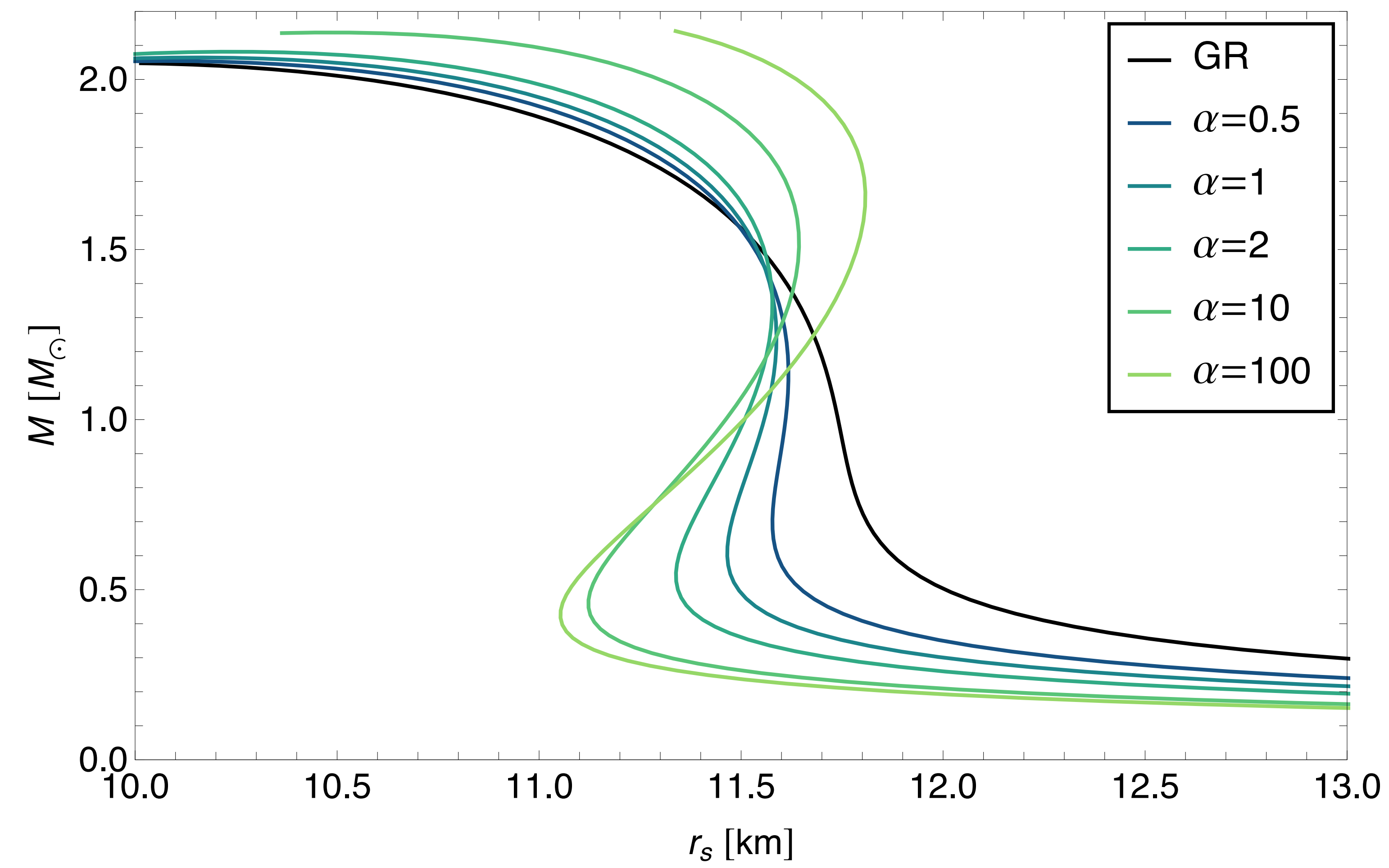


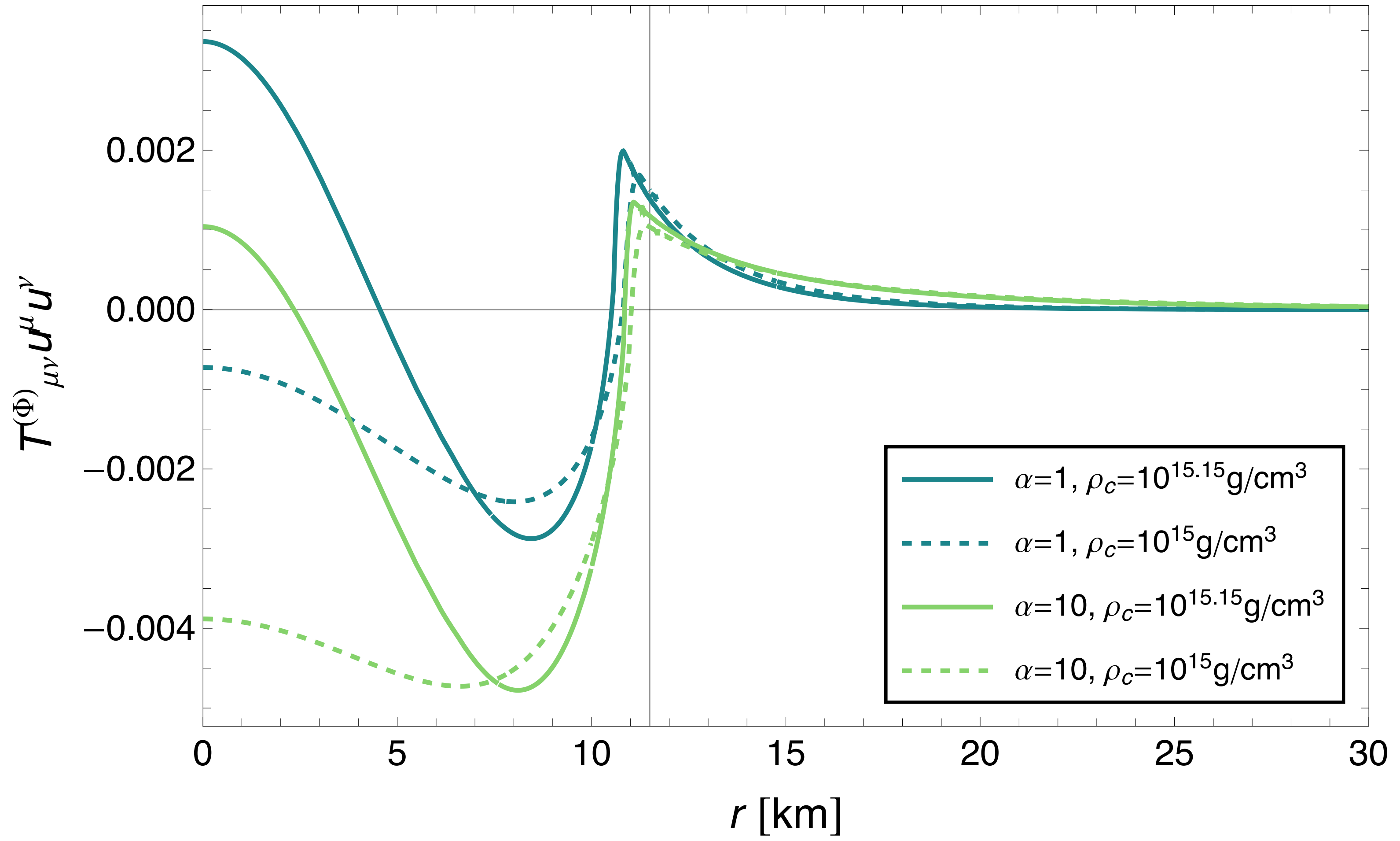
Summary

- We revisit the TOV problem in R^2 gravity enlightening the behavior of the additional scalar field.
- Stars gain mass for high central energy density region, while losing mass for low energy region.
- The scalar field becomes quintessential for interior region, while it satisfies all energy-condition at exterior. This effect is significant for lower ρ_c solutions.



Future works

- Scalar effect on tidal deformation (Behavior around core or surface? Which is responsible?)
- Scalar effect on thermal evolution



B-Slides

Relation of B.C. and $F(R)$ Model

[K. Numajiri+, 2021]

Interior: hydrostatic sol.

EOS: Polytropic $p = K\rho^{1+\frac{1}{n}}\ (0.5 \leq n \leq 1)$

Pressure profile (around surface):

$$p(r) \sim p_0 \left(1 - \frac{r}{r_s}\right)^m \left[1 + p_1 \left(1 - \frac{r}{r_s}\right) + \cdots\right]$$

m: constant, r_s: surface radius

Exterior: Schwarzschild sol.

To be Matched

Reconstructed $F(R)$: $F(R) = R + aR^{1+\beta}\ (0 < \beta < 1)$

Contraposition

Non-trivial exterior geometry would be realized for other $F(R)$ models.

System and Equations

[K. Numajiri+, 2022]

Equations: Sols: $\lambda, \nu, R, p\ (\epsilon)$ $\quad ds^2 = -\mathrm{e}^{2\nu}dt^2 + \mathrm{e}^{2\lambda}dr^2 + r^2\Omega_{AB}dx^A dx^B$

Metric:

$$\lambda' = \frac{\mathrm{e}^{2\lambda}\left\{2\kappa^2 r^2 \epsilon + \alpha R(r^2R-4) - 2\right\} + 4\alpha\left(r^2R'' + 2rR' + R\right) + 2}{4r\left(\alpha(rR'+2R)+1\right)} \qquad \nu' = \frac{\mathrm{e}^{2\lambda}\left\{2\kappa^2 r^2 p - \alpha R(r^2R-4) + 2\right\} - 2\left(4\alpha rR' + 2\alpha R + 1\right)}{4r\left(\alpha(rR'+2R)+1\right)}$$

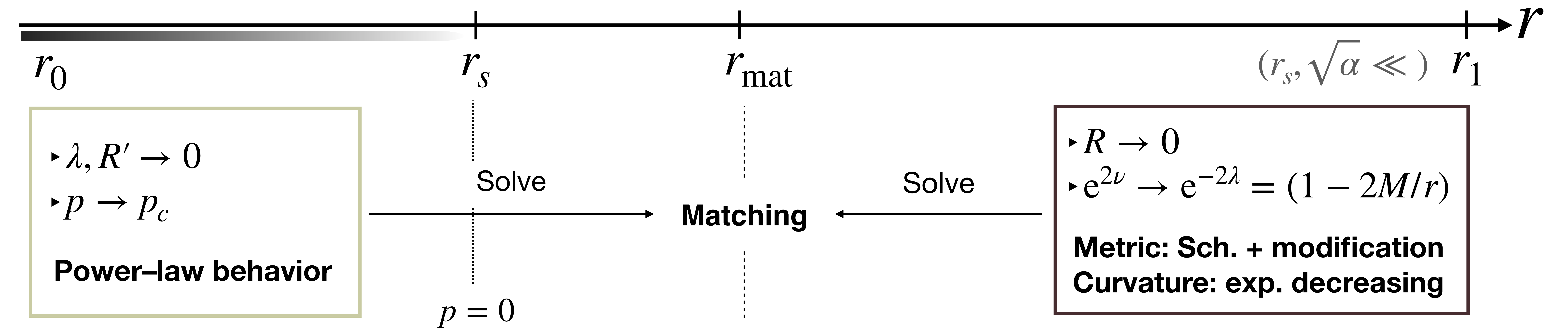
Curvature (scalar):

$$R'' = \frac{1+2\alpha R}{2\alpha}\left[\frac{1}{r}\left(3\nu' - \lambda' + \frac{2}{r}\right) + \mathrm{e}^{2\lambda}\left(\frac{1}{2}R - \frac{2}{r^2}\right)\right] + \left(\lambda' + \frac{1}{r}\right)R'$$

Matter:
(Perfect fluid)

$$p' = -\nu'\left(\epsilon + p\right) \qquad p = p(\epsilon)$$

(EOS: tabulated SLy/APR4 [J. S. Read+, 2009])



Discussion for BC under R^2 Model

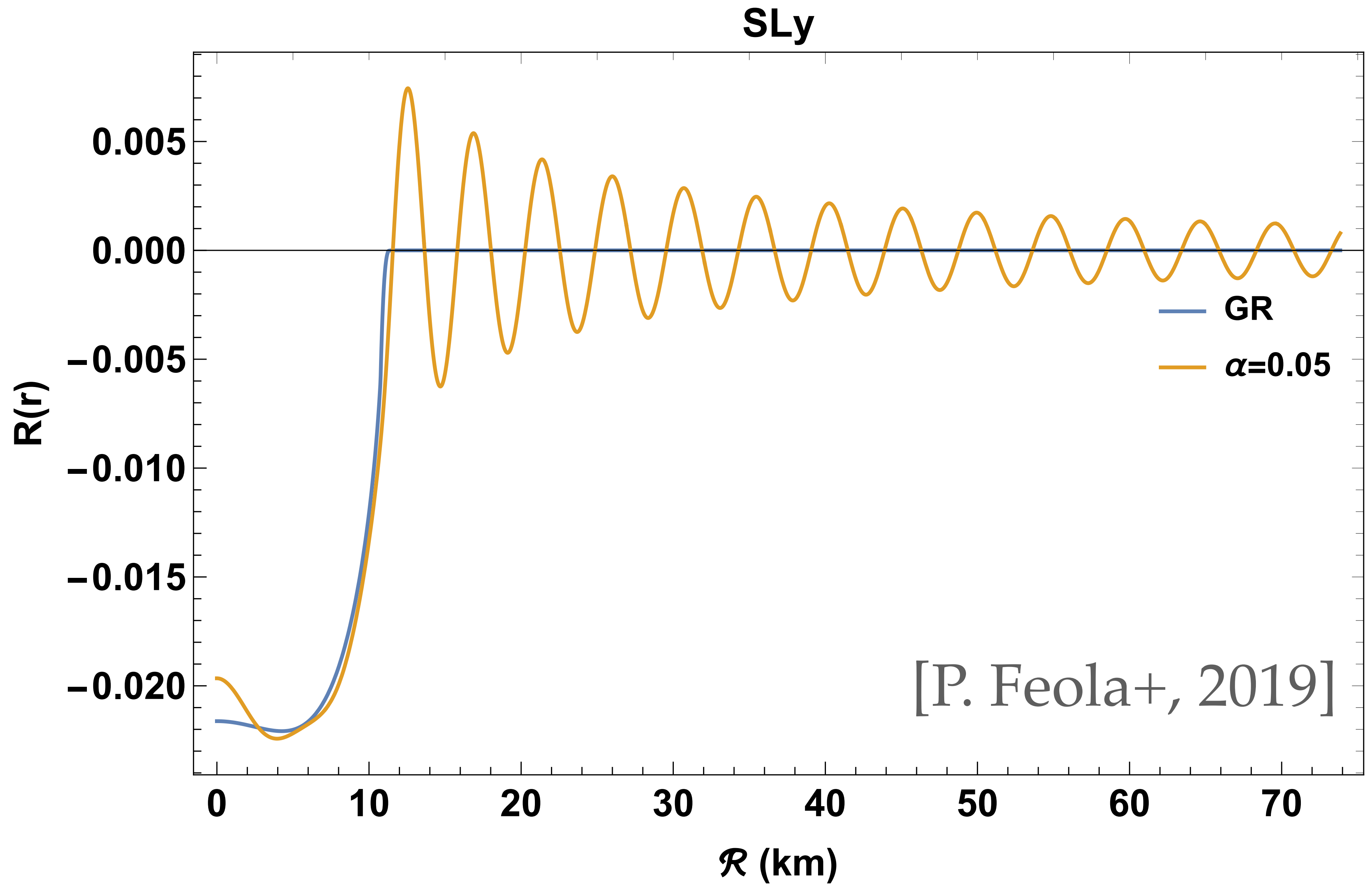
- **Decreasing oscillation?** [M. A. Resco+, 2016] etc…

  Correspond to $\alpha < 0$

  → Tachyonic due to $m_\Phi^2 < 0$

  - Dynamical instability
  - Stellar mass is not-well defined

  [A. V. Astashenok+, 2018] etc…



- **EXACT Schwarzschild?** [A. Ganguly+, 2014], [W.-X. Feng+, 2017] etc…

  Guessed from BH no-hair theorm [B. Whitt, 1984], [S. Mignemi+, 1992]

  → Strongly depend on the existence of horizon/vacuum system

  → Non-trivial in the system composed of matter without conformal invariance

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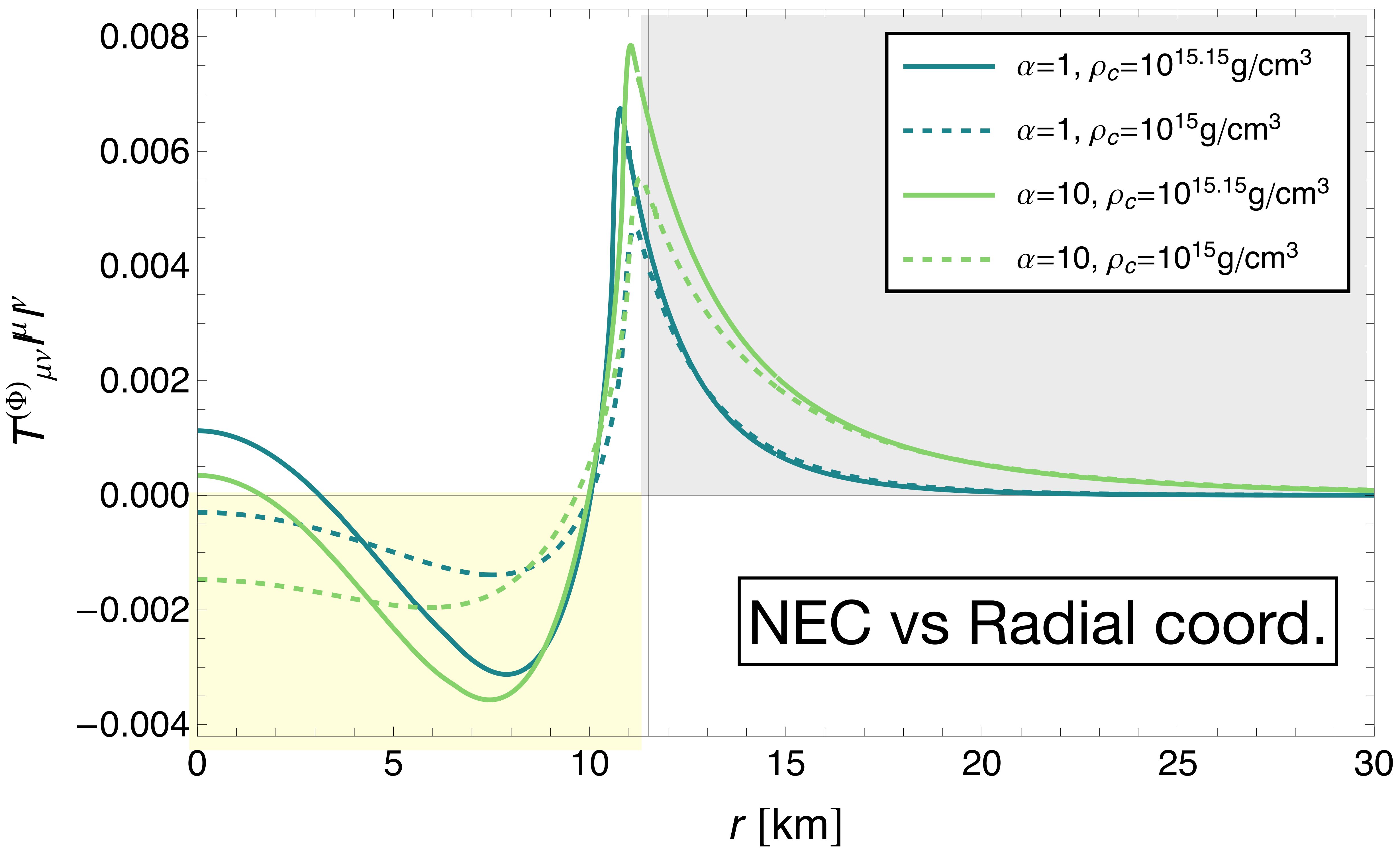

[K. Numajiri+, 2022]