Slowly-rolling scalars around rotating black holes

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Motivations

- Primordial black holes could play an important role in cosmological history
- Few analytic models of black holes in cosmological environments
- Exploring how scalar fields respond to black hole environments
- Extended thermodynamics allows for variations in Λ - scalar evolving in a potential can provide this dynamically

Scalar fields in cosmology

- FLRW metric and scalar field (assuming homogeneity and isotropy)

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2), \quad \phi = \phi(t)$$

- Scalar field equation of motion

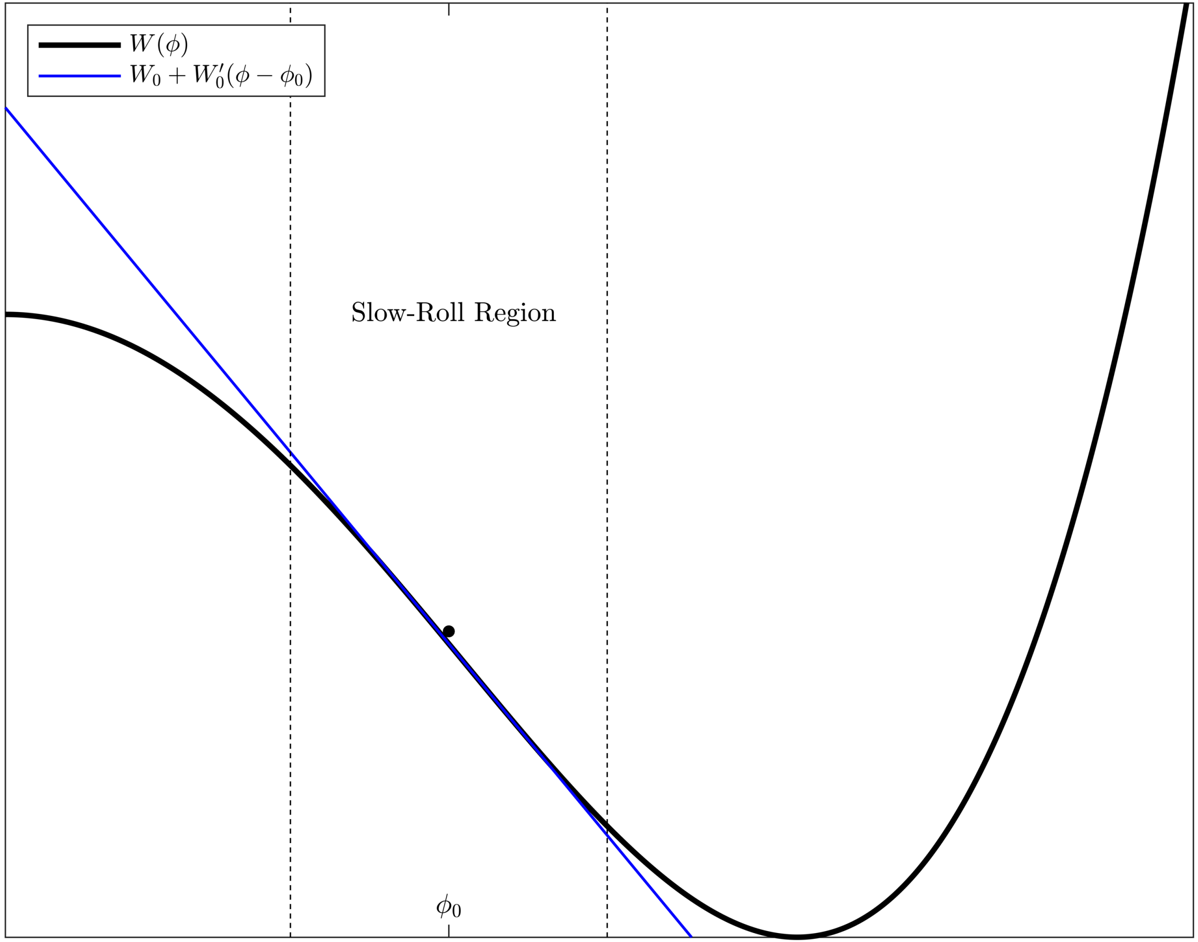
$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dW}{d\phi}$$

Slow-roll in cosmology

- Hubble friction can keep the scalar at constant velocity

$$3H\dot{\phi} \approx -\frac{dW}{d\phi}$$

- Test field approximation – fix the background cosmology, treat H as constant and solve for scalar evolution


$W(\phi)$
$W_0 + W_0'(\phi - \phi_0)$
Slow-Roll Region
ϕ_0

Validity of cosmological slow-roll

- Can only treat H as constant if scalar velocity small, so evolution is slow

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + W(\phi)\right), \quad \dot{H} = -\frac{1}{2M_{\rm pl}^2}\dot{\phi}^2$$

- $W'(\phi)$ shouldn't change too much, otherwise can't neglect scalar acceleration

- Leads to slow-roll parameters

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{W'(\phi)}{W(\phi)}\right)^2, \quad \delta = M_{\rm pl}^2\frac{W''(\phi)}{W(\phi)}$$

Adding a black hole

- FLRW coordinates less convenient for describing black holes

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

- Schwarzschild-de Sitter (SdS) metric, in static patch coordinates

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \quad f(r) \equiv 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2$$

Horizons in Schwarzschild-de Sitter

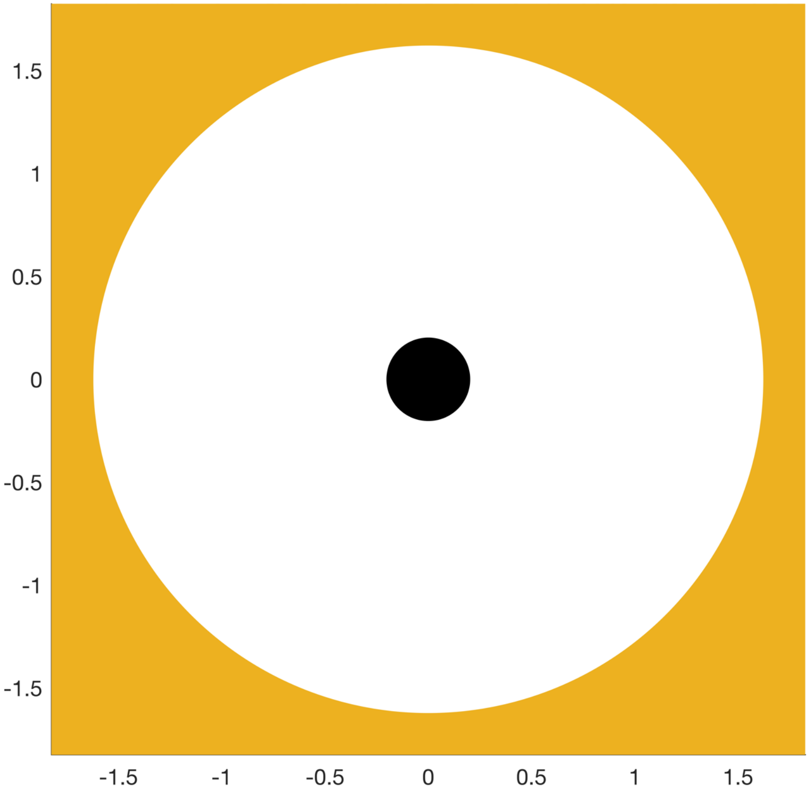
- Horizons located where $r-r$ metric component diverges

- For certain region of parameter space, gives two physical horizons

$$0 = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 = -\frac{\Lambda}{3r}(r - r_b)(r - r_c)(r - r_n)$$

- Have black hole horizon located at $r = r_b$ and cosmological horizon at $r = r_c$

Horizons for $GM = 0.1$ and $\Lambda = 1$



Slow-roll in Schwarzschild-de Sitter

- Black hole breaks homogeneity, so scalar likely to be radially dependent

- Look for "time" that scalar follows

$$\phi = \phi(T), \quad T = t + h(r)$$

- Assuming test field approximation, scalar equation of motion is

$$\left(\frac{1-\eta^2}{f}\right)\ddot{\phi} - \frac{1}{r^2}(r^2\eta)'\dot{\phi} = -\frac{dW}{d\phi}, \quad \eta(r) \equiv f(r)h'(r)$$

[Gregory, Kastor, Traschen, 1804.03462]

Boundary conditions

- Want T to be regular at both horizons, so that ϕ makes sense there
- Acceleration term could diverge at horizons, avoiding this gives regularity boundary conditions

$$\eta(r_b) = 1, \quad \eta(r_c) = -1$$

- This choice ensures the metric is also regular at the horizons

$$\text{As } r \to r_b, \quad dT \to dt + f(r)^{-1} dr = dv$$
$$\text{As } r \to r_c, \quad dT \to dt - f(r)^{-1} dr = du$$

[Gregory, Kastor, Traschen, 1804.03462]

Separating variables

- As before, ignore scalar acceleration for slow-roll

$$-\frac{1}{r^2}(r^2\eta)'\dot{\phi} = -\frac{dW}{d\phi}$$

- Note the radial dependence only appears on the left, so must be constant

$$\frac{1}{r^2}(r^2\eta)' = -3\gamma, \quad 3\gamma\dot{\phi} = -\frac{dW}{d\phi}$$

[Gregory, Kastor, Traschen, 1804.03462]

The scalar time T

- Radial equation can be integrated

$$\eta(r) = -\gamma r + \frac{\beta}{r^2}, \quad \beta = \frac{r_b^2 r_c^2 (r_b + r_c)}{r_c^3 - r_b^3}, \quad \gamma = \frac{r_b^2 + r_c^2}{r_c^3 - r_b^3}$$

- We could solve for $h(r)$ and integrate to get T

$$T = t + \int \frac{dr}{f(r)} \left(-\gamma r + \frac{\beta}{r^2} \right)$$

[Gregory, Kastor, Traschen, 1804.03462]

Validity of test field approximation

- Provided slow-roll conditions satisfied, metric will only have small corrections
- As scalar slowly evolves, sources metric corrections via energy-momentum

$$\dot{M} = 4\pi\beta\dot{\phi}^2, \quad \dot{\Lambda} = -3\gamma\frac{\dot{\phi}^2}{M_{\rm pl}^2}$$

- Backreaction is a next-to-leading order effect

[Gregory, Kastor, Traschen, 1804.03462]

Rotating black holes

- Kerr-de Sitter (KdS) metric, Boyer-Lindquist type coordinates

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\varphi\right)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Xi}d\varphi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2$$

$$\Delta_r = (r^2+a^2)\left(1 - \frac{\Lambda}{3}r^2\right) - 2mr, \qquad \Delta_\theta = 1 + \frac{\Lambda a^2}{3}\cos^2\theta,$$

$$\rho^2 = r^2 + a^2\cos^2\theta, \qquad \Xi = 1 + \frac{\Lambda a^2}{3}.$$

Slow-roll in Kerr-de Sitter

- Scalar time T could now be θ-dependent

$$T = t + h(r, \theta)$$

- Equation of motion splits as before

$$\partial_r \left(\Delta_r(r)\partial_r h\right) + \frac{1}{\sin\theta}\partial_\theta \left(\sin\theta \Delta_\theta(\theta)\partial_\theta h\right) = -3\gamma(r^2 + a^2\cos^2\theta)$$

$$3\gamma\dot{\phi} = -\frac{dW}{d\phi}$$

(r, θ) separation of variables

- Equation permits an additive separation of variables

$$h(r,\theta) = h_1(r) + h_2(\theta)$$

- Gives another separation constant

$$\begin{aligned} \partial_r \left(\Delta_r(r)\partial_r h_1(r)\right) &= -3\gamma r^2 - \gamma a^2 \\ \frac{1}{\sin\theta}\partial_\theta \left(\sin\theta \Delta_\theta(\theta)\partial_\theta h_2\right) &= -3\gamma a^2\cos^2\theta + \gamma a^2 \end{aligned}$$

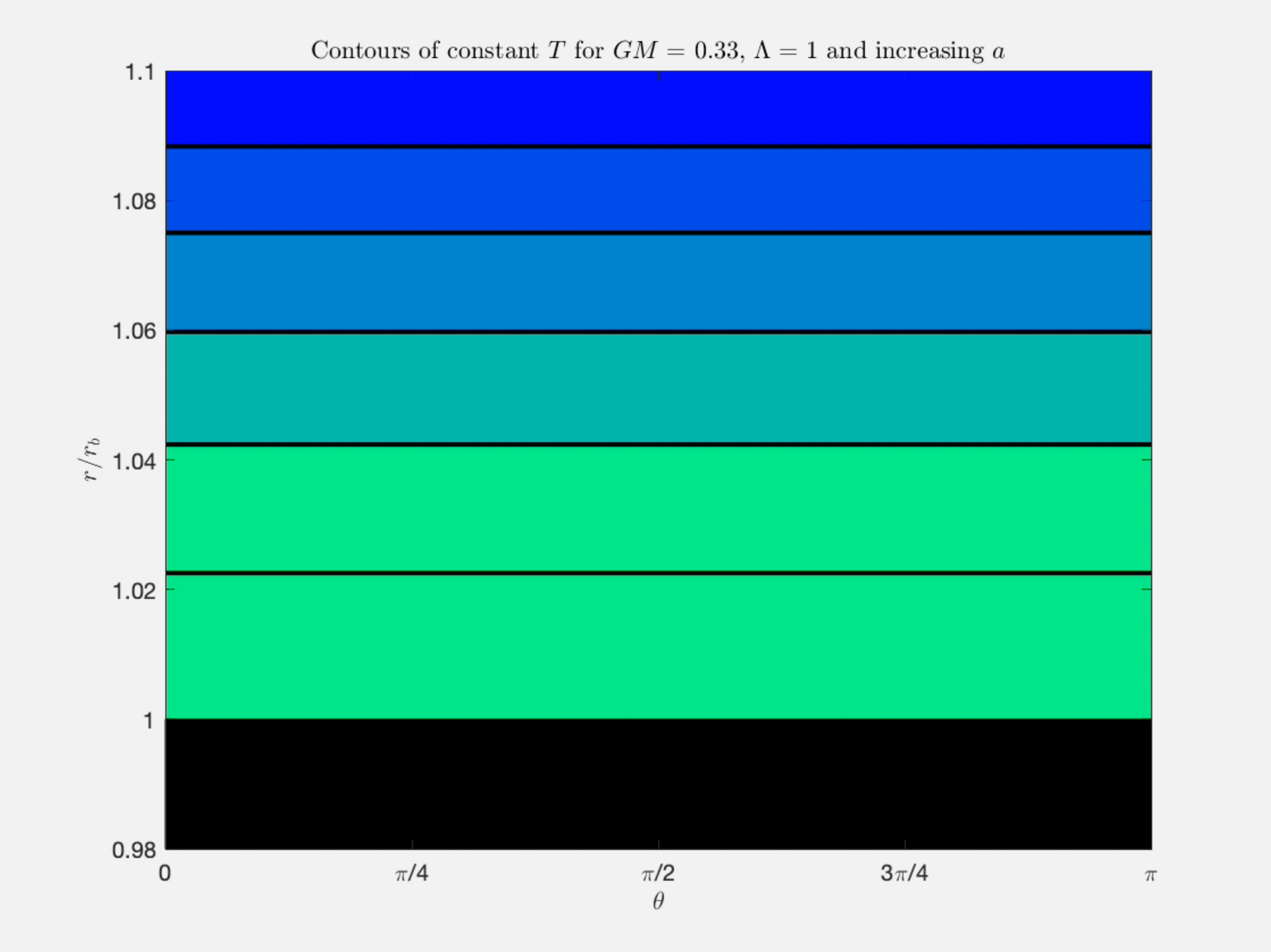
Scalar time T in KdS

- Get a similar form for $\eta(r)$

$$\eta(r) \equiv \frac{\Delta_r}{r^2 + a^2} h_1'(r) = -\gamma r + \frac{\beta}{r^2 + a^2}$$

- We can get T by integrating to get $h_1(r)$ and $h_2(\theta)$

$$T = \; t + \int dr \, \frac{r^2 + a^2}{\Delta_r} \eta(r) + \frac{3\gamma}{2\Lambda} \ln \Delta_\theta(\theta)$$


Contours of constant T for $GM = 0.33$, $\Lambda = 1$ and increasing a
1.1
1.08
1.06
1.04
1.02
1
0.98
r/r_b
0
$\pi/4$
$\pi/2$
$3\pi/4$
π
θ

Conclusions

- We can analytically treat black holes in some cosmological environments
- Scalars in slow-roll can be extended to include the impact of black holes
- Identified contours of slowly-rolling scalar fields around rotating black holes

- Next steps:
  - Backreaction for rotating black holes
  - Coasting scalars in a cosmological-black hole system

Thanks for listening!