Vainshtein screening of time and spatial scalar-field derivatives

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Ongoing work

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YITP long-term workshop “Gravity and Cosmology 2024” 5th week

Ⅰ. Introduction

Dark Energy

Unknown source of late-time cosmic acceleration

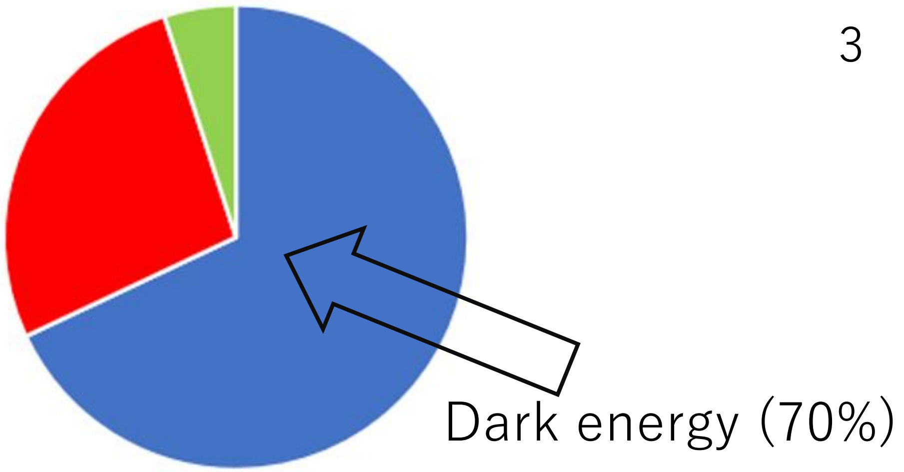


Fig 1. Component of our universe

Scalar-tensor theories

- The theories that include scalar field ϕ coupled to gravity
- They contains many candidate theories of dark energy

⇩ We will focus on…

Theories in which scalar field and Ricci scalar R are directly coupled (= nonminimal coupled)

Example of nonminimal coupling theory

- Dilaton : Low-energy effective action of superstring theory (P.G. Bergmann, 1968)
- $f(R)$ gravity : Theory rewritten as $R \to f(R)$ in the action of GR (M. gasperini and G.Veneziano, 1993)

Brans-Dicke theory

(C. Brans and R. H. Dicke, 1961)

Theory of gravity that contains the above theories

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}\boldsymbol{F(\phi)}R + (1-6Q^2)F(\phi)X - V(\phi)\right] + \mathcal{S}_m$$

ϕ : Scalar field
$X = -[g^{\mu\nu}(\nabla_\mu\phi)(\nabla_\nu\phi)]/2$: Kinetic energy of scalar field
$V(\phi)$: Potential of scalar field
$F(\phi) = e^{-2Q\phi/M_{pl}}$: Functions which represent nonminimal coupling
Q : Constants that characterise the strength of the coupling
$\mathcal{S}_m$: Action of matter field

Typically, $\boldsymbol{Q \sim \mathcal{O}(1)}$

Dilaton : $Q^2 = 1/2$
$f(R)$ gravity : $Q = -1/\sqrt{6}$

Brans-Dicke theory

- Cosmology : Dark energy can be explained

  ⇕

- Solar system : $|Q| \leq 2.5 \times 10^{-3}$ when $V = 0$

  Difficult to distinguish observably from $Q = 0$ case

  (C.D. Hoyle et al, 2004)

Vainshtein mechanism

Screening nonminimal coupling at solar system scale

(R. Kimura et al, 2012)

Although the Vainshtein mechanism can relaxes constraints on Q…

Effective gravitational constant has time variation by the presence of F

→Inconsistent with LLR experiment (S. Tsujikawa, 2019)

Our aim

Investigate the validity of this analysis by numerical calculation

Ⅱ. Vainshtein screening and constraint from experiment (Analytical solution)

Setup

Spherically symmetric metric

⇧ We consider spherically symmetric star (ex. sun) as a matter

$$ds^2 = -e^{2\Psi(t,r)}dt^2 + a(t)^2 e^{2\Phi(t,r)}[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)]$$

Action

Brans-Dicke action+ self interaction term of scalar field

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}F(\phi)R + (1-6Q^2)F(\phi)X + \underline{\boldsymbol{c_3 X \Box \phi}} - V(\phi)\right] + \mathcal{S}_m$$

$\Psi(t,r), \Phi(t,r)$: function of t, r
$a(t)$: scale factor
$\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$
c_3 : coefficient of self-interacting term

⇧ Necessary for Vainshtein mechanism

Assumptions in analysis of previous work (R. Kimura et al, 2012)

- $\frac{\boldsymbol{d}}{\boldsymbol{dt}} \sim \boldsymbol{H}, \frac{d}{dr} \sim 1/r$ ($H = \dot{a}/a$: Hubble parameter)

  →Assuming that **scale of time variation is cosmological scale**

- We consider perturbation on solar-system scale ($ar \ll 1/H$)

→**We ignore time-derivative terms**

- In principle, only linear terms are considered

  but some non-linear terms proportional to c_3 are taken into account

We derive **analytical solution** under these assumptions and **compare with numerical solution**

Perturbation equations

$$\begin{cases} \beta_1 \delta\phi' + 2M_{pl}^2 F \Phi' = -\dfrac{M}{4\pi a r^2} \\ \Psi' + \Phi' = \dfrac{2Q}{\mathrm{M}_{pl}} \delta\phi' \\ 2c_3 \dfrac{\delta\phi'^2}{a^2 r} + \beta_2 \delta\phi' - \beta_1 \Psi' + 4Q M_{pl} F \Phi' = 0 \end{cases}$$

$' = \partial/\partial r$

$M_{pl} = \dfrac{1}{\sqrt{8\pi G}}$

$\delta\phi$: scalar field perturbation

$M = 4\pi a^3 \int \delta\rho\, r^2 dr$: **Mass of the star**

β_1, β_2, A, B : functions of background variables

Order of the ratio of the first and second terms : $\dfrac{\delta\phi}{M_{pl}(arH)^2}$

⇨ **If r is small, we can't ignore first term** $(ar \ll 1/H)$

Correspond to acquire heavier effective mass on small scales

What is the order of criteria of r?

Vainshtein radius r_V

- Typical order : $r_V \sim 10^{20}$cm (Solar-system scale $< 10^{14}$cm) (A. De Felice et al, 2012)
- The behaviour of the solution depends on the relationship between r and r_V

In solar-system scale ($r \ll r_V$)

$$\delta\phi' = \frac{GMA}{Far^2}\left(\frac{r}{r_V}\right)^{\frac{3}{2}}, \Phi' = \frac{GM}{Far^2}\left[1 + \frac{2\left(c_3\dot{\phi}^4 - 4F^2Q^2M_{pl}^2\right)}{\beta}\left(\frac{r}{r_V}\right)^{\frac{3}{2}}\right]$$

Screening of nonminimal coupling
→Vainshtein mechanism

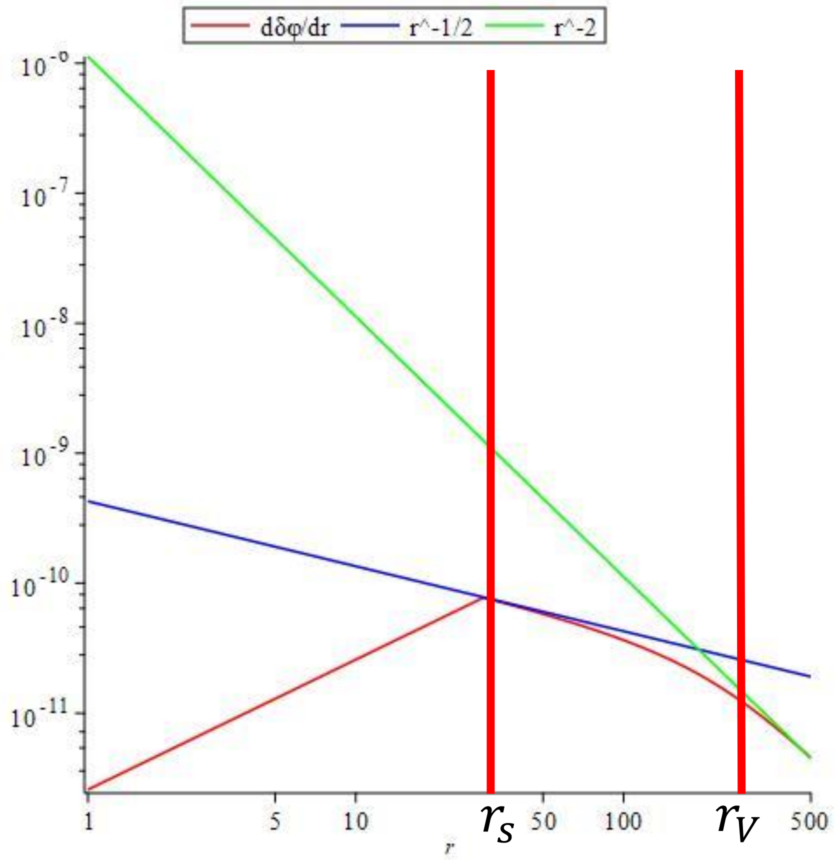


Fig.2 r dependence of ϕ'

⇨ Constraints on Q are **drastically relaxed**

A : function of background variable

$Q \sim \mathcal{O}(1)$ is allowed if we consider typical parameter

Effective gravitational constant G_{eff}

Defined by $-\boldsymbol{\Phi}' \equiv \frac{\boldsymbol{G_{\mathrm{eff}} M}}{\boldsymbol{a r^2}}$ ($G_{\mathrm{eff}} = G$ in GR)

$\boldsymbol{G_{\mathrm{eff}} \cong G/F = G e^{2Q\phi/M_{pl}}}$ in our model in $r \ll r_V$

Constraint on time variation of gravitational constant

If we apply constraint on time variation of G_{eff} by LLR experiment to our model...

$$-3.5 \times 10^{-5} \left(\frac{0.7}{h}\right) \le \frac{\dot{G}_{\mathrm{eff}}}{2H_0 G_{\mathrm{eff}}} = \frac{Q\dot{\phi}}{M_{pl} H_0} \le 1.03 \times 10^{-3} \left(\frac{0.7}{h}\right) \quad (H_0 = 100h \,\mathrm{km\,s^{-1}\,Mpc^{-1}})$$

$\sim\mathcal{O}(10^{-2})$ if scalar field is dark energy

⇨ **Inconsistent with LLR experiment** (S. Tsujikawa, 2019)

III. Validation of analytical solution by numerical calculation

Spatial derivative are screened

⇕

Time derivative are not screened→LLR experiment ❌

Is this analysis, obtained by ignoring time derivative, correct (especially for deep inside the Vainshtein radius) ?

⇨ **Validation by numerical calculation**

In particular, it is important to confirm **time scale of each variable**

⇧

Why?

By using Hamiltonian constraint, G_{eff} can be written as

$$G_{\mathrm{eff}} = \frac{G}{F}\left(1 + \frac{4\pi a r^2 \beta_1 \delta\phi'}{M}\right) + \cdots$$

screened

Time derivative

$$\frac{\dot{G}_{\mathrm{eff}}}{H_0 G_{\mathrm{eff}}} \cong -\frac{\dot{F}}{H_0 F} + \frac{4\pi a r^2 \beta_1 \delta\phi'}{M} \times \frac{1}{H_0}\left(\frac{\dot{a}}{a} + \frac{\dot{\beta}_1}{\beta_1} + \frac{\delta\dot{\phi}'}{\delta\phi'} - \frac{\dot{M}}{M}\right) + \cdots$$

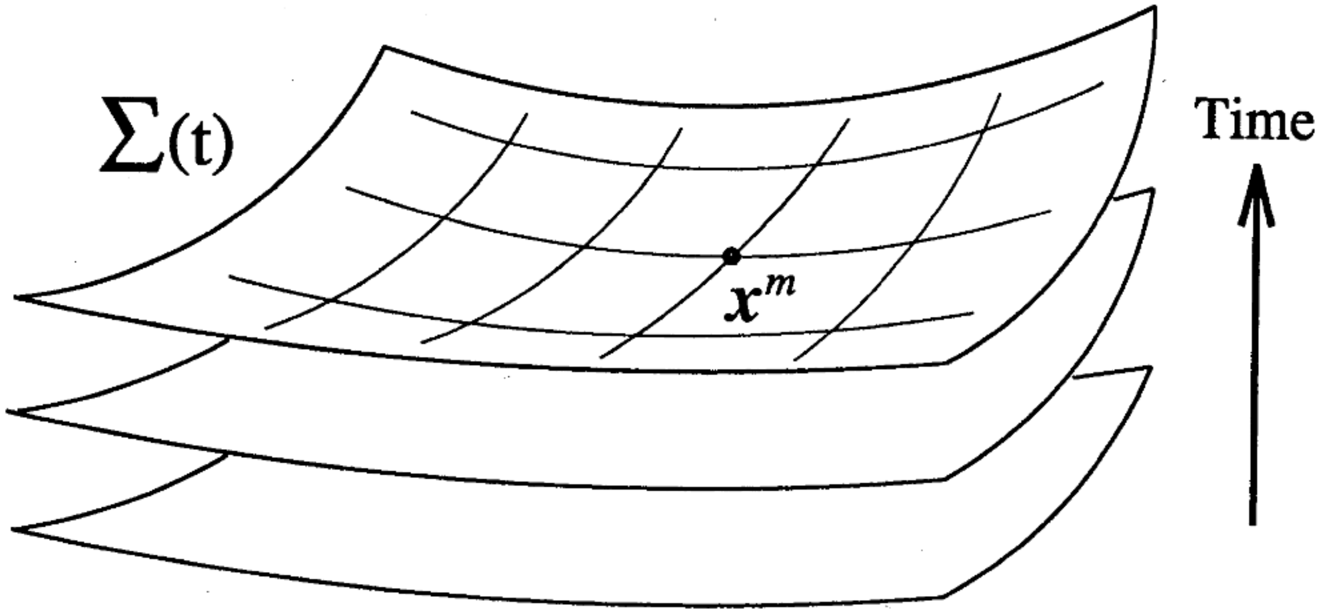
Analytical solution

Hubble scale or not ?

If time scale of these terms are not Hubble scale,
second term could change the constraint on G_{eff}

Method : ADM formalism

- It means the canonical formalism of gravity
- Slicing of 4D spacetime with time-constant 3D hypersurface $\Sigma(t)$ (Fig.3)
- <u>Appropriate for tracking the dynamics of 4D spacetime</u>



We give hypersurface $\Sigma(t)$
at some time as initial condition

⇩

By solving gravitational equations,
$\Sigma(t + dt)$ at the next time
can also be determined sequentially

Fig.3 Slicing of 4D spacetime with 3D hypersurface $\Sigma(t)$
(細谷 暁夫, 永谷 幸則, 丸 信人, 量子重力(講義ノート), 1995)

Preliminary results : Violation of Hamiltonian constraint

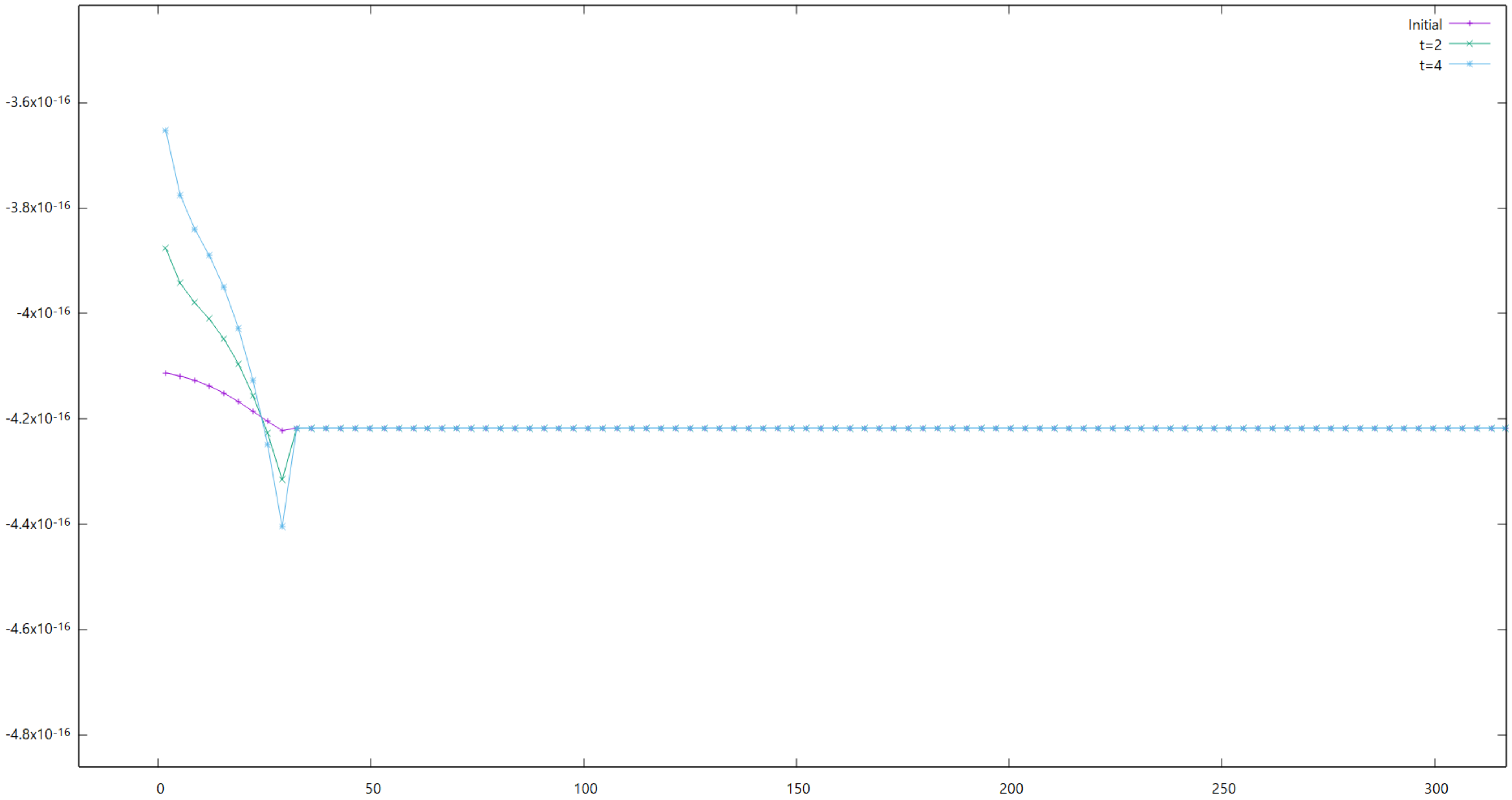


Fig.4 Violation of Hamiltonian constraint (In principle, it must be 0)

Preliminary results : $\delta\dot{\phi}/H_0\delta\phi$

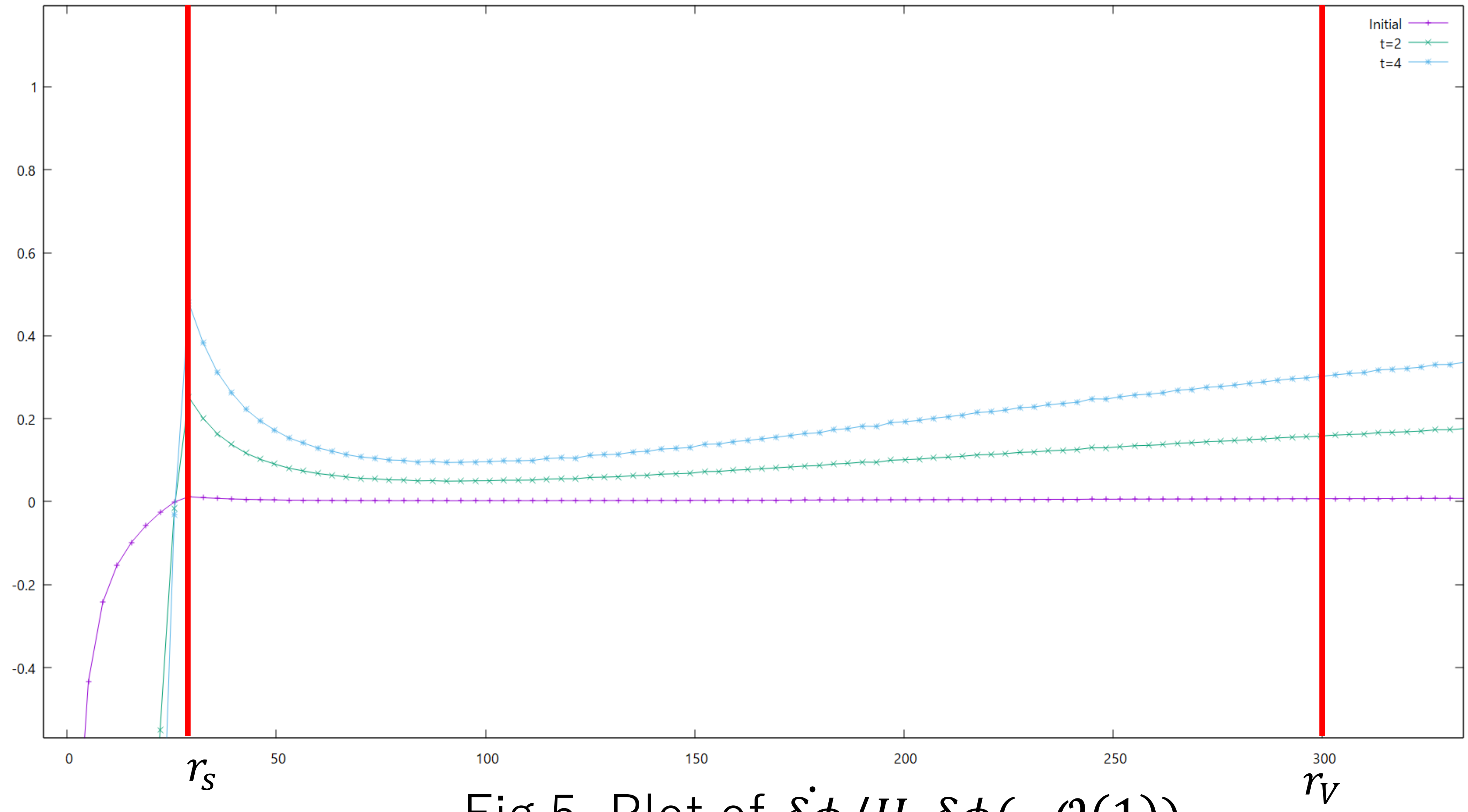


Fig.5 Plot of $\delta\dot{\phi}/H_0\delta\phi(\sim\mathcal{O}(1))$

Preliminary results : $\dot{M}/H_0M$

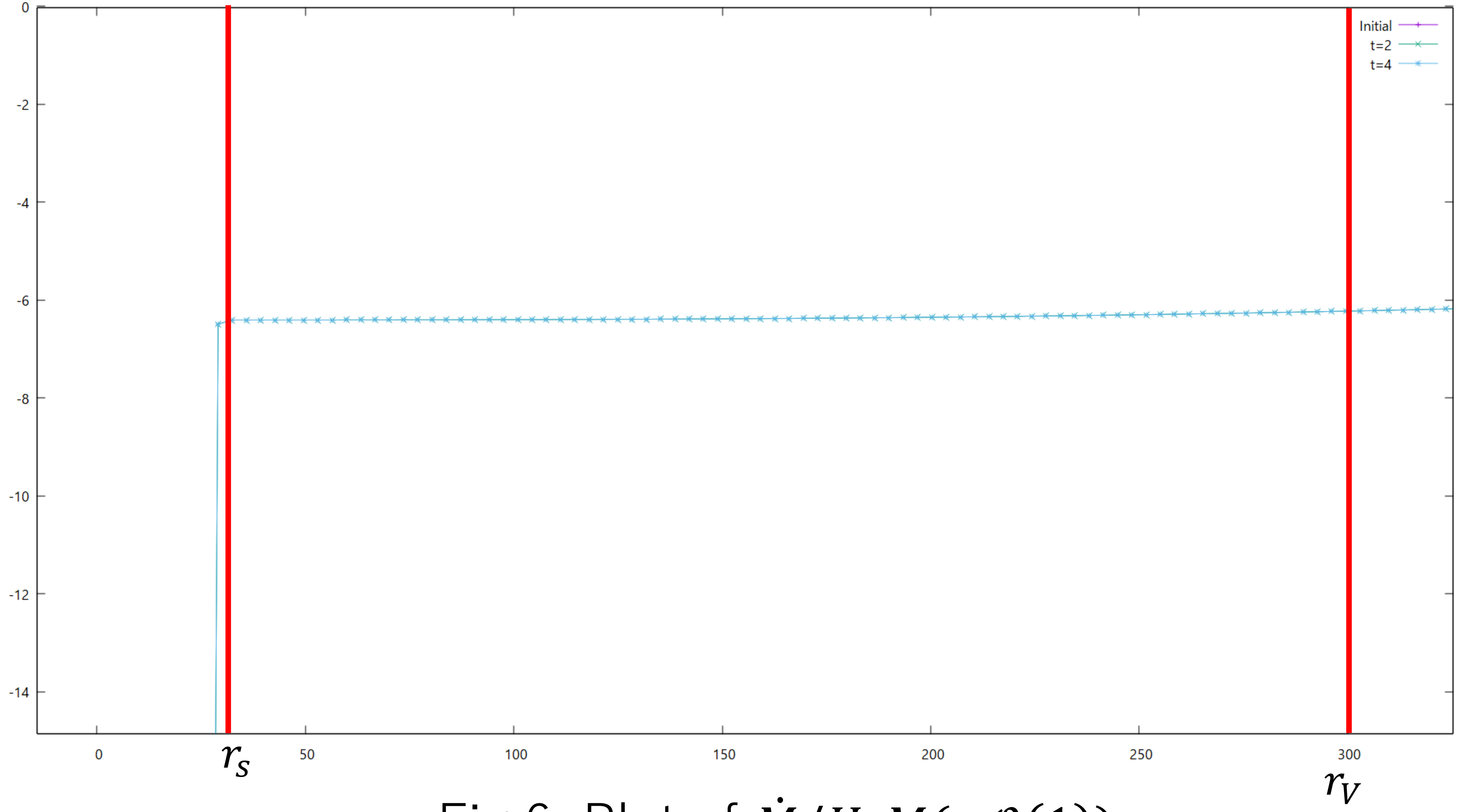


Fig.6 Plot of $\dot{M}/H_0M(\sim\mathcal{O}(1))$

Preliminary results : $\dot{G}_{\rm eff}/H_0 G_{\rm eff}$

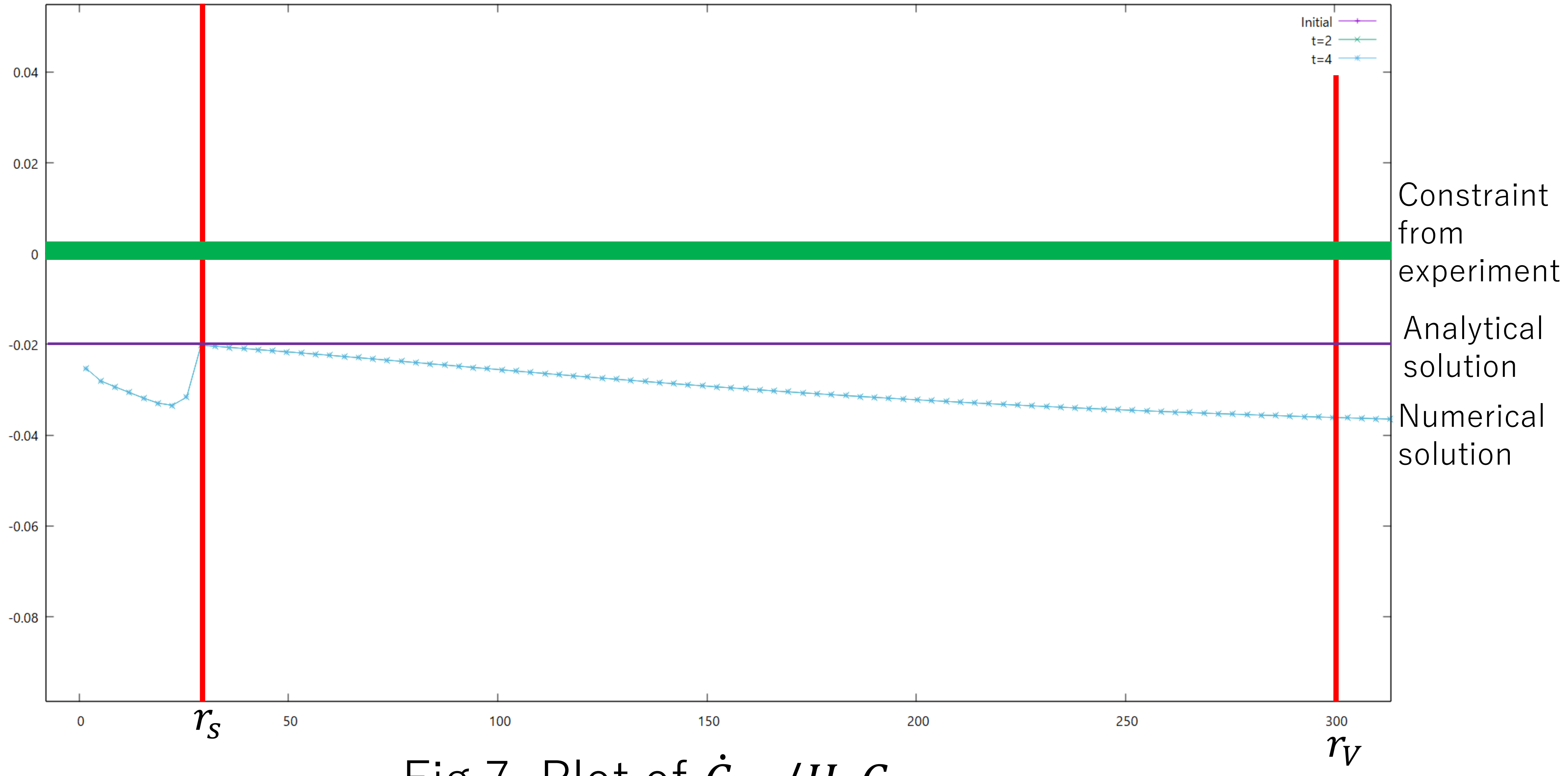


Fig.7 Plot of $\dot{G}_{\rm eff}/H_0 G_{\rm eff}$

We choose larger H_0 than observed value for numerical stability

⇨ There are small discrepancy from analytical solution,
but it is expected to be **very small if we use observed H_0 value**

$$\frac{\dot{G}_{\rm eff}}{H_0 G_{\rm eff}} \cong -\frac{\dot{F}}{H_0 F} + \underbrace{\frac{4\pi a r^2 \beta_1 \delta\phi'}{M}}_{\propto H_0 \text{ in } r \ll r_V} \times \underbrace{\frac{1}{H_0}\left(\frac{\dot{a}}{a} + \frac{\dot{\beta}_1}{\beta_1} + \frac{\delta\dot{\phi}'}{\delta\phi'} - \frac{\dot{M}}{M}\right)}_{H_0 \text{ independent (confirmed by numerical calculation)}} + \cdots$$

⇨ Consistent with analytical solution (preliminary results)

⇨ **Dark energy** theories which have **nonminimal coupling** (with $\boldsymbol{Q \sim \mathcal{O}(1)}$) are **inconsistent with LLR experiment**

*This result may be different in case of interior of star or different time scale event

Ⅳ. Conclusions and future work

- Our numerical results are consistent with analytical solution derived in previous works
- We confirmed that **dark energy theories which have nonminimal coupling* are inconsistent with solar-system experiments**

preliminary result

- We will consider smaller H_0 value (but there are technical problem)
- We can also consider matter or scalar field varying on different time scale →We will also try it

*We only consider cases that are observably distinguishable from the case without nonminimal coupling in the late universe